The inflation risk premium on government debt in an overlapping generations model[1]

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Abstract

This paper presents a general equilibrium model in which nominal government debt pays an inflation risk premium. The model predicts that the inflation risk premium will be higher in economies which are exposed to unanticipated inflation through nominal asset holdings. In particular, the inflation risk premium is higher when government debt is primarily nominal, steady-state inflation is low, and when cash and nominal debt account for a large fraction of consumers' retirement portfolios. These channels do not appear to have been highlighted in previous models or tested empirically. Numerical results suggest that the inflation risk premium is comparable in magnitude to standard representative agent models. These findings have implications for management of government debt, since the inflation risk premium makes it more costly for governments to borrow using nominal rather than indexed debt. Simulations of an extended model with Epstein-Zin preferences suggest that increasing the share of indexed debt would enable governments to permanently lower taxes by an amount that is quantitatively non-trivial.

Keywords: government debt; inflation risk premium; overlapping generations.

"*The real question with respect to whether indexed debt will save the taxpayer money really gets down to an evaluation of the size and persistence of the so-called inflation risk premium that is associated with the level of nominal interest rates*."

Alan Greenspan
Remarks at a Joint Economic Committee hearing on "Inflation Indexing of Government Securities", May 14, 1985[2]

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[2] Cited by Campbell and Shiller (1996, p. 160).

1. Introduction

The inflation risk premium – the compensation demanded by risk-averse bondholders for bearing inflation risk – is of clear practical importance. For instance, the question of whether it is optimal to issue indexed government debt will depend in part on the expected cost of financing debt repayments. Other things being equal, a positive inflation risk premium implies that it is more costly for the government to borrow using nominal debt. Under these circumstances, the government could reduce its real borrowing costs by issuing debt that is indexed to the price level. This would enable the government to keep spending unchanged in real terms while permanently lowering taxes, or to increase government spending for any given path of taxes. In addition, estimates of the inflation risk premium are useful because they allow policymakers to make inferences about market inflation expectations using break-even inflation rates, as argued by Bernanke (2004). Hence, the inflation risk premium matters.

In a recent survey of the literature, Bekaert and Wang (2010) note that empirical estimates of inflation risk premia are generally positive but vary somewhat across studies, ranging from 0 to over 200 basis points depending on the economy and maturity of debt considered. To better understand the factors that drive risk premia, several recent papers have solved for bond yields in New Keynesian models which are approximated to second-order (e.g. De Paoli et al., 2010; Hördahl et al., 2008). Because nominal prices are sticky in these models, monetary policy has real effects. As a result, the inflation risk premium – the covariance between the stochastic discount factor of the representative agent and inflation – depends crucially on the shocks that hit the economy and the response of the central bank to these shocks, as described by a Taylor rule. These papers also demonstrate that the strength of real and nominal rigidities is important for inflation risk premia in New Keynesian models.

This paper makes two main contributions. First, using a general equilibrium model with flexible-prices, it highlights several alternative factors that matter for the inflation risk premium. In particular, an overlapping generations (OG) model is solved for a second-order accurate closed-form expression for the inflation risk premium. The key transmission mechanism in the model has previously been emphasised by Champ and Freeman (1990): in OG models, unanticipated monetary innovations have real effects, because unanticipated inflation erodes the real value of nominally denominated government debt. The contribution here is to show that this transmission mechanism matters for the inflation risk premium. This feature of the model is intuitively appealing since we would expect compensation for inflation risk to be higher in economies which are more exposed to unanticipated inflation through substantial holdings of nominal assets. Doepke and Schneider (2006) show that, in the postwar period, the US economy has been quite exposed to such fluctuations: a moderate episode of unanticipated inflation implies a substantial wealth loss for older agents, the main bondholders in the economy.[3] Likewise, the old in Canada lose out significantly during

[3] For instance, based on 1989 data, a fully unanticipated inflation of 5% that lasts 10 years produces a wealth loss relative to average net worth of 7% for the 66-75 age group and 10% for the over 75s.

periods of unanticipated inflation due, in part, to their substantial holdings of nominal government debt (Meh et al., 2010).

The model predicts the inflation risk premium will be higher in economies where government debt is primarily nominal, steady-state inflation is low, and where money and nominal debt are important sources of retirement consumption. Intuitively, economies in which nominal assets are a large fraction of private sector wealth are more vulnerable to inflation risk, because variations in the real returns on these assets imply fluctuations in retirement consumption. And since episodes of unexpectedly high inflation will squeeze consumption in retirement – raising the marginal utility of additional consumption – risk-averse agents will hold nominal debt only if it pays a premium over indexed debt. The mechanism by which nominal asset holdings matter for the inflation risk premium does not appear to have been highlighted in previous theoretical models or tested empirically. Numerical results indicate that the inflation risk premium is of a comparable magnitude to standard representative agent models, and that it depends crucially on the importance of nominal bonds as a source of retirement consumption and the share of the government debt that is nominal. These mechanisms may help to explain cross-country differences in empirical estimates of inflation risk premia. Empirical work testing the model predictions would be a feasible extension because the risk premium is related to observable macro variables.

The second contribution of the paper is to show that the inflation risk premium has quantitatively relevant implications for fiscal policy and government debt management. Some general analytical results are first presented which show that the inflation risk premium is an important determinant of the cost of issuing nominal versus indexed debt. The analysis then turns to the case of a government that has a positive outstanding debt which it rolls over continuously while maintaining the level of real government spending and satisfying its budget constraint. The aim is to understand whether the implications of this policy for taxes are quantitatively relevant, and whether the answer hinges critically on the share of indexed government debt as a result of the inflation risk premium. Analytical results suggest that the share of indexed government debt is an important determinant of the level of taxes, but the sign of this effect is ambiguous because it depends on a precautionary savings effect as well as the inflation risk premium. However, numerical simulations from an extended model with Epstein-Zin preferences suggest that shifting from nominal to indexed debt would enable the government to permanently lower taxes by a non-trivial amount, because it can avoid paying the inflation risk premium. This finding suggests an interesting avenue for research on optimal management of government debt. It also has policy relevance given the relatively low shares of indexed debt in developed economies (Campbell et al. 2009; Kitamura, 2008).[4]

The literature on optimal government debt management is neatly summarized by Barro (2003). A useful starting point is to note that if Ricardian equivalence does not hold because of distortionary taxes, tax-smoothing is optimal and pins down a desirable level of

[4] The 2008 share of indexed government debt was around 10% in the US, slightly less than 30% in the UK, and around 1.5% in Japan. See the two cited papers for these figures.

government debt (Barro, 1979). However, distortionary taxes alone do not pin down the optimal composition of government debt by maturity or debt type. The composition of government debt only matters when, in addition, there is uncertainty about future government spending, the tax base, or asset prices. Several notable papers have studied optimal debt policy under these conditions, including Lucas and Stokey (1983), Persson et al. (1987) and Bohn (1988). As argued by Barro (2003), if there is uncertainty about government expenditures or the future tax base, it is optimal to issue long-term government debt. The reasoning is that short-term debt is subject to rollover risk (because future interest rates are uncertain), which leads to unexpected variations in real borrowing costs and so deviations from tax-smoothing. In addition, it would appear to be optimal from a tax-smoothing perspective to issue only indexed government debt, because the real borrowing rate on nominal debt will vary with unanticipated inflation.

Issuing nominal government debt can, however, be justified on tax-smoothing grounds if inflation and government expenditure positively covary, as in Bohn (1988). Intuitively, a positive covariance implies a partial default (in real terms) on nominal debt at times when government spending is unexpectedly high, so that taxes will have to move less to bridge the gap between spending and revenue. The conventional wisdom on optimal maturity has also been challenged, most recently by Greenwood et al. (2010). Interestingly, the theoretical literature on optimal debt policy does not appear to have formally investigated the implications of the inflation risk premium. As Campbell and Shiller (1996) point out, these implications could be important because the government can avoid paying the inflation risk premium by issuing indexed debt, and so potentially lower its real borrowing rate. The simulation results in this paper provide evidence from a general equilibrium model that the fiscal implications of the inflation risk premium are quantitatively non-trivial. A full analysis of the welfare implications is beyond the scope of this paper but of obvious interest for future research.

The paper proceeds as follows. Section 2 sets out the model. In Section 3, a closed-form analytical solution for the inflation risk premium is derived and its main predictions are discussed. Section 4 then compares the numerical predictions of the solution with the existing literature. In Section 5 the fiscal implications of the inflation risk premium are investigated both analytically and quantitatively. Finally, Section 6 concludes.

2. Model

Consider a two-period overlapping generations (OG) model in the spirit of Diamond (1965).[5] Each generation supplies one unit of labour in the first period, and retires in the second period. Utility is derived from consumption in both periods, and there is no bequest motive. The number of generations born each period is normalized to 1. Consumption by the young is denoted c_Y. The young are subject to a lump-sum tax T. Their after-tax wage income can used

[5] Diamond studies economic growth in a real economy where consumers can hold indexed government debt or capital and there is no aggregate uncertainty. In his model, the population grows at an exogenous rate. I abstract from population growth here because it seems unlikely to add any additional insights.

for consumption in the same period or allocated to three assets – capital, indexed government bonds and nominal government bonds – in order to finance consumption in old age, c_O. In addition, there is fiat money, M, in the economy, and each generation holds a positive amount of real money balances, $m \equiv M/P$. There are two sources of aggregate uncertainty: a productivity shock and a money supply shock. Each period lasts N years.

Capital, k, is retained (without depreciation) and used an input in the production process next period, yielding a risky return r^k. Bonds take the form of long-term government debt with a maturity of N years. Indexed bonds (b^i) pay a riskless real return r^f next period, and nominal bonds (b^n) a riskless nominal return R next period. The returns r^f and R are endogenously determined so that demand and supply for bonds are equated. Because inflation cannot be forecast with certainty, nominal bonds are risky in real terms, with a real return of $r^n = R/\Pi$, where Π is the gross rate of inflation between youth and old age. The real return on money balances is $r^m = 1/\Pi$. Money is therefore a dominated asset if $R > 1$. A positive demand for money is motivated by a legal requirement to hold real money balances of at least $\delta > 0$, so that $m \geq \delta$. This constraint is assumed to bind with equality, i.e. $m = \delta$.

Champ and Freeman (1990) previously used this constraint to show that unanticipated money innovations affect real variables in an OG model, because unanticipated inflation lowers the real value of nominally denominated government debt.[6] The same transmission mechanism plays a crucial role here, because government debt will pay an inflation risk premium only if unanticipated inflation erodes old generations' consumption. Like Champ and Freeman, I assume money offers no transaction services to show that the main results follow from the effects of inflation on the real value of government debt, and not its effects on the mechanics of exchange. In order to satisfy the reserve requirement $m_t = \delta$, the young born each period must hold the entire money stock. To achieve this, they use their after-tax wage income to purchase the previous money stock, $M_{t-1,}$ from the current old (who consume the proceeds) and the current period money injection, $M_t - M_{t-1,}$ from the government. This leaves the young holding nominal money balances of M_t, and hence real balances of $m_t = \delta$.[7]

2.1 Consumers

The budget constraints faced by the young born in period t are

[6] Champ and Freeman do not include indexed government debt in their model or study the inflation risk premium. In fact, the inflation risk premium is zero in their model because utility is additively separable and linear in old age consumption (ie consumer preferences are risk-neutral over retirement consumption).

[7] The standard approach in OG models is to assume that the current money injection is a lump sum transfer to the old, who then sell the entire money stock to the young. This approach implies that money holdings of the old are not subject to the inflation tax, a reasonable assumption only in economies where the monetary base is a small share of GDP. The alternative approach used here means that the money holdings of the old are hit by the inflation tax, and so is applicable more generally. If the standard approach were adopted instead, money holdings and the steady-state inflation rate would not matter for the inflation risk premium.

$$c_{t,Y} = w_t - T_t - k_{t+1} - b^i_{t+1} - b^n_{t+1} - m_t \tag{1}$$

$$\begin{aligned} c_{t+1,O} &= r^k_{t+1}k_{t+1} + r^f_t b^i_{t+1} + r^n_{t+1}b^n_{t+1} + r^m_{t+1}m_t \\ &= r^k_{t+1}k_{t+1} + [vr^f_t + (1-v)r^n_{t+1}]b_{t+1} + r^m_{t+1}m_t \end{aligned} \tag{2}$$

where v is the constant share of indexed bonds in the total bond portfolio, $b_{t+1} = b^i_{t+1} + b^n_{t+1}$.

Consumers have CRRA preferences with a discount factor β and coefficient of relative risk aversion γ. The young of period t solve the following problem:

$$\max_{\{k_{t+1},b^i_{t+1},b^n_{t+1}\}} U_t = \frac{c^{1-\gamma}_{t,Y} - 1}{1-\gamma} + \beta E_t\left[\frac{c^{1-\gamma}_{t+1,O} - 1}{1-\gamma}\right] \qquad \text{s.t. (1), (2) and } m_t = \delta$$

The first-order conditions are as follows:

$$1 = E_t[sdf_{t+1}r^k_{t+1}] \qquad \text{for capital, } k \tag{3}$$

$$1 = R_t E_t[SDF_{t+1}] \qquad \text{for nominal bonds, } b^n \tag{4}$$

$$1 = r^f_t E_t[sdf_{t+1}] \qquad \text{for indexed bonds, } b^i \tag{5}$$

where $sdf_{t+1} \equiv \beta(c_{t+1,O}/c_{t,Y})^{-\gamma}$ and $SDF_{t+1} \equiv sdf_{t+1}/\Pi_{t+1}$.

2.2 Firms

The production sector consists of a representative firm that produces output by combining capital and labour in a Cobb-Douglas production function. The share of capital in output is equal to α and the labour share is $1–\alpha$. The firm hires capital and labour to maximise current profits. Total factor productivity is denoted A_t. Assuming competitive markets, the real wage and the return on capital are:

$$w_t = y_t - r^k_t k_t = (1-\alpha)A_t k^\alpha_t \tag{6}$$

$$r^k_t = \alpha y_t / k_t = \alpha A_t k^{\alpha-1}_t \tag{7}$$

Total factor productivity is stochastic and follows

$$A_t = (A_{t-1})^{\rho_A} \exp(e_t) \tag{8}$$

where e_t is an IID-normal innovation with mean zero and standard deviation $\sigma_{e,A}$.

2.3 Government

The government conducts fiscal policy and commits to a money supply rule and a bond supply rule. The total supply of government bonds must satisfy $b = b^i + b^n > 0$, and the shares of indexed and nominal bonds in the total bond portfolio are constant and equal to v and $1–v$ respectively. Government debt is assumed to be stationary and in positive net supply, but no particular bond supply rule is specified at this stage to highlight the generality of the results that follow. Likewise, no specific assumptions are made at this stage about taxes and government spending, except that the implied path of primary deficits is consistent with the government budget constraint.

The government budget constraint is given by

$$\begin{aligned} g_t &= T_t + b^i_{t+1} - r^f_{t-1} b^i_t + b^n_{t+1} - r^n_t b^n_t + m_t - r^m_t m_{t-1} \\ &= T_t + b_{t+1} - [v r^f_{t-1} + (1-v) r^n_t] b_t + m_t - r^m_t m_{t-1} \end{aligned} \tag{9}$$

where g_t is government spending in real terms.

Government spending is exogenous and used up in projects that have no effect on utility. Nevertheless, it is important to include the government budget constraint to assess the fiscal implications of the inflation risk premium, as is done in Section 5.

The government sets the nominal money supply $M_t \equiv P_t m_t$ according to

$$M_t = \Pi^* M_{t-1} \exp(\varepsilon_t) \tag{10}$$

where $\Pi^* > 0$ is the target money supply growth rate and ε_t is a money supply shock.

The money supply shock is assumed to follow an AR(1) process, $\varepsilon_t = \rho_M \varepsilon_{t-1} + u_t$, where u_t is an IID-normal innovation with mean zero and standard deviation $\sigma_{u,\Pi}$. The target money supply growth rate is denoted Π^* because it plays the role of a constant inflation target in the model. In fact, since money market equilibrium and the legal constraint on cash holdings imply that $P_t = M_t/\delta$, inflation is equal to the money supply growth rate:

$$\Pi_t = M_t / M_{t-1} = \Pi^* \exp(\varepsilon_t) \tag{11}$$

Notice that, in the absence of money supply innovations, inflation would be stabilized at the inflation target. Hence, monetary innovations are the sole source of inflation variations. This result depends on the assumption that monetary policy attempts to keep inflation at target, without any concern for consumption deviations. However, this is a natural assumption given the long horizon in the model and is consistent with the stated objectives of central banks – to stabilize in the short run and provide a stable nominal anchor in the long run (e.g. Bank of England, 1999). It is also consistent with long run empirical evidence on monetary neutrality and inflation (see Bullard,1999).

2.4 Aggregate resource constraint

Capital is assumed to depreciate fully within a period. It follows that investment in period t is given by $i_t = k_{t+1}$. The economy's aggregate resource constraint in period t is therefore

$$y_t = c_t + k_{t+1} + g_t \tag{12}$$

where $c_t \equiv c_{t,Y} + c_{t,O}$ is aggregate consumption.

It is easy to verify using (1), (2), (6), (7) and (9) that this equation is satisfied in equilibrium.

3. The inflation risk premium: analytical solution and discussion

3.1 Solution

A second-order approximation of the Euler equations (4) and (5) leads to

$$\hat{r}_t^f = -E_t[s\hat{d}f_{t+1}] - \frac{1}{2}\operatorname{var}_t[s\hat{d}f_{t+1}] + O[2] \tag{13}$$

$$\hat{R}_t = -E_t[S\hat{D}F_{t+1}] - \frac{1}{2}\operatorname{var}_t[S\hat{D}F_{t+1}] + O[2] \tag{14}$$

where 'hats' denote log deviations from steady-state and $O[2]$ terms of order higher than two.

Given that $SDF_{t+1} \equiv sdf_{t+1} / \Pi_{+1}$ and ignoring terms higher than second-order, we have the following relationship between nominal and real interest rates:

$$\begin{aligned}\hat{R}_t &= -E_t[S\hat{D}F_{t+1}] - \frac{1}{2}\operatorname{var}_t[S\hat{D}F_{t+1}] = -E_t[s\hat{d}f_{t+1} - \hat{\Pi}_{t+1}] - \frac{1}{2}\operatorname{var}_t[s\hat{d}f_{t+1} - \hat{\Pi}_{t+1}] \\ &= \hat{r}_t^f + \left(E_t[\hat{\Pi}_{t+1}] - \frac{1}{2}\operatorname{var}_t[\hat{\Pi}_{t+1}]\right) + \operatorname{cov}_t[s\hat{d}f_{t+1}, \hat{\Pi}_{t+1}]\end{aligned} \tag{15}$$

The term in the big brackets in (15) is expected inflation. The covariance term is the inflation risk premium. It tells us that if inflation is high when the marginal utility of consumption is high, nominal bonds will pay a higher equilibrium interest rate to consumers to compensate for the fact that their real payoff will tend to be low at times when extra consumption is valued highly. Notice that since the covariance term in (15) is conditional on period-*t* information, it is only the component of marginal utility that is correlated with unanticipated inflation that matters for the inflation risk premium.[8] The reason is simply that, as (15) makes clear, predictable changes in future inflation are reflected in a higher nominal yield through the expected inflation term. As usual, if the inflation risk premium is zero we end up with a Fisher equation relating nominal rates to real rates and expected inflation.[9]

Denoting the inflation risk premium *IRP* we have

$$IRP = \operatorname{cov}_t[s\hat{d}f_{t+1}, \hat{\Pi}_{t+1}] = \operatorname{cov}_t[-\gamma(\hat{c}_{t+1,O} - \hat{c}_{t,Y}), \hat{\Pi}_{t+1}] = -\gamma\operatorname{cov}_t[\hat{c}_{t+1,O}, \hat{\Pi}_{t+1}] \tag{16}$$

And by Equation (2), and using the fact that $m_t = \delta$,

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[8] Consequently, the persistence parameters ρ_M and ρ_A will not matter for the inflation risk premium.

[9] That is, $\hat{R}_t = \hat{r}_t^f + E_t[\hat{\Pi}_{t+1}] - (1/2)\operatorname{var}_t[\hat{\Pi}_{t+1}]$.
The term in $\operatorname{var}_t[\hat{\Pi}_{t+1}]$ is a correction that arises due to Jensen's inequality because $E_t[\Pi_{t+1}^{-1}] \neq [E_t\Pi_{t+1}]^{-1}$.

$$\hat{c}_{t+1,O} = \theta_k(\hat{r}^k_{t+1} + \hat{k}_{t+1}) + \theta_b[v\hat{r}^f_t + (1-v)\hat{r}^n_{t+1} + \hat{b}_{t+1}] + \theta_m \hat{r}^m_{t+1} \tag{17}$$

where $\hat{r}^n_{t+1} = \hat{R}_t - \hat{\Pi}_{t+1}$ and $\hat{r}^m_{t+1} = -\hat{\Pi}_{t+1}$.

Here, $\theta_k \equiv \alpha k^{\alpha}/c_O$ is the steady-state share of capital income in retirement consumption, $\theta_b \equiv r^f b/c_O$ is the steady-state share of bond income in retirement consumption, and $\theta_m \equiv r^m m/c_O = \mathrm{m}(\Pi^*)^{-1}/c_O$ is the steady-state share of cash holdings in retirement consumption. Notice that steady-state values are indicated by the absence of time subscripts.

Using (17) in (16), the inflation risk premium is equal to[10]

$$\begin{aligned} IRP &= -\gamma \operatorname{cov}_t[-(\theta_m + (1-v)\theta_b)\hat{\Pi}_{t+1}, \hat{\Pi}_{t+1}] \\ &= \gamma[(1-v)\theta_b + \theta_m]\operatorname{var}_t[\hat{\Pi}_{t+1}] \\ &= \gamma[(1-v)\theta_b + \theta_m]\sigma^2_{u,\Pi} > 0 \end{aligned} \tag{18}$$

Finally, since each period in the model lasts N years, the annualised inflation risk premium is[11]

$$IRP_{ann} \equiv \frac{IRP}{N} = \frac{\gamma}{N}[(1-v)\theta_b + \theta_m]\sigma^2_{u,\Pi} \tag{19}$$

3.2 Discussion

Equations (18) and (19) show that the inflation risk premium depends positively on the share of nominal government debt, $1 - v$, and the steady-state shares of bond income and cash holdings in consumer retirement portfolios, θ_b and θ_m respectively. These channels do not appear to have been discussed in previous theoretical literature. Increases in these shares raise the inflation risk premium because the old are more exposed to unanticipated inflation if a larger portion of their retirement portfolio is nominal. In fact, the term in square brackets, $(1-v)\theta_b + \theta_m$, is simply the total share of retirement consumption funded by nominal asset holdings.

To bring out the economic factors that matter for the inflation risk premium more clearly, it is instructive to rewrite the expression in (19) as follows:

$$IRP_{ann} = \frac{\gamma}{N\mu_C}\left[(1-v)r^f\left(\frac{b}{y}\right) + \frac{1}{\Pi^*}\left(\frac{m}{y}\right)\right]\sigma^2_{u,\Pi} \tag{20}$$

where $\mu_c \equiv c_O/y$ is the steady-state fraction of old age consumption in GDP and the fact that $r^m = 1/\Pi^*$ at steady-state has been used.

[10] This matches exactly the numerical solution from a second-order perturbation of the model in Dynare.

[11] Division by N follows from the assumption that annual yield = (N-year yield)$^{1/N}$. This conversion is common in the OG literature – see e.g. Constantinides and Mehra (2002, p. 285) and Olovsson (2010, p. 369).

The expression shows that the inflation risk premium depends on several macro variables. It rises with the steady-state debt-GDP ratio and the share of nominal debt in the government bond portfolio, both of which depend on government debt policy.[12] The inflation risk premium will also rise if there is a fall in the steady-state rate of inflation Π^* or if the steady-state ratio of cash to GDP rises, two factors which depend upon monetary policy. Intuitively, lower trend inflation implies less erosion in the value of money holdings held into old age and so leaves consumers with a larger stock of nominal wealth vulnerable to unanticipated inflation. The intuition for the second effect is simply that, for any given trend inflation rate, a larger stock of nominal assets is carried into old age if the reserve requirement for money holdings is strengthened.

It is worth pointing out some additional features of the analytical solution. First, notice that the inflation risk premium is unambiguously positive. The reason is that consumers hold nominal assets but have no nominal liabilities. In theory, introducing nominal liabilities could lead to a negative inflation risk premium, but only if these liabilities had a more important role in retiree portfolios than nominal assets. However, that seems unlikely because the main nominal liability for households is mortgages, and these are typically paid off before retirement. In support of this, Doepke and Schneider (2006) report that net nominal position of US households over 65 is strongly positive, while Meh and Terajima (2008) find the same result for Canada. It is also worth noting that empirical evidence points to an average inflation risk premium that is robustly positive at the long horizons relevant for the model here (see Bekaert and Wang, 2010).

A second interesting feature of the analytical solution is that it implies that the inflation risk premium does not depend on productivity risk.[13] Intuitively, productivity variations matters for the inflation risk premium only to the extent that the resulting fluctuations in marginal utility are correlated with the unpredictable component of inflation. In the model at hand, there is no such correlation because unanticipated variations in inflation result from money supply innovations, which are uncorrelated with innovations to productivity. In turn, this result is driven by the assumption that monetary policy does not attempt to stabilize consumption but instead aims at a constant inflation target. This assumption seems well founded, however, given the long horizon in the model. In particular, it is consistent with the widely held view that the best central banks can do in the long run is provide a stable nominal anchor for the economy.

4. The inflation risk premium: numerical results

To get an idea of how the model performs, it is instructive to plug appropriate numbers for a developed economy into (20). This is done in this section. The predicted inflation risk

[12] The risk premium also rises with the risk-free rate r^f, but this channel is not discussed here.

[13] Hördahl et al. (2008) reach a similar result in a special case of their model in which prices are fully-flexible. However, in De Paoli et al. (2010) the inflation risk premium is positive under flexible prices in an economy with only productivity risk.

premium is compared with the empirical and theoretical literature. The analysis then provides an assessment of which macro variables are likely to be most important for inflation risk premia.

4.1 Numerical solution

To compute the inflation risk premium, the parameters and ratios in (20) need to be assigned a numerical value. Canada was chosen for this purpose because it is fairly representative developed economy and has the advantage that detailed information is publicly available on the distribution of government debt by maturity and the importance of nominal asset positions. The calibration is summarised in Table 1.

Table 1 – Calibrated values in the numerical analysis

Parameter	Value
Number of years per period, N	30
Coeff. of relative risk aversion, γ	4
30yr Money supply innovation std, $\sigma_{u,\Pi}$	0.14
30yr Debt-to-GDP ratio, b/y	0.07
Share of indexed govt. debt, v	0.25
Old age consumption-GDP ratio, μ_c	0.30
Cash-to-GDP ratio, m/y	0.02
Inflation target, Π^*	1.81
Risk-free real interest rate, r^f	1.65

The number of years per period N was set equal to 30, implying that bonds have a maturity of 30 years. The coefficient of relative risk aversion was set equal to 4. This calibration is relatively high, but still plausible, to give the model a better chance of matching inflation risk premia in the data. The variance of the money supply innovation was set at 0.0196 (=30*0.025^2), which implies a 30-year standard deviation of 0.14 (=0.0196$^{1/2}$). This calibration implies a standard deviation of *annual inflation* of 0.025 under the assumption of base-level drift in the price level.[14] Consistent with this figure, the standard deviation of annual CPI inflation in Canada over the period 1980-2012 was approximately 2.5% (see Statistics Canada website). McCallum (1997) and Dittmar et al. (1999) have previously used the random walk assumption to compute inflation risk at long horizons.

[14] Annual inflation in the model is defined as the log of $(\Pi_t)^{1/30}$.

The share of indexed government debt v was set at 0.25, which is similar to the share of 30-year Real Return Bonds in the total stock of 30-year government bonds (see Department of Finance Canada 2011, Table V and Chart 2). The debt-GDP ratio is set at 0.07 because total marketable government debt was 35% of GDP in 2011 and around 1/5th of this total was in 30-year bonds; see Department of Finance (2011, Table 2 and Chart 2) and Bank of Canada (2012, Table H1). The steady-state ratio of old age consumption to GDP was set at 0.30, because aggregate consumption is around 60% of GDP, and this is assumed to be split equally between young and old agents. The steady-state share of money holdings in GDP is set at 0.02 and the inflation target Π^* at 1.81 ($=1.02^{30}$), based on the annual inflation target of 2%.[15] Finally, the steady-state risk-free real rate was set at 1.65, consistent with an annual real interest rate of 1.7% and an annual nominal interest rate of 3.7%.

Plugging these values into (20) implies an inflation risk premium of 8.6 basis points. This is economically non-trivial but roughly an order of magnitude lower than most empirical estimates in the literature. Of course, the difficulties that theoretical models face in matching asset prices and risk-premia are well-known and apply to both bonds and equity (Mehra and Prescott, 1985; Backus et al., 1989; Rudebsuch and Swanson, 2008), so it is perhaps not surprising that the OG model fails to match the magnitude of inflation risk premia in the data. It is nevertheless of interest to compare the numerical solution with the literature to get an idea of how it compares with representative agent models. This is done in the next section after a brief review of the empirical literature.

4.2 Comparison with the literature

Empirical studies

In a recent survey of the empirical literature, Bekaert and Wang (2010) show that there is no clear consensus on the magnitude of the inflation risk premium. In particular, while empirical estimates of the inflation risk premium are generally positive, they vary somewhat across studies, ranging from 0 to over 200 basis points depending on maturity and the economy considered. [16] A standard approach in the empirical literature has been to estimate no-arbitrage affine models of the term structure using nominal yields and inflation data. More recently, however, several studies have included additional information from index-linked yields, inflation surveys or inflation swaps, or have combined a reduced-form model of the term structure with structural equations from DSGE models. The literature has focused mainly on three developed economies: the US, the Euro Area and UK.

US studies have generally found a strongly positive average inflation risk premium. For example, D'Amico et al. (2009) found an inflation risk premium on 10-year bonds of 64 basis points on average, while Ang et al. (2008) and Campbell and Viceira (2001) report a higher

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[15] Currency outside banks was around 3.3% of GDP in 2011, so this calibration may be on the conservative side.

[16] Of the ten recent studies reviewed by Bekeart and Wang, one reports a negative inflation risk premium. However, this is likely to be driven by the presence of a significant 'liquidity premium' in the US TIPS market from its creation in 1997 up until 2004.

average risk premium of around 110 basis points. The largest estimate in the recent literature appears to be the 201 basis points reported by Chernov and Mueller (2012), while the lowest is the average of zero reported by Christensen et al. (2010). In the Euro Area, by contrast, inflation risk premia appear to be much lower. For instance, Garcia and Werner (2010) report an inflation risk premium at a 5-year maturity of around 25 basis points, while Hördahl and Tristani (2012) find an average risk premium at a 10-year maturity of just over 20 basis points. A recent study on Canada by Feunou and Fontaine (2012) also finds a modest inflation risk premium on 5-year bonds. In the UK the picture is more mixed, with Risa (2001) reporting a large average inflation risk premium of 184 basis points on 5-year bonds, compared to around 100 basis points in Joyce et al. (2010), and around 50 in Andreasen (2012). To the author's knowledge, the only study to estimate the average inflation risk premium on 30-year bonds is the US study by Haubrich et al. (2008), who report an estimate of 101 basis points.

While there is no clear consensus on the magnitude of the inflation risk premium, the bulk of empirical evidence points to a premium that is robustly positive and increasing at long maturities. As Bekaert and Wang (2010) note, much of the variation across studies is likely due to different sample periods or differences in information used in estimation. Economic factors may also play a role, particularly in explaining cross-country differences such as the relatively low inflation risk premium in the Euro Area. However, most of the studies in the empirical literature struggle to shed light on the economic fundamentals that matter for inflation risk premia due to their reliance on reduced-form models.[17] As a result, theoretical models have also played an important role in the recent literature.

Theoretical models

To better understand the economic fundamentals drive bond prices and inflation risk premia, several researchers have turned to theoretical models of the economy. For instance, Hördahl et al. (2008) set up a small-scale New Keynesian model with habit formation and show that it can resolve several bond pricing puzzles while providing a fairly good fit to key macroeconomic variables. To better understand the economic mechanisms at work, they provide analytical solutions for bond prices based on second-order perturbation approximations. These solutions show that a strong degree of interest rate smoothing is crucial. They also shed light on the importance of nominal rigidities, as a flex-price version of the model cannot replicate the same results as under sticky prices. The inflation risk premium in the model is positive but modest at less than 5 basis points for maturities up to 1 year and virtually zero for longer maturities up to 10 years.

More recently, De Paoli et al. (2010) set up a small-scale New Keynesian model with real rigidities and solve it numerically using a second-order perturbation method. They show that the impact of nominal rigidities on risk-premia depends on whether the economy is

[17] The exceptions are Andreasen (2012), Hördahl and Tristani (2012) and Buraschi and Jiltsov (2005). Andreasen estimates a full structural general equilibrium model. Hördahl and Tristani and Buraschi and Jiltsov add equations from structural economic models into estimated affine models of the term structure.

dominated by productivity or monetary policy shocks. For instance, with perfectly flexible prices, the inflation risk premium is zero if there are monetary policy shocks only, but this rises to 35 basis points when productivity shocks are introduced. With sticky prices, the inflation risk premium is lower at 9 basis points with only productivity shocks, and it is negative in an economy dominated by monetary policy shocks.

Finally, Andreasen (2012) builds on these earlier findings by estimating a medium-scale New Keynesian model of the UK economy with several different shocks and rigidities using a third-order approximation that allows for time-varying risk premia. He concludes that there was a substantial fall in nominal term premia in the 1990s caused mainly by a reduction in inflation risk premia and driven by preference, investment, and fixed cost shocks, as well as a more aggressive response to inflation by the Bank of England. The 5-year inflation risk premium from the model averages around 50 basis points, which is of the same order of magnitude as empirical studies and somewhat higher than in small-scale calibrated models like De Paoli et al. (2010) and Hördahl et al. (2008).[18]

These results suggest that standard DSGE models may soon be able to provide a reasonable fit to asset risk-premia and key macroeconomic variables. It should be noted, however, that the shocks and rigidities that drive risk premia in the above models do not always admit an easy real-world economic interpretation, because they are not directly unobservable. In this regard, it is interesting that the analytical solutions in the current paper relate the inflation risk premium to observable macro variables. It is also notable that the predicted inflation risk premium is of similar size to other small-scale calibrated models like Hördahl et al. (2008) and De Paoli et al. (2010). The next section uses the analytical solution for the inflation risk premium to shed light on which macro variables are likely to be most important for understanding inflation risk premia in developed economies.

4.3 Which variables matter for the inflation risk premium?

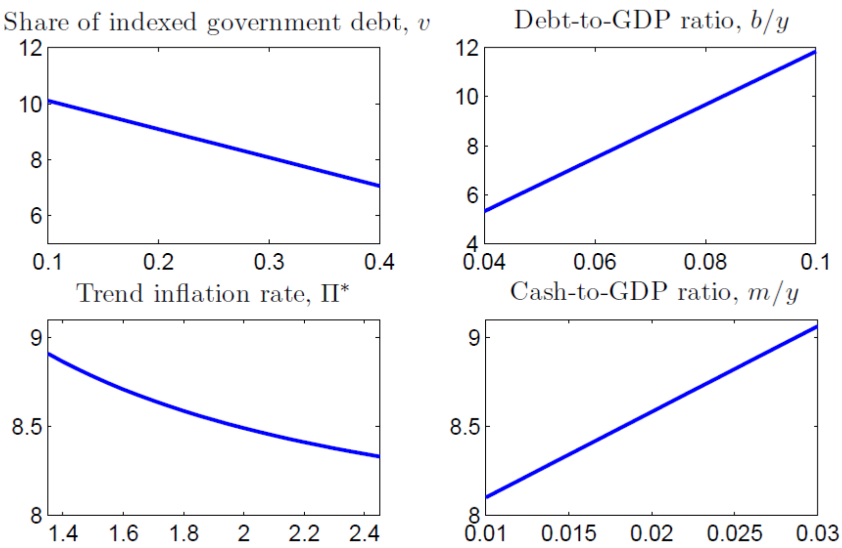
The analytical solution for the inflation risk premium shows that it depends on several different macro variables under the control of policymakers, namely, trend inflation; the ratios of money balances and government debt to GDP; and the share of indexed government debt. This section investigates which of these factors are likely to be important for explaining inflation risk premia in developed economies. To do so, a simple sensitivity analysis is conducted using the calibrated analytical expression from the previous section. In particular, a single share or parameter of interest is varied with all others held constant at their baseline values. The results are shown in Figure 1.

The inflation risk premium is quite sensitive to both the share of indexed debt and the debt-GDP ratio, suggesting that these are potentially important channels by which government policy could have an effect. For example, increasing the 30-year debt ratio from the baseline value of 0.07 to 0.10 increases the inflation risk premium from 8.6 basis points to 11.9 – an

[18] As Andreasen notes, his risk-premia results rely on high risk aversion through Epstein-Zin preferences.

increase of well over one quarter. On the other hand, lowering the debt ratio to 0.04 lowers the risk premium to less than 6 basis points. Increasing the share of debt that is indexed has the effect of lowering the inflation risk premium. The effect here is also relatively strong: increasing the indexation share from 0.25 to 0.40 lowers the inflation risk premium by around 1.6 basis points.

Fig 1 – Inflation risk premium and key macro variables



Notes: Vertical axis is measured in basis points per annum

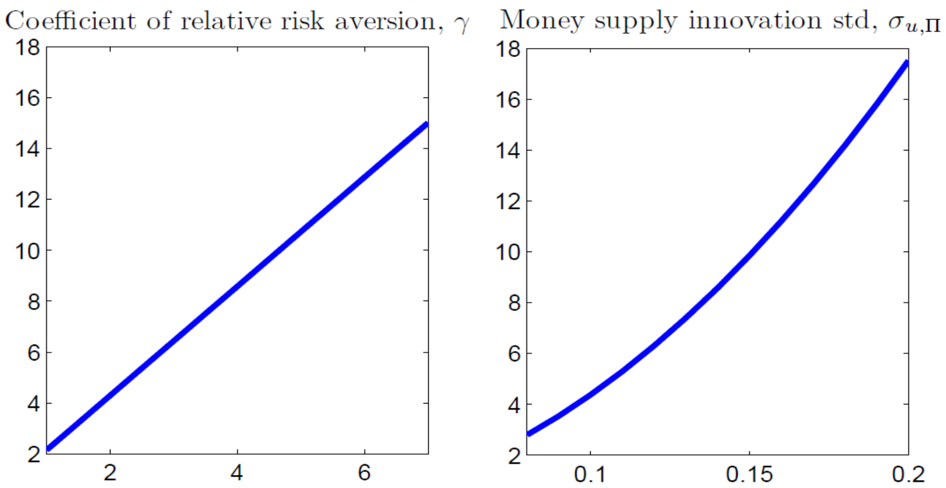
By contrast, the monetary variables have relatively little impact. For instance, increasing the ratio of cash to GDP by one-half to 0.03 raises the inflation risk premium by approximately 1 basis point. The effect from the inflation target is even smaller due to its non-linear impact on the inflation risk premium: raising trend inflation from 1% per annum (*x*-axis intercept) to the baseline value of 2% lowers the inflation risk premium by around 0.3 basis points, and a further increase to 3% per annum (*x*-axis end point) reduces the inflation risk premium by only an additional 0.25 basis points. While these numerical findings are clearly conditional on the calibration for Canada, they are likely to generalise somewhat given that developed economies have similar levels of average inflation and a monetary base that is small relative to GDP and the stock of government debt.

4.4 Sensitivity of the inflation risk premium

The inflation risk premium of 8.6 basis points depends on the coefficient of relative risk aversion, the quantity of inflation risk, and the share of retirement consumption funded by nominal assets, $(1–v)\theta_b + \theta_m$. In this section I consider sensitivity to these calibrated values.

Figure 2 shows the how the inflation risk premium varies with risk aversion and the quantity of inflation risk, where the latter is defined by the standard deviation of the money supply innovation. Unsurprisingly, the inflation risk premium is sensitive to these calibrated values. For instance, if the coefficient of relative risk aversion is reduced to 1 (ie the case of log utility), then the inflation risk premium falls to only 2 basis points per annum, while it rises to over 15 basis points under a relatively high risk aversion coefficient of 7. Sensitivity to the quantity of inflation risk is even greater: the inflation risk premium falls to around 3 basis points with a standard deviation of 0.08, but a standard deviation of 0.2 raises the inflation risk premium to more than 17.5 basis points.[19] Despite this sensitivity, the inflation risk premium is comparable under low calibrations to bond risk premia in canonical DSGE models such as Rudebusch and Swanson (2008).[20] This finding suggests the transmission mechanism in the OG model may be of help in generating non-trivial bond risk premia.

Fig 2 – Inflation risk premium: risk aversion and the quantity of inflation risk



Notes: Vertical axis is measured in basis points per annum

It is also of interest to consider the implications of an alternative calibration for the total share of retirement consumption funded by nominal assets, $(1\text{–}\nu)\theta_b + \theta_m$. We can make such a calibration using the data on Canadian nominal portfolios in Meh et al. (2010). In particular, they report that the net nominal position of consumers in 66-75 age group is 28% of their net worth, and this figure rises to 32% for the over 75s. Taking the mid-point of 30% implies a calibrated value of $(1\text{–}\nu)\theta_b + \theta_m = 0.30$.[21] The implied inflation risk premium is 7.8 basis points. This value is lower than the baseline of 8.6 basis points, but the difference is fairly small since the implied share of nominal assets under the baseline calibration is similar at 0.33. Therefore, the numerical results appear to be robust to alternative ways of calibrating the nominal asset share in the model.

[19] The low standard deviation of 0.08 corresponds to an annual inflation standard deviation of around 1.5%, and the high standard deviation of 0.2 to an annual standard deviation of around 3.7%.

[20] Rudebusch and Swanson show that a benchmark New Keynesian model produces a 10-year term premium on nominal bonds of only 1.4 basis points.

[21] The share in net worth can be interpreted as the consumption share because the old consume all their wealth.

5. Fiscal implications of the inflation risk premium

The results of the previous section suggest that government debt policy could have important implications for the inflation risk premium and that this conclusion does not hinge on a specific calibration of the model. With this in mind, the fiscal implications of the inflation risk premium are investigated in this section. The analysis concentrates on the the share of indexed debt in the government bond portfolio since – as the quote at the start of this paper suggests – the inflation risk premium may be an important cost consideration for governments who can issue both indexed and nominal debt, because issuing indexed debt avoids paying the inflation risk premium.

The analysis starts by presenting some general analytical results that clarify the meaning of ‘fiscal implications’ of the inflation risk premium. It then turns to the example of a government that has a positive amount of debt that it wishes to roll over while maintaining the real level of government spending; it does this by allowing taxes to adjust to ensure that its budget constraint is satisfied. The question addressed here is: are the tax implications of this policy quantitatively relevant, and does the answer hinge critically on the share of government debt that is indexed as a result of the inflation risk premium? Some analytical results are first derived. The analysis then turns to numerical results from a simulated model.

5.1 Analytical results

5.1.1 A general result

As shown in Section 3, the inflation risk premium introduces a wedge in the Fisher equation. As a result, it will generally have implications for government borrowing costs. We can see this formally by taking a second-order Taylor expansion of the real return on nominal bonds and subtracting our second-order accurate expression for the risk-free rate given by (13).

As is shown in Appendix A, this leads to the following real return differential:

$$\hat{r}_t^n - \hat{r}_{t-1}^f = (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) + IRP + \frac{1}{2}(E_{t-1}[\hat{\Pi}_t^2] - \hat{\Pi}_t^2) + O[2] + t.i.i.r. \tag{21}$$

where *t.i.i.r.* denotes ‘terms independent of inflation risk.’[22]

The first term on the right hand side shows that the real return on nominal debt will fall when inflation is unexpectedly high. Lucas and Stokey (1983) showed that, for this reason, the government has an incentive to inflate away its nominal debt if it cannot commit to a monetary policy that delivers a predetermined path for prices. This ability to reduce nominal liabilities *ex post* through surprise inflation could be one reason that governments appear to favour nominal debt over indexed debt. However, (21) shows us that the real return payable on nominal debt may exceed that on indexed debt even at times when inflation is

[22] To ease notation, $\hat{r}_t^n$ denotes the second-order accurate solution here. See Appendix A for the full derivation.

unexpectedly high, because it also includes the positive inflation risk premium.[23] Moreover, (21) tells us that the *expected differential* in real returns – which is what matters for a government committed to achieving an inflation target – will tend to be positive due to the inflation risk premium. We can see this formally from the conditional expectation of (21):

$$E_{t-1}[\hat{r}_t^n - \hat{r}_{t-1}^f] = IRP + O[2] + t.i.i.r. \tag{22}$$

To consider the full fiscal implications of (21) and (22), we need the government budget constraint and some assumptions about government policy. The next section sets out these assumptions. Analytical and quantitative results are then reported for the example at hand.

5.1.2 Tax implications of inflation risk premium

Let us suppose that the government has a positive amount of outstanding debt that it wishes to roll over (under the assumption that it can commit to money supply rule) while maintaining the real level of government spending and satisfying its budget constraint. It implements this policy by allowing lump-sum taxes T_t to adjust to ensure that the government budget constraint holds with equality in every period.

Taxes are therefore given by

$$\begin{aligned} T_t &= g_t + [v r_{t-1}^f + (1-v) r_t^n] b_t - b_{t+1} + r_t^m m_{t-1} - m_t \\ &= \bar{g} + [v r_{t-1}^f + (1-v) r_t^n - 1]\bar{b} + [r_t^m - 1]\delta \end{aligned} \tag{23}$$

where $\bar{g}$ ($\bar{b}$) is the constant level of real government spending (the total bond supply) and the fact that $m_t = \delta$ has been used.

The expression in (23) makes clear that a government wishing to hold government spending constant and roll over its debt must adjust taxes to cover the borrowing rate it faces on its overall debt portfolio, minus any contribution from the inflation tax on money. In turn, the overall borrowing rate depends on the real rates payable on indexed and nominal debt and the constant fractions of each type of debt in the government bond portfolio, as given by the shares v and $1–v$. As (21) and (22) indicate, the inflation risk premium will be an important factor affecting the real return payable on nominal versus indexed debt. However, issuing indexed debt to avoid paying the inflation risk premium is generally *not* enough to guarantee an unambiguous reduction in the level of taxes.

In fact, as shown in Appendix B, a second-order accurate expression for taxes is

$$\hat{T}_t + \frac{1}{2}\hat{T}_t^2 = (1-v)\phi_{1,t}(IRP - JI) - \phi_{2,t} PS_{\Pi} + \phi_3 \hat{\Pi}_t^2 + O[2] + t..i.i.r. \tag{24}$$

[23] The additional term $(1/2)(E_{t-1}[\hat{\Pi}_t^2] - \hat{\Pi}_t^2)$ arises due to Jensen's inequality and has an ambiguous sign.

where $JI \equiv \frac{\sigma_{u,\Pi}^2}{2} > 0$, $PS_\Pi \equiv \frac{\gamma^2[(1-v)\theta_b + \theta_m]^2 \sigma_{u,\Pi}^2}{2} > 0$, $\phi_3 > 0$ and $\phi_{1,t}$, $\phi_{2,t} > 0$ under standard calibrations.

In (24) the term JI is the Jensen's inequality correction term in nominal interest rates, while PS_Π is the precautionary savings effect due to inflation risk – that is, additional saving to guard against the possibility that unexpectedly high inflation next period will push consumption below its optimal level.[24] An increase in precautionary saving pushes down the equilibrium risk-free rate, so that the level of taxes needed to satisfy the government budget constraint is lower. Notice also that, intuitively, the incentive to engage in precautionary saving falls as the share of indexed government debt v increases. Turning to the inflation risk premium, it has been written as $IRP - JI$, since this term will be positive for a standard calibration of risk aversion. Hence, (24) tells us that the effect of inflation risk on the level of taxes depends on two opposing forces: the inflation risk premium term and the precautionary savings effect. In addition, there is a squared term in inflation deviations which pushes up taxes. The coefficient on this term, ϕ_3, falls as the indexation share is increased. Since this last term is time-varying, it is instructive to focus on the average level of taxes.

Taking the conditional expectation of (24) gives

$$E_{t-1}\left[\hat{T}_t + \frac{1}{2}\hat{T}_t^2\right] = (1-v)\phi_{1,t}(IRP - JI) - \phi_{2,t}PS_\Pi + \phi_3\sigma_{u,\Pi}^2 + O[2] + t.i.i.r. \tag{25}$$

This equation shows that taxes will be higher, on average, if the inflation risk premium and inflation variance term dominate the precautionary savings effect. Moreover, it is clear that the impact on taxes will depend on the share of indexed government debt v, though the effect of increasing v will generally be ambiguous due to the precautionary savings effect. For instance, if the share of indexed debt is increased, this will lower level of taxes through a fall in the inflation risk premium and a reduction in the coefficients on the inflation risk premium and inflation variance terms in (25), but there will be a simultaneous increase in taxes due to the fall in precautionary saving that results when future consumption is less vulnerable to inflation variations. It is not possible in general to say whether a rise in the share of indexed debt will lower taxes, or whether the inflation risk premium will play an important role. Numerical analysis is needed to settle this issue.

The next section therefore uses numerical simulations of a calibrated model. To produce a plausible inflation risk premium, the model is augmented with Epstein-Zin preferences. These preferences help the model to match inflation risk premia in the data as they allow the coefficient of relative risk aversion and elasticity of intertemporal substitution to be calibrated separately.

[24] The definition of precautionary saving used here corresponds to that in De Paoli and Zabczyk (2013).

5.2 Simulated results: Tax implications of the inflation risk premium

In this section, the example in 5.1.2 is investigated quantitatively using a simulated version of the model with Epstein-Zin preferences. The main advantage of these preferences is that the elasticity of intertemporal substitution and the coefficient of relative risk aversion are calibrated separately, so that a high coefficient of relative risk aversion need not imply an implausibly low elasticity of intertemporal substitution. Andreasen (2012) and Rudebusch and Swanson (2012) show that these preferences enable otherwise standard New Keynesian models to produce plausible bond risk premia without compromising their ability to fit key macro variables. These preferences also imply that the second-order accurate solution for the inflation risk premium is the same as under CRRA preferences.

5.2.1 The model with Epstein-Zin preferences

Consumers

With Epstein-Zin preferences, consumers solve a maximization problem of the form

$$\max_{\{k_{t+1},b^i_{t+1},b^n_{t+1}\}} U_t^{EZ} = \frac{1}{1-\gamma}\left[c^{\varepsilon}_{t,Y} + \beta\left[E_t c^{1-\gamma}_{t+1,O}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \qquad \text{s.t. (1), (2) and } m_t = \delta \tag{26}$$

where γ is the coefficient of relative risk aversion and $1/(1–\varepsilon)$ is the elasticity of intertemporal substitution.

The first-order conditions are still given by (3)–(5) but, as shown in Appendix C, the stochastic discount factor is now given by

$$sdf^{EZ}_{t+1} = \beta\left(\frac{c_{t,Y}}{c_{t+1,O}}\right)^{1-\varepsilon}\left(\frac{c_{t+1,O}}{(E_t c^{1-\gamma}_{t+1,O})^{1/(1-\gamma)}}\right)^{1-\gamma-\varepsilon} \tag{27}$$

The remainder of the model is unchanged. It is worth briefly considering the implications of Epstein-Zin preferences for the inflation risk premium. Since only the stochastic discount factor has changed, it follows that a second-order accurate expression for the inflation risk premium is given by

$$IRP^{EZ} = \text{cov}_t[\hat{sdf}^{EZ}_{t+1}, \hat{\Pi}_{t+1}] \tag{28}$$

However, it is shown in the appendix that

$$\text{cov}_t[\hat{sdf}^{EZ}_{t+1}, \hat{\Pi}_{t+1}] = \text{cov}_t[\hat{sdf}_{t+1}, \hat{\Pi}_{t+1}] \tag{29}$$

Hence, the analytical solution for the inflation risk premium is equal to that under CRRA preferences for any given calibration of the coefficient of relative risk aversion:

$$IRP^{EZ} = IRP = \gamma[(1-\nu)\theta_b + \theta_m]\sigma^2_{u,\Pi} \tag{30}$$

Thus, the analytical solution for the inflation risk premium derived in Section 3 remains valid for understanding the risk premium in the extended model with Epstein-Zin preferences.

Government

It is worth briefly clarifying the role of the government. As discussed above, the government uses lump-sum taxes to ensure that it satisfies its budget constraint while making a constant level of real government purchases each period and rolling over a constant level of government debt. This situation is chosen because it is broadly representative of developed economies around the world: debt levels are positive and governments are under political pressure to maintain spending in real terms.

With government spending constant at $\bar{g}$ and debt constant at $\bar{b}$, taxes are given by

$$T_t = \bar{g} + [v r_{t-1}^f + (1-v) r_t^n - 1]\bar{b} + [r_t^m - 1]\delta \tag{31}$$

The assumption of constant government spending is made for convenience. This assumption could easily be relaxed in favour of a specification where spending followed an exogenous stochastic process, but this would not provide any additional insights. On the other hand, the assumption that government debt is constant is important, since the aim of the analysis is to isolate the *ceteris paribus* implications of the inflation risk premium for taxes – and this necessitates varying the share of indexed government debt v while holding the total amount of debt constant.

It is important to note that although the bond supply is a constant, it cannot simply be assigned any desired numerical value because it must be consistent with a steady-state solution of the model.[25] Here, steady-state bond supply rules of the following form are considered:

$$1 = (\chi/\beta) sdf^{EZ} \tag{32}$$

where χ is a positive parameter and sdf^{EZ} is the steady-state real stochastic discount factor.

A rule of this kind was chosen because it implies a unique steady-state that can be solved for analytically. The constant bond supply implied by this rule is derived in Appendix D, and the full steady-state solution of the model is listed in Appendix E. Under the bond supply rule in (32), the steady-state real interest rate is equal to χ/β and there is a simple linear relationship between consumption in youth and consumption in old age, which depends on χ. This relationship is used to choose an appropriate calibrated value, as discussed in the next section.

[25] The difficulty is that the steady-state Euler equation will not collapse to $1 = \beta r^f$ as in a representative agent model. Instead, it is given by $1 = r^f sdf^{EZ} = \beta r^f (c_O/c_Y)^{-\gamma}$, which is difficult to solve analytically.

5.2.2 Calibrating the model

The model is calibrated for Canada. The full calibration is listed in Table 2. As in the numerical analysis of Section 4, the number of years per period N is 30, the inflation target Π^* is set at 1.81 ($=1.02^{30}$), the variance of the money supply innovation is 0.0196, and the share of indexed government debt is 0.25. Because Epstein-Zin preferences break the link between risk aversion and the elasticity of intertemporal substitution, the coefficient of relative risk aversion was assigned a much higher value than in the numerical analysis of Section 4. In particular, γ was set at 16 in order to give an inflation risk premium of around 20 basis points. This value is consistent with Euro Area studies such as Garcia and Werner (2010) and Hördahl and Tristani (2012), as well as the results for Canada in Feunou and Fontaine (2012) which likewise suggest a positive but modest average inflation risk premium. The parameter ε was set at –0.35, which implies an elasticity of intertemporal substitution of 0.74. The risk aversion and intertemporal elasticity calibrations are close to those in Olovsson (2010), who also studies an OG model with Epstein-Zin preferences. The discount factor β was set at 0.64, which corresponds to an annual discount factor of 0.985.

Table 2 – Calibrated values in the simulated model

Parameter	Value
Number of years per period, N	30
Private discount factor, β	0.64
Coeff. of relative risk aversion, γ	16
Elasticity of intertemporal substitution, $1/(1-\varepsilon)$	0.74
Share of indexed govt. debt, v	0.25
Real government spending, $\bar{g}$	0.096
30-yr government debt, $\bar{b}$	0.026
Bond supply parameter, χ	0.90
Inflation target, Π^*	1.81
Money supply innovation persistence, ρ_M	0.50
30yr Money supply innovation std, $\sigma_{u,\Pi}$	0.14
Share of capital in output, α	0.24
Productivity persistence, ρ_A	0.80
30yr Productivity innovation std, $\sigma_{e,A}$	0.10
Reserve requirement for money holdings, δ	0.01

The degree of persistence in the money supply innovation ρ_M was set at 0.5 because there is evidence that inflation persistence has fallen in the Great Moderation period (e.g. Benati, 2008). The productivity shock is assumed to be strongly persistent (ρ_A = 0.80) and to have an innovation variance of 0.010. Together, these values imply an unconditional 30-year standard deviation of productivity of around 16.7%. The parameter α was set equal to 0.243, implying that capital income accounts for 24.3% of GDP and wage income for 75.7%. This value is a little on the low side compared to standard calibrations but helps the model to match the investment-GDP and debt-GDP ratios in the data. The parameter χ in the bond supply equation (32) was set equal to 0.90 because this implies that, at the deterministic steady-state, consumption by the young is slightly higher than consumption by the old, consistent with the retirement consumption puzzle.

On the fiscal side, real government spending was fixed at 0.096, which implies that a plausible share of GDP is accounted for by public expenditure. The steady-state bond supply implied by the bond supply rule is 0.026, which implies a reasonable ratio of 30-year government debt to GDP. Finally, the parameter δ was set at 0.01 since this implies a ratio of real money holdings to GDP of around 2%, which is fairly similar to the actual ratio.

5.2.3 Steady-state solution

The model was solved using a second-order perturbation in Dynare (Julliard, 2001). The steady-state solution is reported in Table 3, with target values based on Canadian data. The values reported come from the deterministic steady-state solution, with the exception of the inflation risk premium, which is based on the theoretical mean of the stochastic solution.[26]

Table 3 – Steady-state solution of the model

	Model	**Target**
Consumption:Output	**0.66**	**0.60**
C_Y:Output	**0.34**	**0.32**
C_O:Output	**0.32**	**0.28**
Investment:Output	**0.17**	**0.17**
Govt. spending:Output	**0.17**	**0.23**
30yr Govt. debt:Output	**0.05**	**0.07**
Real returns (r^k, r^n, r^f)	**1.14**	**1.50**
Nominal interest rate *R*	**3.11**	**3.50**
Inflation risk premium	**19.0 basis points**	**20.0 basis points**

Notes: Interest rates are annualised and in percentage points.

[26] The numerical solution for the inflation risk premium matches the analytical solution in (30) exactly.

Overall, the model does a fairly good job of matching the data. The ratios of consumption, investment and government expenditure are plausible, though the model struggles to match the relatively low consumption-GDP ratio in Canada. The ratio of 30-year debt to GDP undershoots its target value of 7%, but the model still generates an inflation risk premium close to the target of 20 basis points with a calibration of Epstein-Zin preferences similar to previous work in OG models (see Olovsson 2010). The real returns on capital, indexed bonds, and nominal bonds are equal at the deterministic steady-state at 1.1% per annum, implying a steady-state nominal interest rate of 3.1% given the annual inflation target of 2%. The target nominal interest rate of 3.5% is based on the yield on nominal government debt in Canadian data (see Bank of Canada, 2012) and the target real interest rate of 1.5% was calculated by subtracting 2% from the target nominal rate to account for the inflation target.

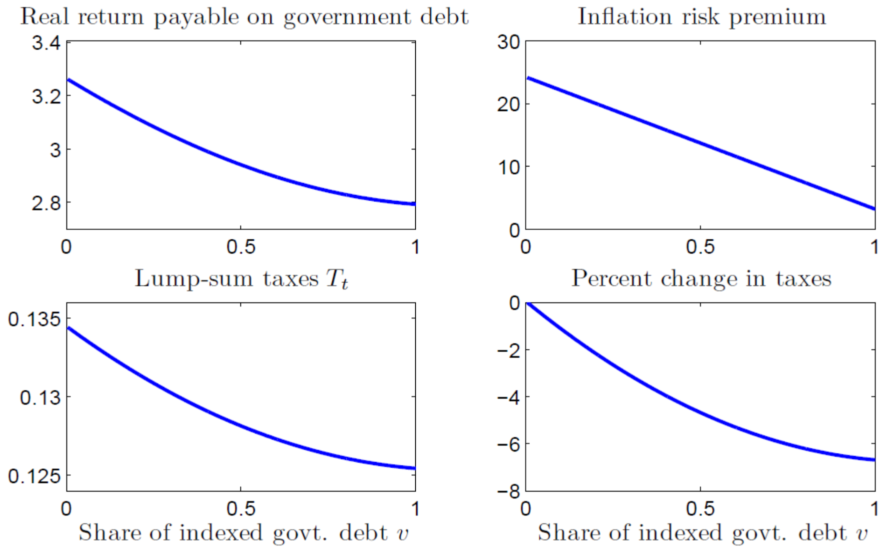
5.2.4 Results

The baseline results from the second-order stochastic solution are shown in Figure 3. The first panel shows that the average real return on government debt, $vr^f+(1-v)r^n$, falls as the share of indexed debt is increased. The effect is quantitatively quite large: moving from nominal debt only to a case where all debt is indexed reduces the average real return by around 35 basis points. As shown by the second panel, this reduction is driven by the inflation risk premium, which falls by around 20 basis points as we move from an economy with only nominal debt to one with only indexed debt. It is notable that the reduction in the real borrowing rate exceeds the reduction in the inflation risk premium alone.

The reason is that a fall in the inflation risk premium lowers borrowing costs in two ways. First, a fall in inflation risk premium has a direct impact on the nominal interest rate, as can be seen from (15). This first effect lowers the real borrowing rate on nominal government debt relative to that on indexed debt, as (21) and (22) show. Second, as Equation (B11) of the Appendix indicates, a fall in the inflation risk premium will generally lower the equilibrium real return on indexed debt because it implies a fall in expected consumption in old age. Hence, real borrowing rates on both nominal and indexed government debt fall as the share of indexed debt is increased, so that the overall reduction in borrowing costs exceeds the reduction in the real cost of nominal debt.

Due to the reduction in the real borrowing rate, the average level of taxes necessary to meet the government spending target falls as the share of indexed debt is increased. The effect on taxes is surprising large given the magnitude of the inflation risk premium. For example, moving from an economy with only nominal debt to one with only indexed debt lowers lump-sum taxes from around 0.134 to less than 0.126 – a reduction of more than 6.5%. It should also be emphasised that this reduction is permanent. These baseline results suggest that the share of indexed government debt has non-trivial implications for taxes due to the inflation risk premium. It is clearly important for this conclusion that the model can produce a plausible risk premium through Epstein-Zin preferences.

Fig 3 – Tax implications of the inflation risk premium



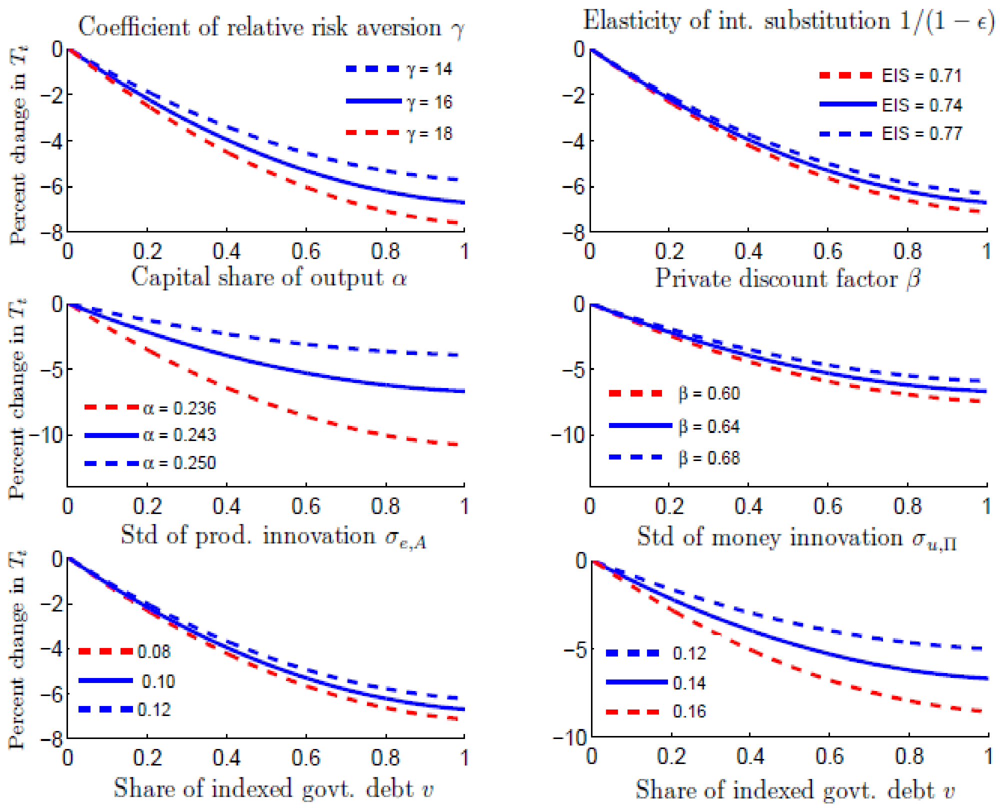
Notes: Real return is annualised and in percentage points. It is defined as log[(mean return)$^{1/30}$].

5.2.5. Sensitivity analysis

As the baseline results rely on a particular calibration, it is instructive to consider whether we reach the same conclusion under alternative calibrations of key parameters. The results of a sensitivity analysis of this kind are reported in Figure 4 for six parameters: the coefficient of relative risk aversion; the output share of capital; the elasticity of intertemporal substitution; the discount factor; and the standard deviations of the innovations to money supply and productivity. Each panel shows the percentage reduction in lump-sum taxes as the share of indexed government debt is increased from 0 to 1. The baseline reduction in taxes is shown by the solid blue line in each panel.

The percentage reductions in lump-sum taxes are fairly similar to the baseline case, except for the capital share of output and the standard deviation of the money supply innovation. Relatively small changes in the former have quite a large impact on the reduction in taxes because this parameter is crucial for matching the ratio of 30-year debt to GDP. For instance, setting a slightly higher value than the baseline calibration pushes down the debt-to-GDP ratio and leads to an increase in capital holdings, so that the model overshoots the investment-GDP ratio while undershooting the debt-GDP ratio by more than in the baseline case. Since overall nominal asset holdings fall somewhat, the inflation risk premium does also, reducing the potential savings from moving to indexed debt. On the other hand, the money supply standard deviation is important because it determines the quantity of inflation risk in the model.

Fig 4 – Tax implications of the IRP: sensitivity analysis



Overall, the baseline conclusion that shifting from nominal to indexed debt would allow taxes to be reduced by a non-trivial amount appears to be robust. Indeed, the percentage reduction in taxes exceeds 3.5% in all cases in Figure 4, and it is considerably higher in all cases but one. Therefore, the inflation risk premium appears to be of quantitative importance. In particular, the potential costs savings from avoiding the inflation risk premium by issuing indexed government debt appear to be non-trivial, both in terms of the impact on the real borrowing rate and the implied impact on taxes through the government budget constraint. An interesting question is whether these cost savings are important for social welfare.

6. Conclusion

This paper presents a general equilibrium model in which nominal government debt pays an inflation risk premium. In contrast to standard representative agent models, the model predicts that nominal asset holdings matter for this premium. This feature of the model is intuitively appealing since we would expect the inflation risk premium to be higher in economies where holdings of nominal assets are substantial, since they will be more vulnerable to unanticipated variations in inflation. Doepke and Schneider (2006) show that, as the main bondholders in the US economy, older agents have been quite exposed to episodes of unanticipated inflation, and Meh et al. (2010) show that the same is true of Canada. In order to focus in on the implications for old agents, the model contains overlapping generations that live for only two periods: youth and old age. Since unexpectedly high inflation pushes down the *ex post* real return on nominal retirement assets, high marginal utility goes hand-in-hand with unexpectedly high inflation. Consequently, the inflation risk premium is positive.

The model makes several predictions about the macro variables that matter for the inflation risk premium, as highlighted using a closed-form analytical solution. In particular, it is higher in economies where government debt is primarily nominal, steady-state inflation is low, and where money and nominal debt account for a large share of retirement consumption. The nominal asset channel by which the above factors matter for the inflation risk premium does not appear to have been highlighted in previous theoretical literature or tested empirically. To assess whether these channels are likely to be of quantitative importance, numerical values for a developed economy were plugged into the analytical solution. The implied inflation risk premium was 8.6 basis points and sensitivity analysis showed that both the total supply of government debt and share of indexed debt are likely to be important for the magnitude of the premium. Although the predicted inflation risk premium is somewhat lower than most estimates in the empirical literature, it is comparable to those obtained from small-scale representative agent models such as Hördahl et al. (2008) and De Paoli et al. (2010).

These findings suggest that further work to understand inflation risk premia in overlapping generations models may be beneficial. Indeed, since the key transmission mechanism in the model – the nominal asset channel by which unanticipated inflation has real effects – has already received attention in quantitative overlapping generations models used to study the distributional effects of unanticipated inflation (Dopeke and Schneider, 2006; Meh et al., 2010), these models could be extended to provide a more reliable quantitative assessment of the channels that matter for the inflation risk premium. Empirical work could also test the predictions of the model for inflation risk premia – a task which is feasible given that the inflation risk premium is related to observable macro variables such as the share of indexed government debt and the steady-state inflation rate.

The paper also contributes to the literature on optimal government debt management. First, some general analytical results were presented showing that the inflation risk premium is an important determinant of the cost of issuing nominal versus indexed debt. Second, the case of a government with outstanding debt and constant real government spending was considered to focus on the fiscal implications of the inflation risk premium. Analytical results were used to show that, as a result of the inflation risk premium, the share of indexed government debt is likely to be an important determinant of the level of taxes. Third, in support of these results, simulations were used to assess the quantitative impact of the indexation share on the level of taxes. In order to make this analysis more meaningful, the model was augmented with Epstein-Zin preferences to enable it to produce plausible inflation risk premia. The main finding was that shifting from nominal debt to indexed debt would enable governments to permanently lower taxes by a non-trivial amount, due to the cost saving from avoiding the inflation risk premium. A topic of obvious interest for future research would be a welfare analysis of the optimal share of indexed debt in the presence of the inflation risk premium.

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Technical Appendix

A – Second-order accurate expression for the real returns differential

The real return on nominal government debt is given by

$$r_t^n = \frac{R_{t-1}}{\Pi_t} \tag{A1}$$

Taking a second-order Taylor series approximation on both sides leads to

$$\hat{r}_t^n + \frac{1}{2}\hat{r}_t^{n\,2} + O[2] = \hat{R}_{t-1} + \frac{1}{2}\hat{R}_{t-1}^2 - \hat{\Pi}_t + \frac{1}{2}\hat{\Pi}_t^2 - \hat{\Pi}_t \hat{R}_{t-1} + O[2] \tag{A2}$$

where 'hats' denote log deviations from the deterministic steady-state.

Using the solution for the nominal rate in main text, cancelling common terms and ignoring any terms of higher than second order, we have:[27]

$$\begin{aligned}
\hat{r}_t^n + \frac{1}{2}\hat{r}_t^{n\,2} &= \hat{r}_{t-1}^f + (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) - \frac{1}{2}(\text{var}_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t^2) + IRP + \frac{1}{2}\hat{R}_{t-1}^2 - \hat{\Pi}_t \hat{R}_{t-1} \\
&= \hat{r}_{t-1}^f + (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) - \frac{1}{2}(\text{var}_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t^2) + IRP \\
&\quad + \frac{1}{2}[\,(\hat{r}_{t-1}^f)^2 + (E_{t-1}[\hat{\Pi}_t])^2 + \hat{r}_{t-1}^f E_{t-1}[\hat{\Pi}_t]\,] - \hat{\Pi}_t E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t \hat{r}_{t-1}^f \\
&= \hat{r}_{t-1}^f + (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) - \frac{1}{2}(E_{t-1}[\hat{\Pi}_t^2] - \hat{\Pi}_t^2) + IRP + t.i.i.r.
\end{aligned} \tag{A3}$$

where *t.i.i.r.* denotes 'terms independent of inflation risk' and $\text{var}_{t-1}[\hat{\Pi}_t] = E_{t-1}[\hat{\Pi}_t^2] - (E_{t-1}[\hat{\Pi}_t])^2 = E_{t-1}[\hat{\Pi}_t^2] + t.i.i.r.$ has been used.

Finally, subtracting the risk-free rate (which is second-order accurate) from both sides gives us a second-order accurate real returns differential of

$$\hat{r}_t^n + \frac{1}{2}\hat{r}_t^{n\,2} - \hat{r}_{t-1}^f = (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) - \frac{1}{2}(E_{t-1}[\hat{\Pi}_t^2] - \hat{\Pi}_t^2) + IRP + t.i.i.r. \tag{A4}$$

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B – Second-order accurate expression for taxes in Section 5.1.2

Rearranging the government budget constraint for taxes under the assumption of constant government spending and constant government debt, we have

$$T_t = \bar{g} + (v r_{t-1}^f + (1-v) r_t^n - 1)\bar{b} + (r_t^m - 1)\delta \tag{B1}$$

where the fact that $m_t = \delta$ has been used.

[27] Note that cross-product and quadratic terms are independent of inflation risk because Schmitt-Grohé and Uribe (2004) show that the coefficients on terms linear and quadratic in the state vector in a second-order expansion are independent of the volatility of exogenous shocks.

Taking a second-order Taylor expansion of this equation gives

$$\hat{T}_t + \frac{1}{2}\hat{T}_t^2 = v\theta_{b,T}\hat{r}_{t-1}^f + (1-v)\theta_{b,T}\hat{r}_t^n - \delta\left[\hat{\Pi}_t - \frac{1}{2}\hat{\Pi}_t^2\right] + \frac{1}{2}[\theta_{b,T}\hat{r}_{t-1}^{f\,2} + (1-\theta_{b,T})\hat{r}_t^{n\,2}] + O[2] \quad \text{(B2)}$$

where $\theta_{b,T} \equiv \bar{b}/T$ and the substitutions $\hat{r}_t^m = -\hat{\Pi}_t$ and $\hat{r}_t^{m\,2} = \hat{\Pi}_t^2$ have been used.

Using our second-order accurate solution for $\hat{R}$ in the main text, we can further say that[28]

$$\hat{r}_t^n = \hat{r}_{t-1}^f + (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) + IRP - JI \quad \text{(B3a)}$$

$$\begin{aligned}\hat{r}_t^{n\,2} &= \hat{r}_{t-1}^{f\,2} + (E_{t-1}[\hat{\Pi}_t])^2 + \hat{\Pi}_t^2 - \hat{\Pi}_t E_{t-1}[\hat{\Pi}_t] + (IRP - JI)^2 \\ &\quad + 2[\hat{r}_{t-1}^f(IRP - JI) + \hat{r}_{t-1}^f(E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t) + (E_{t-1}[\hat{\Pi}_t] - \hat{\Pi}_t)(IRP - JI)] \quad \text{(B3b)} \\ &= \hat{\Pi}_t^2 + O[2] + t.i.i.r.\end{aligned}$$

where $IRP - JI \equiv [\gamma\theta_{nom} - (1/2)]\sigma_{u,\Pi}^2$, $\theta_{nom} \equiv (1-v)\theta_b + \theta_m$, $\hat{\Pi}_t = \varepsilon_t$, and $E_{t-1}[\hat{\Pi}_t] = \rho_M \varepsilon_{t-1}$. Here, as above, *t.i.i.r.* denotes 'terms independent of inflation risk'.

Using (B3a) and (B3b), the expression in (B2) can be written in the form

$$\begin{aligned}\hat{T}_t + \frac{1}{2}\hat{T}_t^2 &= \hat{r}_{t-1}^f + (1-v)\theta_{b,T}(IRP - JI) + (1-v)\theta_{b,T}\rho_M\varepsilon_{t-1} - \left[\delta + (1-v)\theta_{b,T}\right]\hat{\Pi}_t \\ &\quad + \frac{1}{2}\left[\delta + (1-v)\theta_{b,T}\right]\hat{\Pi}_t^2 + O[2] + t.i.i.r.\end{aligned} \quad \text{(B4)}$$

Notice that the only endogenous variable on the RHS of (B4) is the lagged risk-free rate. Using the solution in the main text, the risk-free rate is given by

$$\begin{aligned}\hat{r}_t^f &= -E_t[\hat{m}_{t+1}] - \frac{1}{2}\mathrm{var}_t[\hat{m}_{t+1}] + O[2] = \gamma E_t[\hat{c}_{t+1,O}] - \gamma\hat{c}_{t,Y} - \frac{\gamma^2}{2}\mathrm{var}_t[\hat{c}_{t+1,O}] + O[2] \\ &= \gamma\theta_k(\hat{k}_{t+1} + E_t[\hat{A}_{t+1}]) + \gamma v\theta_b\hat{r}_t^f + \gamma(1-v)\theta_b E_t[\hat{r}_{t+1}^n] + \gamma\theta_m E_t[\hat{r}_{t+1}^m] \\ &\quad - \gamma(a_w\hat{w}_t - a_T\hat{T}_t - a_k\hat{k}_{t+1}) - \frac{\gamma^2}{2}(\theta_k^2\sigma_A^2 + \theta_{nom}^2\sigma_{u,\Pi}^2) + O[2] \quad \text{(B5)} \\ &= \gamma(\theta_k + a_k)\hat{k}_{t+1} + \gamma\theta_k\rho_A\hat{A}_t + \gamma v\theta_b\hat{r}_t^f + \gamma(1-v)\theta_b\hat{R}_t - \gamma\theta_{nom}\rho_M\varepsilon_t \\ &\quad - \gamma(a_w\hat{w}_t - a_T\hat{T}_t) - PS + O[2]\end{aligned}$$

where $a_k \equiv k/c_Y$, $a_w \equiv w/c_Y$, $a_T \equiv T/c_Y$, $PS \equiv (\gamma^2/2)(\theta_k^2\sigma_{e,A}^2 + \theta_{nom}^2\sigma_{u,\Pi}^2)$ is the precautionary savings effect, θ_k and θ_b are as in the main text, and the fact that $E_t[\hat{\Pi}_{t+1}] = \rho_M\varepsilon_t$ has been used.

[28] Again, cross-product and quadratic terms are independent of inflation risk because Schmitt-Grohé and Uribe (2004) show that the coefficients on terms linear and quadratic in the state vector in a second-order expansion are independent of the volatility of exogenous shocks.

Substituting for the nominal rate using the second-order accurate solution in the main text gives

$$\begin{aligned}\hat{r}_t^f &= \gamma(\theta_k + a_k)\hat{k}_{t+1} + \gamma\theta_k \rho_A \hat{A}_t + \gamma\theta_b \hat{r}_t^f - \gamma\theta_m \rho_M \varepsilon_t + \gamma(1-v)\theta_b (IRP - JI) \\ &\quad - \gamma(a_w \hat{w}_t - a_T \hat{T}_t) - PS + O[2]\end{aligned} \tag{B6}$$

We can solve for capital using a second-order approximation of the investment Euler equation:

$$\begin{aligned}(\alpha-1)\hat{k}_{t+1} &= -E_t[\hat{m}_{t+1} + \hat{A}_{t+1}] - \frac{1}{2}\mathrm{var}_t[\hat{m}_{t+1} + \hat{A}_{t+1}] \\ &= \hat{r}_t^f - \rho_A \hat{A}_t - \frac{\sigma_A^2}{2} - \mathrm{cov}_t[\hat{m}_{t+1}, \hat{A}_{t+1}] = \hat{r}_t^f - \rho_A \hat{A}_t + EQP - JI\end{aligned} \tag{B7}$$

where $EQP - JI \equiv [\gamma\theta_k - (1/2)]\sigma_{e,A}^2$ is the equity premium minus the Jensen's inequality correction.

Solving this equation for capital and substituting the result into (B6) gives

$$\begin{aligned}&\hat{r}_t^f \left(1 + \frac{\gamma[\theta_k + a_k - \theta_b(1-\alpha)]}{1-\alpha}\right) = \gamma\left(\frac{a_k + \alpha a_w + \theta_k(2-\alpha)}{1-\alpha}\right)\rho_A \hat{A}_t - \gamma\theta_m \rho_M \varepsilon_t \\ &+ \gamma(1-v)\theta_b(IRP - JI) + \gamma\left(\frac{\alpha a_w - (\theta_k + a_k)}{1-\alpha}\right)(EQP - JI) + \frac{\gamma a_w \alpha}{1-\alpha}\hat{r}_{t-1}^f + \gamma a_T \hat{T}_t - PS + O[2]\end{aligned} \tag{B8}$$

The only variable left to eliminate in (B8) is taxes, which to first-order are given by[29]

$$\begin{aligned}\hat{T}_t &= v\theta_{b,T}\hat{r}_{t-1}^f + (1-v)\theta_{b,T}\hat{r}_t^n - \delta\hat{\Pi}_t + O[1] \\ &= \hat{r}_{t-1}^f + (1-v)\theta_{b,T}E_{t-1}[\hat{\Pi}_t] - [\delta + (1-v)\theta_{b,T}]\hat{\Pi}_t + O[1] \\ &= \hat{r}_{t-1}^f + (1-v)\theta_{b,T}\rho_M \varepsilon_t - [\delta + (1-v)\theta_{b,T}]\hat{\Pi}_t + O[1]\end{aligned} \tag{B9}$$

where $O[1]$ denotes terms of order higher than 1.

Substituting for (B9) in (B8) and solving for the risk-free rate, we have:

$$\begin{aligned}\hat{r}_t^f &= \frac{\gamma}{\varphi}[a_w\alpha + (1-\alpha)a_T]\hat{r}_{t-1}^f + \frac{\gamma}{\varphi}[a_k + \alpha a_w + \theta_k(2-\alpha)]\rho_A \hat{A}_t + \frac{\gamma(1-\alpha)}{\varphi}[(1-v)\theta_{b,T} - \theta_m]\rho_M \varepsilon_t \\ &+ \frac{\gamma}{\varphi}(1-\alpha)(1-v)\theta_b(IRP - JI) + \frac{\gamma}{\varphi}[\alpha a_w - (\theta_k + a_k)](EQP - JI) \\ &- \frac{\gamma}{\varphi}(1-\alpha)[\delta + (1-v)\theta_{b,T}]\hat{\Pi}_t - \frac{(1-\alpha)}{\varphi}PS + O[2] \\ &= \frac{\gamma}{\varphi}[a_w\alpha + (1-\alpha)a_T]\hat{r}_{t-1}^f + \frac{\gamma}{\varphi}(1-v)(1-\alpha)\theta_b(IRP - JI) - \frac{(1-\alpha)}{\varphi}PS_\Pi + O[2] + t.i.i.p.\end{aligned} \tag{B10}$$

[29] It is enough to approximate taxes to first-order because, as noted by Hördahl et al. (2008), first-order accurate solutions are sufficient to obtain second-order accurate solutions for asset prices.

where $\varphi \equiv 1-\alpha+\gamma(\theta_k + a_k - (1-\alpha)\theta_b)$ and $PS_\Pi \equiv \gamma^2\theta_{nom}^2\sigma_{u,\Pi}^2/2$ is the part of precautionary savings effect due to inflation risk.

Since (B10) is recursive in the risk-free rate, we can simplify further. Assuming that the initial period is period 0, we have the following expression:

$$\hat{r}_t^f = \frac{\gamma(1-\rho_{rf}^{t+1})}{\varphi(1-\rho_{rf})}(1-\nu)(1-\alpha)\theta_b(IRP-JI) - \frac{(1-\alpha)}{\varphi}\frac{(1-\rho_{rf}^{t+1})}{(1-\rho_{rf})}PS_\Pi + O[2] + t.i.i.r. \quad \text{(B11)}$$

where $\rho_{rf} \equiv \dfrac{\gamma}{\varphi}[a_w\alpha + (1-\alpha)a_T]$ is the coefficient on the lagged risk-free real rate in (B10).

Notice that if $\varphi > 0$, the inflation risk premium will tend to raise the risk-free rate, while additional precautionary saving driven by inflation risk will tend to lower it. The overall effect of inflation risk on the risk-free real interest rate is therefore ambiguous and will depend upon which effect dominates. Consequently, the impact of inflation risk on taxes will generally also be ambiguous.

We can see this formally by substituting for the lagged value of (B11) in the second-order accurate expression for taxes in (B4) and collecting terms:

$$\hat{T}_t + \frac{1}{2}\hat{T}_t^2 = (1-\nu)\phi_{1,t}(IRP-JI) - \phi_{2,t}PS_\Pi + \phi_3\hat{\Pi}_t^2 + O[2] + t..i.i.r. \quad \text{(B12)}$$

where $\phi_{1,t} \equiv \left(\theta_{b,T} + \dfrac{(1-\alpha)\gamma\theta_b(1-\rho_{rf}^t)}{\varphi(1-\rho_{rf})}\right)$, $\phi_{2,t} \equiv \dfrac{(1-\alpha)}{\varphi}\dfrac{(1-\rho_{rf}^t)}{(1-\rho_{rf})}$ and $\phi_3 \equiv \dfrac{1}{2}[\delta + (1-\nu)\theta_{b,T}]$.

Notice that $\phi_{1,t}$ and $\phi_{2,t}$ will be positive so long as $\rho_{rf} < 1$. The parameter ϕ_3 is unambiguously positive.

C – Stochastic discount factor and the inflation risk premium under Epstein-Zin preferences

Derivation of the stochastic discount factor

Under Epstein-Zin preferences the lifetime utility function is given by

$$U_t^{EZ} = \frac{1}{1-\gamma}\left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma}{\varepsilon}} \quad \text{(C1)}$$

The stochastic discount factor is defined by

$$sdf_{t+1}^{EZ} \equiv \frac{\partial U_t^{EZ}/\partial c_{t+1,O}}{\partial U_t^{EZ}/\partial c_{t,Y}} \quad \text{(C2)}$$

The partial derivatives of the utility function are as follows:

$$\frac{\partial U_t^{EZ}}{\partial c_{t,Y}} = \left[c_{t,Y}^{\varepsilon} + \beta\left[E_t c_{t+1,O}^{1-\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}\right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} c_{t,Y}^{-(1-\varepsilon)} \quad \text{(C3)}$$

$$\frac{\partial U_t^{EZ}}{\partial c_{t+1,O}} = \left[c_{t,Y}^{\varepsilon} + \beta \left[E_t c_{t+1,O}^{1-\gamma} \right]^{\frac{\varepsilon}{1-\gamma}} \right]^{\frac{1-\gamma-\varepsilon}{\varepsilon}} \beta \left[E_t (c_{t+1,O}^{1-\gamma}) \right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma} \tag{C4}$$

Dividing (C4) by (C3) gives the expression stated in Equation (27) of the main text:

$$sdf_{t+1}^{EZ} = \frac{\beta \left[E_t (c_{t+1,O}^{1-\gamma}) \right]^{\frac{\varepsilon-(1-\gamma)}{1-\gamma}} c_{t+1,O}^{-\gamma}}{c_{t,Y}^{-(1-\varepsilon)}} = \beta \left(\frac{c_{t,Y}}{c_{t+1,O}} \right)^{1-\varepsilon} \left(\frac{c_{t+1,O}}{(E_t c_{t+1,O}^{1-\gamma})^{1/(1-\gamma)}} \right)^{1-\gamma-\varepsilon} \tag{C5}$$

Derivation of the inflation risk premium

Proposition: The second-order accurate analytical solution for the inflation risk premium is the same with CRRA and Epstein-Zin preferences for any calibration of the coefficient of relative risk aversion.

Proof.

The inflation risk premium under Epstein-Zin preferences is given by

$$IRP^{EZ} = \operatorname{cov}_t [s\hat{d}f_{t+1}^{EZ}, \hat{\Pi}_{t+1}] \tag{C6}$$

We thus need an expression for $s\hat{d}f_{t+1}^{EZ}$. Defining $z_t \equiv E_t c_{t+1,O}^{1-\gamma}$ and taking logs of (C5) gives

$$\begin{aligned} \ln sdf_{t+1}^{EZ} &= \ln \beta + (1-\varepsilon)(\ln c_{t,Y} - \ln c_{t+1,O}) + (1-\gamma-\varepsilon) \left(\ln c_{t+1,O} - \frac{1}{1-\gamma} \ln z_t \right) \\ &= \ln \beta + (1-\varepsilon) \ln c_{t,Y} - \gamma \ln c_{t+1,O} - [(1-\gamma-\varepsilon)/(1-\gamma)] \ln z_t \end{aligned} \tag{C7}$$

Hence (C7) implies that

$$s\hat{d}f_{t+1}^{EZ} = (1-\varepsilon)\hat{c}_{t,Y} - \gamma \hat{c}_{t+1,O} - [(1-\gamma-\varepsilon)/(1-\gamma)] \hat{z}_t + O[1] \tag{C8}$$

where 'hats' denote log deviations from steady-state and $O[1]$ terms of higher order than 1 .

A first-order Taylor series approximation of z_t gives

$$\hat{z}_t = (1-\gamma) E_t [\hat{c}_{t+1,O}] + O[1] \tag{C9}$$

Hence, ignoring terms of higher than order 1, we have

$$s\hat{d}f_{t+1}^{EZ} = (1-\varepsilon)\hat{c}_{t,Y} - \gamma \hat{c}_{t+1,O} - (1-\gamma-\varepsilon) E_t [\hat{c}_{t+1,O}] \tag{C10}$$

It follows that

$$IRP^{EZ} = \operatorname{cov}_t [s\hat{d}f_{t+1}^{EZ}, \hat{\Pi}_{t+1}] = -\gamma \operatorname{cov}_t [\hat{c}_{t+1,O}, \hat{\Pi}_{t+1}] = IRP \tag{C11}$$

Hence, the inflation risk premium is the same as with CRRA preferences for any given calibration of the coefficient of relative risk aversion γ. **Q.E.D.**

Appendix D – The bond supply rule in the simulated model with Epstein-Zin preferences

As discussed in Section 5.2, the bond supply is given by a steady-state rule of the form $1 = (\chi/\beta)sdf^{EZ}$, where $\chi > 0$. In this Appendix, the constant bond supply implied by this rule is derived.

First, note that $1 = (\chi/\beta)sdf^{EZ}$ implies that $r^f = \chi/\beta$ at steady-state and that

$$c_O = \chi^{1/(1-\varepsilon)} c_Y \tag{D1}$$

where (27) has been used.

Using (2), the LHS of (D1) is equal to

$$\begin{aligned} c_O &= r^k k + (\nu r^f + (1-\nu) r^n)\bar{b} + r^m m \\ &= r^k k + r^f \bar{b} + r^m m \end{aligned} \tag{D2}$$

where the fact that $r^f = r^n$ at steady-state has been used.

And using (3), the RHS is given by

$$\begin{aligned} c_Y &= w - T - k - \bar{b} - m \\ &= w - \bar{g} - r^f \bar{b} - r^m m - k \end{aligned} \tag{D3}$$

where the fact that $T = \bar{g} + (r^f - 1)\bar{b} + (r^m - 1)m$ has been used.

Substituting (D2) and (D3) into (D1) and rearranging for $\bar{b}$ we have the implied bond supply:

$$\bar{b} = \frac{\chi^{1/(1-\varepsilon)}}{r^f (1 + \chi^{1/(1-\varepsilon)})}(w - \bar{g}) - \frac{r^m}{r^f} m - \frac{(r^k + \chi^{1/(1-\varepsilon)})}{r^f (1 + \chi^{1/(1-\varepsilon)})} k \tag{D4}$$

Appendix E – Deterministic steady-state of the simulated model with Epstein-Zin preferences

The deterministic steady-state is given by

$$c_Y = w - \bar{g} - r^f \bar{b} - r^m m - k \tag{E1}$$

$$c_O = r^k k + r^f \bar{b} + r^m m \tag{E2}$$

$$\bar{b} = \frac{\chi^{1/(1-\varepsilon)}}{r^f (1 + \chi^{1/(1-\varepsilon)})}(w - \bar{g}) - \frac{r^m}{r^f} m - \frac{(r^k + \chi^{1/(1-\varepsilon)})}{r^f (1 + \chi^{1/(1-\varepsilon)})} k \tag{E3}$$

$$A = 1 \tag{E4}$$

$$w = (1 - \alpha) k^\alpha \tag{E5}$$

$$r^f = \chi / \beta \tag{E6}$$

$$r^n = r^k = r^f \tag{E7}$$

$$k = (\alpha / r^{f})^{1/(1-\alpha)} \tag{E8}$$

$$r^{k} = \alpha k^{\alpha-1} \tag{E9}$$

$$R = \Pi * r^{f} \tag{E10}$$

$$r^{m} = 1/\Pi * \tag{E11}$$

$$m = \delta \tag{E12}$$

$$g = \bar{g} \tag{E13}$$

$$T = \bar{g} + (r^{f} - 1)\bar{b} + (r^{m} - 1)m \tag{E14}$$

$$\Pi = \Pi * \tag{E15}$$