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## Lattice results for $D/D_s$ leptonic and semileptonic decays

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This review article summarizes recent lattice QCD results for  $D$  and  $D_s$  meson leptonic and semileptonic decays. Knowing the meson decay constants and semileptonic form factors from theory, one can extract CKM elements  $V_{cd}$  and  $V_{cs}$  from experimental results. At present, the most accurate results for decay constants are from the Fermilab Lattice and MILC Collaborations [1]:  $f_D = 212.5 \pm 0.5_{\text{stat}}^{+0.6}_{-1.5}|_{\text{syst}}$  MeV and  $f_{D_s} = 248.9 \pm 0.2_{\text{stat}}^{+0.5}_{-1.6}|_{\text{syst}}$  MeV, giving  $V_{cd} = 0.2184 \pm 0.009_{\text{expt}}^{+0.0008}_{-0.0016}|_{\text{lattice}}$  and  $V_{cs} = 1.017 \pm 0.02_{\text{expt}}^{+0.002}_{-0.007}|_{\text{lattice}}$ . The shapes of the semileptonic form factors from lattice QCD agree very well with experiment, and the accuracy is currently at the 2 – 5% level for  $D \rightarrow \pi \ell \nu$  and 1 – 2% for  $D \rightarrow K \ell \nu$ . Extracting the CKM elements from the semileptonic decays yields  $V_{cd} = 0.225(6)_{\text{expt}}(10)_{\text{lattice}}$  (HPQCD Collaboration, from [2]) and  $V_{cs} = 0.963(5)_{\text{expt}}(14)_{\text{lattice}}$  (HPQCD Collaboration, from [3]). These lattice calculations also revealed that the semileptonic form factors are insensitive to whether the spectator quark is a light or strange quark.

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# 1 Motivation

$D$  and  $D_s$  meson decays are a very interesting research area at the moment. On the experimental side, BES III and Belle have presented preliminary results from their recent runs; on the theory side, lattice QCD is able to provide non-perturbative, precise calculations of meson decay constants and semileptonic form factors from first principles. Combining the experimental and theoretical results allows us to determine elements  $|V_{cs}|$  and  $|V_{cd}|$  of the quark mixing matrix (the CKM matrix). Several processes can be used to extract the same CKM matrix element, which allows for cross checks and consistency tests of the Standard Model and constraints/test for new physics. Similar methods can be used to study  $B$  and  $B_s$  meson decays, so charm decays are an excellent test environment for these lattice QCD tools.

The aim of this review is to summarize recent lattice QCD results for the leptonic and semileptonic decays, i.e. decay constants  $f_D$  and  $f_{D_s}$ , and form factors for  $D \rightarrow K\ell\nu$  and  $D \rightarrow \pi\ell\nu$ . The article is divided into three parts: a general introduction, lattice results and CKM elements  $V_{cd}$  and  $V_{cs}$ .

## 2 Introduction

### 2.1 Leptonic and semileptonic decays

In a leptonic decay a meson (here  $D$  or  $D_s$ ) decays to a lepton and its neutrino via a virtual  $W$  boson. The decay rate is given by

$$\Gamma^{D_s \rightarrow \ell\nu} = \frac{G_F^2}{8\pi} m_\ell^2 M_{D_s} \left(1 - \frac{m_\ell^2}{M_{D_s}^2}\right)^2 f_{D_s}^2 |V_{cs}|^2. \quad (1)$$

Hence  $f_{D_s}^2 |V_{cs}|^2$  can be cleanly extracted from experiment. The decay constant  $f_{D_s}$  (or  $f_D$  for a  $D$  meson decay) is a property of the hadron and can thus be calculated in lattice QCD.

On the other hand, consider a semileptonic decay where a  $D$  meson decays to a  $K$  meson (or a pion), a lepton and its neutrino via a virtual  $W$  boson. If both the initial and final state mesons are pseudoscalars, the partial decay rate can be written as

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{cs}|^2 |f_+^{D \rightarrow K}(q^2)|^2. \quad (2)$$

Here  $p = |\vec{p}|$  is the momentum of the  $K$  meson in the rest frame of the  $D$ , and  $q^2$  is the four-momentum transfer between the two mesons,

$$q^2 = (M_D - E_K)^2 - p^2. \quad (3)$$

Again, experiment can tell us  $|V_{cs}|^2|f_+^{D \rightarrow K}(q^2)|^2$  as a function of  $q^2$ . The form factor  $f_+$  is a QCD quantity, and again this can be determined in a lattice QCD calculation. Note that the same CKM element appears in both cases.

## 2.2 Lattice QCD

At the moment lattice QCD is the only known method that can provide a precise, non-perturbative theoretical determination of form factors and decay constants. In a lattice calculation space-time is discretized to make a 4 dimensional box with lattice spacing  $a$  to allow numerical integration of the QCD path integral. There are many details involved in such a calculation, but a rough sketch of one would be:

1. Generate sets of gluon fields for Monte Carlo integration of the path integral (including effects of sea quarks).
2. Calculate averaged “hadron correlators” from valence quark propagators calculated on these gluon fields.
3. Fit the correlators as a function of time to obtain masses and simple matrix elements.
4. Determine lattice spacing  $a$  and fix quark masses using experimental information (often meson masses) to get results in physical units.
5. Extrapolate to  $a = 0$  and the physical  $u/d$  quark mass for real world.

Lattice calculations have had to work with heavier than physical  $u/d$  masses because of numerical cost. Lattices with physical  $m_{u,d}$  are now being generated and the extrapolation to physical light quark masses is becoming just a small correction.

## 3 Lattice results

### 3.1 Leptonic decays, decay constants

The current status of calculations of the  $D$  and  $D_s$  meson decay constants is shown in Fig. 1, tagged by the names of the lattice groups. The results are from [4, 5, 6, 7, 8, 9, 10, 11, 1]. Note that some of the results are still preliminary.  $n_f$  denotes the number of flavors used in the calculation:  $n_f = 2$  is two light quarks in the sea ( $u$  and  $d$  quarks that both have the same mass),  $n_f = 2 + 1$  is two light plus strange quarks and  $n_f = 2 + 1 + 1$  has in addition charm quarks in the sea. The tags “HISQ”, “twisted mass”, “Fermilab” and “clover” denote different discretizations of the Dirac equation for quarks. These different discretizations should all agree in the continuum

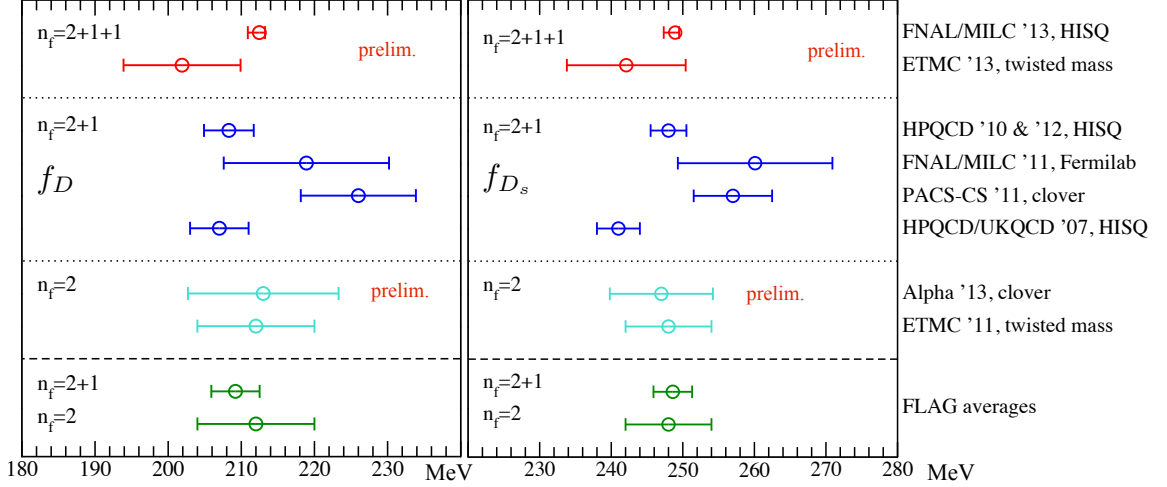


Figure 1: Decay constants:  $f_D$  on the left,  $f_{D_s}$  on the right. The best results at the moment, i.e. results with smallest errors and most modern lattice configurations ( $n_f = 2 + 1 + 1$ , physical pion mass), are from Fermilab Lattice and MILC Collaborations (FNAL/MILC '13):  $f_D = 212.5 \pm 0.5_{\text{stat}}^{+0.6}_{-1.5}|_{\text{syst}}$  MeV and  $f_{D_s} = 248.9 \pm 0.2_{\text{stat}}^{+0.5}_{-1.6}|_{\text{syst}}$  MeV.

limit, and as can be seen in the figures the agreement is good. For completeness, averages from Flavor Lattice Averaging Group (FLAG) [12] are also shown.

It is also interesting to look at the history of the  $D_s$  meson decay constant and see how the value has evolved over the years. This is shown in Fig. 2. A few years ago there was disagreement between the values from experiment and lattice, but that has now mostly gone away leaving a tension of  $2\sigma$ . The very precise (1%) value from lattice QCD has been confirmed by two separate groups and looks solid.

### 3.2 Semileptonic decays, form factors

Let us turn to  $D$  and  $D_s$  meson semileptonic decays and their form factors. In fact there are two form factors, a scalar form factor  $f_0$  and a vector form factor  $f_+$ , associated with a pseudoscalar to pseudoscalar semileptonic decay. In experiment the scalar form factor is suppressed by the lepton mass and thus not accessible. However, on the lattice it is quite straightforward to consider two currents, a scalar and a vector current, and calculate both form factors  $f_0$  and  $f_+$ . There is also a useful kinematic constraint that  $f_+(0) = f_0(0)$ .

Here we will only consider lattice results for decays  $D \rightarrow K\ell\nu$  and  $D \rightarrow \pi\ell\nu$ . Several groups have calculated the shape of the  $D \rightarrow K$  form factors – see Refs. [3, 13, 14]. Fig. 3 shows results by HPQCD from different lattice spacings [coarse ( $a = 0.12$  fm) and fine ( $a = 0.09$  fm) lattice] and extrapolation to continuum and physical

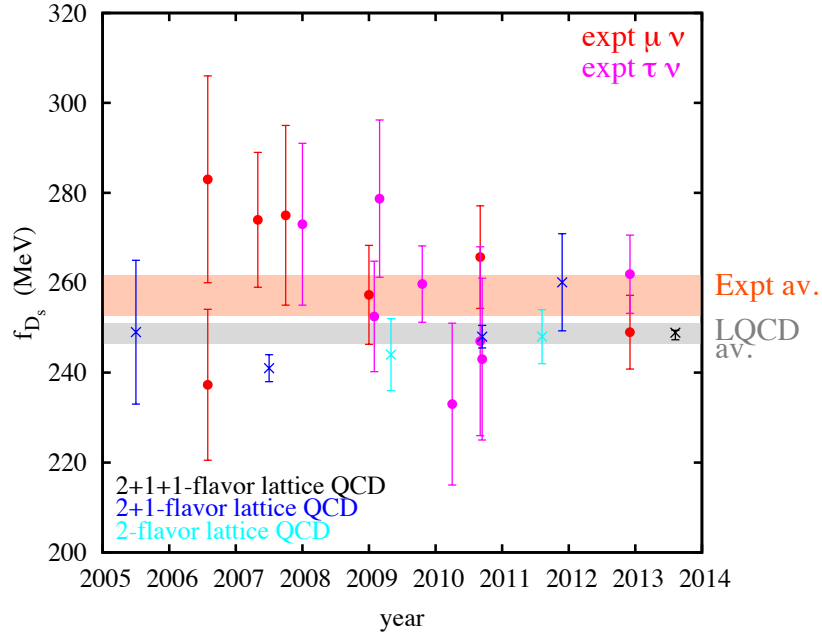


Figure 2: The history of  $f_{D_s}$ . The experimental values have been obtained using the unitarity value for  $|V_{cs}|$  from the PDG (i.e. unitarity of the CKM matrix is assumed). The darker red data points are for the decay channel  $D_s \rightarrow \mu\nu$  and the lighter red for  $D_s \rightarrow \tau\nu$ . Light blue, blue and black crosses are values from 2 flavor, 2 + 1 flavor and 2 + 1 + 1 flavor lattice QCD, respectively. This figure is an update of Fig. 18 in [6].

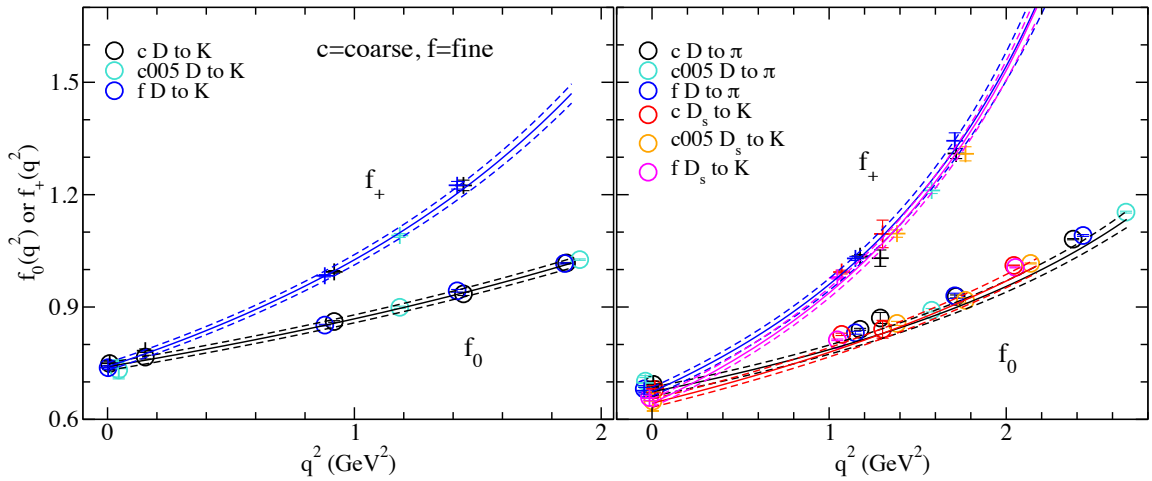


Figure 3: On the left: Scalar and vector form factors of  $D \rightarrow K$  semileptonic decay [3]. On the right: Form factors of  $D \rightarrow \pi$  and  $D_s \rightarrow K$  semileptonic decays. Note that the shape of the form factors is insensitive to the mass of the spectator quark.

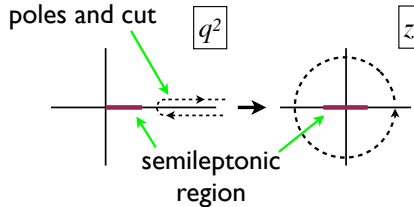


Figure 4: Converting the semileptonic region in  $q^2$ -space to  $z$ -space.

light quark mass (more details of the extrapolation are in Section 3.3).

The study by HPQCD [3] revealed that the semileptonic decay form factors are insensitive to spectator quark mass. This is shown very clearly in Fig. 3: the form factors for  $D \rightarrow \pi \ell \nu$  and  $D_s \rightarrow K \ell \nu$  are the same within few percent, and even within 2% for most of the  $q^2$  range. These decays are both  $c$  to  $d$  decays, and the difference is the spectator quark: a light quark in the  $D \rightarrow \pi$  case, and strange in  $D_s \rightarrow K$ . This has been shown to hold for  $B \rightarrow D \ell \nu$  and  $B_s \rightarrow D_s \ell \nu$  as well [15]. The same lattice methods can be used to study decays that involve vector mesons, like weak decay  $D_s \rightarrow \phi \ell \nu$  or charmonium radiative decay  $J/\psi \rightarrow \eta_c \gamma$  – see for example [16].

### 3.3 The $z$ -expansion, continuum and chiral extrapolation

It is beneficial to do the continuum and chiral extrapolation in  $z$ -space instead of  $q^2$ -space. In  $z$ -space the semileptonic region is inside the unit circle, formed by the region with poles and cut – see Fig. 4. A simple conversion from  $q^2$  to  $z$  is done as follows: First remove the poles

$$\tilde{f}_0^{D \rightarrow K}(q^2) = \left(1 - \frac{q^2}{M_{D_{s0}^*}^2}\right) f_0^{D \rightarrow K}(q^2), \quad \tilde{f}_+^{D \rightarrow K}(q^2) = \left(1 - \frac{q^2}{M_{D_s^*}^2}\right) f_+^{D \rightarrow K}(q^2), \quad (4)$$

then convert to  $z$  variable

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (M_D + M_K)^2, \quad (5)$$

see e.g. [17]. Remembering the constraint  $f_+(0) = f_0(0)$  one can choose  $t_0 = 0$ . The lattice results plotted in Fig. 3 as a function of  $q^2$  are shown in Fig. 5 as a function of  $z$ , which makes the advantage of working in  $z$ -space very clear. The results from different lattice ensembles are then fit as power series in  $z$ :

$$\tilde{f}_0^{D \rightarrow K}(z) = \sum_{n \geq 0} c_n(a) z^n, \quad \tilde{f}_+^{D \rightarrow K}(z) = \sum_{n \geq 0} b_n(a) z^n, \quad c_0 = b_0. \quad (6)$$

Note that the fit parameters depend on lattice spacing and quark masses. In the end one takes  $a = 0$  and  $m_q = m_q^{\text{phys}}$  to get the result in the continuum and at

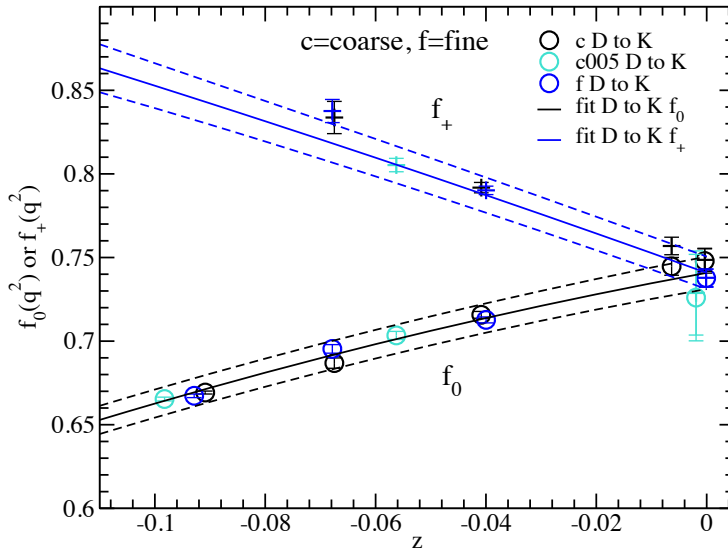


Figure 5:  $D \rightarrow K l \nu$  form factors and fit in  $z$ -space.

physical light quark masses. Comparison with experiment of the parameters from the  $D \rightarrow K l \nu$  fit that determine the form factor shape is shown in Fig. 6.

## 4 $|V_{cs}|$ and $|V_{cd}|$

Now we have the needed input, decay constants and form factors, from lattice QCD to determine CKM elements from leptonic and semileptonic decays. In the case of a semileptonic decay, we can integrate the form factor calculated in lattice QCD over the experimental  $q^2$  bins and determine the CKM element from each bin: the experimental result divided by the lattice result for a given bin is  $V_{cs}^2$  (or  $V_{cd}^2$ ). This is shown in Fig. 7. One can then do a weighted average fit to these values, including bin to bin correlations. This is more accurate compared to earlier calculations that extracted CKM elements from experimental knowledge of  $|f_+(0)|^2 |V_{cs}|^2$  (or  $|f_+(0)|^2 |V_{cd}|^2$ ) and a lattice determination of the form factor at  $q^2 = 0$ , since this uses more information.

The current status of  $V_{cd}$  and  $V_{cs}$  from leptonic and semileptonic decays is shown in Fig. 8. The tags are the same as for the decay constants in Section 3.1: the name of the lattice group, and the fermion discretization and number of sea quark flavors that were used in the calculation. Note that the experimental averages used here to calculate the CKM elements are from 2012 [22, 23, 24] – more recent experimental results have not been included. The vertical lines in the plots show the unitarity value. Leptonic decays tend to give a higher value for  $V_{cs}$  than the unitarity value, but note that all data points in the plot would shift to left or right, if the experimental average changed. All lattice results agree with each other very well, and the semileptonic



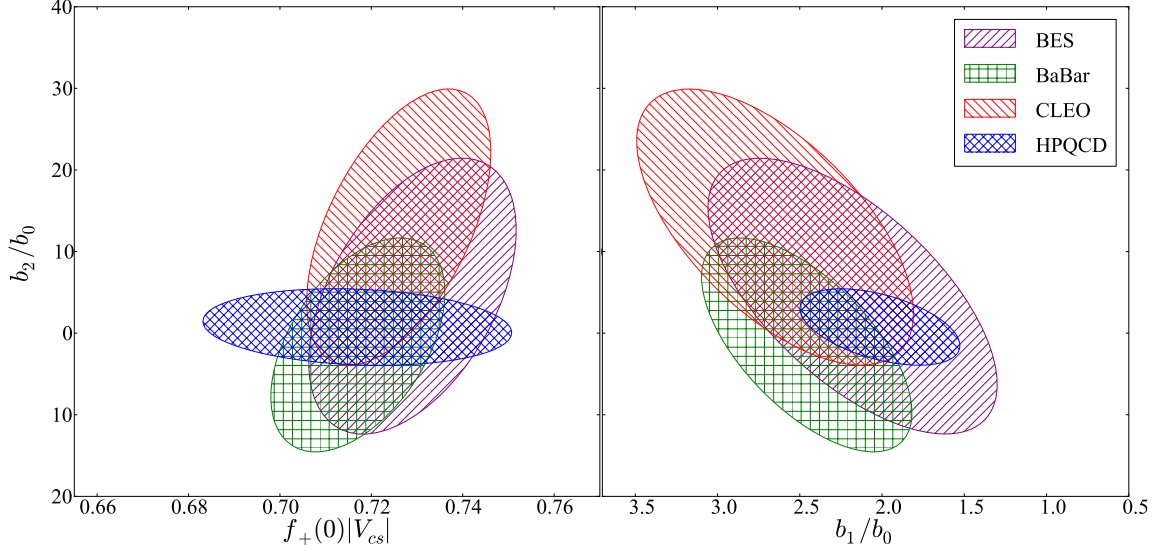


Figure 6: To compare the shape of the  $D \rightarrow K l \nu$  form factors calculated in lattice QCD with experiment we use exactly the same  $z$  expansion as the experimental groups [18, 19, 20, 21] (a more complicated outer function than just a simple pole, and a specific choice of  $t_0$  in Equations 4, 5). Shown here are the  $1\sigma$  ellipse contours of the fit results from Eq. 6 for  $f_+(0)|V_{cs}|$  and  $b_1/b_0$  against  $b_2/b_0$ . The agreement is very good.  $|V_{cs}| = 0.963(5)_{\text{expt}}(14)_{\text{lattice}}$  was used here for normalisation [3].

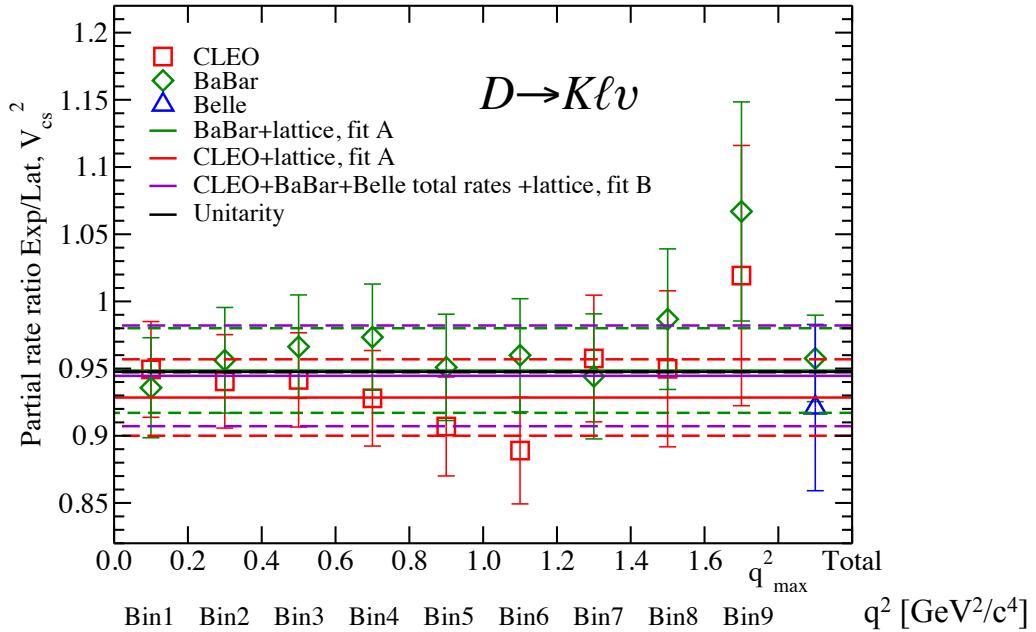


Figure 7:  $|V_{cs}|$  extracted from  $D \rightarrow K l \nu$  decay using all experimental  $q^2$  bins.

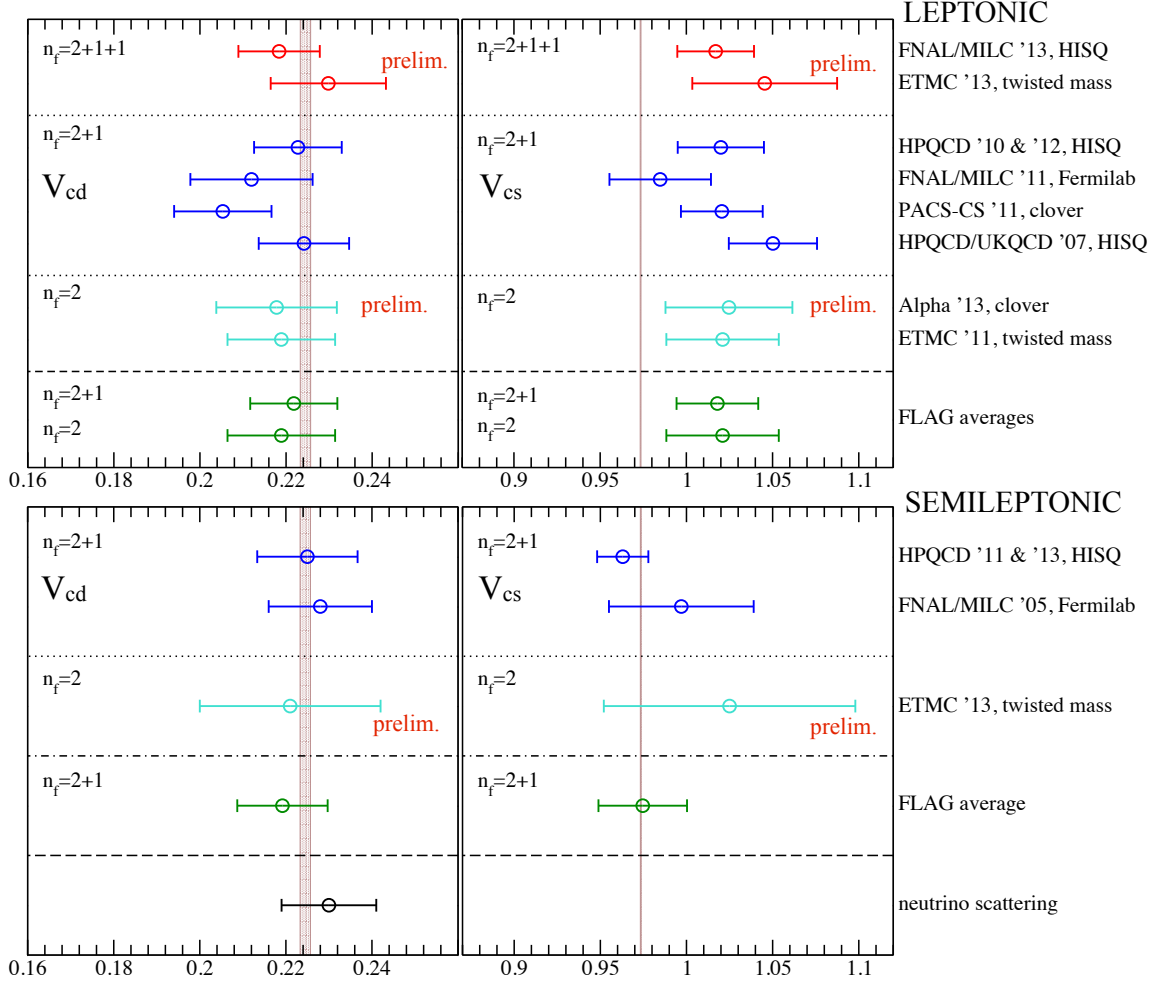


Figure 8: Summary of CKM elements. Top row from left to right:  $|V_{cd}|$  and  $|V_{cs}|$  from leptonic decays. Bottom row from left to right:  $|V_{cd}|$  and  $|V_{cs}|$  from semileptonic decays. Vertical error bands show the unitarity value for reference. The best values using the latest lattice results (most modern lattice configurations with  $n_f = 2 + 1 + 1$ , and smallest errors) are:  $V_{cd}$  (leptonic) =  $0.2184 \pm 0.009_{\text{expt}}^{+0.0008}_{-0.0016}|_{\text{lattice}}$  and  $V_{cs}$  (leptonic) =  $1.017 \pm 0.02_{\text{expt}}^{+0.002}_{-0.007}|_{\text{lattice}}$ , taking decay constants from [1] (FNAL/MILC '13 in Fig. 1);  $V_{cd}$  (semileptonic) =  $0.225(6)_{\text{expt}}(10)_{\text{lattice}}$  from [2] and  $V_{cs}$  (semileptonic) =  $0.963(5)_{\text{expt}}(14)_{\text{lattice}}$  from [3]. Experimental averages used here are (taken from [12]): leptonic decays:  $f_D|V_{cd}| = 46.40(1.98)$  MeV and  $f_{D_s}|V_{cs}| = 253.1(5.3)$  MeV [22]; semileptonic decays:  $f_+^{D \rightarrow \pi}(0)|V_{cd}| = 0.146(3)$ ,  $f_+^{D \rightarrow K}(0)|V_{cs}| = 0.728(5)$  [23]. The latest experimental results (2012 or after) are not included. [3] is the only calculation so far to use all experimental  $q^2$  bins to extract a CKM element from a semileptonic decay.

determination of  $V_{cs}$  and both leptonic and semileptonic determinations of  $V_{cd}$  agree with the assumption of CKM matrix unitarity. For comparison, averages from Flavor Lattice Averaging Group (FLAG) [12] are also shown in the plots, as well as the result for  $V_{cd}$  from neutrino scattering experiments [24]. The lattice results are from [4, 25, 5, 2, 7, 8, 9, 3, 10, 11, 1].

## 5 Summary

Precision tests of Standard Model and searches for new physics can be done by extracting CKM elements from  $D$  and  $D_s$  meson leptonic and semileptonic decays. In addition to precise experimental results input from theory is also needed: decay constants  $f_D$ ,  $f_{D_s}$ , and form factors for  $D \rightarrow K\ell\nu$ ,  $D \rightarrow \pi\ell\nu$ ,  $D_s \rightarrow K\ell\nu$ . These can be calculated in lattice QCD, and the current best results for the decay constants are listed in Fig. 1 along with the corresponding CKM elements in Fig. 8. Results from independent lattice calculations show good agreement, and the extracted  $|V_{cd}|$  and  $|V_{cs}|$  are in agreement with CKM matrix unitarity. We also compare the shape of the form factors from lattice QCD with experimental results and find good agreement.

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