Moon-Tracking Orbits using Motorized Tethers for Continuous Earth-Moon Payload Exchanges

Christopher Murray [1] and Matthew P. Cartmell [2]
School of Engineering, University of Glasgow [3]

For human colonization of the Moon to become reality an efficient and regular means of exchanging resources between the Earth and Moon must be established. One possibility is to pass and receive payloads at regular intervals between a symmetrically laden motorized momentum exchange tether orbiting about Earth and a second orbiting about the Moon. There are significant challenges associated with this method; amongst the greatest of which is the development of a system which incorporates the complex motion of the Moon into its operational architecture in addition to having the capability of conducting these exchanges on a per-lunar-orbit basis. One way of achieving this is a method by which a motorized tether orbiting Earth can track the nodes of the Moon's orbit and allow payload exchanges to be undertaken periodically with the arrival of the Moon at either of these nodes. Tracking the nodes using this method is achieved by arranging the tether to orbit Earth with a critical inclination thus rendering its argument of perigee stationary in addition to utilizing the precession effects resulting from an oblate Earth. Using this in conjunction with preemptive adjustments to its angle of right ascension the tether will periodically realign itself with these nodes simultaneously with the arrival of the Moon.

[1] Post-doctoral Researcher, AIAA non-member.
[2] James Watt Professor of Mechanical Engineering, AIAA senior member.
[3] University of Glasgow, University Avenue, Glasgow, UK, G12 8QQ.

Nomenclature

a	= semi-major axis of EMMET's orbit, km.
A	= cross sectional area of tether sub-spans, mm^2.
e	= eccentricity of EMMET's orbit.
h	= specific angular momentum of EMMET's orbit, km^2s^{-1}.
i	= inclination of EMMET's orbit, radians.
I	= 3x3 identity matrix.
J_2	= Earth's second zonal harmonic.
LSI	= Lunar Sphere of Influence.
m	= orbital variable between the EMMET's and the Moon's orbital periods.
r	= radial distance of EMMET central facility, km.
$\vec{r}_{eq}$	= geocentric position vector of EMMET's orbit, km.
R	= radius of Earth, km.
$\vec{r}_f$	= geocentric position vector of EMMET in final orbit, km.
R_G	= perifocal to geocentric equatorial transformation matrix.
$\dot{R}_G$	= first derivative of perifocal to equatorial frame transformation matrix.
$R_K(\Delta\Omega)$	= rotation matrix for EMMET's ascending node adjustment.
$\vec{r}_i$	= geocentric position vector of EMMET in initial orbit, km.
$\vec{r}_{per}$	= perifocal position vector of EMMET's orbit, km.
S	= 3x3 skew symmetric matrix of unit vector along rotation axis.
T	= orbital period of the EMMET's central facility, seconds.
T_m	= orbital period of the Moon about Earth, seconds.
$\vec{v}_{eq}$	= geocentric velocity vector of EMMET's orbit, kms^{-1}.
$\vec{v}_f$	= geocentric velocity vector of EMMET in final orbit, kms^{-1}.
$\vec{v}_i$	= geocentric velocity vector of EMMET in initial orbit, kms^{-1}.
$\vec{v}_{per}$	= perifocal velocity vector of EMMET's orbit, kms^{-1}.

x_f = x-axis component of geocentric position of final orbit, km.

x_i = x-axis component of geocentric position of initial orbit, km.

y_f = y-axis component of geocentric position of final orbit, km.

y_i = y-axis component of geocentric position of initial orbit, km.

$\Delta\Omega$ = change in EMMET's angle of right ascension, rad.

$\Delta\vec{v}$ = change in EMMET's velocity vector required to adjust angle of right ascension, kms^{-1}.

θ = true anomaly of EMMET's orbit, rad.

$\dot{\theta}$ = angular velocity of EMMET's orbit, rads^{-1}.

θ_f = true anomaly of EMMET in final orbit, rad.

θ_i = true anomaly of EMMET in initial orbit, rad.

μ = Earth's gravitational parameter, $\mathrm{km}^3\mathrm{s}^{-2}$.

ω = argument of right ascension of EMMET's orbit, rad.

Ω = angle of right ascension of EMMET's orbit, rad.

$\dot{\Omega}$ = regression rate of EMMET's angle of right ascension, rads^{-1}.

Ω_f = angle of right ascension of EMMET in final orbit, rad.

Ω_i = angle of right ascension of EMMET in initial orbit, rad.

I. Introduction

At present considerable interest surrounds a possible return to the Moon and the establishment of a permanent lunar colony. This proposal has several advantages, for example it will admit access to the wealth of untapped mineral resources on the Moon in addition to its utility as an interplanetary launch facility. For this concept to be realized an energy efficient means of transporting the extracted resources from the Moon back to Earth in addition to the replenishment of the colony's food, water and oxygen is a priority. Furthermore, this would allow materials for the construction of the colony and for the fabrication of interplanetary spacecraft to be easily transported.

A possible means of achieving this is by utilising the phenomenon of momentum exchange between tethered satellites in orbit. This exploits the propensity of tethered satellites to orbit a planet at the distance and angular velocity of their common centre of mass, where they become naturally aligned along the local gravity gradient. This results in the satellite at the tethered end furthest from the host planet having a greater linear velocity and momentum than the satellite at the tethered end closest to the planet. The lower satellite effectively transfers part of its momentum to the upper satellite. If at this instant the two satellites are disconnected from the tether, the upper satellite has a velocity too great to remain on its current orbit and traverses a trajectory to a point further from the planet; whereas the lower satellite has too little velocity to stay on its current orbit and traverses a trajectory to a point closer to the planet. The upper satellite's orbit has been effectively raised and the lower body has been de-orbited.

To increase the amount of momentum exchanged; librating and motor-driven spinning tether configurations have been conceived. The concept of the addition of a torque to spin the tether was first published by Puig-Suari *et al* [1] in 1995 when it was suggested that the orbital velocity required for interplanetary exploration could be imparted to a tethered satellite by the continuous application of a solar power generated continuous torque to a tether sling attached to a massive hub. Advantages of this method were cited as the lack of propulsive manoeuvre and virtually inexhaustible capacity. However issues arose concerning this configuration for example; the connection between the hub and tether is not located at the centre of mass resulting in a precession of the spin axis and to rectify this it was suggested that a second identical tether was attached symmetrically to the hub.

Independently of this, Cartmell and Zeigler [2] proposed a symmetrically laden motorised momentum exchange tether for application to an interplanetary two-way exchange concept. This system was devised to utilise the increased rotational speed resulting from a motor torque being applied to the tether sub-spans such that momentum exchange between payloads is maximised and results in an increased range for payload raising post-release. This motorized momentum exchange tether was described by Cartmell and Zeigler [3] as follows; the two tether sub-spans which are symmetrically attached to the rotor shaft of the motorized system are termed the propulsion tethers with their free ends attached to the payloads between which the momentum exchange will take place. The rotor shaft is mechanically coupled to the stator of the motor, the back rotation of which is controlled by the addition to two further symmetrically attached tether sub-spans with constant masses attached to their free ends. These are termed the outrigger tethers and provide the reaction mass against which the propulsion tethers can be accelerated. The motor itself in addition to the outrigger tethers are masses are collectively termed the central facility of the motorized system. The dry mass of this central facility has been estimated to be 1.5 tonnes [3] however no power consumption data has yet been published. Additionally, Spectra material was shown to give the highest gain in kinetic energy to the payloads when using currently available materials [4] and this occurred at a tether sub-span length of approximately 100km. Spectra has a characteristic velocity of 2.6 km/s and using this data in addition to a cross sectional area of 65mm^2 and a material density of 0.97 g/cm^3 the tether sub-spans each have a mass of 6305 kg.

By ensuring a simultaneous payload release the orbital altitude of the central facility could be retained. Additionally, by ensuring that the propulsion tethers are aligned along the local gravity gradient at the instant of release, maximum velocity gain of the upper payload relative to Earth is achieved. By utilising this configuration with the central facility at the perigee of its orbit about Earth, the optimum configuration for payload release is achieved. Furthermore, by employing a staged tether system, as conceived by Hoyt and Forward [5] in 1997 for payload transport from sub-Earth orbit to the point of orbital injection; the velocity for injection onto lunar and interplanetary trajectories could be imparted to the satellite furthest from Earth at the instant of release. The symmetrically laden tether concept is further enhanced [2] by proposing the configuration of

a second symmetrically laden tether system in orbit at the destination planet. It is proposed that this tether could capture the incoming payload from Earth and at a later time reverse the operation and return a payload from the destination planet back to Earth. With this architecture the concept of a continuous two-way interplanetary exchange [2] was conceived. The application of rotating asymmetric tethers for a cislunar transport system had previously been shown to be theoretically possible, in 1991, by Forward [6] using little or no propellant. This system utilised a 89km long rotating tether in an elliptical, equatorial orbit about Earth and exchanged payloads with a second 200km long momentum exchange tether in a low lunar orbit. However, this used many simplifying assumptions and so the system was re-investigated in 1999 by Hoyt and Uphoff [7] and an architecture for the system was developed.

To explore further the viability of these symmetrically laden motorised momentum exchange tethers; a series of terrestrial scale model tests were conducted by Cartmell and Zeigler [3] in 2001 and amongst other things showed that a small impulse was generally experienced by the tethers at the instant of payload release but then they quickly returned to their required configuration; and payload release asymmetry was extremely significant and had the potential to induce large overall system displacements. This payload release asymmetry has important consequences in the event of asymmetric payload release by the motorised momentum exchange tether (MMET) and would result in a change in the position of the system's centre of mass relative to the central body which would result in the MMET undertaking a trajectory about the body which differs from the required trajectory. Development of the concept continued with the work of Cartmell, McInnes and McKenzie [8] in 2004 in which a preliminary design architecture for the system was proposed; and with the work of Murray and Cartmell [9] in 2008 with *A Continuous Earth-Moon Payload Exchange Using Symmetrically Laden Motorised Momentum Exchange Tethers* which began a preliminary investigation into the logistical and trajectory designs required for a continuous two-way exchange between the Earth and Moon. It is at this point that the staged system is abandoned here, and potentially in the future, as a result of the inherent complexity in the system when introducing orbital perturbations. This complexity resulted from the differing variation rates of the orbital elements of the inner MMET orbit with those of the MMET in the outer orbit, which is dependent

upon differences in semi-major axis and orbital eccentricity, and when taking these into account the opportunities for payload passage between them, occurring when both MMETs arrive at their mutually aligned Earth perigees, became even more limited. The focus therefore turned to a single motorized system in orbit about Earth imparting maximum rotational velocity and increased orbital velocity at its geocentric perigee which results from increasing the semi-major axis of the central facility's orbit.

The focus of the following is the development of a means by which a motorized tether in orbit about Earth can continuously re-configure its orbit such that it can effectively track the nodes of the Moon's orbit. This allows the tether to launch payloads to the node of the Moon's orbit each time the Moon arrives at this point simplifying the transfer trajectory design and allowing per-lunar-orbit payload exchanges. Finally, preliminary data is obtained for the magnitude of the orbital adjustments required to maintain the required configuration for both Earth and Moon orbiting systems.

II. Continuous Exchange Concept

The principle mode of operation of the system is a two-way payload exchange between two symmetrically laden motorised momentum exchange tethers repeated synchronously with the Moon's orbit about Earth. The exchanges occur between a *MMET in a prograde orbit about Earth* (EMMET) and a MMET in a retrograde orbit about the Moon, termed the *Lunavator* using the terminology of Moravec [10], when the Moon crosses or is close to the ascending or descending node of its orbit about Earth. At the beginning of each operational phase, the EMMET is fully laden with the two payloads attached symmetrically to its upper and lower tips and spinning about the central facility in the same direction as the orbital motion. At the same instant the Lunavator is completely unladen with its tether sub-spans spinning about its central facility in the same direction as its orbital motion.

Payloads are released symmetrically from the upper and lower tips of the EMMET to preserve mass symmetry and prevent de-orbit of the central facility. This occurs at the *perigee* of the EMMET's orbit with its tether sub-spans aligned along the local gravity gradient. As a result of the direct addition of the tether sub-spans' rotational velocity to the orbital velocity this is the best performing configuration for payload orbit raising, and the instant at which greatest velocity can be imparted to the payload. The payload released from the upper tip of the EMMET has the same orbital angular velocity as the central facility at perigee, which is coincident with the system's centre of mass, and this is greater than the orbital angular velocity required to stay on the same trajectory after release. In addition to this, as the direction of rotation of the tether sub-spans are in the same prograde direction as the orbital motion it gains the largest additional velocity increment when it is at the upper tip position relative to the central facility. After release the payload embarks upon a large prograde trajectory bound for the Moon with its position at the upper tip coinciding with the perigee of its outbound orbit. The payload released from the lower tip of the EMMET also has the same orbital angular velocity as the central facility, which is lower than the orbital angular velocity required to stay on the same trajectory were it not attached to the tether. As the direction of rotation of the tether sub-spans is in the same prograde direction as the orbital motion it loses the same magnitude of velocity, and subsequently the same translational momentum relative to the

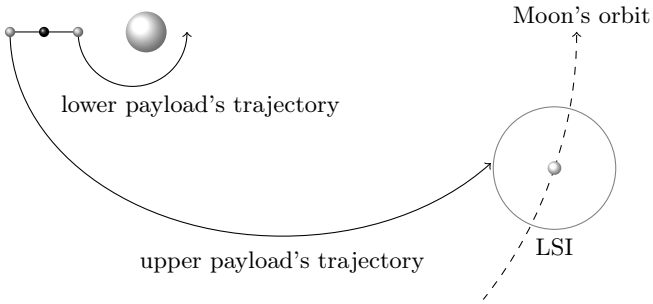


Fig. 1: Earth-Moon elliptic transfer phase

central facility, that the upper tip has gained. After release, this payload embarks upon a prograde trajectory to a position closer to Earth with its position at the lower tip coinciding with the *apogee* of its trajectory. The configuration of the system at the instant of release of the payloads from the EMMET is shown in Figure 1. At the perigee of its orbit it is proposed that the payload released from the lower tip would be captured by a space-plane and returned to Earth. A system similar to the reverse operation of the HASTOL system [11] is a possibility.

When the system is first established the tether tips will be fully laden. The payload bound for the Moon will be attached to the upper tip whilst the payload attached to the lower tip is bound for Earth. The payload bound for Earth is a 'dummy' payload and is used for system initialization. Rather than expend fuel capturing this dummy payload using a space-plane it will be allowed to burn up within Earth's atmosphere after release. All payloads released from the EMMET's lower tip after this will be captured by a space-plane. In the event that there exists velocity differences between the payload release velocity from the EMMET tip and the launch velocity required to undertake the optimum trajectory to the Lunavator tip; the payloads can be equipped with chemical propulsion and guidance systems and are effectively fully equipped spacecraft. These can be used to make small trajectory alterations at launch and capture to minimise any mechanical shocks occurring along the tether sub-spans originating from velocity differences, and to make minor course corrections. Additionally this can be used to minimise mechanical shocks during Lunavator capture and launch operations by closely matching payload to tip velocities.

An elliptical trajectory is the lowest energy orbit for the payload released from the upper tip to reach the Moon and this is undertaken until the payload reaches the boundary between the

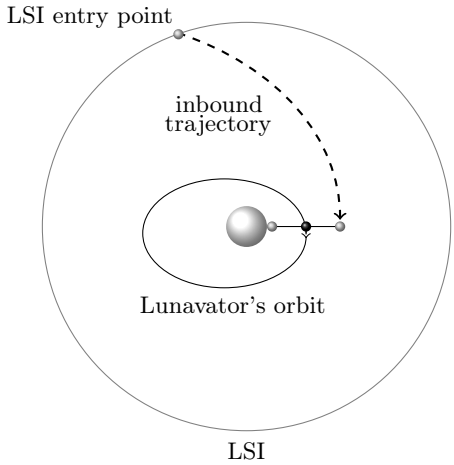


Fig. 2: Earth-Moon hyperbolic transfer phase

dominating influence of Earth's gravity and the Moon's gravity. This boundary is denoted the *lunar sphere of influence* which is abbreviated to LSI. Upon entry into the LSI, the payload's velocity relative to the Moon results in it undertaking a retrograde hyperbolic trajectory to its closest point of approach to the Moon, denoted the *perilune* of the orbit and located on the far side of the Moon from Earth, at which point it is captured by the upper tip of the Lunavator, also at the perilune of its orbit.

In the event of any failed payload captures by the tethers the payload propulsion systems can again be utilised but in both the Lunavator and EMMET cases the payloads will always be returned to Earth. In the case of a failed Lunavator capture at its upper tip, the payload from Earth will continue on its course about the Moon, rather than performing a large propulsive manoeuvre to keep it in lunar orbit, and with the aid of orbital corrections made by the propulsion and guidance system will exit the LSI and return to Earth. To minimise asymmetry across the Lunavator the lower payload must be instantly released to prevent Lunavator de-orbit as a result of asymmetry. In the event of a failed capture at its lower tip the payload at the upper tip must also be instantly released and return to Earth. In the event of failed capture at the upper tip of the EMMET, the lower payload would again be released and return to Earth whilst the payload bound for the EMMET's upper tip would traverse a trajectory about Earth for another capture opportunity at a subsequent arrival at perigee. In the event that this occurred at the lower tip, the payload would

return to Earth whilst the payload bound for the upper tip would again traverse a trajectory about Earth for a capture opportunity at a later time.

The Lunavator is designed such that it is in an elliptical orbit about the Moon, and when it is at the perilune of its orbit and the tether sub-spans are aligned along the local gravity gradient vector the lower tip touches the Moon's surface, and here is the similarity to Moravec's concept [10], hence the term Lunavator. This elliptical orbit allows a greater range of orbital angular velocities to be imparted to the MMET so that the Lunavator's upper tip speed can be more closely matched to that of the incoming payload at perilune. The configuration of the system at the instant of capture of the payloads by the Lunavator is shown in Figure 2. At the instant of capture, the incoming payload from Earth is captured at the upper tip whilst the lower tip picks up a payload from the Moon's surface to preserve mass balance across the system and prevent de-orbit. A waiting period between the Lunavator's capture and launch operations ensues which is designed to allow the Lunavator's upper and lower tips to exchange positions. This results from the tether sub-span rotation about the central facility, and facilitates the return payload's launch to take place from the upper tip position. After the waiting period, the payload previously picked up from the surface is released from the upper tip position on a retrograde hyperbolic LSI escape trajectory whilst the lower payload previously from Earth is placed onto the Moon's surface. The configuration of the system at the instant of release of the payloads by the Lunavator is shown in Figure 3. When the

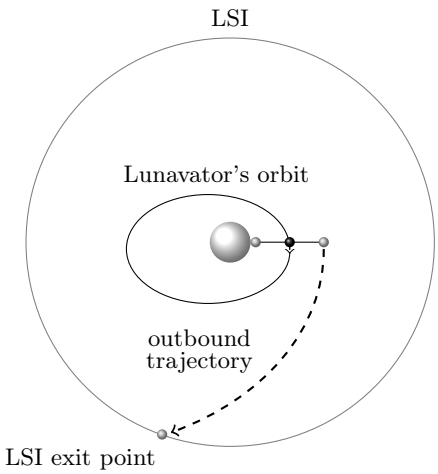


Fig. 3: Moon-Earth hyperbolic transfer phase

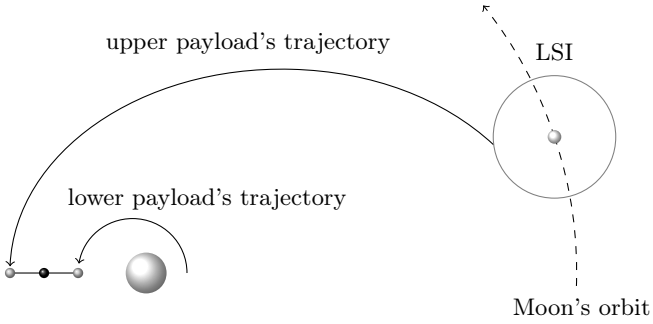


Fig. 4: Moon-Earth elliptic transfer phase

outbound payload reaches the boundary of the LSI, it has a lower velocity relative to Earth and follows a prograde elliptical trajectory back to perigee. At perigee, the payload is captured by the upper tip of the EMMET which is at the perigee of its orbit with the tether sub-spans aligned along the local gravity gradient. At the instant of capture at the upper tip, the lower tip also captures a payload to preserve the mass balance and central facility altitude. The configuration of the system at the instant of capture of the payloads by the EMMET is shown in Figure 4. At this point the lower payload is at the apogee of its orbit, having previously been brought up from Earth's surface and released at the perigee of its orbit a half orbital period prior to capture, in this case an identical system to HASTOL may be useful. When the Moon again returns to the same initial position relative to the Earth, after a time period of 27.32 days, the procedure is repeated.

III. Coordinate Systems

The following is a brief description of the rationale behind the development of the motorized tether kinematics and the coordinate systems used to aid this. We consider the tether sub-spans as part of the overall kinematic system and as they are geometrically symmetrical about the central facility we only have to consider the system's centre of mass which is coincident with the central facility. Therefore it is assumed that the EMMET orbits Earth as if all of its mass is concentrated at its centre of mass and we similarly consider the Lunavator to orbit the Moon in this way. Subsequently we need only develop the kinematics of the EMMET central facility relative to Earth using a method which is as equally applicable to the Lunavator kinematics about the Moon. To simplify this development further we consider a MMET orbiting Earth in an elliptical orbit using only the two-body problem with the position and velocity of the central facility of the MMET initially defined relative to a perifocal frame.

The perifocal frame is utilised with the aim of converting the central facility's kinematic quantities into the geocentric equatorial frame, which we consider to be an inertial frame. The perifocal frame uses the MMET's orbital plane as the fundamental plane and has axes denoted by $(P,\ Q,\ W)$ with its origin coinciding with the centre of Earth. The P axis of the perifocal frame points towards the perigee of the MMET's orbit relative to Earth. The central facility's position vector relative to the perifocal frame is defined in terms of radial distance of the central facility from Earth, r, and the true anomaly, θ, through which the orbit has traversed:

$$\vec{r}_{per} = \begin{bmatrix} rcos\theta & rsin\theta & 0 \end{bmatrix}^T \tag{1}$$

Additionally, we can define the EMMET central facility's velocity vector relative to the perifocal frame as:

$$\vec{v}_{per} = \left(\frac{\mu}{h}\right) \begin{bmatrix} -sin\theta & e + cos\theta & 0 \end{bmatrix}^T \tag{2}$$

where μ is defined as Earth's gravitational parameter, h is the specific angular momentum of the central facility and e is the orbital eccentricity of the EMMET's orbit.

The central facility's kinematic quantities in the perifocal frame are transformed into the geocentric equatorial frame which also has its origin located at the centre of Earth. The axes in this

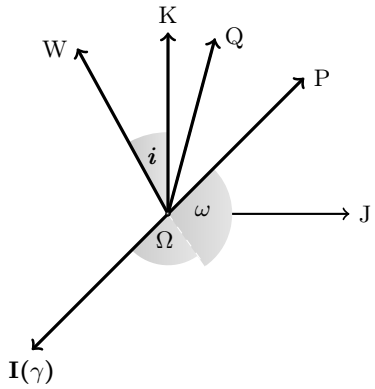


Fig. 5: Perifocal and equatorial frame orientation

frame are denoted $(I,\ J,\ K)$ with the I axis pointing towards the vernal equinox, defined as the ascending node of the Sun as viewed from Earth [12], the J axis lies perpendicular to this, and the K axis points towards the North Pole.

The configuration of the perifocal and equatorial frames is shown in Figure 5 and the conversion from the perifocal to the equatorial frame is achieved by the application of a coordinate transformation matrix populated by the orbital elements of the MMET's orbit relative to Earth. The orbital elements used are the right ascension of the ascending node of the MMET's orbit denoted by Ω, the argument of perigee denoted by ω, and orbital inclination denoted by i. The perifocal to equatorial transformation matrix is defined as [12]:

$$R_G = \begin{bmatrix} \cos\Omega cos\omega - sin\Omega sin\omega cosi & -cos\Omega sin\omega - sin\Omega cos\omega cosi & sin\Omega sini \\ sin\Omega cos\omega + cos\Omega sin\omega cosi & -sin\Omega sin\omega + cos\Omega cos\omega cosi & -cos\Omega sini \\ sin\omega sini & cos\omega sini & cosi \end{bmatrix} \tag{3}$$

The conversion from perifocal position, $\vec{r}_{per}$, to geocentric equatorial position, $\vec{r}_{eq}$, is then achieved by the application of the perifocal to equatorial transformation matrix, R_G, to the perifocal position:

$$\vec{r}_{eq} = R_G.\vec{r}_{per} \tag{4}$$

The conversion from perifocal velocity, $\vec{r}_{per}$, to geocentric equatorial velocity, $\vec{r}_{eq}$, is obtained by

differentiation of equation (4) with respect to time:

$$\vec{v}_{eq} = R_G.\vec{v}_{per} + \dot{R}_G.\vec{r}_{per} \tag{5}$$

IV. Orbital Configuration

To provide an EMMET system which allows payload transfers between the Earth and Moon over an extended time period; a point in the Moon's orbit is chosen such that suitable transfer configurations are attained periodically with the Moon's orbit about Earth. However, due to the third body perturbations of the Sun and other planets, the Moon's orbit is extremely complex and cannot be treated using the precessing inclined ellipse model [13]. This results in variations in the Moon's argument of perigee and angle of right ascension which affects all possible transfer configurations. To conduct successive transfers to the perigee or apogee of the Moon's orbit under these conditions would be extremely difficult as its latitude would vary in addition to its longitude. Similarly to the method used by Hoyt and Uphoff [7], by conducting transfers to either the ascending or descending node of the Moon's orbit; only variations in the nodes' longitude would have to be accounted for.

To account for this longitudinal variation in the lunar orbit's node position; the EMMET's orbit must be configured such that it returns to the required transfer configuration relative to Earth and the Moon, periodically with the Moon's own motion about Earth, whilst altering its orbital elements so that it accounts for these variations in node position. To begin a description of the methods designed to track the nodes of the Moon's orbit, a short description of the Moon's motion is warranted. The motion of the Moon relative to both the geocentric equatorial and ecliptic frames of reference can be summarised from Roncoli [13] as follows:

1. The principal perturbation acting on the Moon is due to the third body gravitational attraction of the Sun, the result of this is that the Moon's motion cannot be described by the precessing inclined ellipse model.

2. The Moon's orbit has a nearly constant inclination with respect to an Earth centred ecliptic frame but a varying inclination relative to Earth's equatorial frame.

3. Relative to the ecliptic frame, the longitude of the ascending node has a secular rate with a period of 18.6 years. However, relative to the equatorial frame it has no secular rate and oscillates about a right ascension of 0° with an 18.6 year period.

4. The Moon's argument of perigee precesses with a period of 8.85 years relative to both the

ecliptic and equatorial reference frames.

As a result of this complex motion, no configuration which utilises only the perturbing effects of an oblate Earth's J_2 zonal harmonic are sufficient to allow the EMMET to be correctly located to transfer and receive payloads with the Moon on a regular (per-lunar-orbit) basis. Additional perturbations resulting from an oblate Earth, and those resulting from external influences, have been neglected at present and will be considered in future research. Orbital manoeuvres will therefore be undertaken by the EMMET and will require either fuel to be brought up from Earth, via the payloads bound for the Moon; or produced by solar cells located on the central facility.

Focusing on the Moon's ascending node, the most promising method of tracking this motion relative to Earth's equatorial frame consists of configuring the EMMET's orbit with a constant critical inclination relative to Earth's equatorial plane, denoted i_c, of either 63.4° or 116.6° which renders its argument of perigee stationary. This orbital inclination of the MMET would be obtained by means of an inclination change upon the initial configuration of the system and would occur between the launch inclination and the critical inclination, the EMMET's inclination would remain constant thereafter. An inclination of 63.4° is prefered to ensure that the released payload undertakes a prograde trajectory to the Moon thus minimising the payload's speed relative to the Moon upon arrival at the lunar sphere of influence. The argument of perigee is set to 0° to ensure that the ascending node and perigee of the EMMET's orbit remain coincident and that the

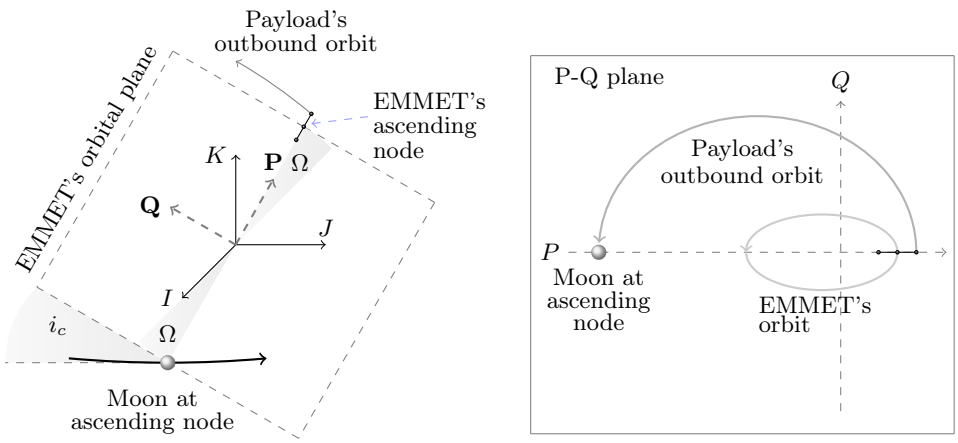


Fig. 6: Critical Inclination Moon-Tracking Configuration

perigee of the payload remains diametrically opposite to the Moon's ascending node at the instant of launch. A result of this configuration is that the angle of right ascension of the EMMET's orbit will have a secular rate, however this can be minimised by careful selection of its semi-major axis and eccentricity. In addition to this, the previously mentioned orbital manoevres are performed to adjust the EMMET's angle of ascending node so that it is repositioned in a manner such that its apse line will regress into alignment with the node line of the Moon's orbit as it crosses the ascending node of its orbit. The EMMET's orbital configuration is shown in Figure 6 with P, Q denoting planar axes of the EMMET's perifocal, or orbit-centred, frame of reference and the orbital plane lying at a critical inclination with respect to Earth's equatorial plane. The magnitude of this manoeuvre is determined from the time period between successive arrivals of the Moon at its node point, the difference in these node positions as a result of perturbations acting on the Moon's orbit and the secular rate of the EMMET's angle of ascending node. It is at this instant of arrival of the Moon at its ascending node and EMMET apse line alignment that ideal payload catch and throw operations are conducted. To clarify this further, the payload leaving the Moon on a return trip to Earth and the payload travelling to the Moon should both reach the boundary of the LSI at this instant. The payload launch operations conducted by the EMMET are therefore performed prior to the Moon's arrival at its ascending node such that the payload bound for the Moon reaches the LSI when the Moon reaches its ascending node. Additionally, the payload bound for Earth is also launched prior to the Moon's arrival at its ascending node such that it also arrives at the LSI when the Moon reaches its ascending node. The waiting time in this case is simply the time period between successive exchange opportunities. By arranging the transfer timings in this way the inbound and outbound trajectories undertaken by the payloads can be arranged to be symmetrical portions of the same trajectory configured between Earth and the location of the Moon's ascending node.

According to Vallado [14], the angle of the ascending node of a circular orbit or an elliptical orbit can be modified by a single impulse manoeuvre occurring at one of two common points of intersection between the initial and final orbits. Impulses occurring at points other than these points of intersection will cause alterations to both the orbital inclination and the ascending node. For the purpose of tracking the ascending node of the Moon's orbit; the impulsive change in velocity

must occur at a point of intersection between the EMMET's initial and final orbits to retain orbital inclination. Furthermore, the final orbit must have an ascending node which takes into account the predicted regression of the EMMET's ascending node during the time interval between payload launch and capture as well as the time interval between payload transfer opportunities. In addition to this, the change in the ascending node angle of the Moon's orbit must be incorporated into the adjustment to ensure correct alignment at the next transfer opportunity. In this case, the ascending node of the final orbit will be positioned such that its apse line will precess into alignment with the Moon's node line in the time period between the adjustment manoeuvre and the succeeding payload transfer opportunity. The change in velocity required to perform the manoeuvre can be calculated by first determining the points of intersection of the initial and final orbits obtained by finding the true anomaly of the trajectories at these points, and secondly by obtaining the velocities of the orbits at the intersection. The change in velocity required is then simply the difference between the velocity vectors of the initial and final orbits at their point of intersection.

V. Manoeuvre Position

A method to calculate the true anomaly at which two orbits intersect when they are identical in every respect, other than by an increment in the angle of their ascending nodes, will now be outlined; we begin by defining the ascending node in the final orbit, Ω_f, in terms of the ascending node in the initial orbit, Ω_i, plus some increment, $\Delta\Omega$, which we define as:

$$\Omega_f = \Omega_i + \Delta\Omega \tag{6}$$

The location of the EMMET's central facility in each orbit about Earth can now be determined relative to its perifocal frame. We begin by designating the perifocal position vectors of the initial, $\vec{r_i}$, and final, $\vec{r_f}$, trajectories as:

$$\vec{r_i} = \begin{bmatrix} x_i & y_i & 0 \end{bmatrix}^T \tag{7}$$

$$\vec{r_f} = \begin{bmatrix} x_f & y_f & 0 \end{bmatrix}^T \tag{8}$$

where $(x_i,\ y_i)$ denotes the position vectors' components in the initial orbit, and $(x_f,\ y_f)$ denotes those in the final orbit. These perifocal positions are converted into the geocentric equatorial frame, the coordinate frame in which the adjustments to the ascending node are to be made.

For the initial and final orbits to intersect their equatorial position vectors must be equal, as must be their vector components at these points. From the conversion of the position vectors at the point of intersection we have three scalar equations, one for each component in the equatorial frame. Here the (x, y) coordinates of intersection in the perifocal frames of the initial, (x_i, y_i), and final orbits, (x_f, y_f), are the four unknowns. In this case, the change in ascending node is between two orbits which differ only in their angle of ascending node whilst their inclination and argument of perigee remain identical. Equating the K axis components of the initial and final orbits in the equatorial frame yields:

$$(sin\omega sini)x_i + (cos\omega sini)y_i = (sin\omega sini)x_f + (cos\omega sini)y_f \tag{9}$$

The (x, y) components of the orbits in their perifocal frames can be be obtained from the components of the perifocal position vector, equation (4), as:

$$x = rcos\theta \tag{10}$$

$$y = rsin\theta \tag{11}$$

Applying equations (10) and (11), in addition to the fact that the radial distance from the centre of Earth to the point of intersection is the same for both the initial and final orbits irrespective of the coordinate frame, equation (9) becomes:

$$sin\omega sinicos\theta_i + cos\omega sinisin\theta_i = sin\omega sinicos\theta_f + cos\omega sinisin\theta_f \tag{12}$$

The only unknowns are now the true anomalies at the points of intersection in the initial, θ_i, and final, θ_f, orbits. Canceling the common inclination terms and using trigonometric sum and difference formulas [15] yields:

$$\theta_i = \theta_f \tag{13}$$

The initial and final trajectories unsurprisingly intersect at the same true anomaly in both orbital paths. The true anomaly at which the orbits intersect can be found firstly by equating the I axis components of the initial and final orbits which we obtain as:

$$\begin{aligned}&(cos\Omega_i cos\omega - sin\Omega_i sin\omega cosi)x_i - (cos\Omega_i sin\omega + sin\Omega_i cos\omega cosi)y_i = \\ &(cos\Omega_f cos\omega - sin\Omega_f sin\omega cosi)x_f - (cos\Omega_f sin\omega + sin\Omega_f cos\omega cosi)y_f\end{aligned} \tag{14}$$

By replacing the (x, y) coordinates in the perifocal frame by their polar form, canceling out the common radial distance terms and by utilising equation (13), we obtain:

$$\begin{aligned}&(cos\Omega_i cos\omega - sin\Omega_i sin\omega cosi)cos\theta_f - (cos\Omega_i sin\omega + sin\Omega_i cos\omega cosi)sin\theta_f = \\ &(cos\Omega_f cos\omega - sin\Omega_f sin\omega cosi)cos\theta_f - (cos\Omega_f sin\omega + sin\Omega_f cos\omega cosi)sin\theta_f\end{aligned} \tag{15}$$

By application of the trigonometric sum and difference formulas [15] we obtain:

$$(cos\Omega_i - cos\Omega_f)cos(\theta_f + \omega) = (sin\Omega_i - sin\Omega_f)cosisin(\theta_f + \omega) \tag{16}$$

Further rearrangement and the application of the sum to product rule for trigonometric functions [15] yields the equation for the true anomaly of intersection:

$$tan(\theta_f + \omega) = -tan\left(\frac{\Omega_i + \Omega_f}{2}\right)seci = -tan\left(\Omega_i + \frac{\Delta\Omega}{2}\right)seci \tag{17}$$

At this point the quadrants of the two points of intersection can be determined in the standard manner and their true anomalies are found by subtracting the argument of perigee, ω, from the angles obtained.

VI. Velocity Adjustment

By applying the orbital configuration required to track the Moon's ascending node to the result of equation (4) and its derivatives, we can determine the position, velocity and acceleration of the EMMET's central facility relative to Earth and subsequently the magnitude of the velocity changes required to adjust the angle of right ascension of the EMMET's orbit such that it will realign itself with the Moon's node line at its arrival at the node. From the moon-tracking orbit described, we have a critically inclined orbital inclination denoted by critical inclination, denoted i_c and equal to 63.4° or 116.6°. This renders the argument of perigee stationary and this is set to 0°. The central facility's position relative to the equatorial frame can therefore be defined as:

$$\vec{r} = r \begin{bmatrix} cos\theta cos\Omega - sin\theta sin\Omega cosi_c \\ cos\theta sin\Omega + sin\theta cos\Omega cosi_c \\ sin\theta sini_c \end{bmatrix} \tag{18}$$

where r denotes the radial distance of the EMMET's central facility from Earth. The the EMMET's central facility velocity relative to the equatorial frame is found by differentiating the equatorial position vector with respect to time. It is assumed that the orbital inclination remains constant throughout. We obtain the EMMET central facility's velocity to be:

$$\vec{v} = \begin{bmatrix} -\left(\frac{\mu}{h} + \dot{\Omega} r cosi\right) cos\Omega sin\theta - \left(\frac{\mu}{h} cosi + \dot{\Omega} r\right) sin\Omega cos\theta - \frac{\mu e}{h} sin\Omega cosi \\ -\left(\frac{\mu}{h} + \dot{\Omega} r cosi\right) sin\Omega sin\theta + \left(\frac{\mu}{h} cosi + \dot{\Omega} r\right) cos\Omega cos\theta + \frac{\mu e}{h} cos\Omega cosi \\ \frac{\mu}{h}(e + cos\theta) sini \end{bmatrix} \tag{19}$$

where $\dot{\Omega}$ is the precession rate of the central facility's angle of right ascension resulting from the non-spherical Earth and arising as a result of the angle of right ascension Ω being a time dependent quantity, and the differentiation of the perifocal to equatorial transformation matrix defined by equation (3).

The next step is to determine the central facility's velocity vector in both the initial and final orbits required for the adjustment to the angle of right ascension and the change in velocity required

to perform the manoeuvre between them. To change the angle of ascending node of an orbit relative to Earth's equatorial frame, without affecting the other orbital parameters, it is evident from the manoeuvre occurring at the same true anomaly on both orbits that the manoeuvre required is simply the rotation of the velocity vector of the initial orbit at its point of intersection, about the equatorial K axis, through an angle $\Delta\Omega$ equal to the required change in right ascension. This can be accomplished by means of the Rodriguez Formula [16][17] which expresses, in a kinematic form, the matrix required to transform the initial velocity vector to the final velocity vector in terms of the angle of rotation and a unit vector along the axis of rotation. In this case, the rotation matrix, R_K, can be written [16][17]:

$$R_K(\Delta\Omega) = [I + Ssin\Delta\Omega + S^2(1 - cos\Delta\Omega)] \tag{20}$$

where I is the 3x3 identity matrix and S, in this case, is a particularly simple skew symmetric matrix obtained from the unit vector along the axis of rotation:

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{21}$$

The velocity of the final orbit, $\vec{v}_f$, relative to the equatorial frame can be found by applying equation (20) to the velocity of the initial orbit, $\vec{v}_i$, at the point of intersection:

$$\vec{v}_f = R_K(\Delta\Omega)\vec{v}_i \tag{22}$$

VII. Orbital Manoeuvre Data

Having described a means by which the EMMET can track the Moon's ascending node over an extended time period, the magnitude of the velocity changes required to undertake this will now be determined. This is achieved firstly by obtaining the rate at which the angle of right ascension of the EMMET's central facility varies relative to Earth then by utilising the expression derived for its velocity relative to Earth. The data obtained was produced using code written in the mathematical software programme *MATHEMATICA*™ but not restricted exclusively to this package.

When configured with a critical inclination of 63.4 ° the argument of perigee of the EMMET's orbit will remain stationary whilst its angle of right ascension will regress relative to Earth's equatorial frame. To facilitate the exchange of payloads on every occasion that the Moon arrives at its ascending node, for example, an orbital manoeuvre is implemented to correctly align the EMMET's orbit. This adjustment must take into account the regression rate of the angle of right ascension; the time period between the manoeuvre being performed and the arrival of the Moon at its ascending node; and the oscillation of the Moon's ascending node about a mean angle of 0° relative to Earth's equatorial frame. These considerations ensure that the EMMET's apse line will regress into alignment with the Moon's node line at the correct instant to perform payload exchanges. To obtain the magnitude of the velocity changes required to correctly adjust the ascending node we must first obtain the rate at which this regression occurs. The rate of this regression, denoted by $\dot{\Omega}$, can be found from the following equation [18]:

$$\dot{\Omega} = \frac{d\Omega}{dt} = -\left[\frac{3}{2}\frac{\sqrt{\mu}J_2R^2}{(1-e^2)^2\,a^{7/2}}\right]cosi \tag{23}$$

where μ denotes Earth's gravitational parameter; J_2 is Earth's second zonal harmonic; R is Earth's equatorial radius; e is the central facility's orbital eccentricity; a is the central facility's semi-major axis; and the central facility's inclination is denoted i, which in this case is the critical inclination of 63.4°. From this it is evident that the rate of regression is dependent upon the semi-major axis and eccentricity of the EMMET central facility's orbit. The semi-major axis of the EMMET's orbit can be calculated as a function of its orbital period, T, as follows:

$$a = -\left(\frac{T\sqrt{\mu}}{2\pi}\right)^{2/3} \tag{24}$$

An additional consideration is that the EMMET must arrive at the perigee of its orbit about Earth in preparation for launch on each occasion that the Moon arrives at its ascending node. Therefore, the orbital period of the EMMET about Earth must be arranged such that it is integer harmonic with the Moon's orbit about Earth. Stating this more clearly, multiplication of the EMMET's orbital period by known positive integer m equals the orbital period of the Moon about Earth. This can be defined as:

$$T = \left(\frac{T_m}{m}\right) \tag{25}$$

where T is the EMMET's orbital period about Earth, T_m is the Moon's orbital period about Earth and m is a positive integer which we define as the *orbital variable* m as it is the ratio of two orbital periods. By implementing this condition the central facility's semi-major axis can be defined as:

$$a = \left(\frac{T_m\sqrt{\mu}}{2\pi m}\right)^{2/3} \tag{26}$$

It can be seen from this that the semi-major axis of the central facility's orbit is inversely proportional to the orbital variable m. Therefore as the orbital variable increases the orbital geometry decreases. Additionally, the orbital eccentricity can be obtained as a function of the semi-major axis and central facility perigee distance, r_p, as:

$$e = 1 - \frac{r_p}{a} \tag{27}$$

The EMMET's ascending node regression rate defined in equation (23) therefore simplifies to a function of the orbital variable m and the perigee distance of the central facility's orbit.

The minimum suitable perigee distance of the EMMET's central facility can be determined by considering the following; in order to negate the effects of atmospheric drag on the EMMET's orbit, it must orbit in the exosphere region of Earth's atmosphere which has been estimated to extend up to an altitude of 1000 km [19]. Assuming that the tether sub-spans are composed of Spectra which gives us an optimum tether sub-span length of 100 km, as determined by Murray [4], we obtain an initial estimate for the EMMET's central facility perigee altitude of 1100 km or a distance of 7478 km from Earth's centre. Utilising this estimated minimum perigee altitude in conjunction with equations (26) and (27), the rate of regression of the angle of ascending node of the EMMET's

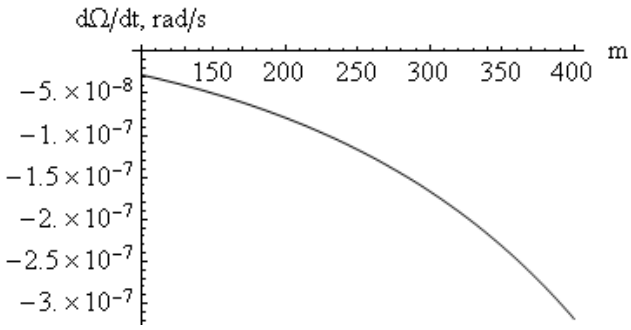


Fig. 7: Ascending node variation with m

central facility was plotted as a function of the orbital variable m and this is shown in Figure 7. The EMMET will undertake a circular orbit for m=366 and elliptical, parabolic or hyperbolic orbits for successively decreasing orbital variable values less than this value. It is clear from this data that the rate of regression of the EMMET's orbit increases with increasing orbital variable m. This is a result of the decreasing orbital geometry resulting in the EMMET spending a greater length of time in regions where the effects of Earth's J_2 zonal harmonic are more significant.

Using the moon-tracking configuration an orbital manoeuvre is performed which adjusts the EMMET's ascending node angle such that it will regress into alignment with the Moon's node line at the instant of launch. Taking the period of oscillation of the Moon's ascending node relative to Earth's equatorial frame to be 18.6 years with an amplitude of 13.5°, this correlates to a maximum variation in the ascending node position of 0.22°(0.00384 radians) for every revolution of the Moon about Earth. The maximum adjustment to the EMMET's ascending node occurs when the variation in the Moon's ascending node is directed opposite to the regression of the EMMET's ascending node. The largest adjustment required will therefore equal the sum of the Moon's own ascending node variation and that of the EMMET's in the time period of a single orbit of the Moon about Earth. To calculate the change in velocity required to configure the EMMET's orbit we must first determine the true anomaly in its current orbit at which the manoeuvre must be undertaken. To simplify matters, and without loss of generality, we assume an initial EMMET ascending node angle of 0° and in conjunction with the argument of perigee set to 0°, and so the true anomaly of intersection

is obtained from equation (17) as:

$$\theta_i = tan^{-1}\left(-tan\left(\frac{\Delta\Omega}{2}\right)seci_c\right) \tag{28}$$

The velocity of the EMMET's central facility at this true anomaly is obtained from equation (19) as:

$$\vec{v}_i = \begin{bmatrix} -\left(\frac{\mu}{h} + \dot{\Omega}r_i cosi\right) sin\theta_i \\ \left(\frac{\mu}{h}cosi + \dot{\Omega}r_i\right) cos\theta_i + \frac{\mu e}{h}cosi \\ \frac{\mu}{h}(e + cos\theta_i)sini \end{bmatrix} \tag{29}$$

where the central facility distance, r_i, is obtained from the equation of an elliptical orbit:

$$r_i = \frac{h^2}{\mu(1 + ecos\theta_i)} \tag{30}$$

and the specific angular momentum, h, of the EMMET's orbit is defined in terms of its semi-major axis and eccentricity as:

$$h = \sqrt{\mu a(1 - e^2)} \tag{31}$$

Utilising equation (22), the change in velocity required to adjust the angle of right ascension of the EMMET's orbit can be expressed solely in terms of the velocity of the initial orbit and the rotation matrix defined in equation (20). The rotation matrix in this case is defined as:

$$R_K(\Delta\Omega) = \begin{bmatrix} cos\Delta\Omega & -sin\Delta\Omega & 0 \\ sin\Delta\Omega & cos\Delta\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{32}$$

The change in velocity required to perform the ascending node adjustment is therefore:

$$\Delta\vec{v} = \begin{bmatrix} -\left(\frac{\mu}{h} + \dot{\Omega}r_i cosi\right) sin\theta_i cos\Delta\Omega - \left(\frac{\mu}{h}cosi + \dot{\Omega}r_i\right) cos\theta_i sin\Delta\Omega - \left(\frac{\mu e}{h}cosi\right) sin\Delta\Omega \\ -\left(\frac{\mu}{h} + \dot{\Omega}r_i cosi\right) sin\theta_i sin\Delta\Omega + \left(\frac{\mu}{h}cosi + \dot{\Omega}r_i\right) cos\theta_i cos\Delta\Omega + \left(\frac{\mu e}{h}cosi\right) cos\Delta\Omega \\ \frac{\mu}{h}(e + cos\theta_i)sini \end{bmatrix} \tag{33}$$

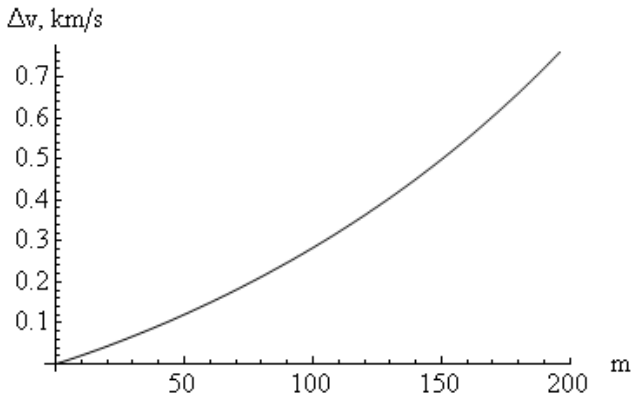
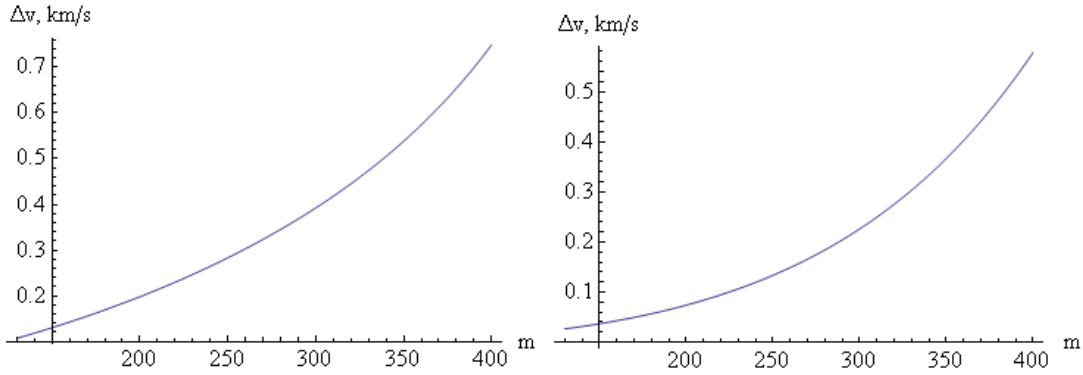


Fig. 8: Velocity change for critical configuration

The largest value of orbital variable m, corresponding to the smallest semi-major axis, of the EMMET's orbit which can generate upper tip velocities capable of launching payloads which reach the Moon's gravitational influence has previously been determined by Murray [20] as occurring at m=196. At orbital variables greater than this value, corresponding to smaller semi-major axes, the combination of orbital speed and maximum tether sub-span rotational speed is insufficient for the upper payload to reach the Moon after release. Having obtained an expression for the change in velocity required to adjust the ascending node of the EMMET's orbit when it has a critical inclination, this change in velocity can be plotted as a function of the orbital variable up to the limiting value of m=196 and this is shown in Figure 8. The velocity change required is seen to increase with an increase in the orbital variable and results from the EMMET's increasing proximity to Earth as its orbital geometry decreases.

Performing a similar analysis on the Lunavator's motion about the Moon yields similar data. As a result of the reduced gravitational attraction of the Moon the Lunavator can alter its orbital elements more readily and this proves useful in accommodating variations into the payload trajectory design. These variations result from the complex motion of the Moon and from the application of a logistical design to the catch and throw operations of the tether systems. This results in the payloads undertaking trajectories between the Earth and Moon in time periods either longer or shorter than the time period required for the optimum trajectory between the two tether systems. This is



(a) Velocity change for angle of right ascension **(b)** Velocity change for argument of perilune

Fig. 9: Velocity change requirements for adjustments to Lunavator's orbital elements

accommodated by the Lunavator adjusting its angle of right ascension, argument of perilune and its orbital inclination such that these match the corresponding quantities of the incoming payloads hyperbola about the Moon and this is useful in reducing velocity differences between the tether tip and payload which reduces mechanical shock upon capture. The velocity changes required to alter each of these elements of the Lunavator's orbit when assuming that there is a 1:1 correspondence between the orbital period of the EMMET and Lunavator's orbital periods were obtained. It can be seen from Figure 9a that the velocity change required to adjust the Lunavator's angle of right ascension is of a comparable magnitude to that required to adjust the corresponding EMMET parameter and this is a consequence of the proximity of the Lunavator's perilune position to the Moon e.g. the lower tether tip is arranged to coincide with the Lunar surface when the Lunavator is at perilune and the tether sub-spans are aligned along the local gravity gradient. Additionally it can be seen from Figure 9b the change in argument of perilune is also of a comparable magnitude. The change in velocity required to alter the orbital inclination of the Lunavator was also plotted between -45^0 and 45^0 and this is shown in Figure 10. Definitive data on the required magnitude of these changes could not be obtained prior to the optimised trajectories being calculated for each transfer opportunity however these variations in inclination are not expected to be large when taking into account small variations to the transfer timings for each opportunity rather than a single large change.

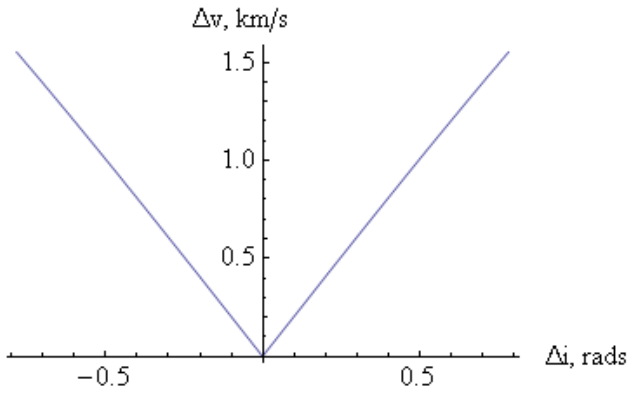


Fig. 10: Velocity change for Lunavator inclination change

VIII. Conclusions

A significant step in the viability of a continuous Earth-Moon payload exchange has been made by the design of a means by which a point in the Moon's orbit can be tracked by the Earth orbiting motorised momentum exchange tether (EMMET) through careful orbit design and by utilising the effects of Earth's oblateness on the tether systems. It has been shown that this can be achieved by means of a critically inclined orbit in conjunction with pre-emptive adjustments to the angle of right ascension of the EMMET's orbit. The maximum magnitude of these adjustment was found to have a value of around 0.7 km/s. Additionally, the adjustments required to alter the Lunavator's angle of right ascension and argument of perilune were found to be of a similar magnitude to those required to alter the EMMET's angle of right ascension. The velocity change required to alter the inclination of the Lunavator's orbit was found between -45^0 and 45^0 with a maximum value of 1.5 km/s however changes to the inclination are expected to be small and certainly not of this magnitude. Assuming that payload velocity changes are negligible and taking the worst case scenario for the velocity adjustment required to the motorised momentum exchange tether (MMET) orbits and a required Lunavator inclination change of 45^0 the required velocity change required for a complete Earth-Moon-Earth transfer each time the Moon arrives at its ascending node is at worst 3.6 km/s and still significantly less than that required using conventional chemical propulsion.

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