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AN ELASTO-PLASTIC MODEL FOR UNSATURATED SOIL INCORPORATING THE EFFECTS OF SUCTION AND DEGREE OF SATURATION ON MECHANICAL BEHAVIOUR

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SYNOPSIS

The paper presents an elasto-plastic model for unsaturated soils that takes explicitly into account the mechanisms with which suction affects mechanical behaviour as well as their dependence on degree of saturation. The proposed model is formulated in terms of two constitutive variables directly related to these suction mechanisms: the average skeleton stress that includes the average fluid pressure acting on the soil pores and an additional scalar constitutive variable ξ related to the magnitude of the bonding effect exerted by meniscus water at the inter-particle contacts. The formulation of the model in terms of variables closely related to specific behaviour mechanisms leads to a remarkable unification of experimental results of tests carried out with different suctions. The analysis of experimental isotropic compression data strongly suggests that the quotient between the void ratio e of an unsaturated soil and the void ratio e_s, corresponding to the saturated state at the same average soil skeleton stress, is a unique function of the bonding effect due to water menisci at the inter-particle contacts. The same result is obtained when examining critical states at different suctions. Based on these observations, an elasto-plastic constitutive model is developed using a single yield surface the size of which is controlled by volumetric hardening. In spite of this simplicity, it is shown that the model reproduces correctly many important features of unsaturated soil behaviour. It is especially remarkable that, although only one yield surface is used in the formulation of the model, the irreversible behaviour in wetting-drying cycles is well captured. Because of the behaviour normalization achieved by the model, the resulting constitutive law is economical in terms of the number of tests required for parameter determination.

INTRODUCTION

Suction has long been recognized as a fundamental variable in the understanding of the mechanical behaviour of unsaturated soils. For this reasons many well-known constitutive models (Alonso et al. 1990, Cui & Delage 1996 and Wheeler & Sivakumar 1995) include suction as a basic stress variable together with the net stress $\overline{\sigma}$ (defined as total stress minus pore air pressure).

In fact suction influences mechanical behaviour of an unsaturated soil in two different ways (Karube & Kato 1994, Wheeler & Karube 1995):

i) modifying the skeleton stress through changes in the average fluid pressure acting in the soil pores;

ii) providing an additional bonding force at the particle contacts often attributed to capillary phenomena occurring in the water menisci.

It is important to realize that, for the two mechanisms, the effects of suction are influenced by the state of saturation of the soil. The relative area over which the water and air pressures act depends directly on the degree of saturation (the percentage of pore voids occupied by water) but the same parameter also affects the number and intensity of capillary-induced inter-particle forces.

Therefore, models using only suction in their formulation are unlikely to be complete. It is necessary to incorporate, through a parameter such as degree of saturation, information regarding over which proportion of the soil suction effects are relevant. Therefore, it is not surprising that constitutive models that use only suction as the unsaturated variable face difficulties in describing important features of unsaturated soil behaviour. Another class of elasto-plastic models for unsaturated soils, such as those proposed by Bolzon et al. (1996), Jommi & Di Prisco (1994), Karube & Kawai (2001), Loret & Khalili (2000), are expressed in terms of a different set of constitutive variables that include explicitly the degree of saturation in their definitions. In these models the stress variable has the form of the Bishop (1959) stress:

$$\sigma'_{hk} = \sigma_{hk} - \delta_{hk}\left[u_a - \chi\left(u_a - u_w\right)\right] \qquad (1)$$

where σ'_{hk} is the Bishop (1959) stress, σ_{hk} is the total stress, u_a is the air pressure, u_w is the liquid pressure, δ_{hk} is Kronecker's delta and χ is a soil parameter ranging between 1 (at saturation) and zero (at dry conditions) and that is a function of degree of saturation. The additional scalar variable is given either by suction or by degree of saturation depending on the specific model, with the exception of the model of Karube & Kawai (2001) where it is a function of both suction and degree of saturation. Although such models introduce explicitly the degree of saturation in the definition of the soil variables, they still present limitations when predicting important aspects of unsaturated soil behaviour unless the additional complexity of multiple yield surfaces is introduced. Two examples of features of behaviour that require adequate modelling are:

1. the irreversible reduction of specific volume that can occur during drying of an unsaturated soil (i.e. during increase of suction);
2. the dependency of the soil response during virgin loading at constant suction on the past history of suction variation.

The first type of behaviour has been observed by Alonso et al. (1995) and Sharma (1998) in laboratory tests involving wetting-drying cycles (i.e. cycles of decrease-increase of suction) on soil samples subjected to oedometric and isotropic conditions respectively. Examples of this type of soil response are shown in Figure 1. The second type of behaviour has been observed by Sharma (1998) during isotropic loading to virgin states of samples at constant suction. In particular, the results by Sharma (1998) suggest that a compacted unsaturated soil shows a different stiffness during virgin loading at the same constant suction depending on whether the sample undergoes a wetting-drying cycle prior to loading or not. Figure 2 shows examples of isotropic tests where the dependency of the soil stiffness during virgin loading on the previous history of suction variation can be observed. Moreover, for some of these models the determination of model parameters requires non-conventional laboratory tests (e.g. tests at constant degree of saturation) or, alternatively, a backanalysis for fitting model predictions to conventional laboratory results.

In this paper a model is described that incorporates explicitly in its formulation the two distinct suction effects mentioned previously, including their dependence on degree of saturation. By staying close to these basic behaviour mechanisms, the proposed elasto-plastic model is capable of reproducing the most important patterns of unsaturated soil mechanical behaviour, including those indicated above, in a rather simple manner employing only a single yield surface. The model also provides an effective way of unifying experimental results of tests performed at different suctions. Apart from the conceptual benefits of such unification, this fact results in economical procedures of parameter determination from the point of view of the number of laboratory tests required.

MODELLING ASSUMPTIONS

In the proposed model the basic stress variable is the “average skeleton stress” (Jommi 2000), that is equivalent to the Bishop (1959) stress where the χ parameter of Equation 1 is equal to the degree of saturation, S_r:

$$\sigma'_{hk} = \sigma_{hk} - \delta_{hk}\left[u_a - \chi\left(u_a - u_w\right)\right] \tag{2}$$

This variable expresses the average stress acting in the soil skeleton, that is the difference between the total stress and the average pressure of the two fluid phases (i.e. gas and liquid) with the degree of saturation as a weighting parameter. It incorporates, therefore, in a direct manner the first of the suction roles noted above. The definition of the average skeleton stress represents the natural extension to the unsaturated domain of the Terzaghi (1936) effective stress for saturated granular materials and it reduces to the Terzaghi's effective stress at saturated condition (i.e. degree of saturation equal to unity).

Laboratory tests have shown, however, that it is not possible to explain important features of the unsaturated soils behaviour, such as the irreversible compression (collapse) during wetting (i.e. during a suction reduction) and the increase of the pre-consolidation pressure with increasing suction, by using the average skeleton stress as the only constitutive variable (see, for example, Jennings & Burland 1962). To account for these phenomena it is necessary to consider the second suction mechanism. Irreversible mechanical response of a granular material is mainly associated to the relative slippage taking place at the interface between soil particles. In an unsaturated soil, the possibility of such slippage is partially prevented by the stabilizing effect of the normal force exerted at the inter-particle contacts by meniscus lenses of water at negative pressure (Wheeler & Karube 1995). Several features of the elasto-plastic behaviour of unsaturated soil are, therefore, likely to be the consequence of bonding and de-bonding phenomena between soil particles due to the formation and vanishing of water menisci at inter-particle contacts and they can not be accounted for by using exclusively the average skeleton stress as a constitutive variable.

Consequently an additional constitutive variable ξ needs to be introduced as a measure of the magnitude of the inter-particle bonding due to water menisci so that the second type of suction effects is properly accounted for. The magnitude of such inter-particle bonding is expected to be the result of two contributions: a) the number of water menisci per unit volume of the solid fraction and b) the intensity of the stabilizing normal force exerted at the inter-particle contact by a single water meniscus. Hence the variable ξ is defined in the present formulation as the product of two factors, namely the degree of saturation of the air ($1-S_r$) and the function of suction $f(s)$:

$$\xi = f(s)\left(1 - S_r\right) \tag{3}$$

The factor ($1-S_r$) accounts for the number of water menisci per unit volume of solid fraction. The existence of a unique relationship between the value of ($1-S_r$) and the number of water menisci per unit volume of solid fraction is a physically reasonable assumption, however the uniqueness of such relationship is rigorously true only for the ideal case where the soil is rigid (i.e. when the dimensions and shapes of voids do not change due to particle re-arrangements) and where each value of degree of saturation corresponds to a given arrangement of water within soil pores. The term ($1-S_r$) is equal to zero when the soil is saturated (i.e. S_r=1) and water menisci are absent whereas it assumes positive increasing values when the number of water menisci increases. The number of water menisci per unit volume of solid fraction can therefore be expressed as a monotone increasing function of the term ($1-S_r$). The validity of this definition does not apply to the case of a soil in an extremely dry state, when the water menisci will start to disappear from the particle contacts. Although the extension to the case of extremely dry soils should not present any conceptual difficulty, this is not covered in the present paper because the experimental validation would require experimental results from a test programme conducted on samples at very low degree of saturation, which are currently unavailable.

Clearly the relationship between the number of water menisci per unit volume of solid fraction and the term (1-S_r) is dependent on the specific fabric of the soil (i.e. on the pore size distribution of the soil). For the purposes of this work, it is not necessary however to characterize explicitly such relationship because this information implicit in the definition of the function, introduced later in the paper, that provides the variation of the ratio e/e_s in terms of the bonding variable ξ.

The function of suction *f(s)*, which multiplies the factor (1-S_r), varies monotonically between 1 and 1.5 for values of suction ranging between zero and infinity respectively and it accounts for the increase with increasing suction of the stabilizing inter-particle force exerted by a single meniscus. In particular, it expresses the ratio between the value of stabilizing force at a given suction, *s* and the value of stabilizing force at a suction of zero for the ideal case of a water meniscus located at the contact between two identical spheres (the analytical solution of this problem is due to Fisher 1926). The specific form of the function *f(s)* depends on the size of the spheres and on the value of the water surface tension but the range of variation, between 1 and 1.5, is always the same regardless of dimensions and physical properties. The relationship *f(s)* used in this work is shown in Figure 3 corresponding to the case of two spheres having radii of 1 μ and a value of the surface tension of water corresponding to a temperature of 20° C. Haines (1925) suggested that a material with the texture of a compacted kaolin could be represented by spheres having radii equal to 1 μ. Obviously the shapes of the aggregates are far from being spheres of the same size. In addition, for soils with a multi-modal pore size distribution, the dimension of the spherical grains in the solution of Fisher (1926) should be defined as a variable depending on the average size of the soil pores that include water menisci. At this stage of development of the model, however, the assumption of a simplified relationship, such as the one given in Figure 3, is considered reasonable.

The presence of meniscus water provides a physical explanation to the experimental observation that, at the same value of average skeleton stress, the value of void ratio during virgin loading of unsaturated soil is always greater than the value of void ratio of the same soil subjected to the same load under saturated condition. The existence of water in the form of meniscus lenses within an unsaturated soil makes the inter-particle contacts more stable and, therefore, restrains the reciprocal slippage of soil particles that causes compressive strains during virgin loading. Consistently with such empirical observations, this work introduces a fundamental modelling assumption specifying that during virgin loading of an unsaturated soil the ratio e/e_s between void ratio in unsaturated conditions e and void ratio in saturated conditions e_s at the same average skeleton stress state is a unique function of the bonding variable ξ. This assumption not only provides an essential starting point for the development of the model but it also offers a powerful unifying perspective to examine the results of tests performed at different suctions. This assumption is validated in the next section on the basis of published laboratory test data.

EXPERIMENTAL VALIDATION OF MODELLING ASSUMPTIONS

The validation of the assumption introduced in the previous section has involved the analysis of different sets of data from laboratory tests performed on compacted Speswhite

Kaolin (Sivakumar 1993 and Wheeler & Sivakumar 2000), on a compacted mixture of Bentonite and Kaolin (Sharma 1998) and on compacted Kiunyu gravel (Toll 1990). The first part of this section analyses the data from isotropic virgin compression tests at constant suction (Sivakumar 1993 and Sharma 1998) while, at the end of the section, the analysis of further experimental data from triaxial shear tests on compacted Speswhite Kaolin (Sivakumar 1993 and Wheeler & Sivakumar 2000) and on compacted Kiunyu gravel (Toll 1990) demonstrates that the conclusion achieved for isotropic stress states can also be extended to non-isotropic stress states.

Sivakumar (1993) and Sharma (1998) performed isotropic virgin compression of soil samples at different values of suction, namely 100 kPa, 200 kPa, 300 kPa, as well as of saturated samples. During these tests the corresponding changes of void ratio, e and water ratio, e_w (i.e. the volume of water in a volume of soil containing unit volume of solids) were measured. The analysis of the experimental results indicates that, for the range of stresses considered, the normal compression lines at constant suction follow a linear relationship in the semi-logarithmic planes e-ln $\bar{p}$ and e_w-ln $\bar{p}$ (where $\bar{p}$ is the mean net stress). Each normal compression line is therefore identified by the values of the two parameters that correspond to the slope and to the intercept at a given value of $\bar{p}$. As for the data set by Sharma (1998) there were no virgin loading tests on saturated samples, the slope and intercept of the saturated normal compression line were estimated from the drying branch of a wetting-drying test under isotropic constant load. In this type of test, after an initial wetting that brought the soil to saturation, the sample was subjected to drying that caused significant irreversible changes of void ratio. The principle of the effective stress holds during most of such drying because the sample remained saturated for a large increase of suction due to its high air-entry value. Under saturated conditions the imposed change of suction corresponds to an equivalent change of the effective stress. Hence, by plotting the void ratio against the mean effective stress, it was possible to estimate the slope and the intercept of the saturated normal compression line.

The values of slopes and intercepts of normal compression lines of e and e_w at constant suction were used to re-plot the normal compression lines in terms of the isotropic average skeleton stress p''. Figures 4 and 5 show the normal compression lines at constant suctions of zero (saturated), 100 kPa, 200 kPa, 300 kPa, in the semi-logarithmic plane e-ln p'' for each set of data respectively.

Inspection of Figures 4 and 5 reveals that the normal compression lines at non-zero values of suction are not straight lines in the semi-logarithmic plane e-ln p'' but they are curves with slopes that decrease as they approach the saturated line (zero suction). This is consistent with the experimental observation that degree of saturation increases during isotropic loading to virgin states at constant suction. Indeed, if a soil sample attains saturation during compression at a positive value of suction, the isotropic average skeleton stress coincides with the saturated effective stress and the corresponding value of void ratio should lie on the saturated normal compression line. After saturation, the normal compression line at non-zero suction should therefore have the same slope of the saturated normal compression line. It is therefore to be expected that the slope of the normal compression lines at non-zero suction progressively reduce as they converge towards the saturated line.

From the normal compression lines in the semi-logarithmic plane e-ln p'' shown in Figures 4 and 5, it is possible to calculate the ratio between the e value of the unsaturated soil and that corresponding to the saturated state, e_s at the same average skeleton stress.

Figures 6 and 7 show, for the data sets of Sivakumar (1993) and Sharma (1998) respectively, the value of the ratio e/e_s plotted against the value of the bonding variable, ξ defined by equation (3) (corresponding to the values of S_r and $f(s)$ of the unsaturated soil). The value of the function of suction $f(s)$ has been calculated according to the relationship shown in Figure 3 and it is equal to 1.10, 1.15 and 1.18 for suction values of 100 kPa, 200 kPa and 300 kPa respectively. The relationship shown in Figure 3 refers to the Fisher (1926) solution where the spherical grains have radii equal to 1 μ. This is the order of magnitude of the macro-structural voids of a clay soil compacted dry of optimum, such as the soils investigated by Sivakumar (1993) and Sharma (1998). Porosimetry studies have shown that clay materials compacted dry of optimum present a marked bi-modal pore size distribution (see, for example, Gens and Alonso 1992) with macro-structural and micro-structural voids of the order of magnitude of 1 μ and 0.01 μ respectively. For the range of suctions investigated by Sivakumar (1993) and Sharma (1998) it is reasonable to expect that only macro-voids are affected by de-saturation (and, hence, by the formation of water menisci) while the micro-voids stay saturated.

Inspection of Figures 6 and 7 suggests remarkably that, for all the three values of suction investigated, the data from normal compression are consistent with a unique relationship linking the value of the proportion e/e_s and the bonding variable, ξ. Such bonding variable therefore appears to be uniquely related to the ability of the skeleton to sustain higher void ratios when the soil is under suction. For each of the three curves at constant suction shown in Figures 6 and 7, the value of the proportion e/e_s is expected to attain a value of 1 when ξ is equal to zero (i.e. when the sample achieves saturation) because in this case the normal compression lines at non-zero suctions coincide with the saturated line in the semi-logarithmic plane e-ln p''. The model equation which fits the three e/e_s vs ξ curves at constant suction in Figures 6 and 7, has the following form:

$$\frac{e}{e_s} = 1 - a \cdot \left(1 - \exp\left(b \cdot \xi\right)\right) \qquad (4)$$

where a and b are fitting parameters. Equation 4 predicts a value of e/e_s equal to 1 when ξ is equal to zero, consistent with the physical explanation given above.

For the range of suction investigated the value of the function $f(s)$ varies relatively little in comparison with the variation of the value of the bonding variable ξ. It is therefore reasonable to expect that experimental data might be similarly consistent with a relationship linking the value of the proportion e/e_s during isotropic virgin loading to the value of the degree of saturation of the gas phase (1-S_r).

Sivakumar (1993) and Wheeler & Sivakumar (2000) presented further experimental data from shearing tests to critical state on compacted Speswhite Kaolin under various suction values. These data have been used to investigate whether the relationship between the ratio e/e_s and the bonding variable, ξ could be extended to non-isotropic stress states. The model equation in Figure 7, which had been defined on the basis of isotropic normal compression tests, was therefore used to predict values of void ratio at critical states. The predicted values of void ratio at critical state were computed following the same procedure as in the isotropic case. Firstly the saturated critical state line in the semi-logarithmic plane (e, ln p') was defined (by fixing its slope and intercept) on the basis of shearing tests performed by Sivakumar (1993) and Wheeler & Sivakumar (2000) on saturated samples. Then, for each unsaturated sample sheared to critical state, the corresponding experimental values of mean net stress, degree of saturation and suction at critical state were used to calculate the isotropic average skeleton stress, p'' and the bonding variable, ξ. These values of p'' and ξ were then employed to compute the void ratio, e_s from the saturated critical state line and the ratio, e/e_s from the model equation of Figure 7 respectively. Figure 8 shows the comparison between predicted and experimental values of void ratio at critical state corresponding to different suction levels.

Inspection of Figure 8 indicates remarkably that the relationship established between the ratio e/e_s and the bonding factor ξ for isotropic virgin compression and given by equation (4) can also accurately predict the void ratio values at critical state. This implies that such relationship might be unique for the elasto-plastic loading of an unsaturated soil regardless of the specific stress ratio applied to the sample and that the selected bonding variable closely represents the real effect of suction on inter-granular stress. The different series of Figure 8 correspond to different procedures adopted by Wheeler & Sivakumar (2000) for the compaction of Speswhite Kaolin at the same dry of optimum water content (i.e. for the test series II and III the compaction pressure and method of compaction were different from those employed for preparing the samples from series I shown in Figures 5 and 7). On the basis of their empirical results, Wheeler and Sivakumar (2000) concluded that the behaviour at critical state of a soil compacted at the same dry of optimum water content is not affected by the procedure adopted for compaction. This result is also confirmed by the comparison shown in Figure 8.

Finally, the analysis of the data from undrained (with respect to water phase) triaxial shear tests performed by Toll (1990) on compacted samples of a lateritic gravel from Kenya (Kiunyu gravel) is presented. Samples were compacted by Toll (1990) at different values of water content ranging from 17.0 to 27.7 and then sheared in axial compression to critical state while preventing flow of water. The Author reports the measured values of the void ratio, degree of saturation, suction and net stress state at the end of the tests when the unsaturated samples have attained critical state conditions. Together with the results from unsaturated soil samples, the Author presents a smaller set of data from undrained triaxial shear tests to critical state performed on saturated samples. For the saturated tests, the soil samples were compacted at water content ranging between 18.7 and 31.0 and were then saturated prior to testing. On the basis of the saturated shear tests, Toll (1990) suggested the values of the slope and intercept of saturated critical state line in the semi-logarithmic plane (e, ln p'').

The data presented by Toll (1990) are limited to the critical state values measured at the end of the undrained shearing of each sample. Such data were used in this work to validate the proposed assumption of a unique relationship between the ratio e/e_s (corresponding to a given value of the isotropic average skeleton stress) and the bonding variable ξ at the critical state. The soil tested by Toll (1990) is likely to exhibit a different grading from the soils investigated by Sivakumar (1993) and Sharma (1998). However, due to the absence of precise information on the fabric of the soil tested by Toll (1990), the radii of the spheres in the Fisher (1926) solution (i.e. in the function *f(s)* of equation (3)) were taken equal to 1 μ, the same value as in the previous two analyses. For each experimental data point, the value of the function *f(s)* was then calculated according to the suction measured by Toll (1990) at the critical state.

For the purposes of the study presented here, the unsaturated samples tested by Toll (1990) were classified in two different groups, each one including samples compacted at similar values of water content. The data shown in Figure 9 refer to unsaturated samples whose compaction water content was comprised between 24.9% and 27.7% whereas for the samples shown in Figure 10 the compaction water content ranged between 19.6% and 21.9%. The value of the compaction water content affects significantly the ensuing fabric of the unsaturated sample and this in turn has an effect on the mechanical response of the sample, including the response at critical state (as discussed by Wheeler and Sivakumar 2000). Hence, the purpose of such distinction was to select two homogenous groups of samples, which presented a similar soil fabric and whose mechanical response during laboratory testing could be, therefore, compared. From the examination of Figures 9 and 10, it can be observed that all critical state data points (each of them corresponding to different values of suction, net stress state and degree of saturation at critical state) follow a unique trend when plotted in the plane (e/e_s, ξ). In Figures 9 and 10 the expression of the curve interpolating the experimental data is given by equation (4) (the same expression used for the experimental data of Sharma (1998) and Sivakumar (1993) shown in Figures 6 and 7).

The samples compacted at a water content between 24.9% and 27.7% (Figure 9) had suction values ranging from 2 kPa to 73 kPa on reaching the critical state whereas the suction values at critical state for the samples compacted at water contents between 19.6% and 21.9% (Figure 10) varied between 22 kPa and 537 kPa. The proposed relationship, therefore, is shown to be capable of bringing together results from a very wide range of suction values.

FORMULATION OF THE ELASTO-PLASTIC STRESS-STRAIN MODEL

An elasto-plastic isotropic stress-strain model for unsaturated soils incorporating volumetric-hardening is described in this section. The success of the modelling ideas proposed here will be demonstrated in the next section by comparing the predictions with the experimental results from various types of laboratory tests all performed under isotropic loading. The development of the model is hence limited in this section to isotropic stress states in order to concentrate attention to the basic features of the model. Extension to more general stress states is quite straightforward following standard procedures (Gens, 1995).

The formulation of a constitutive model including volumetric-hardening requires the definition of:

1. a normal compression state surface, which relates the values of void ratio *e*, isotropic average skeleton stress p'' and bonding variable ξ during the irreversible behaviour of the soil;
2. an incremental expression, which relates the elastic part of the change of void ratio, *e* to the changes of the isotropic average skeleton stress, p'' and bonding variable, ξ.

The normal compression state surface is defined here as the product of two factors. The first factor is the equation of the saturated normal compression line relating the variation of the void ratio e_s to the change of the isotropic average skeleton stress p'' and the second factor is the equation that links the variation of the ratio e/e_s to the change of the bonding variable ξ. For the materials studied here, the analytical form of the normal compression state surface is therefore expressed as:

$$e\,(p'',\xi) = \frac{e}{e_s}(\xi)\; e_s(p'') \tag{5}$$

where $e(p'',\xi)$ is the normal compression state surface, $\frac{e}{e_s}(\xi)$ is given by equation (4) and $e_s(p'')$ is the saturated normal compression line (a straight line in the semi-logarithmic plane e-ln p'') having the form:

$$e_s(p'') = \mathrm{N} - \lambda \ln p'' \tag{6}$$

N and λ in equation (6) are the intercept (at p'' = 1 kPa) and the slope of the saturated normal compression line respectively. Note that, for saturated conditions, the isotropic average skeleton stress p'' coincides with the mean effective stress $\bar{p}$ and therefore the parameters N and λ are equal to those that identify the saturated normal compression line in the semi-logarithmic plane e-ln $\bar{p}$. Figure 11(a) shows three examples of normal compression lines that lie on the normal compression state surface and correspond to constant values of the bonding variable ξ. Equations (5) and (6) indicate that the normal compression lines at constant ξ are straight lines in the semi-logarithmic plane e-ln p'' and that their extrapolations at high values of p'' intersect each other at the same point coinciding with a value of the void ratio e_s on the saturated normal compression line equal to zero.

The elastic change of void ratio, Δe^e is assumed equal to:

$$\Delta e^e = -\kappa \ln \frac{p''_f}{p''_i} \tag{7}$$

where κ is the elastic swelling index and p_i'' to p_f'' are the initial and final value of the isotropic average soil skeleton stress respectively. Equation (7) implies that the elastic change of void ratio depends exclusively on the change of the isotropic average skeleton stress p'' (i.e. it is independent of the variation of the bonding variable ξ). This is equivalent to assume that the elastic deformation of the soil skeleton is not affected by the bonding action that the water menisci exert at the inter-particle contacts. It will be shown in the next section that this assumption fits well the elastic behaviour of the laboratory tests considered in this work.

The normal compression state surface defined by equation (5) acts as a limiting surface in the (e, p'', ξ)-space, where it separates the region of the attainable soil states from the region of the non-attainable soil states. The soil response is elastic while the soil follows a path inside the space of the attainable soil states. When the soil path reaches the normal compression state surface, this surface imposes a constraint on further changes of e, p'' and ξ and the soil state can therefore either move back inside the space of the attainable soil states or alternatively follow a path lying on the normal compression state surface. When the latter possibility occurs, irreversible (elasto-plastic) changes of void ratio develop.

Thus the normal compression state surface and the elastic law introduced above (equations (5) and (7) respectively) implicitly define a yield locus that incorporates a volumetric hardening rule. To obtain the analytical form of such yield locus consider the elastic stress path in Figure 11(a) starting from the soil state denoted by 1, at a isotropic average skeleton stress $p_o''(0)$ on the saturated normal compression line, and moving to the soil state denoted by 2, at a isotropic average skeleton stress $p_o''(\xi_1)$ on the unsaturated normal compression line corresponding to $\xi=\xi_1$. The change of void ratio during the path from state 1 to state 2 is computed according to the elastic equation (7):

$$\Delta e = -\kappa \ln \frac{p_o''(\xi_1)}{p_o''(0)} \qquad (8)$$

As the soil states 1 and 2 also belong to the normal compression state surface, they must lie on the same yield locus and an alternative expression for the variation of void ratio during the path from state 1 to state 2 can therefore be obtained by using the normal consolidation state surface of equation (5):

$$\Delta e = e\,(p''(0),0) - e\,(p''(\xi_1),\xi_1) = \mathrm{N} - \lambda \ln p_o''(0) - \frac{e}{e_s}(\xi_1)\left[\mathrm{N} - \lambda \ln p_o''(\xi_1)\right] \qquad (9)$$

By equating equation (8) and equation (9) and then rearranging, the following equation of the yield locus in the isotropic plane ξ-ln p'' is obtained:

$$\ln p_o''(\xi_1) = \frac{\lambda - \kappa}{\frac{e}{e_s}(\xi)\,\lambda - \kappa} \ln p_o''(0) + \frac{\left(\frac{e}{e_s}(\xi) - 1\right)(1+\mathrm{N})}{\frac{e}{e_s}(\xi)\,\lambda - \kappa} \qquad (10)$$

Figure 11(b) shows the yield locus of equation (10) in the isotropic plane ξ-ln p'' together with the two yield points corresponding to the soil states 1 and 2, which are identified by the coordinates ($p_o''(0)$, 0) and ($p_o''(\xi_1)$, ξ_1) respectively.

Figure 11(b) also shows an expanded yield locus, indicated by the broken line, which refers to a soil sample that has experienced additional plastic volumetric strains and whose yield locus has therefore undergone volumetric hardening. The current size of the yield locus is identified by the value of its intercept $p_o''(0)$ with the horizontal axis, which is the yield value of the isotropic average skeleton stress during isotropic compression of a saturated sample. The saturated yield stress $p_o''(0)$ can therefore be assumed as the hardening parameter of the present elasto-plastic model. The irreversible change of void ratio Δe^p associated to the expansion of the yield locus from an initial position identified by $p_o''(0)= p_o''(0)_i$ to a final position identified $p_o''(0)= p_o''(0)_f$ coincides then with the irreversible change of void ratio calculated by the saturated normal compression line for a variation of the isotropic average skeleton stress from $p_o''(0)_i$ to $p_o''(0)_f$:

$$\Delta e^p = -(\lambda - \kappa)\ln\frac{p_o''(0)_f}{p_o''(0)_i} \tag{11}$$

and equation (11) represents thus the volumetric hardening rule of the proposed elasto-plastic model.

The complete model includes a formulation to compute the degree of saturation that must incorporate the effect of hydraulic hysteresis and stress-induced changes of soil fabric. The relationships proposed by Vaunat et al. (2000) and Gallipoli et al. (2002) can be used for this purpose but a detailed description of this component of the model is outside the scope of the paper. For the model computations presented in the next Section, the experimentally observed degrees of saturation have been used. In this way the differences between predictions and observations must be attributed exclusively to the mechanical elasto-plastic model.

MODEL PREDICTIONS

The good performance of the proposed elasto-plastic model is demonstrated here by comparing the results from a selection of experiments performed by Sharma (1998) on a compacted mixture of Bentonite and Kaolin with the corresponding model predictions. In particular, the comparison will show the potential of the proposed model for correctly predicting:

- The initial yield locus of the soil corresponding to the after-compaction state.
- The irreversible change of void ratio occurring during wetting (collapse).

- The irreversible change of void ratio during drying.
- The dependency of the soil response during isotropic virgin loading at constant suction on the previous history of suction variation.

The last two points refer to typical features of unsaturated soil behaviour that are not taken into account by existing elasto-plastic constitutive frameworks formulated in terms of a single yield surface.

The selection process of the model parameter values used for the predictions (see Table 1) has been described in Section 3, except for the value of the elastic swelling index κ that was selected on the basis of elastic isotropic loading-unloading cycles at constant suction. Figures 12, 13 and 14 show the comparison between experimental and predicted behaviour for three isotropic loading tests at constant suction (100 kPa, 200 kPa and 300 kPa respectively) that involve elasto-plastic yielding. Inspection of Figures 12, 13 and 14 reveals that the proposed model correctly calculates the respective yield points by assuming for all three test simulations the same initial yield locus associated to a value of the hardening parameter $p_o''(0)$=17 kPa. Such model prediction is corroborated by the soil response observed by Sharma (1998) during the equalization stage prior to loading, when the suction of the three samples was decreased from the value after compaction to 100 kPa, 200 kPa and 300 kPa respectively. During this stage all three samples experienced exclusively elastic swelling, which indicates that the initial yield curve after compaction had not undergone further expansion associated to plastic volumetric compression (collapse). It is then expected that all three samples would yield on the same locus during isotropic loading and the proposed model indeed correctly predicts this. Therefore, for the test simulations presented in the remainder of this section, the value of the hardening parameter corresponding to the soil after compaction was assumed equal to 17 kPa.

Figure 13 also shows the comparison between model predictions and experimental results for an elastic loading-unloading cycle, which confirms the adequacy of the elastic law given by equation (7). The very good match between experimental and predicted values of void ratio at the beginning of the loading in Figures 12, 13 and 14 provides a further proof of the validity of equation (7) because the predicted initial values of void ratio are computed by means of an elastic path starting from the saturated yield stress state ($p_o''(0)$=17 kPa) according to the following expression:

$$e_i = e_o - \kappa \ln \frac{p_i''}{17\ \text{kPa}} \tag{12}$$

where e_o is the void ratio predicted by the saturated normal compression line for p''= 17 kPa and p_i'' is the isotropic average skeleton stress of the unsaturated sample at the beginning of loading.

Figure 15 shows the comparison between the experimental and predicted behaviour for a wetting-drying cycle performed at constant isotropic net stress of 50 kPa. Inspection of the stress path followed by the soil during the test in Figure 15(b) reveals that the model predicts irreversible changes of void ratio during both the wetting and the drying branch of the test. Yielding of the soil occurs initially during wetting and the consequent development of elasto-plastic strains produces the first expansion of the yield locus from its initial position to the position indicated by (A) (corresponding to the end of wetting). After the reversal of suction the model continues to predict elasto-plastic deformations during the whole drying and this corresponds to a further expansion of the yield locus from the position (A) to the final position (B). This test simulation clearly demonstrates the potential of the present framework to interpret the elasto-plastic volumetric strains that occur during both the wetting and the drying phases as a single mechanical phenomenon that can be modelled by employing only one yield locus. Part of the discrepancy between experimental results and model prediction in Figure 15(a) is due to the incomplete equalisation of suction within the sample during the test. The occurrence of incomplete equalisation is proven by the experimental observation, reported by Sharma (1998), that a significant amount of water flowed into the sample during the stabilisation period at the end of the wetting stage, when the sample was kept at constant suction of 100 kPa for a period of time necessary to equalize suction before subsequent drying (the vertical change of void ratio at constant suction of 100 kPa shown in Figure 15(a) correspond to this stabilisation period). However, despite such experimental limitation, the inspection of Figure 15(a) still indicates a satisfactory agreement between predicted and computed results.

Now the more complex stress paths involving wetting/drying cycles are considered. Figure 16 shows the comparison between the experimental and predicted behaviour during wetting-drying cycles performed at a constant isotropic net stress of 10 kPa. Inspection of Figure 16(b) indicates that the occurrence of elasto-plastic strains occur exclusively during the drying branches whereas elastic swelling takes places during the wetting phases. The irreversible strains generated by the first drying produce an expansion of the yield locus from the initial position to position (A) whereas the second drying originates a further expansion from position (A) to position (B). Note that, as explained above, the discontinuity in the slope of the wetting paths shown in Figure 16(b) is due to the stabilisation phase following incomplete suction equalisation during previous wetting.

The model predictions in Figures 15 and 16 represent a significant improvement over existing elasto-plastic models based on a single yield locus, which would incorrectly predict elastic compression during all the drying phases of the above tests.

Finally, Figure 17 shows the comparison between experimental and predicted behaviour for two constant suction isotropic tests that show different mechanical responses depending on whether the sample has undergone a wetting-drying cycle prior to loading or not. Inspection of Figure 17 reveals that the model is capable of capturing the different stiffness shown by the soil during virgin loading in two cases. This is a significant advance with respect to existing models that would predict instead the same slope of the normal compression lines for both cases regardless of the previous history of suction variation. A further improvement with respect to existing models is that the present framework correctly predicts different values of void ratio at the beginning of loading in the two cases of Figure 15(a). This is due to the irreversible change of void ratio that occurs during drying of the sample subjected to the wetting-drying cycle (see Figure 15(b)) and it also reflects in the different degree of expansion of the yield locus achieved at the end of test in the two cases (position (A) and position (B) respectively).

CONCLUSIONS

The paper proposes an innovative constitutive framework for unsaturated soil that is able to explain the various mechanical features of this material by resorting to a physical description of the different effects of suction on soil straining. In the assumed mechanism, the relative slippage of soil particles is governed by two counteracting actions exerted on the assemblage of soil particles: a) the perturbing action of the average stress state acting on the soil skeleton and b) the stabilizing action of the normal force exerted at the inter-particle contacts by water menisci. The variables controlling each one of these actions (i.e. the average skeleton stress variable $\boldsymbol{\sigma}''$ and the bonding variable ξ respectively) are defined on the basis of the current values of the net stress state, suction and degree of saturation. The introduction of degree of saturation in the definition of the soil constitutive variables is essential to properly represent the contribution of soil suction to the two effects described above.

Based on a physical argument, the present proposal assumes that, during the elasto-plastic loading of a soil element, the proportion e/e_s between the void ratio e under unsaturated conditions and the void ratio e_s under saturated condition at the same average skeleton stress state, is a unique function of the bonding variable ξ. This fundamental assumption is successfully validated in this work by the analysis of several published sets of experimental data for different materials. The analysis of one set of data (for which both isotropic and shearing tests are available) also suggests that the relationship between e/e_s and ξ is unique for a given soil and it is independent of the applied stress ratio.

On the basis of this assumption, a full elasto-plastic stress-strain model for isotropic stress states is formulated and its good performance is demonstrated by the comparison between predicted and laboratory tests results from a comprehensive experimental programme including a wide variety of different stress paths. This comparison confirms the potential of the proposed model for correctly predicting the most important features of the mechanical behaviour of unsaturated soils by retaining at the same time the simplicity of a model formulated in terms of single yield curve. In particular it is able to predict correctly the following two typical responses of unsaturated soils that are not modelled by existing elasto-plastic constitutive frameworks based on a single yield surface:

1. the irreversible change of void ratio during drying;
2. the dependency of the response during virgin compression at constant suction on the previous history of suction variation.

An additional significant advantage is that a reduced number of laboratory tests are necessary for calibrating the proposed model. In particular, the relationship between e/e_s and ξ is the only additional information required for the unsaturated soil behaviour (apart from the parameter values for the saturated model). To define the relationship between e/e_s and ξ, it is possible to choose among alternative testing options that involve irreversible straining of the soil such as virgin loading at constant suction, undrained virgin loading and wetting-drying at constant applied stress.

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REFERENCES

Alonso, E.E., Gens, A. & Josa, A. 1990. A constitutive model for partially saturated soils. *Géotechnique* 40(3): 405-430.

Alonso, E.E., Lloret, A., Gens, A. & Yang, D.Q. 1995. Experimental behaviour of highly expansive double-structure clay. In *Proceedings 1st International Conference on Unsaturated Soils, Paris* 1: 11-16.

Bishop, A.W. 1959. The principle of effective stress. *Teknisk Ukeblad* 106(39): 859-863.

Bolzon, G., Schrefler, B.A. & Zienkiewicz, O.C. 1996. Elastoplastic soil constitutive laws generalized to partially saturated states. *Géotechnique* 46(2): 279-289.

Cui, Y.J. & Delage, P. 1996. Yielding and plastic behaviour of an unsaturated compacted silt. *Géotechnique* 46(3): 405-430.

Fisher, R.A. 1926. On the capillary forces in an ideal soil; correction of formulae given by W.B. Haines. *Journal Agric. Science* 16(3): 492-505.

Gallipoli, D., Wheeler, S.J. & Karstunen, M. 2002. Modelling the variation of degree of saturation in a deformable unsaturated soil. *Submitted to the Géotechnique Symposium in Print on Suction in Unsaturated Soils.*

Gens, A. 1995. Constitutive laws. *Modern issues in non-saturated soils, Springer Verlag*, Wien, 129-158.

Gens, A. & Alonso, E.E. 1992. A framework for the behaviour of unsaturated expansive clays. *Canadian Geotechnical Journal* 29: 1013-1032.

Haines, W.B. 1925. A note on the cohesion developed by capillary forces in an ideal soil. *Journal Agric. Science* 15(4): 529-535.

Jennings, J.E. & Burland, J.B. 1962. Limitation to the use of effective stress in partly saturated soils. *Géotechnique* 12(2): 125-144.

Jommi, C. 2000. Remarks on the constitutive modelling of unsaturated soils. In *Proceedings International Workshop on Unsaturated Soils, Trento*: 139-153.

Jommi, C. & Di Prisco, C. 1994. Un semplice approccio teorico per la modellazione del comportamento meccanico dei terreni granulari parzialmente saturi (in Italian). In *Atti Convegno sul Tema: Il Ruolo deiFluidi nei Problemi di Ingegneria Geotecnica, Mondoví*: 167-188.

Karube, D. & Kato, S. 1994. An ideal unsaturated soil and the Bishop's soil. *Proceedings 13th International Conference on Soil Mechanics and Foundation Engineering, New Delhi,* 1: 43-46.

Karube, D. & Kawai, K. 2001. The role of pore water in the mechanical behavior of unsaturated soils. *Geotechnical and Geological Engineering* 19: 211-241.

Loret, B. & Khalili, N. 2000. A three phase model for unsaturated soils. *International Journal of Numerical and Analytical Methods in Geomechanics* 24: 893-927.

Sharma, R. S. 1998. *Mechanical Behaviour of Unsaturated Highly Expansive Clays*. Ph.D. thesis: University of Oxford, UK.

Sivakumar, V. 1993. *A Critical State Framework for Unsaturated Soil*. Ph.D. thesis: University of Sheffield, UK.

Terzaghi, K. 1936. The shearing resistance of saturated soils and the angle between the planes of shear. In *Proceedings 1st International Conference on Soil Mechanics and Foundation Engineering, Camb., Mass.* 1: 54-56.

Toll, D.G. 1990. A framework for unsaturated soil behaviour. *Géotechnique* 40(1): 31-44.

Vaunat, J., Romero E. & Jommi, C. 2000. An elastoplastic hydro-mechanical model for unsaturated soils. In *Proceedings International Workshop on Unsaturated Soils, Trento*: 121-138.

Wheeler, S.J. & Karube, D. 1995. State of the art report. Constitutive modeling. In *Proceedings 1st International Conference on Unsaturated Soils, Paris*: 1323-1356.

Wheeler, S.J. & Sivakumar, V. 1995. An elasto-plastic critical state framework for unsaturated soil. *Géotechnique* 45(1): 35-53.

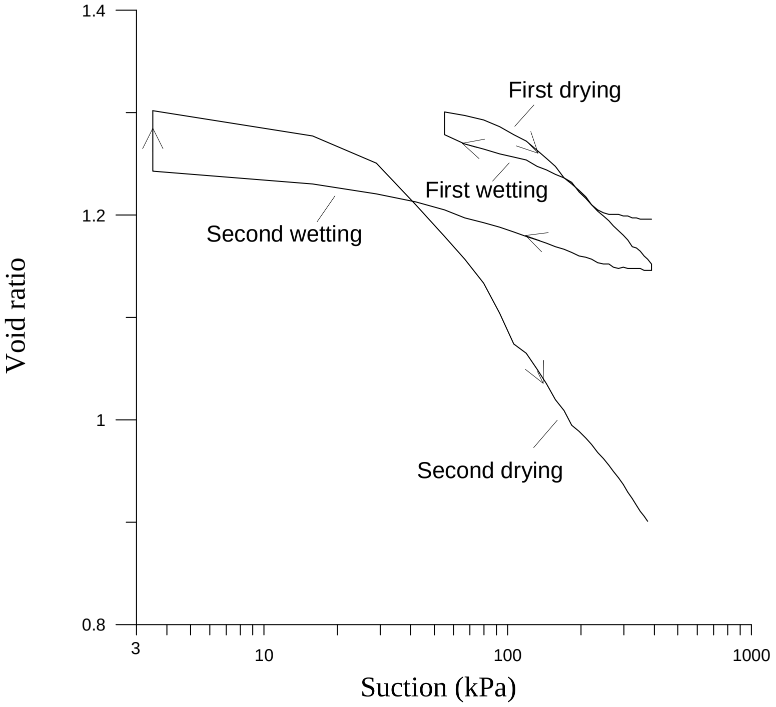
Wheeler, S.J. & Sivakumar, V. 2000. Influence of compaction procedure on the mechanical behaviour of an unsaturated compacted clay. Part2: shearing and constitutive modelling. *Géotechnique* 50(4): 369-376.

TABLE

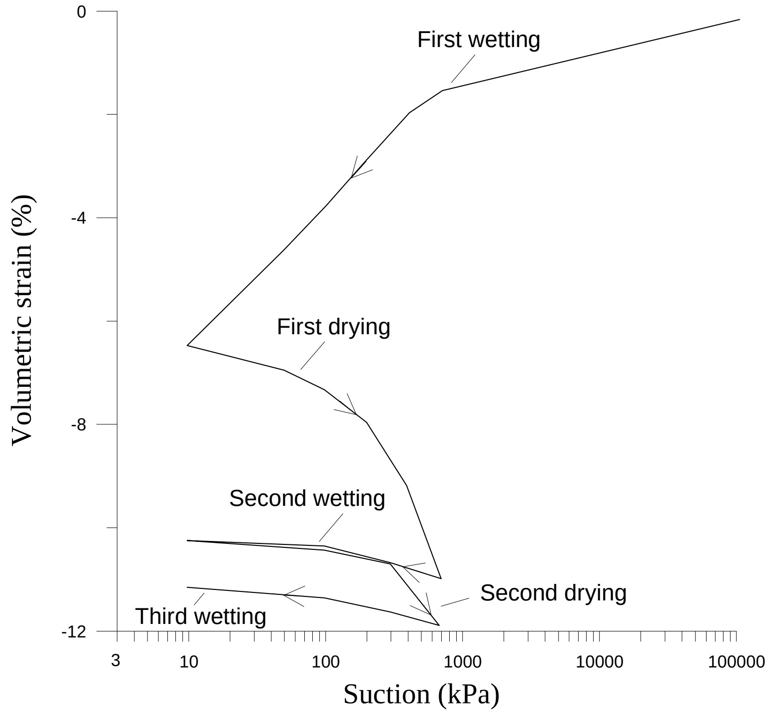
Parameter	Value
λ, slope of NCL at s=0	0.144
N, e on NCL at s=0 for p''=1 kPa	1.759
κ; swelling index for changes of p''	0.040
a, parameter of model equation (4)	0.369
b, parameter of model equation (4)	1.419

Table 1. Parameter values for the proposed elasto-plastic model.

FIGURES

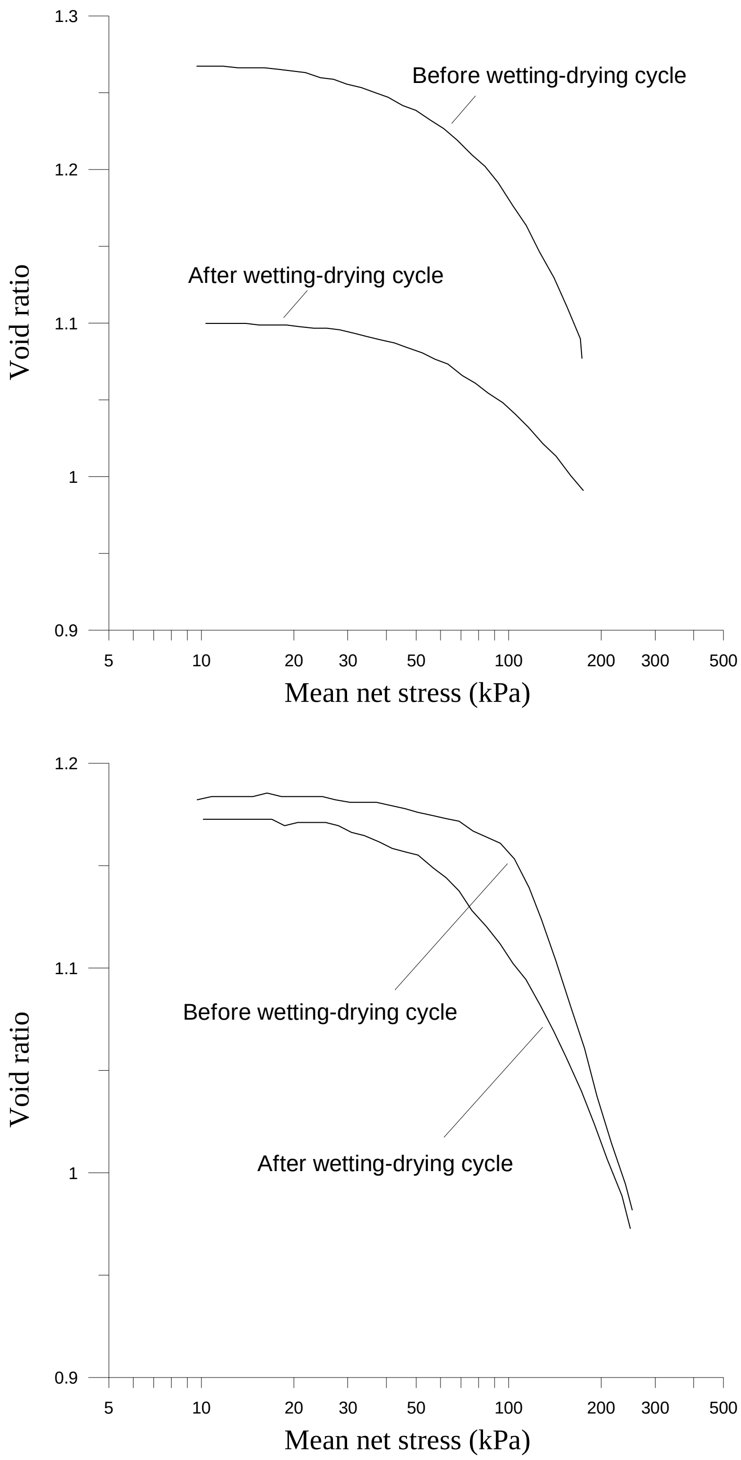


(a)



(b)

Figure 1. Wetting-drying cycle on: (a) compacted mixture of Bentonite and Kaolin under constant mean net stress of 10 kPa (after Sharma 1998), (b) compacted Boom clay under oedometric conditions at constant vertical stress of 400 kPa (after Alonso et al. 1995).



(a)

(b)

Figure 2. Isotropic virgin loading of compacted mixture of Bentonite and Kaolin at: (a) constant suction of 300 kPa (after Sharma 1998), (b) constant suction of 200 kPa (after Sharma 1998).

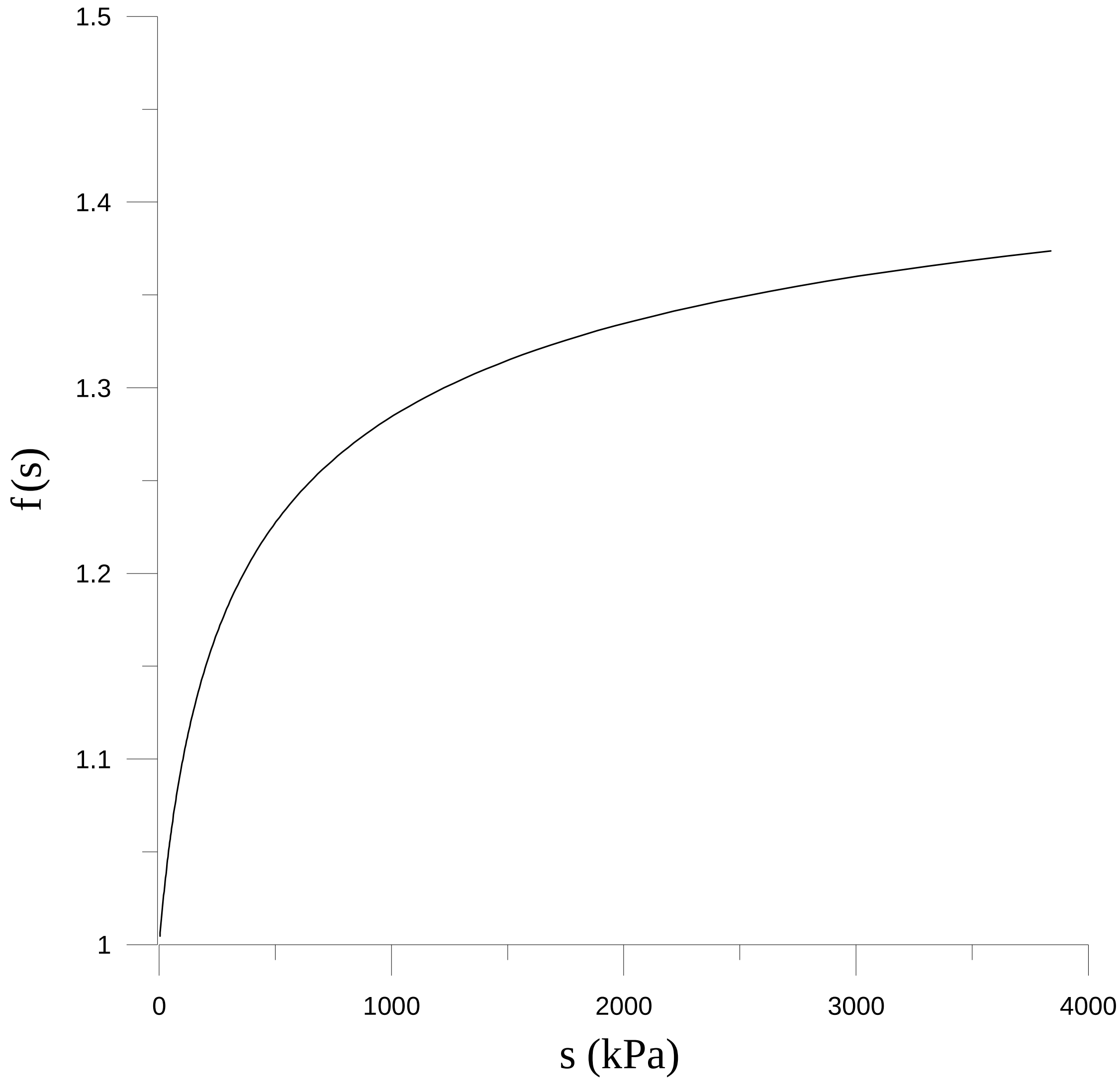


Figure 3. Ratio between inter-particle forces at suction, *s* and at null suction due to a water meniscus located at the contact between two identical spheres (analytical solution by Fisher 1926).

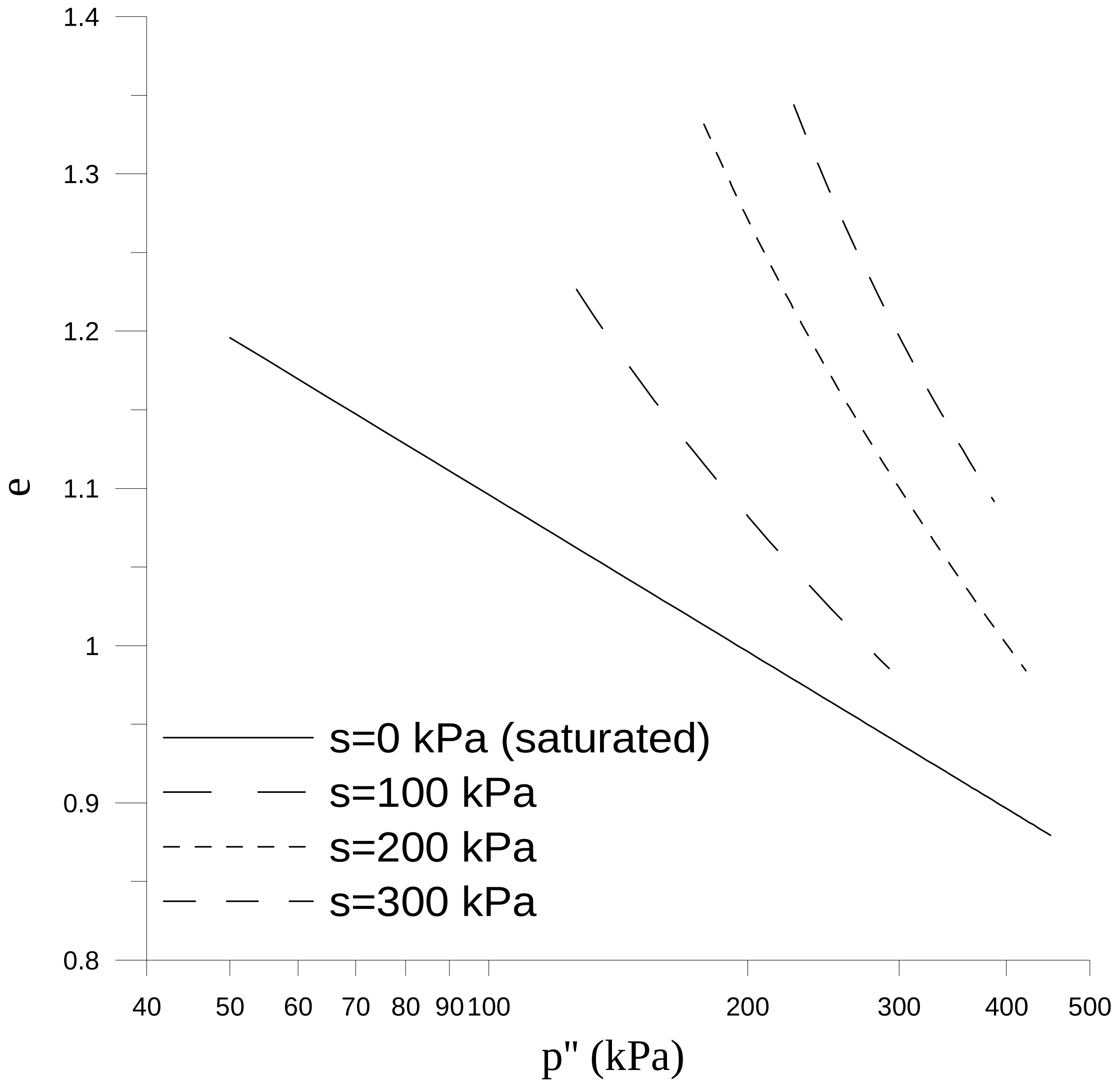


Figure 4. Normal compression lines at constant suction in the plane *e*-ln *p´´* (data by Sharma 1998).

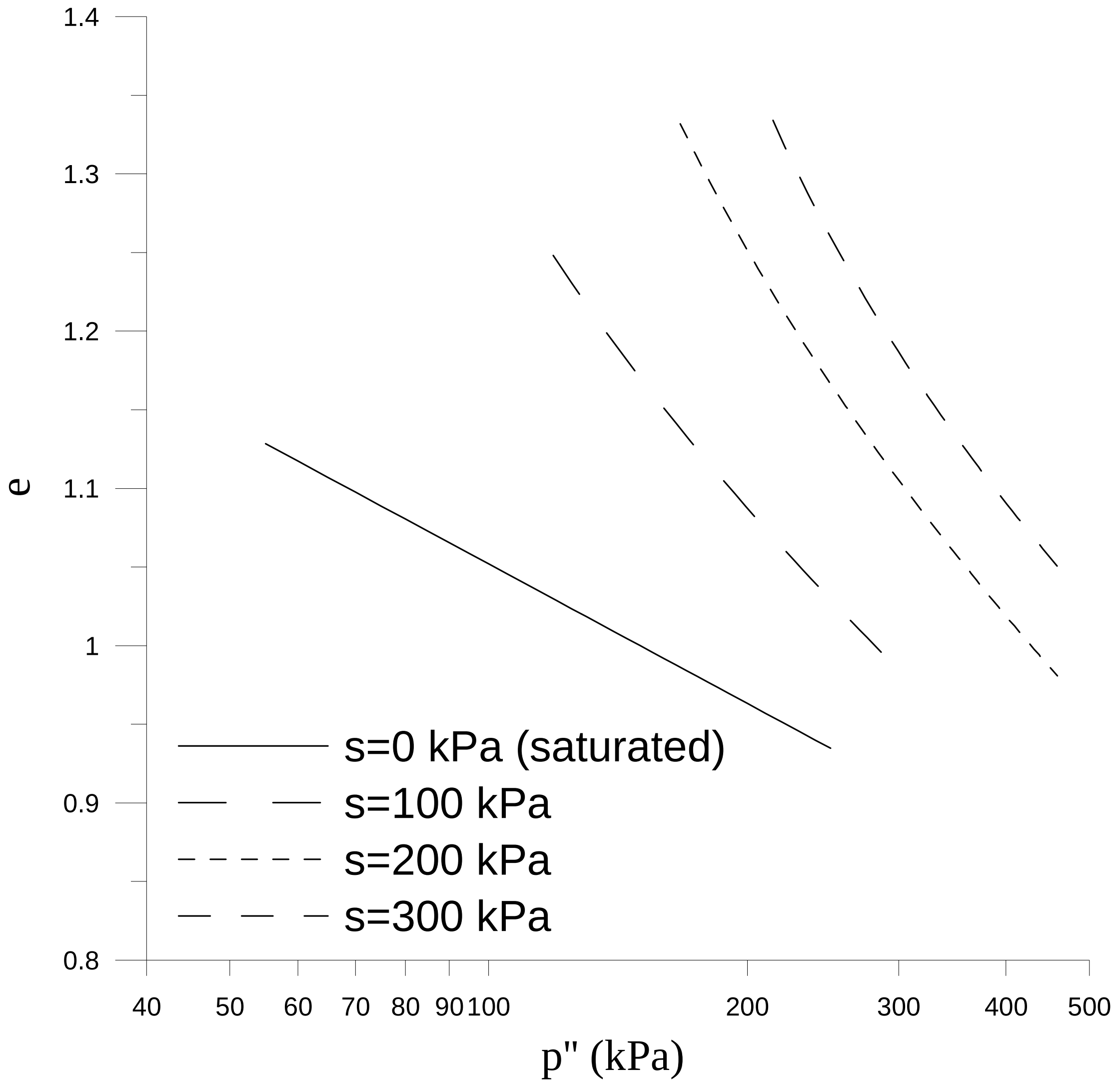


Figure 5. Normal compression lines at constant suction in the plane *e*-ln *p´´* (data by Sivakumar 1993).

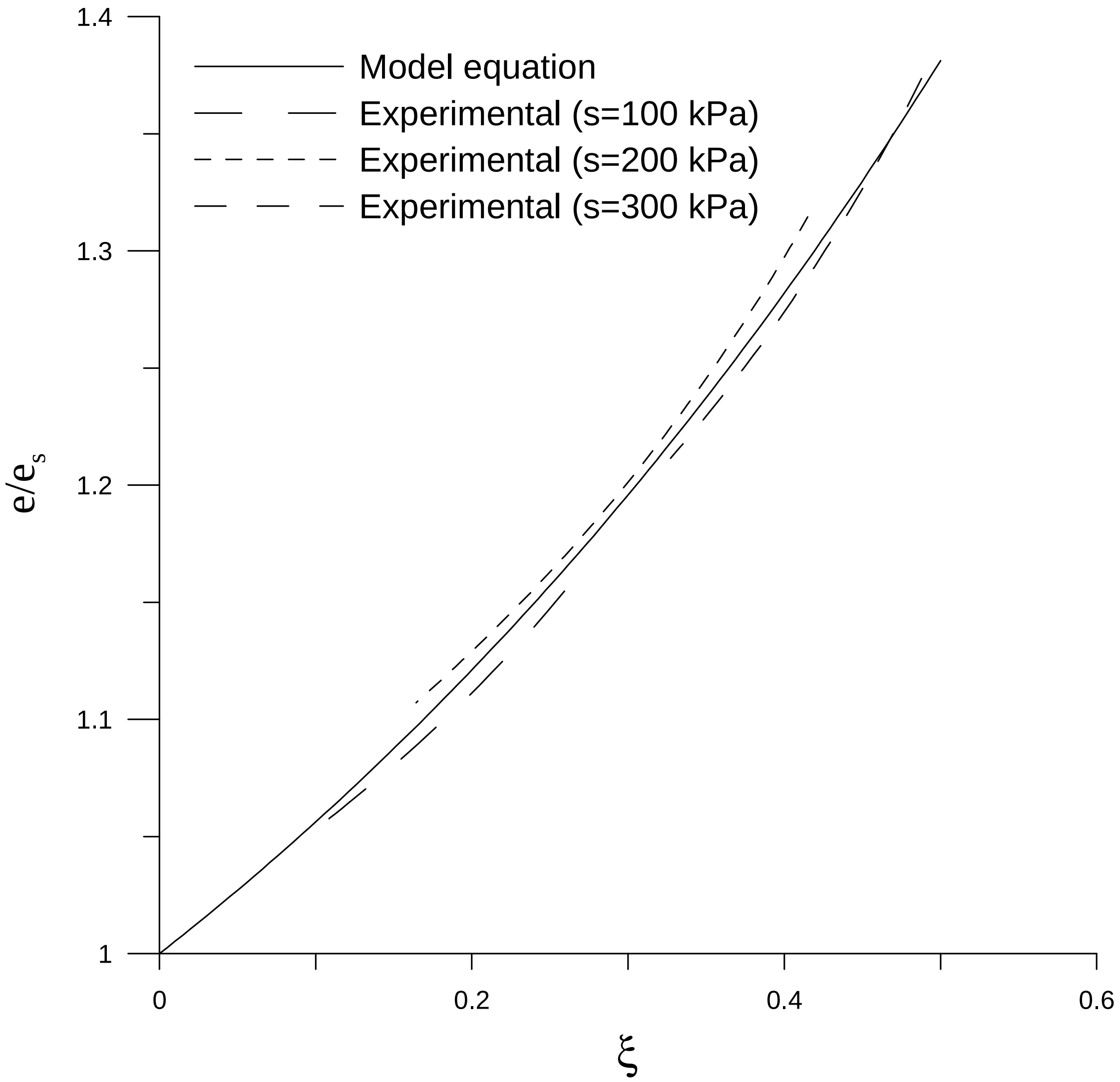


Figure 6. Relationship between ratio e/e_s and bonding factor ξ during isotropic virgin loading at constant suction (data by Sharma 1998).

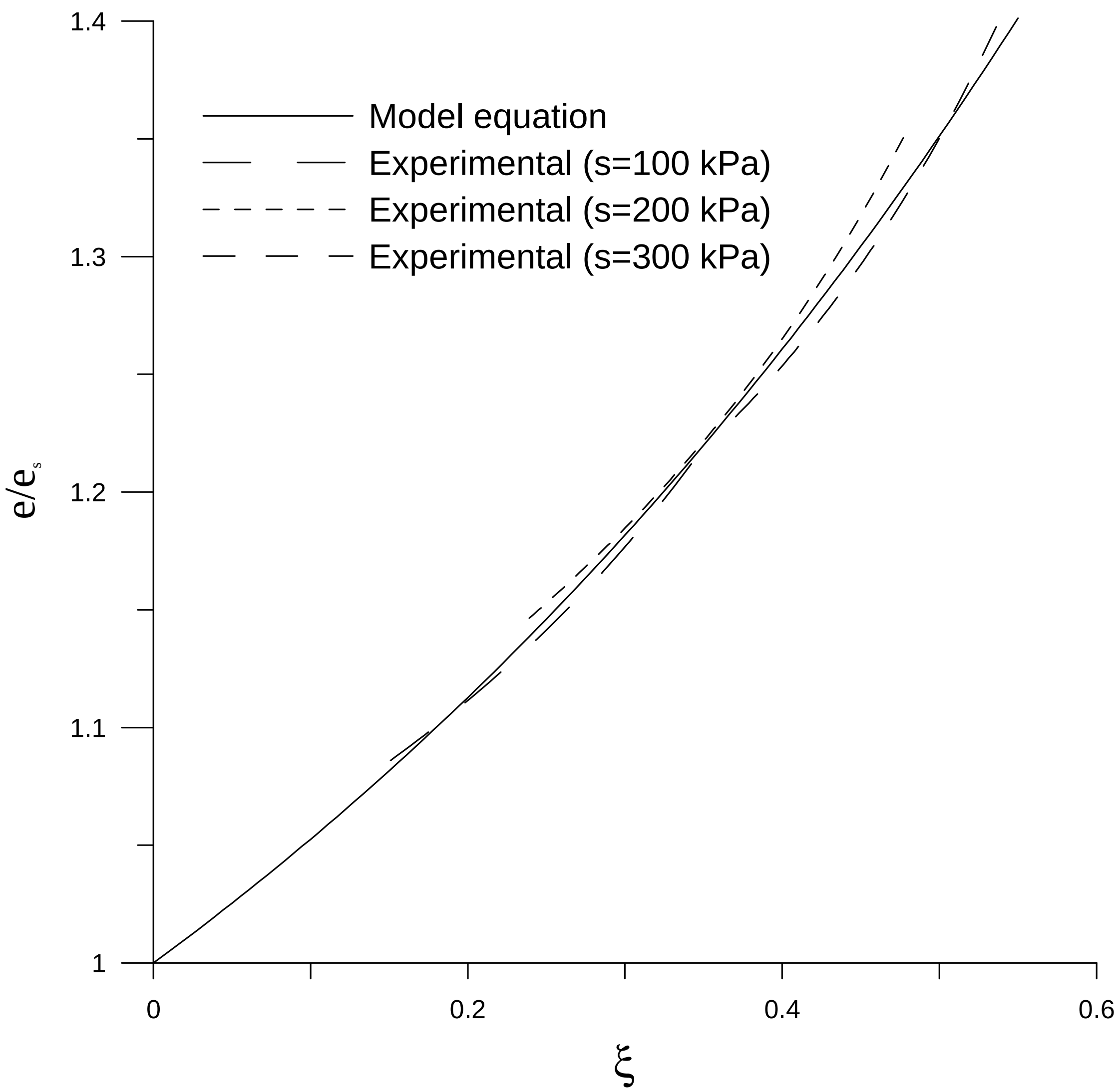


Figure 7. Relationship between ratio e/e_s and bonding factor ξ during isotropic virgin loading at constant suction (data by Sivakumar 1993).

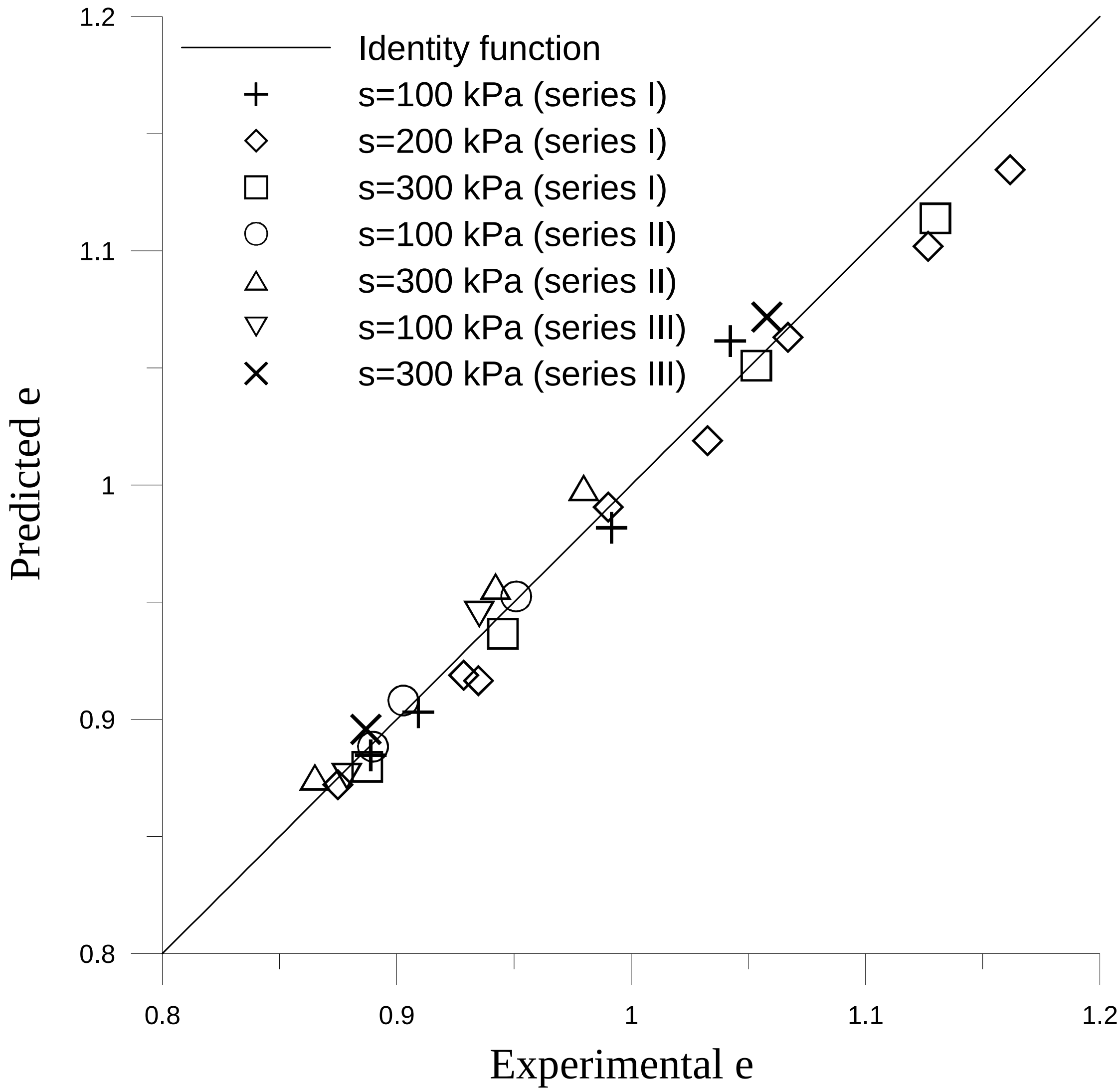


Figure 8. Comparison between experimental and predicted void ratio at critical state (data by Sivakumar 1993 and by Wheeler & Sivakumar 2000).

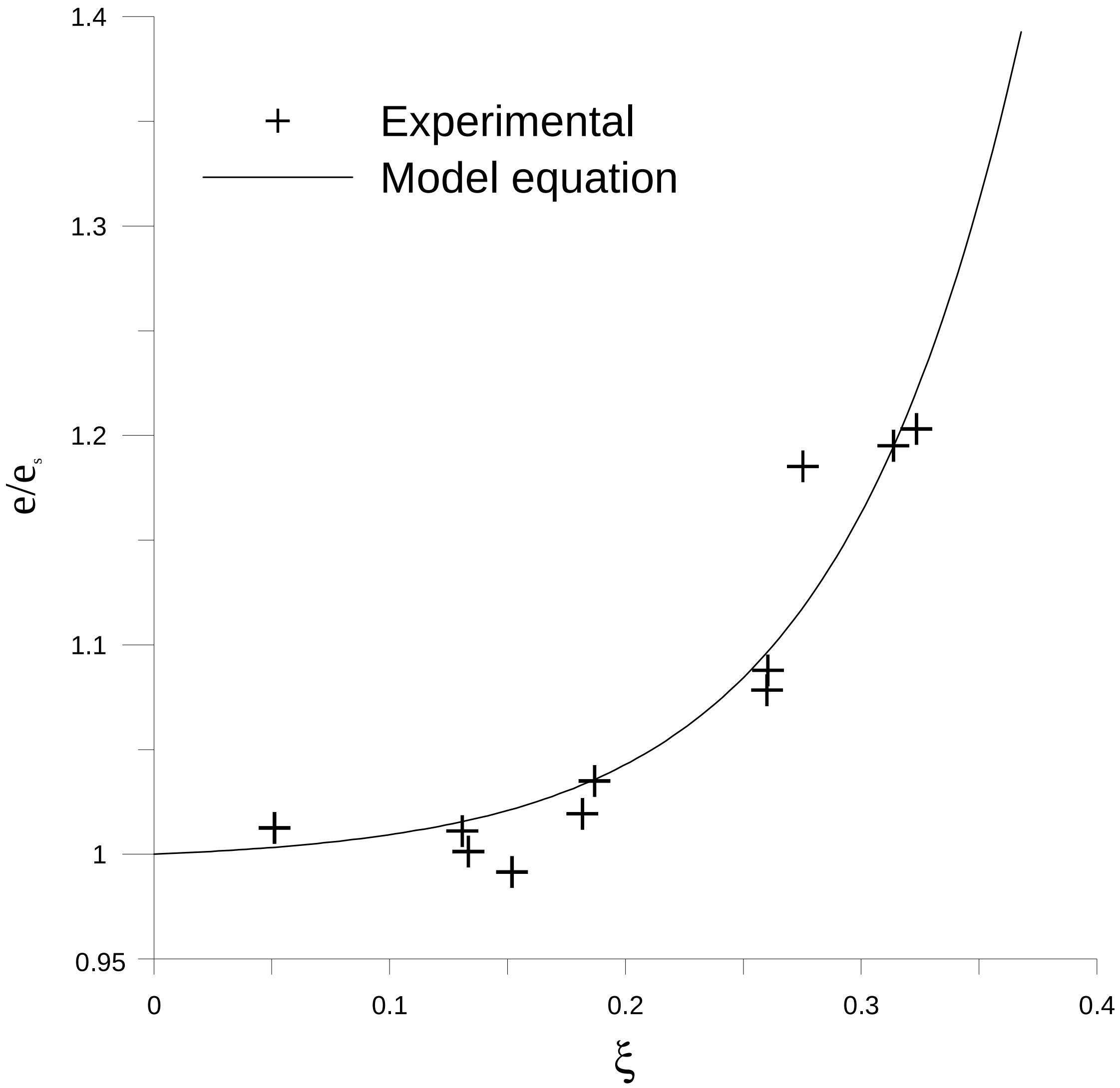


Figure 9. Relationship between ratio e/e_s and bonding factor ξ at critical state for soil samples compacted at water content between 24.9% and 27.7%. The suctions at critical state range from 2 kPa to 73 kPa (data by Toll 1990).

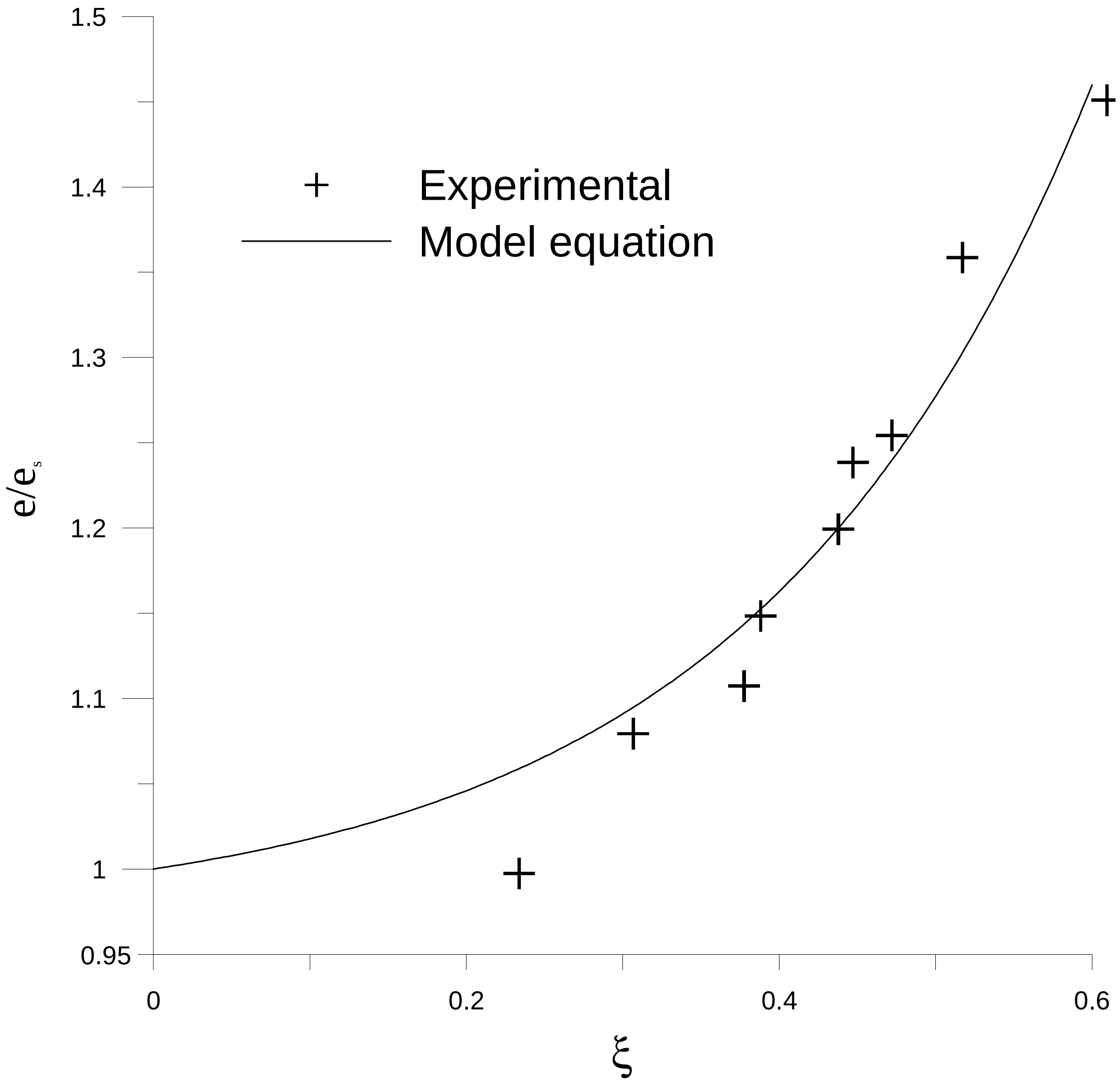


Figure 10. Relationship between ratio e/e_s and bonding factor ξ at critical state for soil samples compacted at water content between 19.6% and 21.9%. The suctions at critical state range from 22 kPa to 537 kPa (data by Toll 1990).

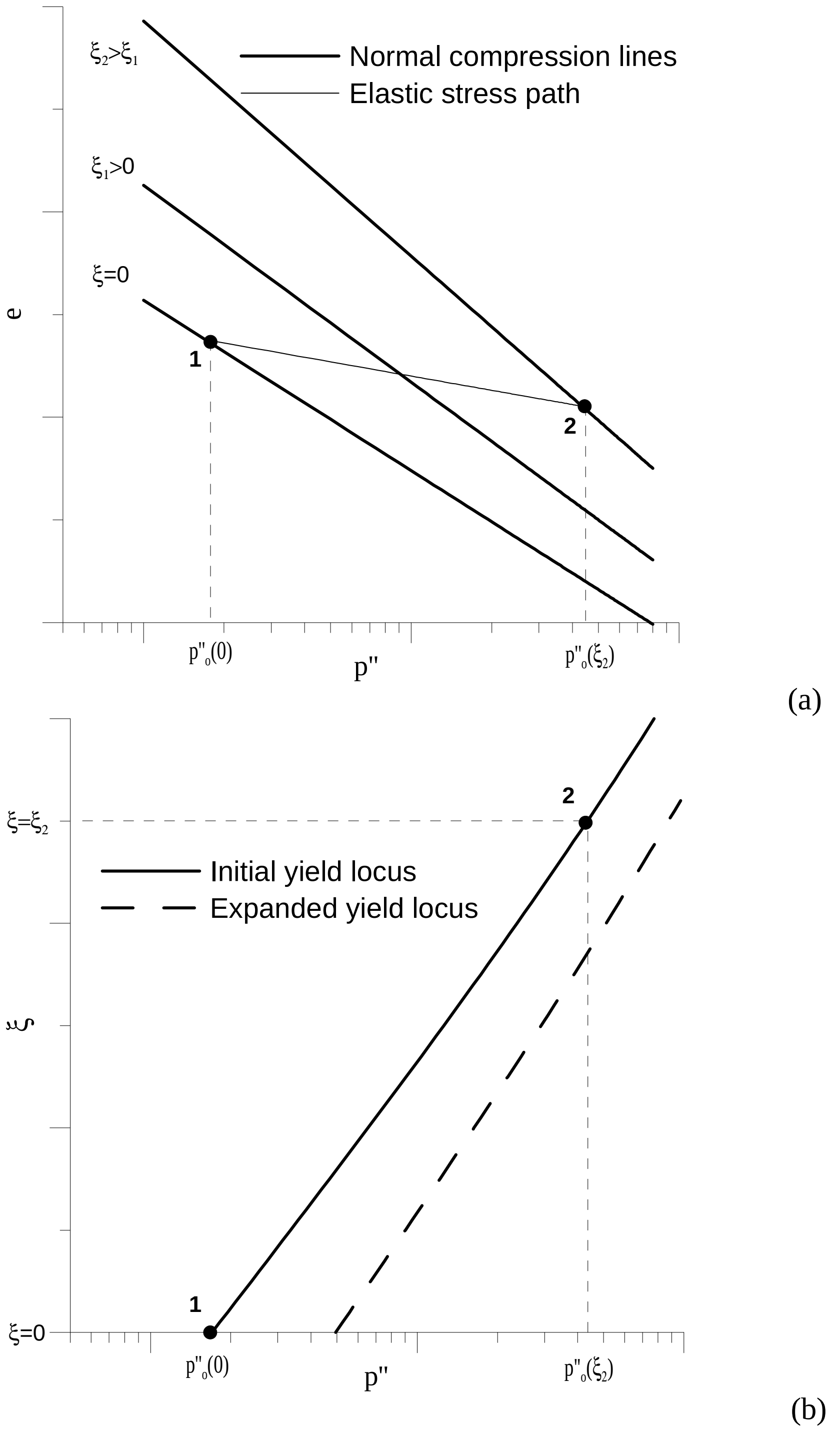


Figure 11. Derivation of the yield locus in the isotropic plane.

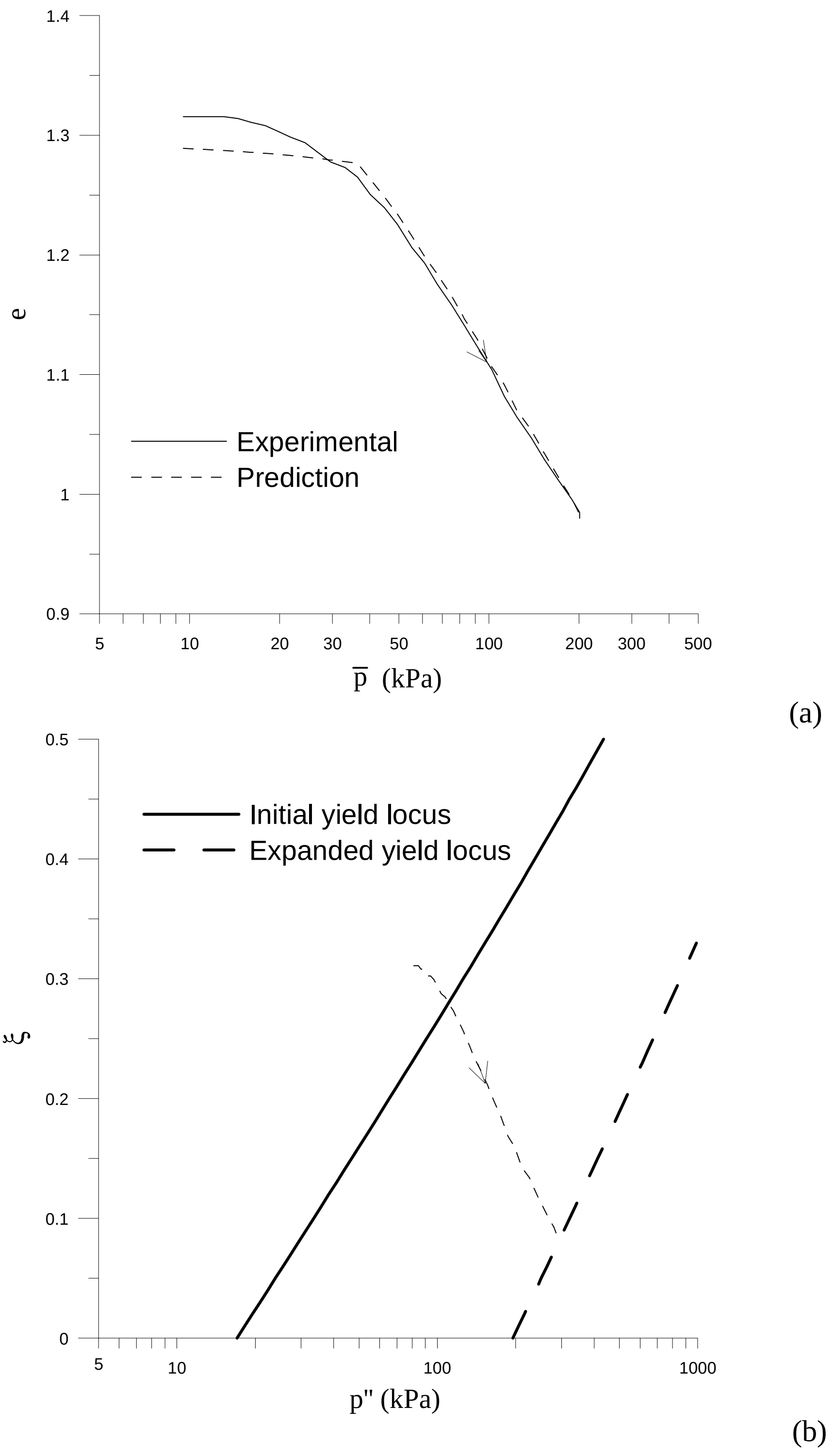


Figure 12. Model prediction for isotropic virgin loading at constant suction of 100 kPa: (a) change of void ratio, (b) stress path (experimental data by Sharma 1998).

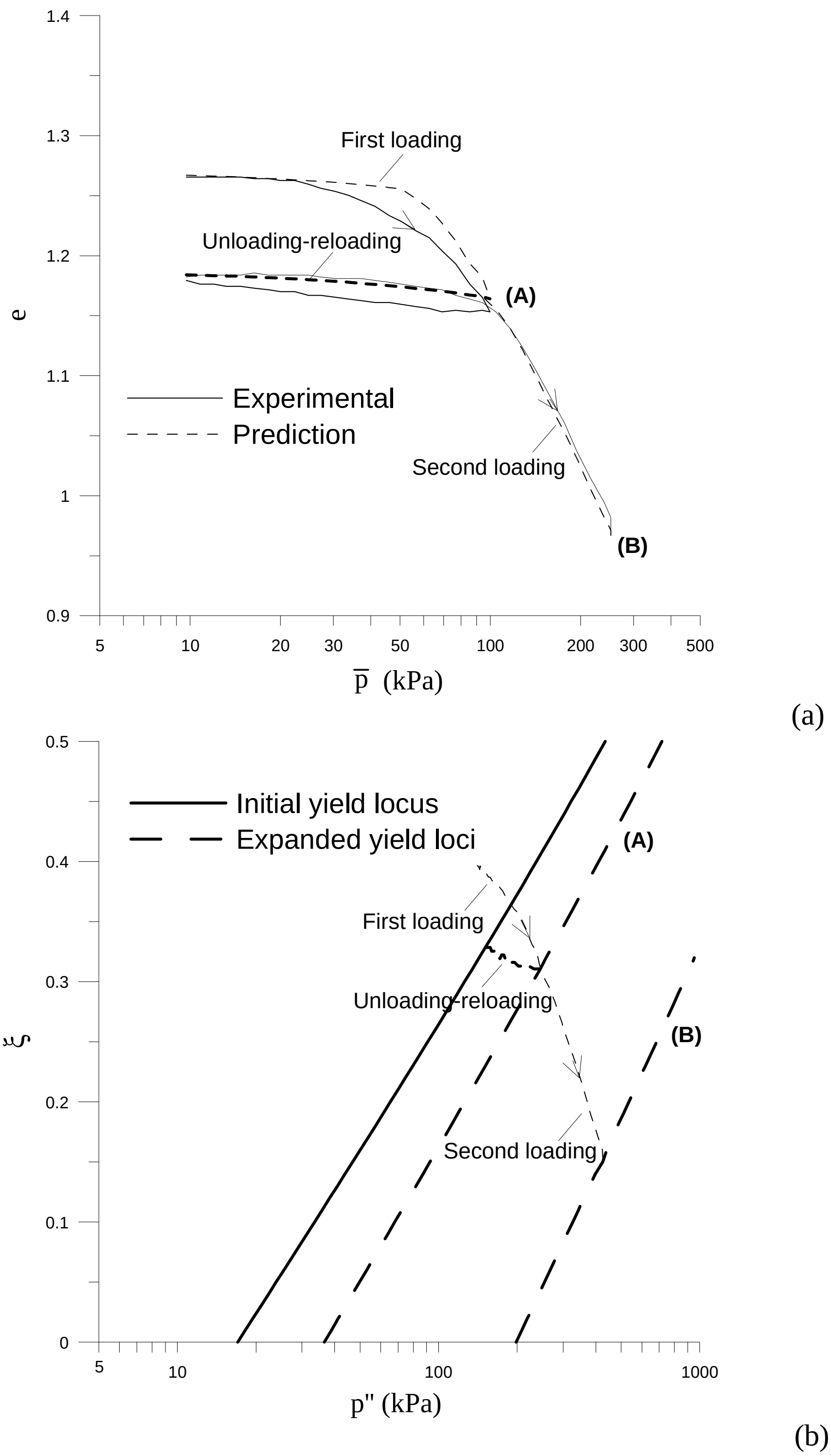
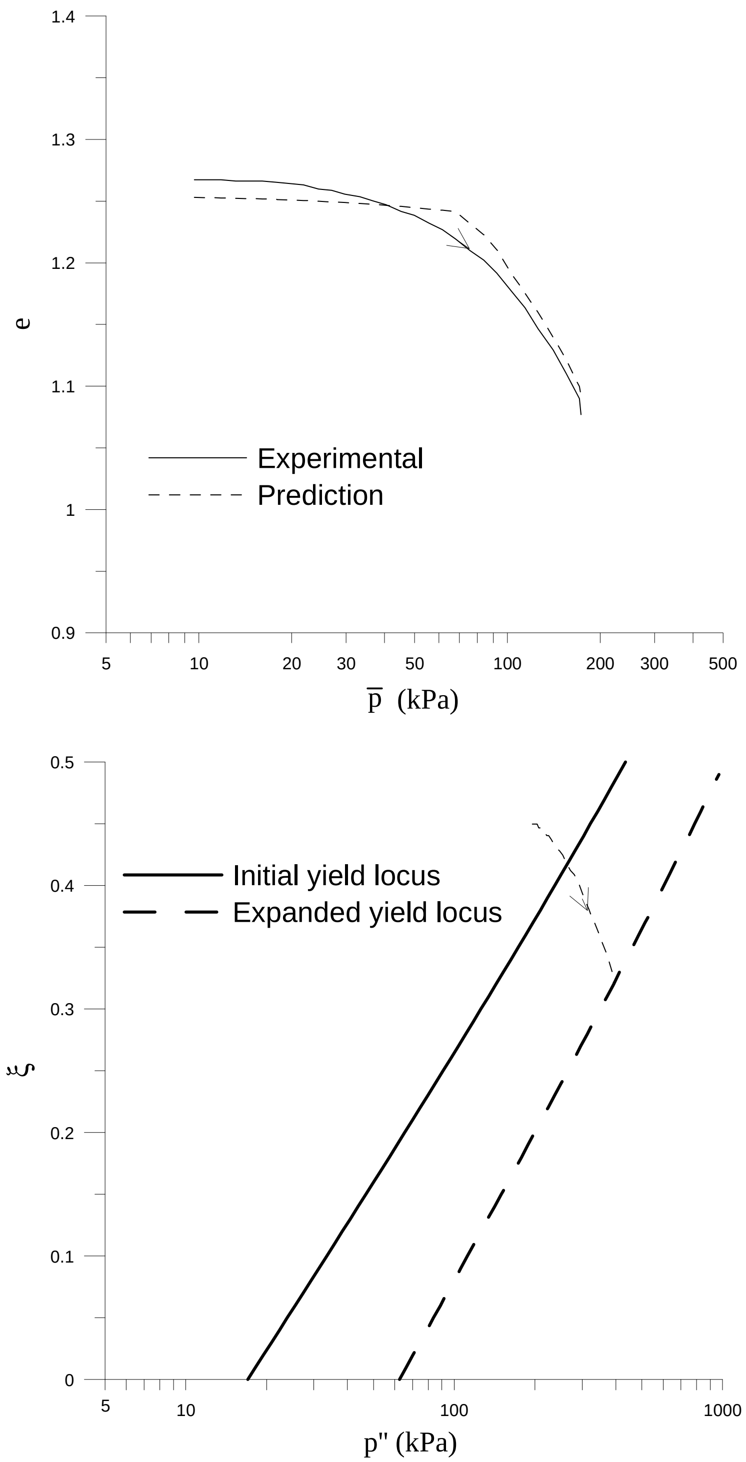


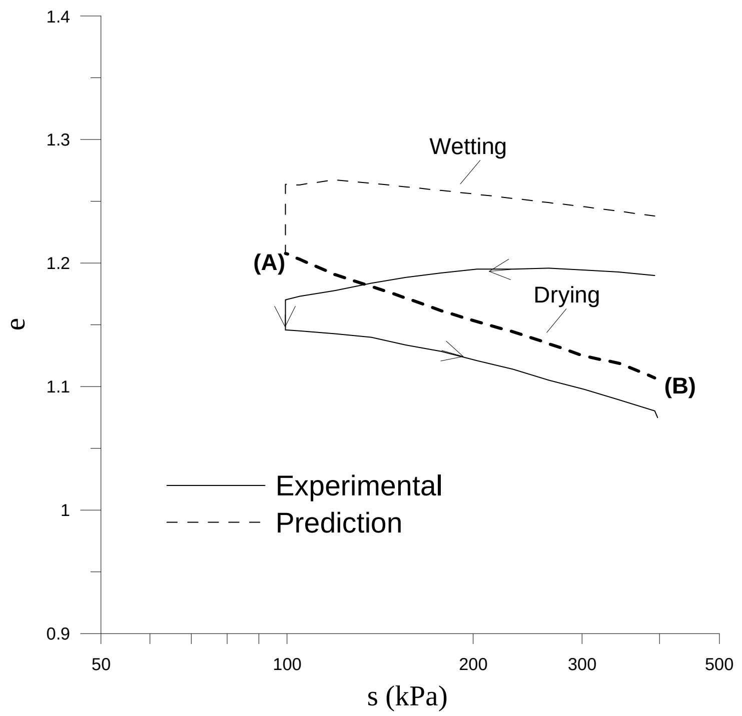
Figure 13. Model prediction for isotropic virgin loading at constant suction of 200 kPa: (a) change of void ratio, (b) stress path (experimental data by Sharma 1998).



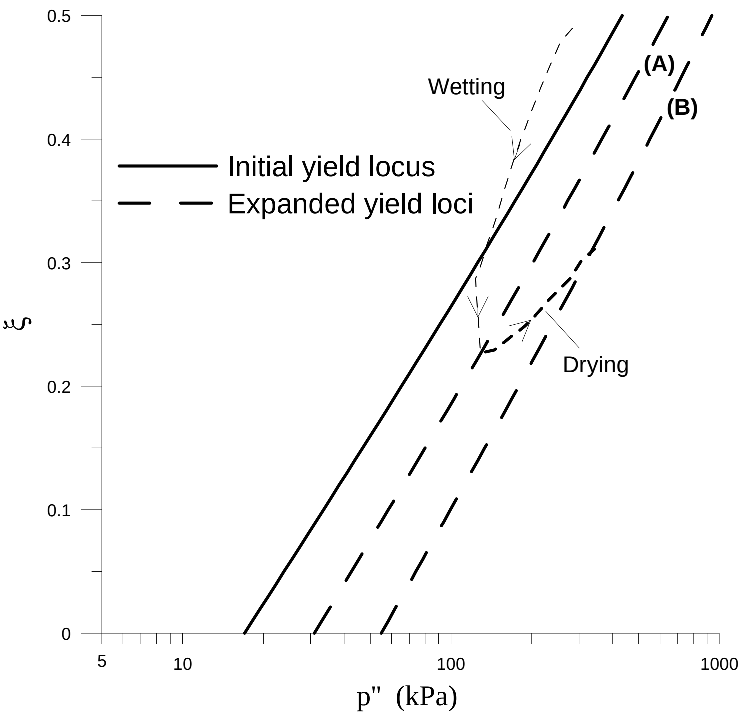
(a)

(b)

Figure 14. Model prediction for isotropic virgin loading at constant suction of 300 kPa: (a) change of void ratio, (b) stress path (experimental data by Sharma 1998).

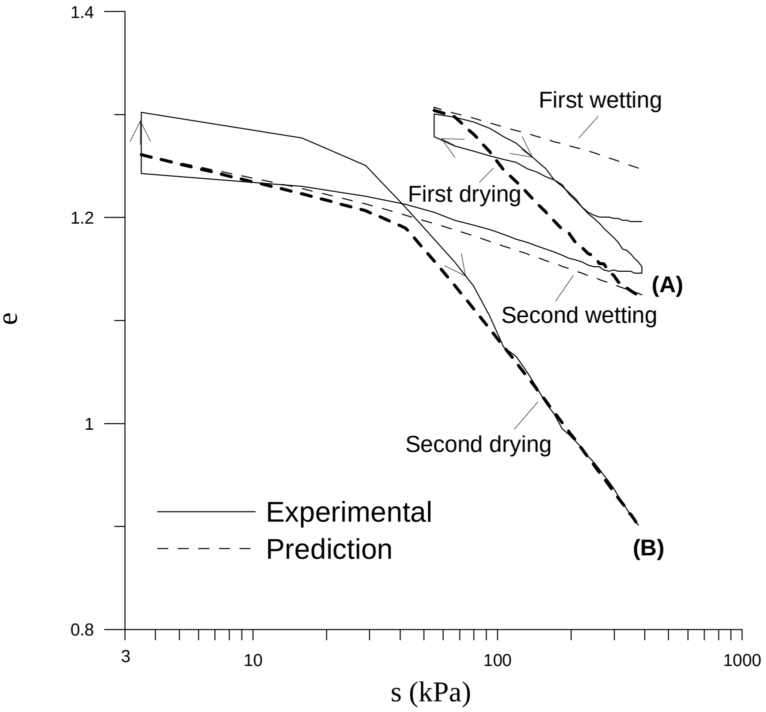


(a)

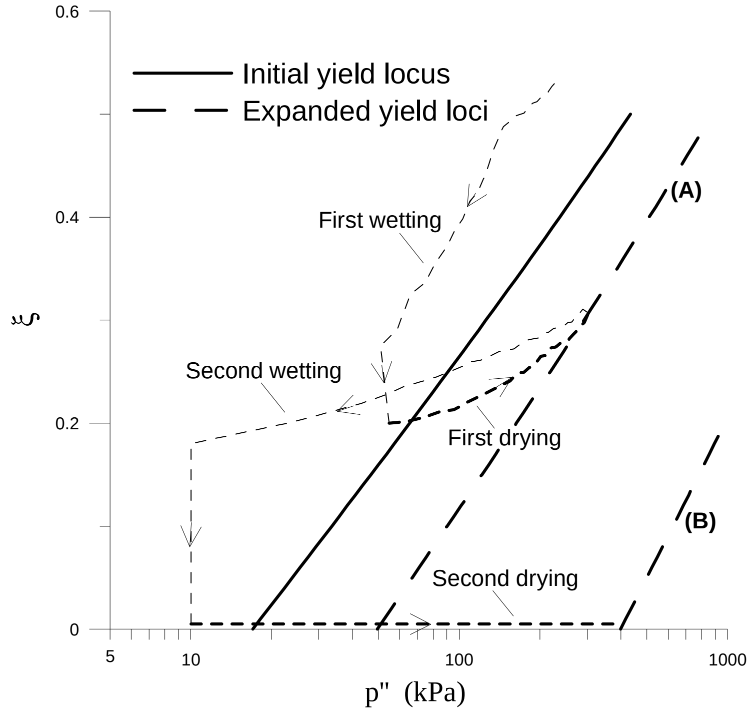


(b)

Figure 15. Model prediction for a wetting-drying cycle of a sample subjected to a constant mean net stress of 50 kPa: (a) change of void ratio, (b) stress path (experimental data by Sharma 1998).



(a)



(b)

Figure 16. Model prediction for wetting-drying cycles of a sample subjected to a constant mean net stress of 10 kPa: (a) change of void ratio, (b) stress path (experimental data by Sharma 1998).

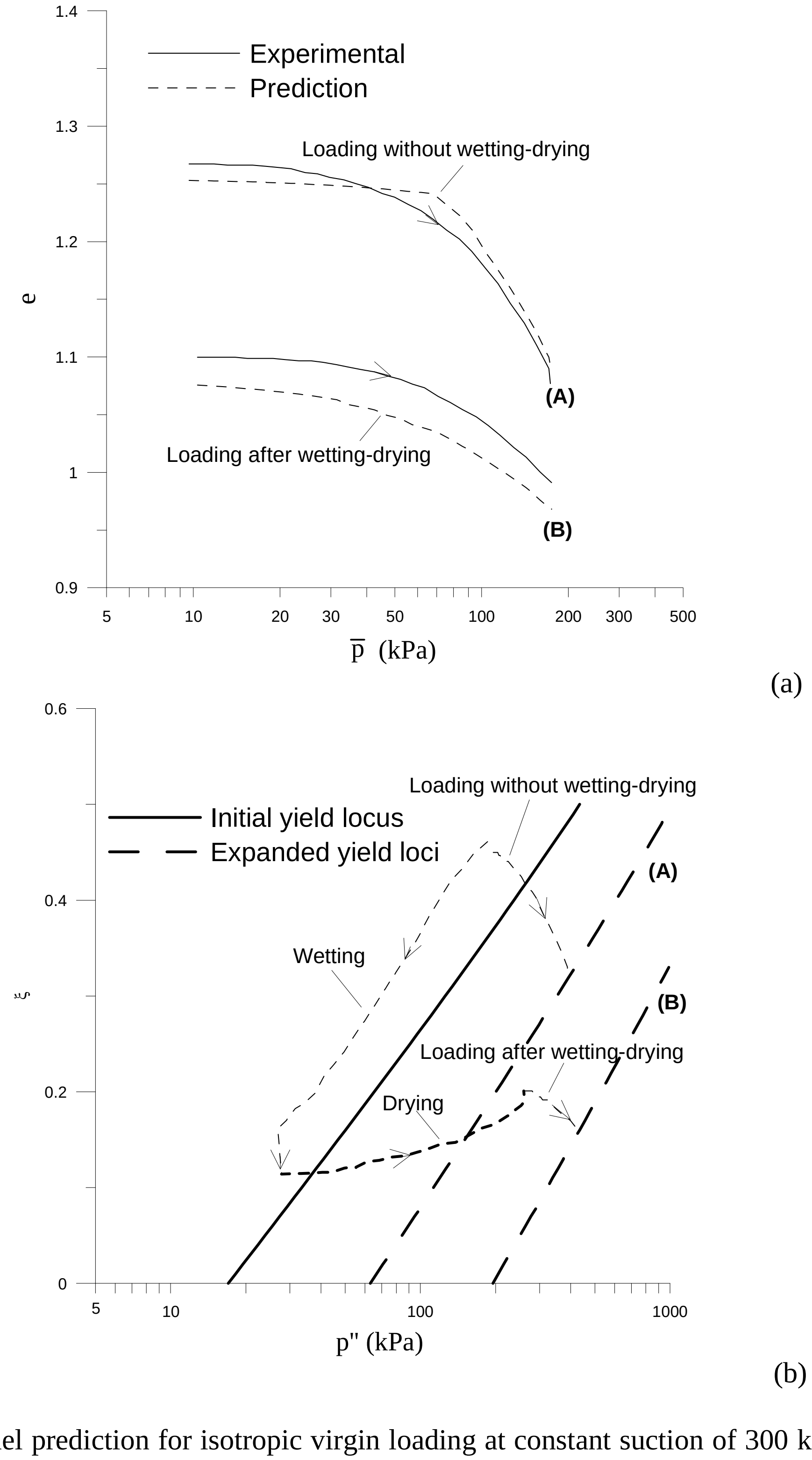


Figure 17. Model prediction for isotropic virgin loading at constant suction of 300 kPa: (a) change of void ratio, (b) stress path (experimental data by Sharma 1998).