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Fiscal Sustainability in a New Keynesian Model

Recent work on optimal monetary and fiscal policy in New Keynesian models suggests that it is optimal to allow steady-state debt to follow a random walk. In this paper we consider the nature of the time inconsistency involved in such a policy and its implication for discretionary policymaking. We show that governments are tempted, given inflationary expectations, to utilize their monetary and fiscal instruments in the initial period to change the ultimate debt burden they need to service. We demonstrate that this temptation is only eliminated if following shocks, the new steady-state debt is equal to the original (efficient) debt level even though there is no explicit debt target in the government's objective function. Analytically and in a series of numerical simulations we show which instrument is used to stabilize the debt depends crucially on the degree of nominal inertia and the size of the debt stock. We also show that the welfare consequences of introducing debt are negligible for precommitment policies, but can be significant for discretionary policy. Finally, we assess the credibility of commitment policy by considering a quasi-commitment policy, which allows for different probabilities of reneging on past promises.

JEL codes: E62, E63
Keywords: New Keynesian model, government debt, monetary policy, fiscal policy, credibility.

In sticky-price New Keynesian models where social welfare is derived from consumers' utility, Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004) show that optimal steady-state debt follows a random walk when policymakers can commit to a time-inconsistent monetary and fiscal policy. In this paper we analyze the nature of the time inconsistency involved and show that

We would like to thank the editor, Paul Evans, three anonymous referees, Javier Andres, Florin Bilbiie, Tatiana Kirsanova, Luisa Lambertini, Ioana Moldovan, Raffaele Rossi, Martin Uribe, and participants at seminars at the Universities of Oxford and Valencia and the Bank of Spain for very helpful comments. We would also like to thank Tatiana Kirsanova and Christoph Himmels for providing us with computer code to facilitate the solution of the model under quasi-commitment. All errors remain our own. We are also grateful to the ESRC, grant nos. RES-156-25-003 and RES-062-23-1436, for financial assistance.

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Received August 3, 2010; and accepted in revised form September 26, 2012.
Journal of Money, Credit and Banking, Vol. 45, No. 8 (December 2013)

the implications of optimal (time-consistent) discretionary policy for debt are quite different. This result has particular empirical relevance today as governments assess the extent to which they need to reverse the large increases in debt caused by the recession, in a context where fiscal policy commitments are often far from credible.

Much of the previous work examining time-consistency problems involving debt has been based on flexible price models, where the issues are very different. Lucas and Stokey (1983) identify three possibilities depending of the initial value of nominal government debt. If debt is positive, the optimal policy is to raise the price level to infinity to deflate the real value of debt. If the initial level of government liabilities is negative, then monetary policy should be consistent with a price level that ensures the government purchases can be financed without recourse to any distortionary taxation. The "only possibility . . . of potential practical interest" (Lucas and Stokey, p. 83) is the case where there is, in the initial period, no outstanding government debt, such that the price level cannot be costlessly manipulated to achieve welfare gains. It is only to the extent that the surprise inflation cannot costlessly and instantaneously be used to manipulate the real value of the debt stock that a potential time-inconsistency problem exists. In light of this, the flexible-price literature makes the discretionary policy problem interesting either by assuming an *ad hoc* cost of inflation in the policymaker's objective function (see, e.g., Obstfeld 1991, 1997) or by introducing a welfare cost to actual inflation through a device such as a cash-in-advance constraint, where money is considered to be a predetermined variable. For example, when monetary injections cannot be fully spent in the period in which they occur (Nicolini 1998), time-consistent policy may actually result in surprise deflations rather than inflations. This and other papers show that the desired long-run level of debt under the time-consistent policy may actually be positive or negative, depending on preferences and whether or not monetary policy follows the Friedman rule (see, e.g., Ellison and Rankin 2007, Diaz-Gimenex et al. 2008, Martin 2009).[1]

In this paper we focus on the time-consistency of policies where the costs of inflation arise from nominal inertia rather than monetary transaction frictions. The results in Schmitt-Grohe and Uribe (2004) show that even a "miniscule" degree of price stickiness means that it ceases to be optimal (under a Ramsey policy) to use surprise inflation in the manner described by Lucas and Stokey (1983). As a result, it may be thought that the time-inconsistency problem in a sticky-price environment may be slight. Our results later show that this is not the case. Our analysis of the time-inconsistency problem under sticky prices reveals that governments following the optimal time-inconsistent policy are tempted to utilize policy instruments to modify the level of debt in the initial period. For a shock that raises debt, we show that the time-inconsistent policy will cut interest rates and government spending in

1. In all these papers the policymaker faces an intertemporal problem in deciding how to allocate distortions over time—since inflation is assumed to be costly due to some kind of cash-in-advance constraint, there is a cost in deflating the real value of debt through inflation surprises such that they must decide to what extent they are willing to trade off such short-term distortions against the distortions caused by having to deal with government debt in the future. Time-consistent policy is such that these intertemporal effects are balanced until there are no incentives, in the absence of new information, to generate additional policy surprises to alter the equilibrium paths of inflation and government debt.

the initial period relative to their new steady-state levels (movements in tax rates are ambiguous), which raises output and inflation. This occurs whether debt is real or nominal. We show analytically and in simulation that under (time-consistent) discretionary policy, debt will always be returned to its initial (efficient) steady state to eliminate this temptation, and debt no longer follows a random walk. The instruments used to stabilize debt under a discretionary policy depend crucially on the degree of nominal inertia and the size of the debt stock, and for plausible debt–GDP ratios and degrees of price stickiness, monetary policy will often bear the greatest burden of fiscal adjustment. In contrast to ongoing fiscal consolidation efforts (International Monetary Fund [IMF] 2012), cuts in government spending are not used heavily in reducing debt levels under optimal discretionary policy.

We also show that the optimal discretionary (time-consistent) policy can involve a very rapid correction to debt after a shock. Partly as a consequence, the welfare consequences of allowing for debt without lump-sum taxes can be significant for discretionary policy, particularly compared to commitment policies. In adding debt to a New Keynesian model, the problem with discretionary policy (relative to commitment) is not that it fails to stabilize the debt stock, but that it is overzealous in doing so.

These large differences between discretionary and commitment policies emphasize the importance of the sustainability of commitment. The costs and benefits of cheating with debt policy are rather different from the more familiar inflationary bias example, because the benefits to cheating come from reduced debt levels in the future rather than a contemporaneous output gain. We show using the notion of quasi- (Schaumburg and Tambalotti 2007), loose (Debertoli and Nunes 2010), or limited (Himmels and Kirsanova 2013) commitment, that relatively modest degrees of policy credibility, as measured by an exogenous probability that the policymaker will not abandon their commitment plan, can bring us close to achieving the welfare levels achieved under commitment.

The organization of the paper is as follows. In Section 1 we outline our model in which consumers supply labor to imperfectly competitive firms that are only able to change prices following random intervals of time. Worker's labor income is taxed. In Section 2, we derive a second-order approximation to welfare for these consumers. This is important since the effective rejection of the Friedman rule in sticky-price models relies on the dominance of the welfare costs of price distortions relative to the costs reducing the inflation tax. In Section 3, we describe the optimal precommitment policy and analyze the time inconsistency inherent in that policy, before computing the discretionary policy in Section 4. This then informs the simulation results in Section 5, which reveal that operating under discretion overturns the usual random walk result and can potentially generate significant welfare costs. Section 6 then assesses the robustness of our key results when considering the case of an inefficient steady state, as well as alternative parameterizations. Finally, Section 7 considers the case of quasi-commitment as a means of evaluating what degree of commitment is required to mitigate the welfare costs associated with discretionary policy. Section 8 concludes.

1. THE MODEL

Our model is a standard New Keynesian model, but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or borrowing. This basic setup is similar to that in Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004) but with some differences. First, we allow for the possibility that the government will vary government spending in the face of shocks in an optimal way, rather than simply treating government spending as an exogenous flow, which must be financed. We feel that it is important to allow government spending to, potentially, contribute to debt stabilization policy since current fiscal consolidation efforts rely heavily on government spending cuts (see IMF 2012) and the macroeconomic consequences of spending cuts can be quite different from tax increases, even if the direct fiscal consequences are similar.[2] Second, we consider discretionary and quasi-commitment policies. In order to do so, for most of the paper, we set aside the usual inflationary bias caused by an inefficiently low level of steady-state output due to imperfect competition and distortionary taxes. This is achieved through a steady-state subsidy (financed by lump-sum taxation), which renders our steady state efficient and implies that the policymaker has no incentive to increase output above its steady-state level.[3]

We utilize this steady-state subsidy as it enables us to formulate a valid linear-quadratic (LQ) approximation to the underlying policy problem (see Woodford 2003, chap. 6 for a discussion), which in turn supports the derivation of some analytical results. It should be noted that there are alternative devices for eliminating the desire to boost output above its steady-state level from the description of the policy problem under discretion/quasi-commitment. For example, in developing optimal policy algorithms as part of the DYNARE project, Levine and Pearlman (2011) simply assume that the policymaker can commit to target the Ramsey steady state, even although that steady state may be inefficient, but that the same policymaker cannot commit to how they will respond to shocks that push the economy away from that steady state. Differently, Levine et al. (2008) balance a consumption habits' externality against the distortions due to taxation and monopolistic competition to render the steady state near efficient, which could be applied to our setup provided the externality was based on current per capita consumption rather than lagged consumption to avoid introducing additional dynamics. Finally, Bilbiie et al. (2008) note that a steady-state tax on leisure could restore efficiency without requiring a lump-sum tax-financed

2. Bi, Leeper, and Leith (2013) consider the impact of a realized fiscal consolidation being based on spending cuts rather than tax increases, when, *ex ante*, there is uncertainty over the composition of state-dependent fiscal consolidations. They show that this can determine whether or not a particular consolidation is expansionary and/or successful in terms of reducing debt levels. More generally, the empirical literature assessing the consequences of observed fiscal consolidations often stresses the importance of identifying the composition of any fiscal adjustment (see, e.g., Alesina and Ardagna 2010).

3. We assume that policymakers do not have access to lump-sum taxes outside of the steady state, such that monetary policy, distortionary taxes, and/or government spending must adjust to stabilize government debt following shocks with fiscal consequences.

subsidy. Any reader uncomfortable with our use of a steady-state subsidy should note that these alternative modeling devices would lead to isomorphic policy problems to the ones we analyze.

Alternatively, we could have used higher order perturbation methods to analyze the time-consistent solution as in Klein, Krusell, and Rios Rull (2008) and Martin (2009). As discussed in Klein, Krusell, and Rios Rull, the problem with such methods lies in obtaining the steady state of the time-consistent solution to the nonlinear policy problem, around which to approximate the economy. The steady-state subsidy, which renders our economy efficient, implies that there is no time-inconsistency problem associated with debt in our initial steady state, which we can describe analytically before log-linearizing our economy. A second alternative would have been to use global projection methods (see Heer and Maussner 2009, chaps. 5 and 6 for a survey) to analyze the nonlinear policy problem. However, the richness of our sticky-price economy (which would contain government debt, price dispersion and the shock processes as state variables) would make such an approach prohibitively complex, particularly given our goal of contrasting the discretionary, commitment, and quasi-commitment solutions.[4]

1.1 Households

There are a continuum of households of size one. We shall assume full asset markets, such that through risk sharing, they will face the same budget constraint. As a result the typical household will seek to maximize the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t{}^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

where C_t, G_t, and N_t are a consumption aggregate, a public goods aggregate, and labor supply, respectively, where the private and public consumption goods aggregates are defined as $C_t = (\int_0^1 C(j)_t^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}}$ and $G_t = (\int_0^1 G(j)_t^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}}$, respectively, with an elasticity of substitution between goods of different varieties given by $\epsilon > 1$. Optimization of expenditure across individual goods implies the household's demand function for good j, $C(j)_t = (\frac{P(j)_t}{P_t})^{-\epsilon} C_t$ with an associated price level of $P_t = (\int_0^1 P(j)_t^{1-\epsilon} dj)^{\frac{1}{1-\epsilon}}$.

The budget constraint at time t is given by

$$P_t C_t + E_t Q_{t,t+1} D_{t+1} = \Pi_t + D_t + W_t N_t (1 - \tau_t) - T_t, \tag{2}$$

4. Anderson, Kim, and Yun (2010) and Wolman and Zandweghe (2010) recently extend the projection approach to the benchmark New Keynesian sticky-price economy under discretionary monetary policy, but without the presence of government debt as a state variable. Niemann, Pichler, and Sorger (2008) consider the case of discretion with government debt. Modeling the commitment and quasi-commitment solutions using such techniques would increase the size of the state vector even further.

where D_{t+1} is the nominal payoff of the portfolio held at the end of period t, Π_t is the representative household's share of profits in the imperfectly competitive firms, W_t are wages, τ_t is an wage income tax rate, and T_t are lump-sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs.

We can then maximize utility (i) subject to the budget constraint and (ii) to obtain the optimal allocation of consumption across time,

$$\beta R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1, \tag{3}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one-period bond paying off a unit of currency in $t+1$. This is the familiar consumption Euler equation, which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

A log-linearized version of (3) can be written as

$$\widehat{C}_t = E_t\{\widehat{C}_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}), \tag{4}$$

where hatted variables denote percentage deviations from steady state, $r_t = R_t - \rho$, where $\rho = \frac{1}{\beta} - 1$, and $\pi_t = \ln(P_t/P_{t-1})$ is price inflation.

The second first-order condition (FOC) relates to their labor supply decision and is given by

$$(1 - \tau_t)(w_t) = N_t^{\varphi} C_t^{\sigma}, \tag{5}$$

where real wages are defined as $w_t = W_t/P_t$. This expression log-linearizes as

$$-\frac{\overline{\tau}}{1-\overline{\tau}}\widehat{\tau}_t + \widehat{w}_t = \varphi \widehat{N}_t + \sigma \widehat{C}_t. \tag{6}$$

1.2 Firms

The production function is linear, so for firm j

$$Y(j)_t = A_t N(j)_t, \tag{7}$$

where $a_t = \ln(A_t)$ is time varying and stochastic, while the demand curve they face is given by

$$Y(j)_t = \left(\frac{P(j)_t}{P_t} \right)^{-\epsilon} Y_t,$$

where $Y_t = [\int_0^1 Y(j)_t^{\frac{\epsilon-1}{\epsilon}} dj]^{\frac{\epsilon}{\epsilon-1}} = C_t + G_t$, after assuming the government attempts to minimize the costs of obtaining a given level of public consumption by allocating expenditure across individual goods in the same manner as households. The objective function of the firm is given by

$$\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t}\left[P(j)_t Y(j)_t - W_t \frac{Y(j)_t(1-\varkappa)}{A_t}\right], \tag{8}$$

where θ_p is the probability that the firm is unable to change its price in a particular period, and $\varkappa$ is a time-invariant employment subsidy that can be used to eliminate the steady-state distortion associated with monopolistic competition and distortionary income taxes. Profit maximization then implies that firms that are able to change price in period t will select the following price:

$$P_t^* = \frac{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t}\left[\epsilon W_t P_t^{\epsilon}\frac{Y_t}{A_t}\right]}{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t}\left[(\epsilon-1)P_t^{\epsilon} Y_t(1-\varkappa)\right]}.$$

Log-linearization of this pricing behavior implies a New Keynesian Phillips curve for price inflation which is given by

$$\pi_t = \beta E_t \pi_{t+1} + \gamma(mc_t),$$

where $\gamma = \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}$, $mc_t = -a_t + \widehat{w}_t$ are the real log-linearized marginal costs of production.

1.3 Equilibrium

Goods market clearing requires, for each good j,

$$Y(j)_t = C(j)_t + G(j)_t, \tag{9}$$

which allows us to write

$$Y_t = C_t + G_t,$$

where aggregate output is defined as $Y_t = [\int_0^1 Y(j)_t^{\frac{\epsilon-1}{\epsilon}} dj]^{\frac{\epsilon}{\epsilon-1}}$. Log-linearizing implies

$$\widehat{Y}_t = \theta\widehat{C}_t + (1-\theta)\widehat{G}_t,$$

where we define $\theta = \frac{\overline{C}}{\overline{Y}}$.

1.4 Government Budget Constraint

Noting the equivalence between factor incomes and national output,

$$P_t Y_t = W_t N_t + \Pi_t - \varkappa W_t N_t,$$

and the definition of aggregate demand, we can map the consumer's flow budget constraint, (2), to that of the government as

$$T_t + W_t N_t(\tau_t - \varkappa) + Q_{t,t+1}D_{t+1} = D_t + P_t G_t - T_t, \tag{10}$$

where the net value of the households' portfolio at time t is $D_t = R_{t-1}B_{t-1}$ and where B_{t-1} is the stock of government bonds at the end of period $t - 1$ and R_{t-1} is the risk-free nominal interest rate.

With borrowing constraints that rule out Ponzi schemes, the sequential series of budget constraints gives the intertemporal budget constraint (see Woodford 2003, chap. 2, p. 69). In order to focus on the time-inconsistency problem associated with the introduction of debt and distortionary taxation to the New Keynesian model we follow Rotemberg and Woodford (1997) and later authors and introduce a steady-state subsidy.[5] This subsidy offsets, in steady state, the distortions caused by distortionary taxation and imperfect competition in price setting, and removes the usual desire on the part of policymakers to raise output above its natural level to compensate for these distortions. In other words, this subsidy ensures that the steady state is efficient. The steady-state subsidy is financed by lump-sum taxation. We shall assume that both the level of the subsidy and the associated level of lump-sum taxation cannot be altered from this steady-state level, so that any changes in the government's budget constraint have to be financed by changes in distortionary taxation, government spending, or debt service costs.[6] This implies that $W_T N_T \varkappa = T_T$ in our economy at all points in time, allowing us to simplify the budget constraint to

$$W_t N_t \tau_t + B_t = R_{t-1}B_{t-1} + P_t G_t - T_t. \tag{11}$$

Defining real debt as $b_t \equiv \frac{B_{t-1}R_{t-1}}{P_{t-1}}$ and its initial steady state by

$$\overline{b} = \frac{\overline{w}\overline{N}\overline{\tau} - \overline{G}}{1 - \beta},$$

we can log-linearize the government's flow budget constraint around this steady state, assuming a steady-state gross inflation rate of 1 and replacing the interest rate with the consumption Euler equation, (4), yields

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma\widehat{C}_t &= \beta E_t\{\widehat{b}_{t+1} - \pi_{t+1} - \sigma\widehat{C}_{t+1}\} \\ &\quad + \left[-\sigma(1-\beta)\widehat{C}_t + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t\right],\end{aligned} \tag{12}$$

as in Benigno and Woodford (2003). The next section defines the steady-state ratios contained in this log-linearization as a function of model parameters and the initial steady-state debt–GDP ratio.

5. Although hardly realistic, this setup allows us to focus attention in the time inconsistency involved in debt policy, under varying degrees of commitment. We discuss alternative approaches to analyzing the policy problem in the introduction to Section 1, and partially relax this assumption in Section 6.1.

6. If we considered the lump-sum financed subsidy to be a policy instrument, which could be varied over time, then there would be no trade-off between fiscal solvency and business cycle stabilization, and the policy problem would be rendered trivial.

2. POLICY OBJECTIVES

In order to derive a welfare function for policy analysis we proceed in the following manner. First, we consider the social planner's problem. We then contrast this with the outcome under flexible prices in order to determine the level of the steady-state subsidy required to ensure the model's initial steady state is socially optimal.[7] Finally, we construct a quadratic approximation to utility in our sticky-price/distortionary tax economy, which assesses the extent to which endogenous variables differ from the efficient equilibrium due to the nominal inertia and tax distortions present in the model. We then recast our model in terms of the "gap" variables contained within our welfare metric.

2.1 The Social Planner's Problem

The social planner is not constrained by the price mechanism and simply maximizes the representative household's utility, (1), subject to the technology, (7), and resource constraint, (9). This yields the following FOCs:

$$(C_t^*)^{-\sigma} = \chi G_t^{*-\sigma},$$

$$(C_t^*)^{-\sigma} - Y_t^{*\varphi} A_t^{-(1+\varphi)} = 0,$$

where we introduce the "*" superscript to denote the efficient level of that variable implying the efficient level of output is given by

$$Y_t^* = (A_t)^{\frac{1+\varphi}{\sigma+\varphi}} (1 + \chi^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma+\varphi}},$$

which log-linearizes as

$$\widehat{Y}_t^* = \left(\frac{1+\varphi}{\sigma+\varphi}\right) a_t = \widehat{C}_t^* = \widehat{G}_t^*.$$

2.2 Flexible Price Equilibrium

Profit-maximizing behavior implies that firms will operate at the point at which marginal costs equal marginal revenues,

$$-\ln(\mu_t) = mc_t,$$

$$\left(1 - \frac{1}{\epsilon}\right) = \frac{(1-\varkappa)}{(1-\tau_t)} (N_t^n)^{(\varphi)} A_t^{-1} (C_t^n)^{\sigma}.$$

7. We shall assume that this subsidy is in place throughout most of the paper. However, in Section 6 we shall explore the robustness of our results to relaxing this assumption. We do so by considering the case where the subsidy is insufficient to replicate the efficient allocation in the initial steady state, such that output is suboptimally low due to tax and monopolistic competition distortions.

By choosing the steady-state subsidy, $\varkappa$, to optimally offset the distortions due to taxation and monopolistic competition,

$$(1-\varkappa)=\left(1-\frac{1}{\epsilon}\right)(1-\overline{\tau}), \tag{13}$$

we obtain a flexible price steady state, which, assuming a steady-state government spending rule, $\frac{\overline{G}}{\overline{Y}}=(1+\chi^{-\frac{1}{\sigma}})^{-1}$, is identical to the optimal level of employment in the efficient steady state chosen by the social planner,

$$\overline{Y}=\overline{N}=\left(1+\chi^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma+\varphi}}=\overline{Y}^{*}.$$

This is turn defines the steady-state real wage, $\overline{w}=\frac{1}{1-\overline{\tau}}$, the steady-state tax rate required to support a given debt-to-GDP ratio,

$$\overline{\tau}=\frac{(1-\beta)\frac{\overline{B}}{\overline{Y}}+\frac{\overline{G}}{\overline{Y}}}{1+(1-\beta)\frac{\overline{B}}{\overline{Y}}+\frac{\overline{G}}{\overline{Y}}},$$

and the ratio of tax revenues to debt,

$$\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}=\frac{\frac{\overline{\tau}}{1-\overline{\tau}}}{\frac{\overline{B}}{\overline{Y}}}.$$

These relationships are sufficient to define all log-linearized relationships dependent on model parameters and an initial debt-to-GDP ratio such that the initial steady state is efficient. We consider the case of an inefficient steady state in Section 6.

2.3 Social Welfare

Appendix A derives the quadratic approximation to utility

$$\Gamma=-\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Big\{\sigma\theta(\widehat{C}_t-\widehat{C}_t^{*})^2+\sigma(1-\theta)(\widehat{G}_t-\widehat{G}_t^{*})^2$$
$$+\varphi(\widehat{Y}_t-\widehat{Y}_t^{*})^2+\frac{\epsilon}{\gamma}\pi_t^2\Big\}+tip+O[2].$$

It contains quadratic terms in price inflation reflecting the costs of price dispersion induced by inflation in the presence of nominal inertia, as well as terms in the consumption, government spending, and output gaps, that is, the difference between the actual value of the variable and its optimal value. The weights attached to each element are a function of deep model parameters. The key to obtaining this quadratic specification, suitable for analyzing the various forms of optimal policy we consider, lies in adopting an employment subsidy that eliminates the steady-state distortions caused by imperfect competition in labor and product markets as well as the steady-state impact of a distortionary income tax. It is important to stress that this subsidy only applies in the steady state such that it cannot be used as a policy instrument to either stabilize the economy or the government's finances in the face of shocks.

Moreover, as discussed at the beginning of Section 1, alternative, but probably less familiar, modeling devices can achieve the same objective function without recourse to such a subsidy.

2.4 Gap Variables

We have derived welfare based on various gaps, so we now proceed to rewrite our model in terms of the same gap variables to facilitate derivation of optimal policy. The consumption Euler equation can be written in gap form as

$$(\widehat{C}_t - \widehat{C}_t^*) = E_t\{(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} - \frac{1}{\sigma}((r_t - r_t^*) - E_t\{\pi_{t+1}\}),$$

where $r_t^* = \sigma \frac{1+\varphi}{\sigma+\varphi}(E_t\{a_{t+1}\} - a_t)$ is the natural/efficient rate of interest. (This comes from the fact that $\widehat{C}_t^* = \widehat{Y}_t^*$ and the definition of the efficient level of output.)

While the New Keynesian Phillips Curve (NKPC) can be written in gap form as

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \left(\varphi(\widehat{Y}_t - \widehat{Y}_t^*) + \sigma(\widehat{C}_t - \widehat{C}_t^*) + \frac{\overline{\tau}}{1-\overline{\tau}}(\widehat{\tau}_t) \right),$$

Appendix B rewrites the budget constraint in gap form as

$$\widehat{b}_t - \pi_t = \beta \widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} + ps_t - f_t + \sigma\beta(\widehat{C}_t - \widehat{C}_t^*)$$

with the primary surplus defined as

$$ps_t = \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}} \left[(1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t) + \sigma(\widehat{C}_t - \widehat{C}_t^*) \right] - \frac{\overline{G}}{\overline{b}}(\widehat{G}_t - \widehat{G}_t^*) \quad (14)$$

and

$$f_t = -(\sigma(1-\beta\rho_a) + (1-\sigma)(1-\beta))\frac{(1+\varphi)}{\sigma+\varphi} a_t,$$

captures the extent to which the technology shocks hitting the economy have fiscal consequences.

3. PRECOMMITMENT POLICY

In this section, we shall consider the precommitment policies for our model. Since the economy is cashless, the monetary policymaker is free to adjust interest rates to achieve a particular level of the consumption gap. Therefore, we can simplify the problem by treating the consumption gap as the monetary policy instrument and dropping the consumption Euler equation as an explicit constraint. It should be noted that by replacing the interest rate with the consumption Euler equation, the government budget constraint still contains the impact of variations in debt service costs. The Lagrangian associated with the policy problem under commitment in the presence of a government budget constraint is therefore given by

$$
\begin{aligned}
L_0^c = E_0 \sum_{t=0}^{\infty} \beta^s \Bigg[& \sigma\theta(c_t^g)^2 + \sigma(1-\theta)(g_t^g)^2 + \varphi(y_t^g)^2 + \frac{\epsilon}{\gamma}\pi_t^2 \\
& + \lambda_t^\pi \left(\pi_t - \beta\pi_{t+1} - \gamma\left(\varphi y_t^g + \sigma c_t^g + \frac{\overline{\tau}}{1-\overline{\tau}}\widehat{\tau}_t \right)\right) \\
& + \lambda_t^y (y_t^g - (1-\theta)g_t^g - \theta c_t^g) \\
& + \lambda_t^b (\widehat{b}_t - \pi_t - \beta(\widehat{b}_{t+1} - \pi_{t+1} - \sigma c_{t+1}^g) + \frac{\overline{G}}{\overline{b}} g_t^g \\
& - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}} \left[(1+\varphi)(y_t^g) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t) + \sigma c_t^g \right] + f_t - \sigma\beta c_t^g \Bigg) \Bigg],
\end{aligned}
$$

where λ_t^π, λ_t^y, and λ_t^b are the Lagrange multipliers associated with the NKPC, the resource constraint, and the government's budget constraint, respectively. To simplify notation we have rewritten the gap variables in the form, $x_t^g = \widehat{X}_t - \widehat{X}_t^*$.

3.1 The FOCs for t > 0

The FOCs from optimization for periods $t > 0$ for c_t^g, y_t^g, π_t, $\widehat{\tau}_t$, and $\widehat{b}_{t+1}$, respectively, are given by

$$
\begin{aligned}
& 2\sigma\theta c_t^g - \gamma\sigma\lambda_t^\pi - \theta\lambda_t^y - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\sigma\lambda_t^b - \sigma\beta\lambda_t^b + \sigma\lambda_{t-1}^b = 0, \\
& 2\varphi y_t^g - \gamma\varphi\lambda_t^\pi + \lambda_t^y - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(1+\varphi)\lambda_t^b = 0, \\
& 2\frac{\epsilon}{\gamma}\pi_t + \Delta\lambda_t^\pi - \Delta\lambda_t^b = 0, \\
& -\frac{\overline{\tau}}{1-\overline{\tau}}\gamma\lambda_t^\pi - \frac{\overline{w}\overline{N}}{\overline{b}}\frac{\overline{\tau}}{1-\overline{\tau}}\lambda_t^b = 0, \\
& E_0\lambda_t^b - \lambda_0^b = 0,
\end{aligned}
$$

while if government spending is considered to be an instrument of policy, its FOC is given by

$$
2\sigma(1-\theta)g_t^g - (1-\theta)\lambda_t^y + \frac{\overline{G}}{\overline{b}}\lambda_0^b = 0;
$$

otherwise, in line with much of the existing literature, government spending would be considered as exogenous.[8]

8. We consider the case where government spending is not considered to be an instrument of optimal policy in the online appendix available at http://eprints.gla.ac.uk/70226/.

The FOC for debt implies that the Lagrange multiplier for debt follows a random walk and this will underpin the random walk for steady-state debt result derived later. Combining this FOC with that for inflation and the tax rate implies that in the absence of new information, inflation is zero. Solving the remaining FOCs implies the following relationships between the gapped consumption variable and the Lagrange multiplier associated with debt:

$$c_t^g = -\frac{(\varphi+\sigma)(\frac{\sigma-\theta(1-\theta)}{\theta\sigma}) - (1-\beta)\frac{\overline{B}}{\overline{Y}}}{2(\varphi+\sigma)\frac{\overline{B}}{\overline{Y}}}\lambda_0^b = -a_2\lambda_0^b. \tag{15}$$

The constancy of the consumption gap implies that monetary policy is set such that interest rates are consistent with the natural rate of interest.

When government spending is utilized as a policy instrument it will satisfy

$$g_t^g = -\frac{(\varphi+\sigma)\frac{\theta}{\sigma} - (1-\beta)\frac{\overline{B}}{\overline{Y}}}{2(\varphi+\sigma)\frac{\overline{B}}{\overline{Y}}}\lambda_0^b = -a_3\lambda_0^b, \tag{16}$$

and the output gap is a linear function of the consumption and government spending gaps, which in the case when government spending is chosen optimally is given by

$$y_t^g = -\frac{(\varphi+\sigma) - (1-\beta)\frac{\overline{B}}{\overline{Y}}}{2(\varphi+\sigma)\frac{\overline{B}}{\overline{Y}}}\lambda_0^b = -a_1\lambda_0^b. \tag{17}$$

It is clear from these definitions that the coefficients, $a_i, i = 1, 2, 3,$ are positive provided the initial steady-state debt stock satisfies the following conditions:

$$\begin{aligned}
&(1-\beta)\frac{\overline{B}}{\overline{Y}} < (\sigma+\varphi)\frac{\theta}{\sigma}\ , \qquad (18)\\
&(1-\beta)\frac{\overline{B}}{\overline{Y}} < (\varphi+\sigma)\left(\frac{\sigma-\theta(1-\theta)}{\theta\sigma}\right),\\
&(1-\beta)\frac{\overline{B}}{\overline{Y}} < (\sigma+\varphi).
\end{aligned}$$

For plausible steady-state debt–GDP ratios all variants of this condition will hold,[9] implying that when $\lambda_0^b > 0$, y_t^g, c_t^g, and g_t^g will all be negative, which implies that $\widehat{\tau}_t > 0$. The converse is true when $\lambda_0^b < 0$. In other words, following a shock that raises debt, the government will commit to permanently raise taxes and cut spending to service that debt. This will lead to permanently lower consumption and output, but the policy mix will ensure that the higher debt level is maintained without generating any inflation.

9. For the paramater values adopted in the simulation section later, the annualized steady-state debt-to-GDP ratio would have to exceed 2812.5% for the strongest of these conditions to be violated.

It is helpful to rewrite the definition of the primary surplus, (14), in terms of the value of the Lagrange multiplier associated with the government's budget constraint by substituting for the tax rule, which applies after the initial period,

$$ps_t = \frac{\overline{y}}{\overline{b}}\left[\left(\frac{\overline{\tau}}{1-\overline{\tau}} - \varphi\right) y_t^g - \sigma c_t^g - (1-\theta) g_t^g\right] - f_t.$$

Using the expressions relating the gap variables to the Lagrange multiplier, this can be rewritten as

$$ps_t = \Psi \lambda_0^b - f_t, \tag{19}$$

where $\Psi = (\frac{\overline{B}}{\overline{Y}})^{-1}[(\varphi - \frac{\overline{\tau}}{1-\overline{\tau}})a_1 + \sigma a_2 + (1-\theta)a_3] > 0$, again for debt not too large.[10]

3.2 *The FOCs for $t = 0$*

In this subsection, we consider the case where the policymaker exploits the fact that expectations are given in the initial period. By contrasting the solution in the initial period to that which follows, we can highlight the nature of the time-inconsistency problem facing policymakers, which will help generate intuition for the outcome under discretion. In the initial period the initial values of the Lagrange multipliers can be set equal to zero as the policymaker is assumed to be formulating a plan to which they will commit from this point on. This implies the following relationship for consumption in the initial period,

$$c_0^g = \left(\frac{(\varphi(1-\theta)+\sigma)\beta}{2(\varphi+\sigma)} - a_2\right)\lambda_0^b, \tag{20}$$

and, when government spending is chosen optimally,

$$g_0^g = -\left(\frac{\varphi\beta}{2(\varphi+\sigma)} + a_3\right)\lambda_0^b, \tag{21}$$

where λ_0^b is the Lagrange multiplier associated with the government's budget constraint under optimal (nontimeless) commitment given the information available at time 0.

Assuming the government spending gap is equal to zero when government spending is not a policy instrument implies the output gap is proportional to the output gap, $y_0^g = (1-\theta)c_0^g$, while when government spending is chosen optimally we have

$$y_0^g = \left(\frac{\sigma\beta}{2(\varphi+\sigma)} - a_1\right)\lambda_0^b. \tag{22}$$

The initial term in each of these expressions captures the extent to which output and consumption gaps are higher and government spending gaps lower in the initial period

10. For the parameter values adopted in the simulation section later, it is not possible for this coefficient to be negative for any positive debt-to-GDP ratio.

as the government attempts to exploit the fact that expectations of the initial period are already formed. Therefore, in the face of a shock with negative fiscal consequences we observe higher inflation and relatively higher output and consumption, but lower government spending in the initial period. This implies that there is a negative interest rate gap as monetary policy accommodates debt in the initial period. Whether or not taxes are relatively higher or lower in the initial period is ambiguous. The net effect of these policies is to raise inflation in the initial period. Given this behavior in the initial period, the initial government surplus is given by

$$ps_0 = (\Psi + \Psi_0)\lambda_0^b - f_0,$$

where $\Psi_0 = \frac{(\sigma+\varphi)(\theta(1-\theta)-\sigma)+\theta\sigma(1-\beta)\frac{\overline{B}}{\overline{Y}}}{2\frac{\overline{B}}{\overline{Y}}\theta(\sigma+\varphi)} > 0$ provided inequality (18) holds. In other words there is an attempt to reduce the fiscal consequences of the shock in the initial period.

3.3 The Time-Inconsistent Response

In order to determine the size of the Lagrange multiplier associated with the government's IBC, we need to substitute these expressions in the intertemporal budget constraint to obtain

$$\widehat{b}_0 - \pi_0 - \sigma\frac{(\varphi(1-\theta)+\sigma)\beta^2}{2(\varphi+\sigma)}\lambda_0^b = \left(\Psi_0 + \frac{\Psi}{\beta^{-1}-1}\right)\lambda_0^b - E_0\sum_{t=0}^{\infty}\beta^t f_t.$$

Using the expression for the initial rate of inflation and solving for the Lagrange multiplier yields

$$\left(\frac{\overline{w}\overline{N}}{\overline{b}} + \gamma + \Psi_0 + \frac{\Psi}{1-\beta} + \sigma\frac{(\varphi(1-\theta)+\sigma)\beta^2}{2(\varphi+\sigma)}\right)\lambda_0^b = \widehat{b}_0 + E_0\sum_{t=0}^{\infty}\beta^s f_0.$$

That is, the Lagrange multiplier is proportional to the sum of the initial debt disequilibrium and the expected discounted value of the fiscal effects of the technology shocks. However, *ceteris paribus*, the value of the multiplier will not be as large when the policymaker exploits fixed expectations in the initial period to raise additional tax revenue and deflate the debt. This implies that in the case of a shock with a higher debt stock, output, consumption, and government spending will not fall by as much, and taxes will not need to rise by as much to support the new steady-state debt stock, which is lower than it would be under a policy that did not exploit the fact that expectations are given in the initial period.

As a result of the fact that some of the fiscal consequences of shocks will be undone in the initial period, the new steady-state debt stock under (nontimeless) commitment can be shown to be given by

$$\overline{b}^C = \left(\frac{\Psi}{1-\beta}\right)\lambda_0^b < \widehat{b}_t + E_0\sum_{t=0}^{\infty}\beta^t f_t.$$

It is important to note that if there was no attempt to behave differently in the initial period then there would be full accommodation of all the fiscal consequences of shocks.

We are now in a position to fully describe the response to shocks under commitment. Shocks only have an effect on welfare-relevant gap variables to the extent that they have fiscal repercussions, the financing of which limits the extent to which monetary and fiscal policy can achieve the first-best solution. Under both full and timeless commitment, inflation beyond the initial period is always zero. Outside of the initial period, policy allows the fiscal effects of shocks to be fully reflected in the debt stock and to only adjust fiscal instruments (government spending gaps and the tax gap) to the extent required to support the new steady-state debt stock. In this sense, optimal policy fully accommodates the debt shock. Debt will slowly evolve until it reaches a new steady-state value consistent with higher taxes, lower government spending, reduced consumption, and output.

The time-inconsistent (nontimeless) optimal commitment policy exploits the Phillips curve in the initial period by raising inflation, thereby reducing the ultimate increase in debt. The extent to which it does this will depend on whether debt is real or nominal: the increase in steady-state debt (and associated changes in other variables) will be less when debt is nominal.

4. DISCRETIONARY POLICY

Following Anderson, Kim, and Yun (2010), we can write the discretionary policy problem in the form of a Lagrangian:

$$
\begin{aligned}
L_t^d &= \sigma\theta(c_t^g)^2 + \sigma(1-\theta)(g_t^g)^2 + \varphi(y_t^g)^2 + \frac{\epsilon}{\gamma}\pi_t^2 + \beta E_t V(\widehat{b}_{t+1}, \xi_{t+1}) \\
&+ \lambda_t^\pi(\pi_t - \beta(\phi 1\widehat{b}_{t+1} + \delta 1 a_t) - \gamma\left(\varphi y_t^g + \sigma c_t^g + \frac{\overline{\tau}}{1-\overline{\tau}}\widehat{\tau}_t)\right) \\
&+ \lambda_t^y(y_t^g - (1-\theta)g_t^g - \theta c_t^g) \\
&+ \lambda_t^b(\widehat{b}_t - \pi_t - \beta(\widehat{b}_{t+1} - (\phi 1\widehat{b}_{t+1} + \delta 1 a_t) - \sigma(\phi 2\widehat{b}_{t+1} + \delta 2 a_t) + \frac{\overline{G}}{\overline{b}} g_t^g \\
&- \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\left[(1+\varphi)(y_t^g) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t^g) + \sigma c_t^g\right] + f_t - \sigma\beta c_t^g).
\end{aligned}
$$

The first key differences between discretion and commitment is that under discretion the policymaker takes expectations as given, although the policymaker recognizes that expectations will be affected by the current state of the economy. Accordingly, given our LQ framework, the expectations of future inflation are set equal to a linear combination of the state variables, $E_t\pi_{t+1} = \phi 1 b_{t+1} + \delta 1 a_t$, as are consumption expectations, $E_t c_{t+1}^g = \phi 2 b_{t+1} + \delta 2 a_t$, where the parameters ϕi and δi, $i = 1, 2$, are still to be determined. This implies that the policymaker cannot commit to future policy actions in order to favorably influence expectations. Instead, they must reoptimize every period, although that reoptimization will take account of the fact that both expectations and the future policy payoffs ($\beta E_t V(\widehat{b}_{t+1}, a_t)$) will be influenced by the value of any endogenous state variables bequeathed to the future.

The FOCs for this optimization for c_t^g, y_t^g , π_t, $\widehat{\tau}_t$, and $\widehat{b}_{t+1}$, respectively, are given by

$$2\sigma\theta(c_t^g) - \lambda_t^\pi \gamma\sigma - \theta\lambda_t^y - \lambda_t^b \sigma \left(\beta + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\right) = 0,$$

$$2\varphi(y_t^g) - \lambda_t^\pi \gamma\varphi + \lambda_t^y - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(1+\varphi)\lambda_t^b = 0,$$

$$2\frac{\epsilon}{\gamma}\pi_t + \lambda_t^\pi - \lambda_t^b = 0,$$

$$-\gamma\frac{\overline{\tau}}{1-\overline{\tau}}\lambda_t^\pi - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\frac{1}{1-\overline{\tau}}\lambda_t^b = 0,$$

$$\beta\frac{\partial E_t V(\widehat{b}_{t+1}, \xi_{t+1})}{\partial b_{t+1}} - \beta\phi 1\lambda_t^\pi - \beta\lambda_t^b(1 - \phi 1 - \sigma\phi 2) = 0,$$

while that for government spending when it is treated as a policy instrument is given by

$$2\sigma(1-\theta)(g_t^g) - (1-\theta)\lambda_t^y + \frac{\overline{G}}{\overline{b}}\lambda_t^b = 0.$$

Conditional on the "guesses" for the expectations equations and the parameterization of the value function, we can solve these FOCs and the original constraints to obtain values for all variables and Lagrange multipliers in terms of the current values of the states. Contrasting this solution with the original guesses can then verify whether or not the guess was correct. If it is, then we have found a time-consistent solution to the policy problem. If not, we must continue to update the guess until it delivers a solution, which is consistent with the original guess, thereby verifying the guess. In solving the policy problem under discretion, we employ the computer codes of Soderlind (1999) who develops an iterative procedure in the manner of Oudiz and Sachs (1985). This yields a solution to the policy problem, which satisfies both the expectations formation equations and the definition of the value function.

Before we turn to explore the numerical solution of this system, it is possible to obtain some further analytical results by partially solving some of the policy instruments as a function of inflation. Doing so casts some light on the optimal assignment of instruments to the control of inflation versus the stabilization of government debt.

Accordingly, solving the first four FOCs implies the following relationship between consumption and inflation:

$$c_t^g = -\frac{(\sigma(\sigma+\varphi-\frac{\sigma+\varphi}{\sigma}\theta(1-\theta))-\frac{\overline{B}}{\overline{Y}}\sigma(\sigma\beta+\theta(1-\beta)+\varphi\beta(1-\theta)))\varepsilon}{(\varphi+\sigma)\sigma\theta((1-\theta)+(1-\beta)\frac{\overline{B}}{\overline{Y}}+1+\frac{\overline{B}}{\overline{Y}}\gamma)}\pi_t, \quad (23)$$

while if government spending is chosen optimally

$$g_t^g = -\frac{\left(\theta(\varphi+\sigma)+\sigma(\beta(1+\varphi)-1)\frac{\overline{B}}{\overline{Y}}\right)\varepsilon}{\sigma(\varphi+\sigma)\left((1-\theta)+(1-\beta)\frac{\overline{B}}{\overline{Y}}+1+\frac{\overline{B}}{\overline{Y}}\gamma\right)}\pi_t. \quad (24)$$

Combining this with the definition of output allows us to obtain the following relationship between the output gap and inflation under discretion, when government spending is chosen optimally (if G_t is set exogenously the output gap under discretion is simply a linear function of the optimal consumption gap and the exogenous government spending gap, $y_t^g = (1-\theta)c_t^g + \theta g_t^g$),

$$y_t^g = -\frac{\left((\varphi+\sigma)-(1+\beta(\sigma-1))\frac{\overline{B}}{\overline{Y}}\right)\varepsilon}{(\varphi+\sigma)\left((1-\theta)+(1-\beta)\frac{\overline{B}}{\overline{Y}}+1+\frac{\overline{B}}{\overline{Y}}\gamma\right)}\pi_t. \quad (25)$$

For plausible labor supply parameters, the government spending gap has the opposite sign to the rate of inflation under discretion, but the signs of the other relationships depend crucially on the size of the debt stock. For small steady-state debt–GDP ratios under discretion consumption, and consequently the output gap, will have the opposite sign to the rate of inflation. Basically, following a shock with negative fiscal consequences, government spending will fall and taxes will rise in order to stabilize the debt stock. As tax rates are an element of marginal costs, the tax rise will fuel inflation. Monetary policy will be tightened in order to control this inflation, and this will serve to reduce consumption (and output). Although the tightening of monetary policy will raise debt service costs, the relatively small size of the initial debt stock ensures that this is not a significant problem.

However, for sufficiently large debt stocks policymakers must recognize the negative effect of monetary policy on debt service costs. This raises the efficacy of using monetary policy to stabilize the debt such that, with a sufficiently large steady-state debt–GDP ratio the output and consumption gaps will move in the same direction as inflation. In other words, despite the fact that government spending falls, consumption (and output) will increase as a result of a relaxation of monetary policy needed to stabilize debt. Whether or not taxes increase or decrease, augmenting or offsetting this inflationary impulse, is ambiguous, but as the steady-state debt–GDP ratio rises the inflationary consequences of taxation becomes more significant (as at higher levels of steady-state taxation a marginal increase in tax rates becomes a greater drag on labor supply) such that taxes are less likely to be used as a tool to stabilize the debt, and are more likely to be utilized to control inflation.[11]

The level of debt at which monetary policy moves from an anti-inflationary stance to one of fiscal accommodation is given by

$$\frac{\overline{B}}{\overline{Y}} > \frac{(\sigma - \theta(1-\theta))(\sigma+\varphi)}{\sigma\beta + \beta(1-\theta)\varphi + (1-\beta)\theta}. \tag{26}$$

For the parameter values considered below this critical value occurs at an annualized debt–GDP ratio of only 30.4%.[12] Given the simple linear relationship between output and inflation under discretion it is also possible to assess the relative volatility of the output gap and inflation in the face of shocks. The relative size of the variances will be given by

$$\frac{\mathrm{Var}(y^g)}{\mathrm{Var}(\pi)} = \left(\frac{\left((\varphi+\sigma) - (1+\beta(\sigma-1))\frac{\overline{B}}{\overline{Y}}\right)\varepsilon}{(\varphi+\sigma)((1-\theta) + (1-\beta)\frac{\overline{B}}{\overline{Y}} + 1 + \frac{\overline{B}}{\overline{Y}}\gamma)}\right)^2. \tag{27}$$

This implies that the volatility of inflation relative to output increases in the the steady-state debt–GDP ratio. It is also the case that raising the degree of price flexibility (raising γ) will reduce the adjustment of output and government spending relative to inflation in responding to shocks under discretion, and in the limit as $\gamma \to \infty$ and prices become flexible, all adjustment is through surprise inflation deflating the nominal debt stock. These results are confirmed in the numerical analysis later.

5. OPTIMAL POLICY SIMULATIONS

In this section, we outline the response of the model to shocks, and illustrate the analytical results established earlier. Following the econometric estimates in Leith and Malley (2005) we adopt the following parameter set, $\varphi = 1$, $\sigma = 2$, $\mu = 1.2$, $\overline{\epsilon} = 6$, $\beta = 0.99$, and, following Gali (1994), the share of government consumption in GDP, $1 - \theta = 0.25$. In our benchmark simulations we assume a degree of price stickiness of $\theta_p = 0.75$, which implies that an average contract length of 1 year, and an initial debt–GDP ratio of 60%. However, we also explore the implications of alternative assumptions regarding the degree of price stickiness and the initial steady-state debt stock. The productivity shock follows the following pattern:

$$a_t = \rho_a a_{t-1} + \xi_t,$$

11. The changing balance between fiscal and monetary stabilization of debt under discretion has echoes of the policies observed under the alternative determinate combinations of simple monetary and fiscal policy rules (see, e.g., Leeper 1991, Leith and Wren-Lewis 2000, 2006, Leith and von Thadden 2008).

12. A numerical analysis of the contributions of various policy instruments to debt stabilization is conducted in the following section.

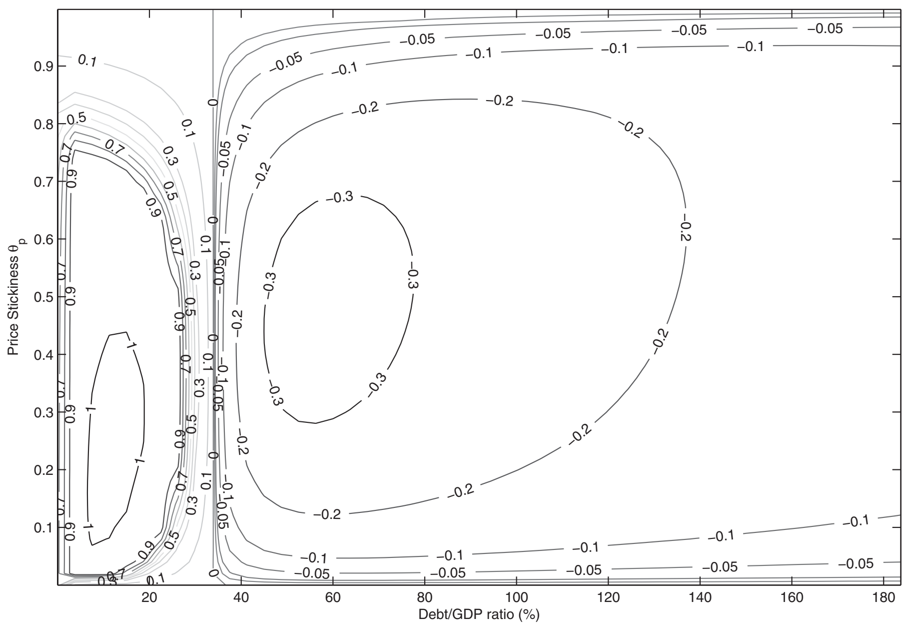

FIG. 1. Evolution of Debt under Discretion as a Function of Price Stickiness and the Debt–GDP Ratio.

where we adopt a degree of persistence in the productivity shock of $\rho_a = 0.99$ as in Ireland (2004).

5.1 Debt under Commitment and Discretion

We begin by confirming that there typically is no random walk in debt under discretion. Figure 1 plots contour lines for the coefficient on the lagged value of debt in the state-space solution under discretion, as a function of the degree of price stickiness and steady-state debt–GDP ratio.[13] This confirms that our benchmark solution is stationary since this coefficient is less than one. It is only at very low levels of the debt–GDP ratio that this coefficient tends to one. The reason is implicit in the analysis mentioned above and will be explored in the following section. In essence, at very low debt–GDP ratios surprise inflation and monetary policy accommodation are less effective in stabilizing debt so the temptation to use them weakens. Instead, adjustment takes place through fiscal instruments, implying a far more gradual stabilization of debt.

13. In this particular figure, government spending is considered to be an instrument in the policy optimization, which relies on monetary and tax policy only. A very similar plot emerges if, instead, government spending is excluded as a policy instrument (see the online appendix available at:http://eprints.gla.ac.uk/70226/).

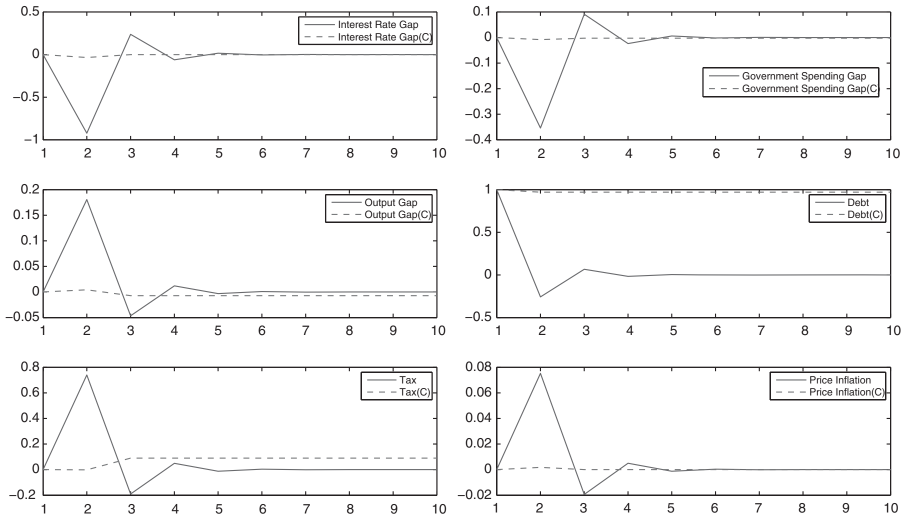

FIG. 2. Response to a Debt Shock under Commitment (C) and Discretion—Endogenous G.

As we increase the size of the debt stock, we find that at relatively modest debt–GDP ratios the sign on the lagged debt stock in the discretionary solution switches from being positive to negative, implying a potential overshooting in debt correction, which will be explored later. This tends to imply greater volatility in the various gap variables in face of shocks, with negative implications for welfare that we note later. It also implies that the response to any deviation of debt from its efficient level will be particularly aggressive under discretion.

This can be seen in Figure 2, which plots the different responses to an unexpected increase in debt under commitment and discretion in the case where government spending is used as an instrument in the optimization.[14] Commitment allows policymakers to exploit the fact that expectations are given in the initial period, and we observe a (small) initial cut in the interest rate gap and government consumption, and increases in taxes and the output gap. This is an example of our finding that in the first period the commitment policy moves to reduce the steady-state increase in debt. This fuels inflation despite the moderating effects of a fall in taxation. As a result, debt is reduced in the initial period, which allows lower increases in taxation and lower falls in output, government spending, and consumption than would otherwise be the case beyond the initial period.

Under discretion we observe that debt returns to its initial (= efficient) level. This requires a more substantial response for all variables. Our analytical results demonstrated that the direction of response of variables depends crucially on the

14. The case where government spending is not utilized as an instrument in the fiscal stabilization is considered in the online appendix available at http://eprints.gla.ac.uk/70226/.

steady-state debt–GDP ratio. In our simulations our chosen debt–GDP ratio of 60% constitutes a "high" level of debt when considering the analytical results derived earlier. This implies that, initially, interest rates are cut. On top of this taxes are also initially increased and this further fuels inflation in the initial period. This serves to reduce the debt stock below its steady-state value in the initial period. Since under discretion governments are performing a period-by-period optimization, this drop in debt results in incentives to move policy instruments in the opposite direction in the following period, but with the same basic pattern outlined in the section on discretion earlier. Instruments then follow a damped cycle until the debt stock has returned to its initial value.[15]

The overshooting of debt under discretion is an interesting result, which stems from the inability of the policymaker to influence expectations except by influencing the levels of endogenous state variables left to the future. As a result, by overshooting when reducing debt, the policymaker is able to mitigate the impact on the initial period's rate of inflation of a given reduction in real interest rates, since the lower debt gives rise to expectations of a mild deflation in the next period. Given the New Keynesian Phillips curve lower inflation expectations in the future reduce current inflation, *ceteris paribus*. Therefore, the policymaker manages to reduce debt service costs at a lower cost of inflation by using debt as a means to influence expectations, which are otherwise outside their control.

Although Figure 2 highlights the nature of the response to an unexpected increase in debt, in our model deviations of debt from its efficient level are the result of the fiscal consequences of the technology shocks introduced in our model.[16] Figure 3 details the paths of key endogenous variables following a positive persistent technology shock under the two policy regimes. The pattern of the gap variables under both commitment and discretion are the same as mentioned above. Under commitment there is an initial decline in debt, and gap variables are then held constant at the levels consistent with the new higher steady-state level of debt. However, in contrast to Figure 2, the eventual return of debt to its steady-state value under discretion is relatively slow. This is a result of the persistence of the technology shock itself, which renders the efficient level of debt equally persistent such that the discretionary solution eliminates any deviations of debt from its efficient level very quickly in both cases.

In terms of welfare, the costs of technology shocks under commitment are 0.000173% of steady-state consumption, while under discretion those costs are 42 times higher at 0.0074%. This is despite the fact that the commitment policy involves a permanent increase in tax and spending distortions in support of a higher level of government debt, while discretionary policy eliminates such distortions

15. An inefficient initial steady-state would create an additional incentive to drive debt to a lower level consistent with efficiency, but the burden of adjustment among instruments under discretion would be similar to that identified here. We consider this case in Section 6.

16. Since in the absence of a need to satisfy the government's budget constraint, the combination of monetary and tax policy could offset the deviations from the flex price solution caused by typical economic shocks such as labor supply, markup, and preference shocks, adding such shocks to our model only matter for optimal policy to the extent that they have fiscal repercussions.

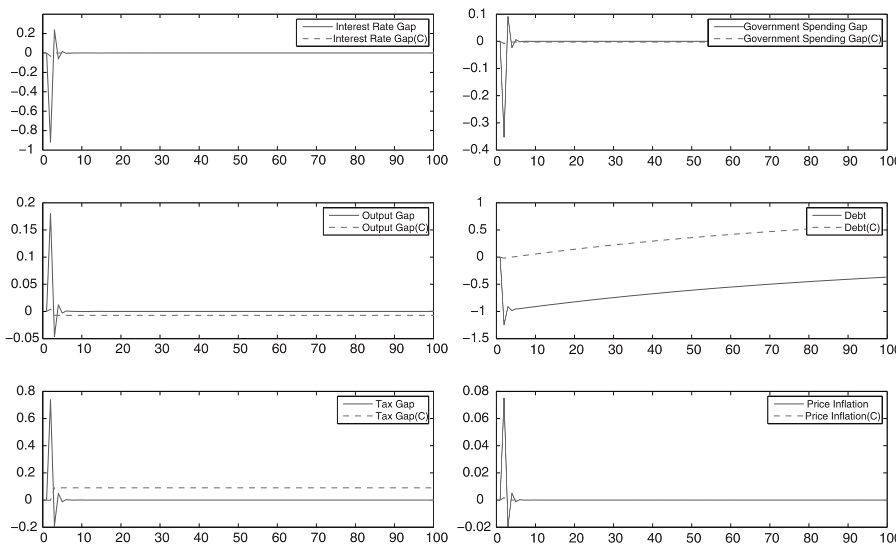

FIG. 3. Response to a Persistent Technology Shock under Commitment (C) and Discretion.

rapidly. The relatively small welfare costs of technology shocks under commitment reflects the fact that such shocks are only costly to the extent that they have fiscal consequences. Under commitment there is only a very partial offsetting of the fiscal consequences of the shock—outside of the initial period instruments are adjusted to service the new debt stock that emerges as a result of the shock, but the adjustment required to pay the interest on the increase in steady-state debt is relatively small. In contrast, policy under discretion completely offsets the fiscal consequences of shocks, which requires greater short-term movement in policy instruments, which then generates the welfare differences across the two policies.

5.2 *Controlling Debt under Discretion*

In light of these results it is informative to know which policy instruments bear the brunt of the adjustment. To do this, we calculate the contribution of tax revenues, government spending, surprise inflation, and lower debt service costs (but excluding surprise inflation) to returning debt to its steady-state value following a shock. The computation of the contribution of each instrument is based on the discounted sum of the fiscal impact of the respective instruments relative to the discounted value of the fiscal consequences of the shock, which needs to be offset. These contour lines are plotted as a function of the steady-state debt–GDP ratio and the degree of price stickiness, θ_P.

Figure 4 reveals that it is only at very low debt levels and high degrees of price stickiness that increased tax revenues play a significant role in returning debt to its steady-state value. While reductions in government expenditure do not contribute to fiscal stabilization except at moderate degrees of price stickiness and relatively low

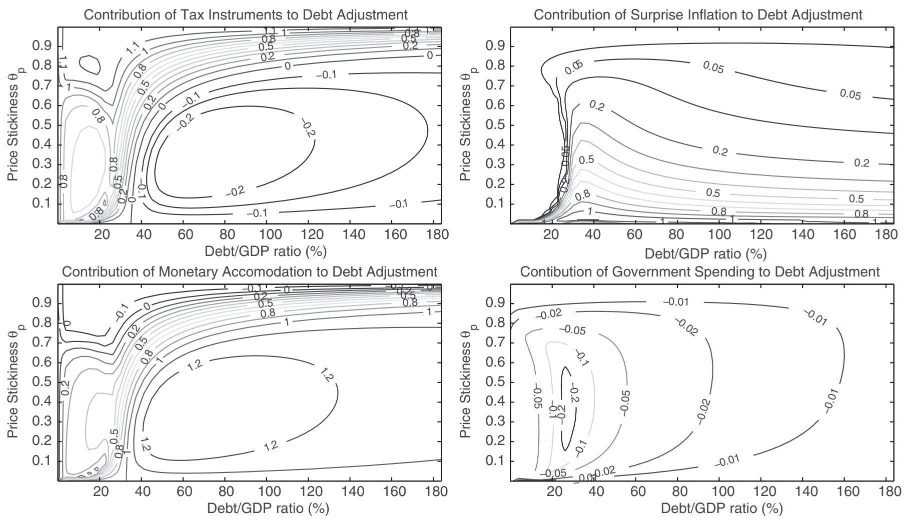

FIG. 4. Contribution of Policy Instruments to Debt Adjustment under Discretion.

levels of steady-state debt and even then only marginally. The lack of government spending adjustment as a means of stabilizing debt is interesting. The IMF (2012) calculates that all of nine major fiscal consolidations currently underway rely heavily on spending cuts as electorates appear to resist tax increases. Our analysis of optimal policy under discretion suggests that such an approach is not optimal and that while there may be some role for tax policy in stabilizing debt, adjusting the government spending gap is not a feature of optimal policy.

With fiscal instruments making little contribution to debt stabilization under discretion for plausible levels of debt and price stickiness, the burden of adjustment must fall on monetary policy. This is confirmed in Figure 4. Here we find that beyond debt–GDP ratios of 30% monetary policy reduces debt service costs in an attempt to return the debt to its initial value. As the debt–GDP ratio rises the efficacy of stabilizing debt in this way increases and it almost single-handedly offsets the fiscal consequences of the shock as debt–GDP ratios rise to 200%. As prices become more flexible, the ability of the monetary authorities to engineer a change in real interest rates is reduced and there is greater reliance on surprise inflation to return debt to its steady state. Interestingly, the use of reduced debt service costs eventually declines as prices become increasingly sticky. However this reflects the fact that at high levels of nominal inertia a relatively modest decline in real interest rates produces a significant rise in the tax base, which boosts tax revenues (this is evident from the increasing importance of tax revenues at high levels of price stickiness).

For our central parameter set with price contracts lasting for 1 year ($\theta_p = 0.75$) and an annualized debt–GDP ratio of 60%, the relative contributions of taxation and government spending to the stabilization of debt are 18.38% and 2.93%, respectively.

Surprise inflation accounts for 7.53% of the required stabilization of debt. However, by far the greatest mechanism for stabilizing the debt comes from reduced debt service costs, which accounts of 78.69% of the adjustment.[17]

6. ROBUSTNESS

In this section, we collect together two exercises that assess the robustness of our results. We begin by relaxing the assumption that our initial steady state is efficient. We then turn to consider the robustness of our results across a wider range of structural model parameters, beyond the debt–GDP ratio and degree of price stickiness emphasized so far.

6.1 Distorted (Initial) Steady State

Up until now we have assumed that a time-invariant steady-state subsidy was in place that ensured our initial steady state (which is also the steady state to which we return under discretion) was efficient. In order to assess the robustness of our results to this assumption, we now assume that the subsidy was not set at a sufficiently high level to completely offset the tax and monopolistic competition distortions. That is, we no longer assume the subsidy is set at the optimal level defined in (13), but that it is set below that level such that there is a wedge, $\Psi = \frac{(1-\varkappa)}{(1-\overline{\tau})}(\frac{\overline{\epsilon}}{\overline{\epsilon}-1}) > 1$, between the marginal utility of consumption and marginal disutility of labor supply under flexible prices,

$$(\overline{C}^n)^{-\sigma} = (\overline{N}^n)^{\varphi}\Psi,$$

such that the steady state under flexible prices is no longer efficient and output will be suboptimally low. As before, we can define all log-linearized relationships dependent on model parameters and the initial debt-to-GDP ratio, but where the steady state around which we log-linearize our sticky-price New Keynesian economy is no longer the efficient one.

Appendix C derives the quadratic approximation to utility when the initial steady state is distorted as

$$\Gamma = -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t \left\{\sigma\Psi\theta(\widehat{C}_t - \widehat{C}_t^* - T_t)^2 + \sigma(1-\theta)\Psi(\widehat{G}_t - \widehat{G}_t^* - T_t)^2 \right.$$
$$\left. + (1-\Psi+\varphi)(\widehat{Y}_t - \widehat{Y}_t^* - T_t)^2 + \frac{\epsilon_t}{\gamma}\pi_t^2\right\}$$
$$+ tip + O[3], \quad (28)$$

17. The policy implications of using interest rates to control debt are considered in Kirsanova, Leith, and Wren-Lewis (2007, 2009). To the extent that government debt maturities exceed the single period considered in this paper, monetary accommodation may be less effective in manipulating the size of the debt stock.

where $T_t = \frac{\Psi-1}{1-\Psi+\varphi+\sigma\Psi}(1 - \frac{(\sigma-1)(1+\varphi)}{\sigma+\varphi}a_t)$ is the "target" for the gap variables. When $\Psi = 1$, this reduces to the welfare measure we derived earlier. Note that in addition to affecting the "target" variable, the degree of inefficiency, Ψ, also affects the relative weights of consumption, government spending, and output. The targets are time varying since they depend on the level of technology, but they also reflect the steady-state inefficiencies, which create a desire on the part of policymakers to permanently increase consumption and output.

We shall see that when the subsidy is insufficient to render the initial steady state efficient, the policymaker will face a temptation to reduce debt levels and the associated costs of distortionary taxation. In fact, they will continue to face such a temptation until debt levels have been reduced by enough to enable the economy to achieve its efficient allocation. It is therefore helpful to derive the efficient level of debt conditional on a given level of subsidy. In other words, since we are assuming that our subsidy is insufficient to achieve efficiency given our initial debt–GDP ratio and supporting income tax rate, we can define the tax rate, which ensures that a given subsidy achieves efficiency. Efficiency requires that $\Psi = 1$, which implies the following steady-state tax rate:

$$\overline{\tau}^* = 1 - (1 - \chi)\left(\frac{\epsilon}{\epsilon - 1}\right).$$

Note that in the absence of a subsidy this requires a negative tax rate.

It is also helpful to define the size of the output gap required to mimic the efficient level of output. From the social planner's problem the efficient level of output is given by

$$\overline{Y}^* = (1 + \chi^{\frac{1}{\sigma}})^{\frac{\sigma}{\delta+\varphi}},$$

while the flexible price steady state is given by

$$\overline{Y}^n = \left(1 + \chi^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma+\varphi}} \Psi^{-\frac{1}{\sigma+\varphi}},$$

such that the percentage increase in output relative to the flex price steady state is given by

$$\Psi^{\frac{1}{\sigma+\varphi}} - 1.$$

Note that the targets derived as part of the social welfare derivation imply, for small distortions, that the efficient steady state is the steady state to which the policymaker acting under discretion will tend to. However, for larger distortions the targets derived later will only be a first-order approximation to the efficient steady state.

Figure 5 plots the response to beginning at the inefficient steady state under discretion and commitment. Under commitment in the initial period there is a mild attempt to reduce the debt level in the initial period when the commitment policy is first formulated. However, beyond that the policymaker commits to maintain the debt level, which requires a suboptimally high tax rate and suboptimally low levels of output. In contrast, under discretion, the policymaker continues to use their policy

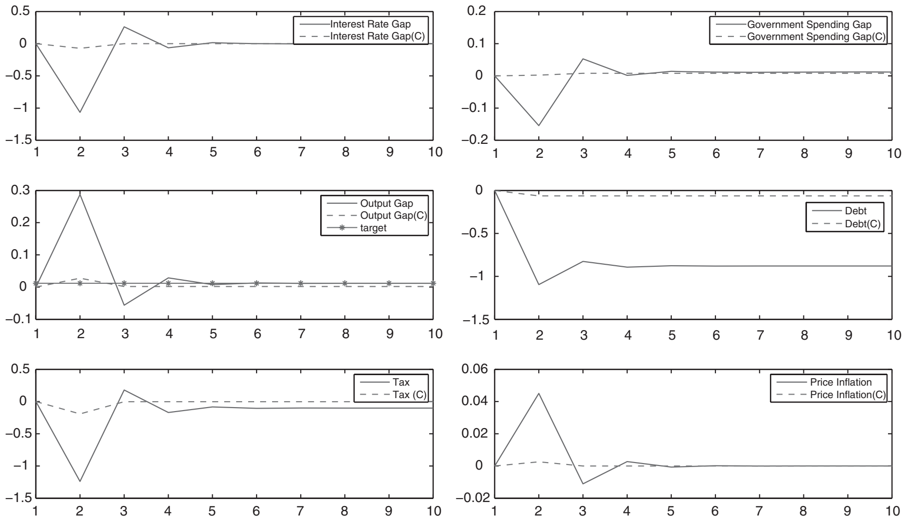

FIG. 5. Impulse Responses Starting from an Initial Distorted Steady State.

instruments to reduce debt to the level at which the tax rate is reduced by enough that the economy enters a new steady state, which now replicates the efficient allocation chosen by the social planner. Again the policy mix used to achieve the desired reduction in debt is very similar to that considered earlier when analyzing the response to shocks, and again it can be shown that this composition will vary depending on debt levels, sticky prices, and other model parameters in the manner described in Section 4. Nevertheless, the key point that emerges is that at plausible debt levels there are very strong incentives for policymakers to undertake policies that would rapidly reduce debt levels to the level that is consistent with an efficient allocation of resources, but that they would ideally commit not to do so due to the short-run costs of achieving this efficient debt level.

6.2 Alternative Parameterizations

While we have systematically varied debt–GDP ratios and the degree of price stickiness in assessing the robustness of results, our model contains two other central parameters, which may affect the magnitude of the effects we describe. Therefore, we assess the implications of adopting alternative values of the intertemporal elasticity of substitution, $\sigma = 1, 2,$ and 3, as well as the inverse of the Frisch elasticity of labor supply, $\varphi = 1, 2,$ and 3.[18] Variations in these parameters do not substantially alter the key results of this paper.

18. These results are contained in an online appendix, which is available online at http://eprints.gla.ac.uk/70226/.

7. QUASI-COMMITMENT

Our results suggest a sharp divergence in the behavior of optimal policy under discretion compared to commitment. However, in the real world, policy may feature an imperfect ability to commit to future policy actions that lies somewhere between the two extremes we have considered so far. In order to assess how much credibility a policymaker must possess in order to come close to mimicking the optimal policy under commitment we employ the notion of quasi-commitment. In the case of quasi-commitment, we nest the cases of both discretion and commitment. Essentially, following Schaumburg and Tambalotti (2007), Debertoli and Nunes (2010), and Himmels and Kirsanova (2013), we allow a policymaker to formulate a commitment plan, but assume with probability $0 \leq \alpha \leq 1$ that the government will renege on that plan. We can think of this as the case where a particular policymaker loses office, at which point a new policymaker is appointed, who will formulate a new plan, but again faces an exogenous probability, α, of not being able to implement that plan in a given period.

When $\alpha = 1$, this reduces to the problem for the discretionary policymaker considered earlier, while when $\alpha = 0$, the policymaker operates under full commitment. For intermediate values the policymaker faces an infinite horizon policy problem, but with probability $(1 - \alpha)$ they will be forced to reoptimize in a given period. The Lagrangian for such a policy problem can be written as

$$
\begin{aligned}
L_0^{qc} = E_0 \sum_{t=0}^{\infty} \Bigg[((1-\alpha)\beta)^t \Bigg[& \sigma\theta(c_t^g)^2 + \sigma(1-\theta)(g_t^g)^2 + \varphi(y_t^g)^2 + \frac{\epsilon}{\gamma}\pi_t^2 \\
& + \alpha\beta V(b_{t+1}, a_t) + \lambda_t^{\pi}\Big(\pi_t - (1-\alpha)\beta\pi_{t+1} - \alpha\beta(\phi 1\widehat{b}_{t+1} + \delta 1 a_t) \\
& - \gamma\Big(\varphi y_t^g + \sigma c_t^g + \frac{\overline{\tau}}{1-\overline{\tau}}\widehat{\tau}_t\Big)\Big) + \lambda_t^y(y_t^g - (1-\theta)g_t^g - \theta c_t^g) \\
& + \lambda_t^b(\widehat{b}_t - \pi_t - \beta(\widehat{b}_{t+1} - (1-\alpha)\pi_{t+1} - \alpha(\phi 1\widehat{b}_{t+1} + \delta 1 a_t) \\
& - \sigma(1-\alpha)c_{t+1}^g - \sigma\alpha(\phi 2\widehat{b}_{t+1} + \delta 2 a_t) \\
& + \frac{\overline{G}}{\overline{b}}g_t^g - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\Bigg((1+\varphi)(y_t^g) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t^g) + \sigma c_t^g\Bigg] + f_t - \sigma\beta c_t^g\Bigg)\Bigg)\Bigg].
\end{aligned}
$$

This implies that although the policymaker is making a commitment, they are factoring into their decision making the knowledge that they may be removed from office and be unable to implement those promises. Accordingly, they are only able to make promises to the extent that economic agents expect them to remain in office to be able to implement those promises. Therefore, their objective function becomes a weighted average of the discounted value of utility under commitment, and the payoff function under discretion, while expectations can only be affected through policy commitments to the extent that those policies are expected to be implemented. Otherwise, expectations depend linearly on endogenous states.

The FOCs from optimization for periods $t > 0$ for c_t^g, g_t^g, y_t^g, π_t, $\widehat{\tau}_t$, and $\widehat{b}_{t+1}$, respectively, are given by

$$2\sigma\theta c_t^g - \gamma\sigma\lambda_t^\pi - \theta\lambda_t^y - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\sigma\lambda_t^b - \sigma\beta\lambda_t^b + \beta\sigma(1-\alpha)\lambda_{t-1}^b = 0,$$

$$2\sigma(1-\theta)g_t^g - (1-\theta)\lambda_t^y + \frac{\overline{G}}{\overline{b}}\lambda_t^b = 0,$$

$$2\varphi y_t^g - \gamma\varphi\lambda_t^\pi + \lambda_t^y - \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(1+\varphi)\lambda_t^b = 0,$$

$$2\frac{\epsilon}{\gamma}\pi_t + \lambda_t^\pi - \lambda_t^b + (1-\alpha)(\lambda_{t-1}^b - \lambda_{t-1}^\pi) = 0,$$

$$-\frac{\overline{\tau}}{1-\overline{\tau}}\gamma\lambda_t^\pi - \frac{\overline{w}\overline{N}}{\overline{b}}\frac{\overline{\tau}}{1-\overline{\tau}}\lambda_t^b = 0,$$

$$\alpha\beta\frac{\partial V(b_{t+1}, a_t)}{\partial b_{t+1}} + \alpha(\beta\phi 1(\lambda_t^\pi + \lambda_t^b) - \sigma\phi 2\lambda_t^b) + E_t\lambda_t^b - \lambda_t^b = 0.$$

These FOCs are very similar to those under commitment, except that the random walk in the Lagrange multiplier for the government's budget constraint is no longer present since debt levels affect both future policy payoffs and expectations to the extent that the policymaker lacks full commitment. Moreover, the influence of past Lagrange multipliers, which captures adherence to past promises made under the commitment plan, are weighted by the probability that the commitment regime remains in place $(1-\alpha)$. Accordingly, in order to solve this policy problem we can make an initial guess of both the parameterization of the value function and the projection equations for expectations of consumption and inflation. Conditional on these guesses we can solve the policy problem under quasi-commitment, before assessing whether or not the guesses are correct. If they are not we update our guesses until they are and we have a quasi-commitment solution, which is consistent with rational expectations over the probability of the policymaker reneging on the commitments they have made.[19]

Figure 6 plots impulse response function (IRF) for debt being 1% above its initial steady-state value, when the probability of reneging on the policy commitments in any period is $\alpha = 0.2$, such that the policymaker's plan is expected to remain in place for 1.25 years. Within the figure we consider three different types of impulse response. In type (i) we do not actually observe any reneging on the policy commitments,

19. A matrix algebra representation of the policy problem, which nests that under commitment and discretion, is given in the online appendix: http://eprints.gla.ac.uk/70226/. The appendix also outlines the solution algorithm developed by Himmels and Kirsanova (2013), which we employ.

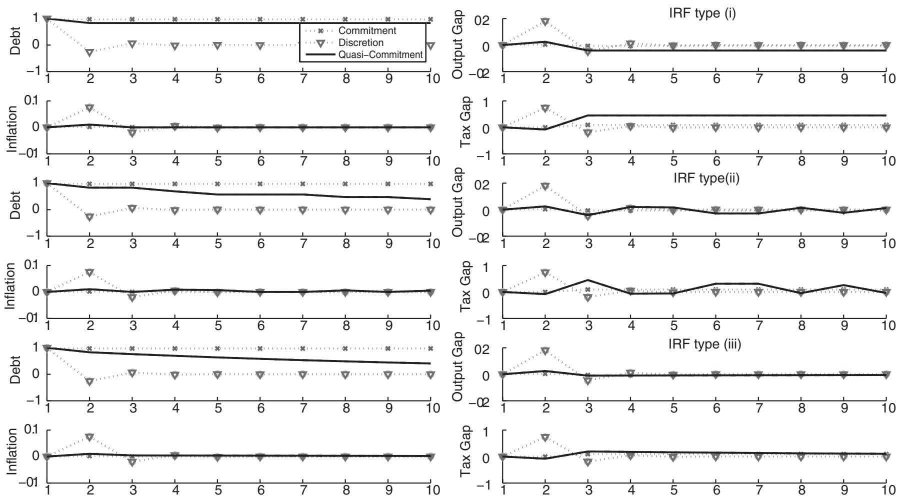

FIG. 6. Impulse Response under Quasi-Commitment—$\alpha = 0.2$.

although both the policymaker and economic agents in the economy anticipate doing so with probability, $\alpha = 0.2$. Under type (ii) impulse responses we allow for observed "defaults" in periods 3, 4, 7, and 9. Finally, in type (iii) impulse responses we plot the *ex ante* average of IRFs across the draws from the reoptimization distribution. Across all types of IRF we also plot the cases of discretion and commitment. Within the type (i) impulse responses, which assume that there is no reneging on commitments, despite the expectation that there would be, the policy looks quite like that under commitment, with the key difference that since future promises are expected to be broken, there is a greater incentive to reduce debt in the first period to reduce the costs of subsequent reoptimizations. This ongoing risk of reneging on previous commitments raises inflationary expectations and the policymaker tightens monetary policy such that *ex ante* real interest rates are unchanged. However, since the reneging on commitment is assumed never to be realized in these subplots there are persistent negative price surprises, which are offset by higher tax rates and lower government spending, such that debt remains constant for as long as the quasi-commitment plan remains in place.

Within the type (ii) IRFs we assume there are realized reoptimizations in periods 3, 4, 7, and 9. Within each reoptimization period the policymaker acts to reduce the inherited debt level before embarking on a new quasi-commitment policy, in much the same way as the policymaker would do in the initial period of commitment. Therefore, each period of reoptimization serves to reduce the debt level until debt disequilibrium is eventually eliminated. Finally, the type (iii) IRF plots the average response to the debt shock under quasi-commitment with a probability of reneging of $\alpha = 0.2$, and we observe a gradual stabilization of debt, which is far slower than

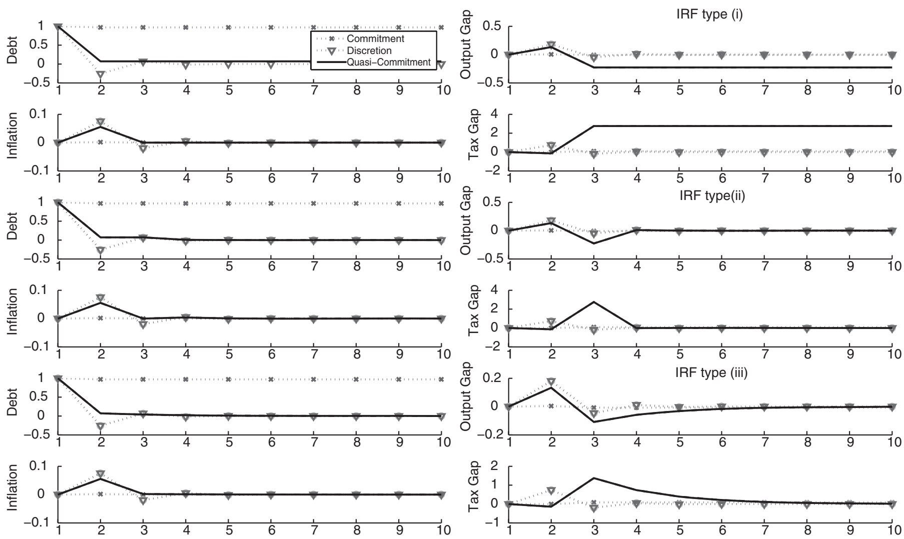

FIG. 7. Impulse Responses under Quasi-Commitment—$\alpha = 0.5$.

under discretion, but which is clearly different from the random walk result found under commitment.

Figure 7 does the same but increases the probability of reneging on previous commitments to $\alpha = 0.5$, implying that any plan is only expected to last for 6 months. As a result in the initial period of the quasi-commitment plan the policymaker comes close to eliminating the debt overhang in a single period. Under the type (i) IRFs we assume that there are no further reoptimizations although economic agents expect there to be so with probability $\alpha = 0.5$. As before, the quasi-commitment plan in the absence of reoptimization has features similar to that of the full-commitment policy, but with the exception that further reoptimizations are expected. Thus, as mentioned above, in the absence of further reoptimizations, inflationary expectations are high and interest rates rise in line with those expectations to leave the *ex ante* real interest rate unchanged and, therefore, consumption constant. However, in the absence of the expected reoptimizations price surprises are negative and taxes must be raised (and government spending cut) to maintain the constant debt levels observed under the quasi-commitment plan. Such a plan, with suppressed levels of private and public consumption, is clearly costly in welfare terms. However, it is really a promise that the government makes in anticipation of default. As the type (ii) IRFs show, as soon as this commitment is overturned such a perverse policy mix can be abandoned and debt is quickly stabilized. As the type (iii) IRFs show the average response is to stabilize debt very quickly and the costly reductions in private and public consumption are eliminated as soon as the quasi-commitment policy is reoptimized.

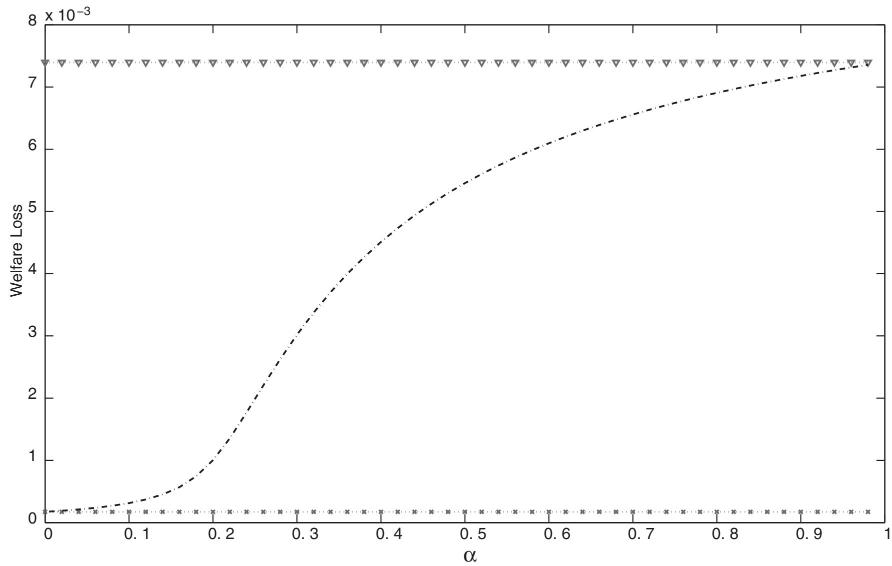

FIG. 8. Welfare Gains from Commitment.

NOTES: Triangles, discretion; crosses, commitment; and the dashed line, quasi-commitment.

The welfare implications of not being able to fully commit to future policy actions are summarized in Figure 8. The relationship between welfare losses and the probability of default against policy promises is monotonic and nonlinear, and how one views the significance of imperfect credibility is a question of interpretation. Since the costs of technology shocks when policymakers have access to both monetary and fiscal policy instruments are so small under commitment, measuring any loss of commitment relative to that base looks significant. For example, a probability of reoptimization of $\alpha = 0.1$ (such that policy promises are expected to be kept for 2.5 years) leads to a doubling of the welfare losses relative to the case of fully credible commitment. However, this is still a long way from the costs observed under discretion, and the welfare costs under quasi-commitment are only closer to those under discretion when policy promises are only expected to last for less than 1 year (approximately $\alpha = 0.35$).

Another way of assessing the importance of commitment is to look at the half-life of debt following a shock. That is, following an increase in debt, how long is it before debt has been reduced by half of that increase? This is reported in Figure 9. Here we see that the random-walk property of debt under commitment is very quickly overturned as we increase the probabilities of reneging on policy promises. Moreover, for any probability of reoptimization $\alpha > 0.3$, over 50% of the debt reduction occurs in the first quarter. Nevertheless, it remains the case that the half-life of debt is longer

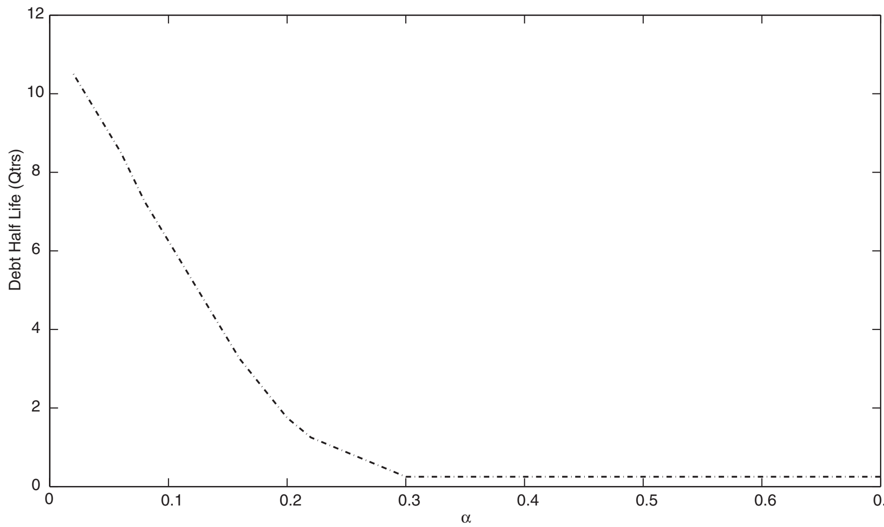

FIG. 9. Half-Life of Debt under Quasi-Commitment.

than the expected time until reoptimization for effective policy horizons in excess of 1 year.

8. CONCLUSIONS

In this paper, we examine the optimal response of government debt to shocks in a New Keynesian model, where welfare is derived from consumers' utility. We analytically derive commitment policy and confirm that steady-state debt would follow a random walk. However, we show that in the initial period under optimal commitment, policymakers exploit the fact that expectations are given by trying to reduce the long-run level of debt. While tax and spending adjustments play a role in this adjustment, for plausible values of the debt–GDP ratio and price stickiness, monetary policy through either inflation surprises or reduced debt service costs also plays a role.

The time inconsistency inherent in commitment policy means that the optimal (time-consistent) discretionary policy for debt is quite different. The random walk result, typically, no longer holds, and instead debt returns to its steady-state level. Only by returning debt to its steady state can the incentive to reduce debt through policy surprises noted under commitment be eliminated. Analytically, we demonstrate that exactly how this is achieved depends crucially on the debt–GDP level and the degree of price stickiness, although will typically not involve any significant adjustment of government expenditure in contrast to the composition of fiscal consolidations currently being undertaken (see IMF 2012). For plausible parameter values, a negative fiscal shock is likely to be unwound in large part by monetary policy engineering a reduction in debt service costs. The time-consistent policy is likely to involve a quite rapid reversal of any fiscal shock. As a result, we show that the welfare consequences of shocks to debt when policy operates under discretion can be significant, which in turn implies that welfare analysis under discretion using models that employ the fiction of lump sum taxes may be incomplete.

Finally, we consider the sustainability of the commitment solution in the context of a policy of quasi-commitment where the policymaker commits to plan, but both she and other economic agents realize that plan maybe abandoned with an exogenous probability α, and a new policymaker draws up a new plan, which the new policymaker may, in turn, renege on. The analysis of this case suggests that relatively modest levels of commitment involving a time horizon in excess of 1 year, can achieve welfare levels closer to those observed under commitment than discretion.

APPENDIX A: POLICY OBJECTIVES

Individual utility in period t is

$$\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}.$$

Before considering the elements of the utility function we need to note the following general result relating to second-order approximations:

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + O[2],$$

where $\widehat{Y}_t = \ln(\frac{Y_t}{Y})$, $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second-order approximation to the first term:

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \overline{C}^{1-\sigma}\left(\frac{C_t - \overline{C}}{\overline{C}}\right) - \frac{\sigma}{2}\overline{C}^{1-\sigma}\left(\frac{C_t - \overline{C}}{\overline{C}}\right)^2 + tip + O[2],$$

where *tip* represents "terms independent of policy." Using the results mentioned above this can be rewritten in terms of hatted variables:

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \overline{C}^{1-\sigma}\left\{\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2 - \sigma\widehat{C}_t\right\} + tip + O[2].$$

Similarly for the term in government spending:

$$\chi \frac{G_t^{1-\sigma}}{1-\sigma} = \chi \overline{G}^{1-\sigma} \left\{ \widehat{G}_t + \frac{1}{2}(1-\sigma)\widehat{G}_t^2 - \sigma \widehat{G}_t \right\} + tip + O[2].$$

The final term in labor supply can be written as

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \overline{N}^{1+\varphi} \left\{ \widehat{N}_t + \frac{1}{2}(1+\varphi)\widehat{N}_t^2 \right\} + tip + O[2].$$

Now we need to relate the labor input to output and a measure of price dispersion. Aggregating the individual firms' demand for labor yields:

$$N_t = \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P(j)_t}{P_t}\right)^{-\epsilon} dj.$$

It can be shown (see Woodford 2003, chap. 6) that

$$\begin{aligned}\widehat{N}_t &= \widehat{Y}_t - a_t + \ln\left[\int_0^1 \left(\frac{P(j)_t}{P_t}\right)^{-\epsilon} dj\right] \\ &= \widehat{Y}_t - a_t + \frac{\epsilon}{2}\operatorname{var}_j\{P(j)_t\} + O[2],\end{aligned}$$

so we can write

$$\begin{aligned}\frac{N_t^{1+\varphi}}{1+\varphi} = \overline{N}^{1+\varphi} &\left\{ \widehat{Y}_t + \frac{1}{2}(1+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2}\operatorname{var}_j\{P(j)_t\} \right\} \\ &+ tip + O[2].\end{aligned}$$

Using these expansions, individual utility can be written as

$$\begin{aligned}\Gamma_t = {} & \overline{C}^{1-\sigma} \left\{ \widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2 \right\} + \chi \overline{G}^{1-\sigma} \left\{ \widehat{G}_t + \frac{1}{2}(1-\sigma)\widehat{G}_t^2 \right\} \\ & - \overline{N}^{1+\varphi} \left\{ \widehat{Y}_t + \frac{1}{2}(1+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2}\operatorname{var}_j\{P(j)_t\} \right\} \\ & + tip + O[2].\end{aligned}$$

Using second-order approximation to the national accounting identity:

$$\theta \widehat{C}_t = \widehat{Y}_t - (1-\theta)\widehat{G}_t - \frac{1}{2}\theta \widehat{C}_t^2 - \frac{1}{2}(1-\theta)\widehat{G}_t^2 + \frac{1}{2}\widehat{Y}_t^2 + O[2].$$

With the steady-state subsidy in place and government spending chosen optimally, the following conditions hold in the initial steady state:

$$\overline{C}^{1-\sigma} = \overline{N}^{1+\varphi}\theta,$$

and

$$\chi \overline{G}^{1-\sigma} = \overline{N}^{1+\varphi}(1-\theta),$$

which allows us to eliminate the levels terms and rewrite welfare as

$$\Gamma_t = \overline{C}^{1-\sigma}\left\{-\frac{1}{2}\sigma\widehat{C}_t^2\right\} + \chi\overline{G}^{1-\sigma}\left\{-\frac{1}{2}\sigma\widehat{G}_t^2\right\} - \overline{N}^{1+\varphi}\left\{\frac{1}{2}\varphi\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2}\mathrm{var}_j\{P(j)_t\}\right\} + tip + O[2].$$

We now need to rewrite this in gap form using the FOCs for the social planner to eliminate the term in the technology shock:

$$\Gamma_t = -\overline{N}^{1+\varphi}\frac{1}{2}\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \sigma(1-\theta)(\widehat{G}_t - \widehat{G}_t^*)^2 + \varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \epsilon\,\mathrm{var}_j\{P(j)_t\}\} + tip + O[2].$$

Using the result from Woodford (2003) that

$$\sum_{t=0}^{\infty}\beta^t\mathrm{var}_i\{p_t(i)\} = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + tip + O[2],$$

we can write the discounted sum of utility as

$$\Gamma = -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \sigma(1-\theta)(\widehat{G}_t - \widehat{G}_t^*)^2 + \varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\gamma}\pi_t^2\right\} + tip + O[2].$$

APPENDIX B: THE BUDGET CONSTRAINT USING GAP VARIABLES

The log-linearized budget constraint is given by

$$\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t) = \beta\widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma(\widehat{C}_{t+1})\} + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t - \sigma(1-\beta)(\widehat{C}_t).$$

Using the labor supply function to eliminate real wages and the definition of efficient output to eliminate the technology shock:

$$\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t) = \beta\widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma(\widehat{C}_{t+1})\} - \sigma(1-\beta)(\widehat{C}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}((1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}\widehat{\tau}_t + \sigma(\widehat{C}_t - \widehat{C}_t^*) + \widehat{Y}_t^*).$$

Gapping the remaining variables and combining shock terms,

$$
\begin{aligned}
\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t - \widehat{C}_t^*) = \beta\widehat{b}_{t+1} &- \beta E_t\{\pi_{t+1} + \sigma(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} - f_t \\
&- \sigma(1-\beta)(\widehat{C}_t - \widehat{C}_t^*) - \frac{\overline{G}}{\overline{b}}(\widehat{G}_t - \widehat{G}_t^*) \\
&+ \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}\Big((1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t) \\
&+ \sigma(\widehat{C}_t - \widehat{C}_t^*)\Big),
\end{aligned}
$$

where

$$f_t = -(\sigma(1-\beta\rho_a) + (1-\sigma)(1-\beta))\frac{(1+\varphi)}{\sigma+\varphi}a_t,$$

captures the fiscal consequences of the technology shocks hitting the economy.

APPENDIX C: DISTORTED STEADY STATE

With the steady-state subsidy in place and government spending chosen optimally, the following conditions hold in the initial steady state:

$$\overline{C}^{1-\sigma} = \Psi\overline{N}^{1+\varphi}\theta,$$

and

$$\chi\overline{G}^{1-\sigma} = \Psi\overline{N}^{1+\varphi}(1-\theta),$$

where $\Psi = 1$ reflects the case where the steady-state subsidy has been chosen to achieve an efficient steady state, while $\Psi > 1$ implies that the subsidy has not been set sufficiently high to offset the distortions due to monopolistic competition and distortionary taxation, which allows us to combine but not eliminate the levels terms and rewrite welfare as

$$
\begin{aligned}
\Gamma_t = {} & \overline{C}^{1-\sigma}\left\{-\frac{1}{2}\sigma\widehat{C}_t^2\right\} + \chi\overline{G}^{1-\sigma}\left\{-\frac{1}{2}\sigma\widehat{G}_t^2\right\} \\
& - \overline{N}^{1+\varphi}\left\{(1-\Psi)\widehat{Y}_t + \frac{1}{2}(1-\Psi+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2}\mathrm{var}_j\{P(j)_t\}\right\} \\
& + tip + O[2].
\end{aligned}
$$

We now need to rewrite this in gap form. To do this recall the log-linearized FOCs for the social planner:

$$-\sigma\widehat{C}_t^* = \varphi\widehat{Y}_t^* - (1+\varphi)a_t,$$

and

$$\widehat{C}_t^* = \widehat{G}_t^*,$$

where the "*" denotes the efficient level of that variable in the face of shocks. From the national accounting identity the latter implies, $\widehat{C}_t^* = \widehat{G}_t^* = \widehat{Y}_t^*$. Using the definition of the optimal level of output in the face of shocks we can eliminate the cross-term in the technology shock,

$$\begin{aligned}
\Gamma_t &= \overline{C}^{1-\sigma}\left\{-\frac{1}{2}\sigma\widehat{C}_t^2\right\} + \chi\overline{G}^{1-\sigma}\left\{-\frac{1}{2}\sigma\widehat{G}_t^2\right\} \\
&\quad - \overline{N}^{1+\varphi}\{(1-\Psi)\widehat{Y}_t + \frac{1}{2}(1-\Psi+\varphi)\widehat{Y}_t^2 - \varphi\widehat{Y}_t^*\widehat{Y}_t \\
&\quad - \sigma\widehat{C}_t^*\widehat{Y}_t + \frac{\epsilon}{2}\mathrm{var}_j\{P(j)_t\} \\
&\quad + tip + O[2].
\end{aligned}$$

This can be factored as

$$\begin{aligned}
\Gamma_t &= -\overline{N}^{1+\varphi}\frac{1}{2}\Big\{\sigma\Psi\theta(\widehat{C}_t - \widehat{C}_t^* - T_t)^2 + \sigma(1-\theta)\Psi(\widehat{G}_t - \widehat{G}_t^* - T_t)^2 \\
&\quad + (1-\Psi+\varphi)(\widehat{Y}_t - \widehat{Y}_t^* - T_t)^2 + \frac{\epsilon}{2}\mathrm{var}_j\{P(j)_t\}\Big\} \\
&\quad + tip + O[2],
\end{aligned}$$

where $T_t = \frac{\Psi-1}{1-\Psi+\varphi+\sigma\Psi}(1 - \frac{(\sigma-1)(1+\varphi)}{\sigma+\varphi}a_t)$.

Using the result from Woodford (2003) relating the discounted sum of price dispersion to quadratic terms in inflation, we can write the discounted sum of utility as

$$\begin{aligned}
\Gamma &= -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Big\{\sigma\Psi\theta(\widehat{C}_t - \widehat{C}_t^* - T_t)^2 + \sigma(1-\theta)\Psi(\widehat{G}_t - \widehat{G}_t^* - T_t)^2 \\
&\quad + (1-\Psi+\varphi)(\widehat{Y}_t - \widehat{Y}_t^* - T_t)^2 + \frac{\epsilon}{\gamma}\pi_t^2\Big\} + tip + O[2],
\end{aligned}$$

where $T_t = \frac{\Psi-1}{1-\Psi+\varphi+\sigma\Psi}(1 - \frac{(\sigma-1)(1+\varphi)}{\sigma+\varphi}a_t)$.

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