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# EFFECT OF WIDTH OF A VERTICAL PARALLEL PLATE CHANNEL ON THE TRANSITION OF THE DEVELOPING THERMAL BOUNDARY LAYER

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**ABSTRACT** Numerical simulations were performed to study the transition of the development of the thermal boundary layer of air along an isothermal heated plate in a large channel which is bounded by an adiabatic plate. In particular, the aim is to investigate the effects of the channel width, *b*, on the transition of the flow under various ambient air and plate temperatures. Three different RANS based turbulent *k*- $\varepsilon$  models, namely standard, RNG and Realizable with an enhanced wall function were employed in the simulations. The channel width was varied from 0.04 m to 0.45 m and the numerical results of the maximum values of the flow velocity, turbulent kinetic energy were recorded along the vertical axis to examine the critical distance of the developing flow. The results show that the transition delays when the width is increased from 0.04 m to 0.08 m, and particularly, the critical distance at *b* = 0.08 m reaches its maximum with the Grashof number of 2.8 × 10<sup>10</sup>. However, when *b* is increased further from 0.08 m to 0.45 m, the critical distance drops, indicating an early transition of the flow. Comparisons of the selected numerical results are made with available experimental data of turbulent flow, and a satisfied agreement is received.

#### NOMENCLATURE

$A_0, A_{S_1} C_1, C_2, C_{1\varepsilon}, C_{2\varepsilon}$ and $C_{3\varepsilon}$ model constants			
b	channel width (m)		
$C_p$	air specific heat capacity (J/kg°C)		
g	gravitational acceleration $(m/s^2)$		
Gr	Grashof number		
$G_k$ , , $G_b$	rate of turbulent kinetic energy and buoyancy force respectively		
k	kinetic energy of turbulence $(m^2/s^2)$		
L	channel height (m)		
'n	mass flow rate (kg/s)		
Nu	average of Nusselt number		
$n_y, n_x$	number of nodes in yand x directions respectively		
$p, p_o$	static and ambient pressure (N/m <sup>2</sup> )		
Pr	Prandtl number		
$q_p$	heat flux of the plate $(W/m^2)$		
Ra	Rayleigh number		
Т	temperature (°C)		

и, v	velocity components in the x and y directionsrespectively (m/s)
<i>x</i> , <i>y</i>	Cartesian coordinates (m)

## **Greek Symbols**

в	thermal expansion coefficient
Γ	exchange coefficient for general transport defined as $\mu/Pr$
ρ	density(kg/m <sup>3</sup> )
v	kinematic viscosity $(m^2/s)$
8	dissipation rate of turbulent kinetic energy $(m^2/s^2)$
$C_{\prime\prime}$	turbulent viscosity (Pa.s)
μ	dynamic viscosity coefficient (Pa.s)
$\mu_t$	turbulent molecular viscosity
$\sigma_t$	constant of turbulent Prandtl number
$\sigma_k, \sigma_\epsilon$	turbulent Prandtl number for k and $\varepsilon$ respectively
α	thermal diffusivity $(m^2/s)$

#### **Subscripts**

Α	air
Р	plate
е	experimental
n	numerical
С	critical
in	inlet

#### **INTRODUCTION**

The fluid flow and heat transfer between two vertical plates have applications in many widely used engineering systems, for example, cooling and heating industrial and electronic equipment such as transistor, mainframe computer, plate heat exchanger, solar energy collectors, and cooling of nuclear reactor fuel element. Therefore, over the past decades numerous numerical as well as experimental researches have been performed to investigate the behaviour of developing heat flow inside channels made by two vertical plates. The mechanism of the flow inside a channel can be divided into three distinct groups: free convection, forced convection and mixed convection. The movement of mass in free convection depends on the density gradient which is driven by the buoyancy force of the flow, while the flow in forced convection is driven by other external forces. On the other hand, mixed convection is driven by both free and forced convections.

Free convection has received much attention due to its wide variety of applications, e.g. by using natural ventilation a building can reduce a conventional heating cost by 30% to 70% as reported in Liping and Angui [2004]. So, this is substantially reducing the energy consumption in a modern building. Specifically, in a vertical parallel plate channel, free convection develops when at least one of the plates is heated. But the transition of flow from laminar to turbulent usually depends on several parameters such as the properties of the flow, the channel geometry, the difference of the temperature between the heated plates, and the ambient conditions.

In 1942 the first experimental work on the buoyancy driven convection flow in a vertical channel was done by Elenbaas [1942]. The work was done on three squares, which had the dimensions of  $5.95 \times 5.95$  cm<sup>2</sup>,  $12 \times 12$  cm<sup>2</sup> and  $24 \times 24$  cm<sup>2</sup> respectively. The plate thickness for the first two cases was 6 mm, while for the last one was 10 mm. The plates were isothermal and the ambient temperature was varied from 283K to 609K. The results were presented for the different ranges of inclination of the plates from 0° to 90° and particular attention was given to the prediction of the heat transfer coefficient of the air flow. A detailed result of the thermal characteristics of cooling by the free convection was also reported.

Numerical analysis of turbulent convection of air flow in a symmetrically heated, vertical parallel plate channel was done by Fedorov and Viskanta [1997]. The heated plate was tested with different values of the heat flux from 80 to 208 W/m<sup>2</sup>. A low Reynolds number k- $\varepsilon$  turbulent model was used in this analysis, and a finite difference numerical solution technique was employed to solve the two dimensional momentum and energy equations. The results of the local heat flux and Nusselt number distributions have been reported to improve understanding of the turbulent flow and compared with the experimental data of Miyamoto et al. [1986]. The results indicate that the turbulent intensity at the inlet of the channel has some effect on the transition of the flow from laminar to turbulent state, and particularly, an increase in the turbulent intensity moves the transition point further.

Two dimensional numerical simulations of turbulent natural convection in a heated channel were recently performed by Said et al. [2005] and Badr et al. [2006]. The governing equations were solved by the finite volume discretization method assuming all the thermal properties of the air to be constant, except the density which was solved by the Boussinesq approximation. The work of Badr et al. [2006] covered a wide range of thermo physical aspects of the problems and geometrical ratios of the model, and they compared their results with the experimental data of Miyamoto et al. [1986].

Another work on natural convection in a vertical channel was reported by Ben-Mansour et al. [2006]. In the study different inlet boundary conditions of the turbulent flow were tested e.g. uniform inlet pressure, adiabatic inlet, and uniform inlet velocity. The numerical results of five different k- $\varepsilon$  turbulent models (standard turbulent model, low Reynolds number k- $\varepsilon$  Launder and Sharma [1974], low Reynolds number k- $\varepsilon$  of Yang and Shih [1993], Renormalization Group (RNG) k- $\varepsilon$  turbulent model, and Reynolds stress turbulent model) were presented. The numerical model was developed as steady state with two dimensional flow, and the results showed that the thermal fields which obtained by the low Reynolds number k- $\varepsilon$  model of Yang and Shih [1993] with a uniform pressure inlet condition was very close to the experimental results of Miyamoto et al. [1986].

As evident from the summary of literature survey, the natural convection between two vertical plates has extensively been studied by numerous researchers, and many published papers presented the main characteristics of flow with different boundary conditions. The majority of these presented the results of velocity, temperature and turbulence parameters in the vertical direction of the flow; but none of these papers provides information about the transition of the flow inside the channel and importantly how the transition is affected by the merging of the two growing boundary layers inside the channel. In this study our particular focus is to investigate the effect of the channel width on the transition stage.

## **MODEL GEOMETRY**

The channel is formed by the two vertical plates with length *L*, and the distance between the plates is denoted by *b*. The wall on the left side is isothermal, while the other is adiabatic. The numerical simulation considered to be two dimensional free convection and steady state. The air is chosen to be the test fluid. The model geometry along with the Cartesian coordinate system is shown in Figure 1. For the comparison with the experimental data, the model geometry is studied under the same physical dimensions of the experimental model of Yilmaz and Fraser [2007], where L = 3.0 m and b = 0.1 m.



Figure 1. The vertical parallel plate channel.

# MATHEMATICAL FORMULATION

**Governing Equations** For a two dimensional incompressible fluid flow the conservation equations of momentum, mass and energy are written in the following forms as reported in Markatos and Pericleous [1984] and Bacharoudis et al. [2007].

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{1}$$

$$\frac{\partial(\rho uu)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y}\right)$$
(2)

$$\frac{\partial(\rho v u)}{\partial x} + \frac{\partial(\rho v v)}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y} \left(\mu \frac{\partial v}{\partial y}\right) + g(\rho - \rho_0)$$
(3)

$$\frac{\partial(\rho uT)}{\partial x} + \frac{\partial(\rho vT)}{\partial y} = \frac{\partial}{\partial x} \left( \Gamma \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma \frac{\partial T}{\partial y} \right)$$
(4)

The conservation equations (1–4) are solved directly for the natural convection flow in the laminar region, while in the turbulent region both the  $\Gamma$  and  $\mu$  are replaced by the effective values,  $\Gamma_{eff}$  and  $\mu_{eff}$ , as presented in Nicholas and Markatos [1978] and Bacharoudis et al. [2007] and defined as  $\mu_{eff} = \mu + \mu_t$  and  $\Gamma_{eff} = \mu/Pr + \mu_t/\sigma_t$  respectively. In this study a compression among the three different k- $\varepsilon$  models (standard, RNG and Realizable) are made. Note that the major differences between these models are the ways to determine the turbulent viscosity  $\mu_t$ , the turbulent dissipations energy  $\varepsilon$ , and the turbulent kinetic energy k, described below.

**Standard** *k*- $\varepsilon$  **Model** The standard *k*- $\varepsilon$  model was initially proposed by Launder and Spalding [1972], and based on the two terms: the dissipation rate  $\varepsilon$ , and the turbulent kinetic energy *k*. The model transport equation for  $\varepsilon$  is obtained by employing physical reasoning, while the turbulent kinetic energy *k* is derived from an exact equation by assuming that the effect of the molecular

viscosity of the flow is negligible and the flow is fully developed. The distribution of the turbulent kinetic energy k and the rate of dissipation  $\varepsilon$  can be determined from the following equations:

$$\frac{\partial(\rho uk)}{\partial x} + \frac{\partial(\rho vk)}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] + G_k + G_b - \rho \varepsilon$$
(5)

$$\frac{\partial(\rho u\varepsilon)}{\partial x} + \frac{\partial(\rho v\varepsilon)}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial y} \right] + G_{1\varepsilon} \frac{\varepsilon}{k} (G_k + G_{3\varepsilon} G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k}$$
(6)

where

 $\sigma_k$ ,  $\sigma_{\varepsilon}$  are the Prandtl number for k and  $\varepsilon$ , taken as 1.0 and 1.3, respectively.

 $G_k$  is the rate of the kinetic energy determined from  $\mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \frac{\partial u_i}{\partial x_j} - \frac{2}{3}\rho k \delta_{ij} \frac{\partial u_i}{\partial x_j}$ .

δ<sub>ij</sub> is the Kronecker delta ( $δ_{ij}$ =1.0 if *i*=*j* and  $δ_{ij}$ =0.0 if *i*≠*j*).  $G_b$  is the buoyancy force =  $g_i β \frac{\mu_t}{\sigma_t} \frac{\partial T}{\partial x_i}$  and  $\mu_t$  is the turbulent viscosity =  $ρC_μ k^2/ε$ .

The adjustable constants are  $C_{1\varepsilon} = 1.44$ ,  $C_{2\varepsilon} = 1.92$ ,  $C_{3\varepsilon} = 1.0$  and  $C_{\mu} = 0.09$ .

**RNG** *k*- $\varepsilon$  **Model** The renormalization group (RNG) was derived by Yakhot et al. [1992] with an extensive analysis of the eddy viscosity model. The RNG *k*- $\varepsilon$  model carries an additional factor in the turbulent dissipation energy ( $\varepsilon$ ) equation, which leads to improving the accuracy of strain in the flow. The RNG *k*- $\varepsilon$  model has a similar form of kinetic and dissipation energy equations compared with the standard model:

$$\frac{\partial(\rho uk)}{\partial x} + \frac{\partial(\rho vk)}{\partial y} = \frac{\partial}{\partial x} \left[ \alpha_k \mu_{eff} \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \alpha_k \mu_{eff} \frac{\partial k}{\partial y} \right] + G_k + G_b - \rho \varepsilon$$
(7)

$$\frac{\partial(\rho u\varepsilon)}{\partial x} + \frac{\partial(\rho v\varepsilon)}{\partial y} = \frac{\partial}{\partial x} \left[ \alpha_{\varepsilon} \mu_{eff} \frac{\partial \varepsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \alpha_{\varepsilon} \mu_{eff} \frac{\partial \varepsilon}{\partial y} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon} C_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} - R_{\varepsilon}$$
(8)

where  $\alpha_k = \alpha_{\varepsilon} \approx 1.393$  are the inverse effective Prandtl numbers, and  $R_{\varepsilon}$  is an additional term related to the mean strain and turbulence quantities. The models constant are  $C_{I\varepsilon} = 1.42$ ,  $C_{2\varepsilon} = 1.68$  and  $C_{3\varepsilon} = 1.0$ .

The turbulent viscosity ( $\mu$ ) is determined from the equation,  $\mu = \rho C_{\mu} k^2 / \varepsilon$  with  $C_{\mu} = 0.0845$ , whereas the effective viscosity is calculate by  $v_v = \mu_{eff} / \mu$ , where  $d\left(\frac{\rho^2 k}{\sqrt{\varepsilon\mu}}\right) = 1.72 \frac{v_v}{\sqrt{(v_v^3 - 1 + C_v)}} dv_v$  and  $C_v \approx 100$ .

**Realizable** k- $\varepsilon$  **Model** Realizable k- $\varepsilon$  model was proposed by Shih et al. [1995], which was derived based on the dynamic equations of the mean square fluctuation at a large range of Reynolds number, so the model has an ability to control the Reynolds stresses. The conservation equations are

$$\frac{\partial(\rho uk)}{\partial x} + \frac{\partial(\rho vk)}{\partial y} = \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial y} \right] + G_k + G_b - \rho \varepsilon$$
(9)

$$\frac{\partial(\rho u\varepsilon)}{\partial x} + \frac{\partial(\rho v\varepsilon)}{\partial y} = \frac{\partial}{\partial x} \left[ \left( \mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \left( \mu + \frac{\mu_t}{\sigma_{\varepsilon}} \right) \frac{\partial \varepsilon}{\partial y} \right] + \rho C_1 S_{\varepsilon} - \rho C_2 \frac{\varepsilon^2}{\varepsilon + \sqrt{v\varepsilon}} + G_{1\varepsilon} \frac{\varepsilon}{k} C_{3\varepsilon} G_b$$
(10)

where,  $C_1 = \max[0.43, \eta/(\eta + 5)]$  and  $\eta = Sk/\varepsilon$ . A new formula is used to compute the eddy viscosity as reported by Shih et al. [1995]; and  $\mu_t = \frac{\rho k^2}{\varepsilon} \left(A_0 + A_S \frac{kU^*}{\varepsilon}\right)^{-1}$  where  $A_0$  and  $A_S$  are the model constants given as  $A_0 = 4.04$  with  $A_s = \sqrt{(6\cos \emptyset)}$  and  $\emptyset = 1/3\cos^{-1}(\sqrt{6W})$ . The formulation of  $U^*$  and W depend on the angular velocity and more details can be found in Shih et al. [1995]. Note that the model constants in this model are  $C_{1\varepsilon} = 1.44$ ,  $C_2 = 1.9$ ,  $C_{3\varepsilon} = 1.0$ ,  $\sigma_k = 1.0$  and  $\sigma_{\varepsilon} = 1.2$ .

#### **BOUNDARY CONDITIONS FOR THE MODEL**

At the inlet, ambient temperature and pressure conditions are applied so that the air velocity is accelerated from the rest to the induced velocity *v*. A uniform temperature is imposed at the heated plate of channel while the other plate is adiabatic. According to Marcondes and Maliska [1999], Zamora and Hernandez [2001] and Bacharoudis et al. [2007], the temperature and pressure at the outlet of the channel are set to the ambient conditions.

No slip boundary condition is imposed on the velocity components at the walls, where  $u_i = 0$ , for  $0 \le y \le L$ , x = 0 and x = b.

The thermal boundary conditions for the heated and the adiabatic plate are defined respectively as T = Tp, for  $0 \le y \le L$ , x = 0 and  $\frac{\partial T}{\partial x} = 0$ , for  $0 \le y \le L$ , x = b.

Moreover, the turbulent kinetic energy vanishes at the wall, so k = 0, for  $0 \le y \le L$ , x = 0 and x = b.

It is important to note here that when a comparison with the experimental data of Yilmaz and Fraser [2007] is presented, the model is studied under the experimental conditions where the inlet and outlet temperature are assumed to be 23°C. The isothermal conditions for the heated plate is set at the temperature of 100°C, and the turbulent intensity at the inlet is measured to be 13%. Moreover, the distributions of the characteristics of the flow have large gradients near the wall so an enhanced wall function is used in the boundary.

#### NUMERICAL TECHNIQUES

The finite volume method is used to discretise the governing equations with the second order upwind scheme to solve the discretised equations, where the unknown quantities at the cell faces are computed by using a multidimensional linear reconstruction approach of Barth and Jespersen [1989] to achieve higher order all the cell faces through a Taylor series expansion of the cell centered solution about the cell centroid.

The SIMPLE algorithm of Patankar [1980] is then employed to solve the pressure based equation which is derived from the momentum and continuity equations such that the velocity and pressure fields are coupled to each other and solved by adopting an iterative solution strategy. In this algorithm, the discretised momentum equations are initially solved by using an assumed pressure field  $(p^*)$  to yield the velocity solutions  $(u^*, v^*)$ . A correction to the pressure (p') is then applied as the difference between the actual and assumed pressures to achieve a better approximation of the pressure field using  $p = p' + p^*$ . Similarly, the velocity components are corrected to get (u, v), then the discretised equations of a scalar quantity e. g. the energy (T), turbulent energy (k) and its dissipation rate  $(\varepsilon)$  are solved using the most updated results of (u, v, p). All these equations are

solved sequentially and iteratively, and the final numerical solutions are achieved when the residuals of the continuity and velocity components become less than  $10^{-6}$ . The residual of the energy, turbulent kinetic energy and its dissipation is reduced to  $10^{-8}$  to avoid any sensitivity to the solutions of the turbulent fluctuating components.

# MESH INDEPENDENCE TEST

Mesh independence study was initially carried out on the Realizable turbulent model to find out a suitable combination of the grid sizes which will be applicable to resolve the flow inside the channel. This mesh independence study was performed by changing the total number of grid nodes in both the vertical ( $n_y$ ) and horizontal ( $n_x$ ) directions by using five different grid resolutions e.g. 22 × 220, 30 × 370, 120 × 370, 200 × 370 and 300 × 370.

Figure 2 shows the results of the heated air velocity (v), turbulent kinetic energy (k) and air temperature in three different vertical locations, y = 0.09 m, 1.5 m and 3.0 m. As can be seen both the coarse (especially in the horizontal direction) grids,  $22 \times 220$  and  $30 \times 370$ , provide somehow satisfactorily results for the velocity and temperature fields, but severely overestimate the turbulent kinetic energy production inside the channel. The next three finer grids,  $120 \times 370$ ,  $200 \times 370$  and  $300 \times 370$ , produce most satisfactorily results since the differences among the results found are very small indeed and almost negligible. Therefore, either of these three could be used, however to avoid any undesirable discrepancies in the numerical results the grid size of  $200 \times 370$  is chosen to perform all the numerical simulations.

# ASSESSMENT OF THE DIFFERENT TURBULENT MODELS

Figure 3 (a) presents the distribution of the temperature of air at the outlet of the channel predicted by the three turbulent models. A comparison with the experimental data of Yilmaz and Fraser [2007] is made; and it is found that the Realizable turbulent model gave the best predicted results compared to the RNG and standard models.

The velocity profiles are presented in Figure 3 (b) at the two different vertical locations, 0.09 m and 2.94 m. The velocity at y = 0.09 m shows a symmetric profile, as this location is very close to the inlet, so the effect of the adiabatic plate on the velocity is negligible. But, this was not the case in the experimental data, which show some asymmetries near the adiabatic plate close to x = 0.09 m, and surprisingly, none of the simulations accurately predicts this behaviour, and the velocity profile is approximately smooth and flat in the centre of the channel. With increase in the vertical distance of the heated plate, at y = 2.94 m, the peak of the velocity profile appears beside the heated plate due to the fact that the buoyancy force beside the heated wall is very large and has a profound effect on the flow development near the wall.

Over all, comparing among the three different numerical models, the velocity profile close to the inlet, at y = 0.09 m, by the RNG turbulent model shows a very good agreement with the experimental data, especially within the range  $0 \le x \le 0.08$  m. However, near the outlet of the channel, e. g. at y = 2.94 m, the Realizable turbulent model gives the numerical results which are close to the experimental data.

The numerical prediction of the turbulent kinetic energy shown in Figure 3 (c) is lower than the experimental data. At y = 2.94 m the turbulent kinetic energy reaches its maximum at the centre of the channel, which has an agreement with the experimental data. And, particularly, the Realizable model predictions are more close to the experiment than the other two models.



Figure 2. Mesh independence test showing on the results of the velocity, turbulent kinetic energy and air temperature at different locations along the channel.



Figure 3. Outlet temperature (a), velocity (b) and turbulent kinetic energy (c).

Over all, seeing all the results in Figure 3, by employing the enhanced wall function with the three different turbulent models (standard, RNG and Realizable), the percentages of error to calculate the velocity near the outlet are found to be 8.97%, 9.7% and 6.3% respectively. The same comparison can be made for the outlet temperature, which are 1.75%, 1.73% and 1.55% respectively. Therefore, it is clear that the enhanced wall function is capable of predicting the distribution of the outlet temperature with all the models of the turbulent flow. However, comparatively the Realizable turbulent model performed best, therefore it is selected to perform all the other numerical simulations for studying transition and the results are presented in the following sections.

# EFFECT OF THE WIDTH OF THE CHANNEL ON THE TRANSITION STAGE

Numerical results of the velocity, turbulent kinetic energy and local heat flux are presented in this section to investigate the effect of the width of the channel on the process of transition occurring inside the channel. The heated plate of the channel is kept as isothermal at 70°C with the inlet air temperature of 15°C, but the width of the channel is varied from 0.04 m to 0.45 m to study the transition effects.

To avoid any ambiguity in presenting the numerical results, the profiles are divided into two groups: the first group presents the results of the width from b = 0.04 m to b = 0.10 m; whereas the second one from b = 0.15 m to b = 0.45 m.

Figure 4 (a) shows that the maximum velocity for the width b = 0.04 m to 0.10 m reaches its maximum at the transition point, then it starts to drop slightly and after that the growth becomes approximately constant towards the downstream of the channel. For the turbulent kinetic energy in Figure 4 (b), its value is initially zero at the inlet as well as at the laminar region of the channel. But the kinetic energy grows rapidly in the region where the transition begins, and the growth remains steady within the whole transition region, followed by a sharp increase seen in the most cases within the region of the turbulent flow. Specifically, for a small width of the channel, e.g. at b = 0.04 m and 0.05 m, the kinetic energy reaches its peak approximately at  $y \approx 1.5$  m, while for b = 0.06 m and 0.10 m the location of peaks becomes later at y = 1.8 m and 2.4 m respectively.

In the second group, the results in Figure 4 (c) show that the growth of the velocity profile at b = 0.15 m to 0.45 m is gradual in the laminar flow region, but just after the transition point it drops slightly and then increases again towards the downstream because of the presence of turbulence. The nature of the growth in the kinetic energy in Figure 4 (d) for b = 0.15 m and 0.2 m remains quite similar, e.g. rapid rise at the beginning of the transition and then steady within the transition region, and finally sharp rise at the downstream. However, the transition region found for b = 0.30 m and 0.45 m is relatively small compared to the other cases, and the growth of the kinetic energy becomes similar to that of the one-heated plate case.

Since the peak of the temperature and velocity distributions from the heated plate can be considered to be a starting point of the transition stage according to Katoh et al. [1991], where the location of a minimum point of the local heat flux should also correspond to the transition point, the three different parameters of the heated air such as the velocity, turbulent kinetic energy and local heat flux are used to predict the transition point inside the channel with different values of the width. The values of the critical distance at the transition point are derived from the velocity and the kinetic energy already presented in Figure 4 and now summarised in Figure 5.

Figure 5 shows that the transition points predicted by the turbulent kinetic energy are very close to those estimated by the velocity distribution in all the cases. In addition, for a large channel width  $b \ge 0.1$  m the local heat flux also predicts the critical distance that agrees well with other two. However, for a small channel width e.g. b = 0.04 m to 0.08 m Figure 5 shows that the local heat flux reaches its minimum further upstream of the channel and does not agree with the predictions of the velocity and the turbulent kinetic energy. The possible reason for that is that both the thermal and the thickness of the boundary layer increase sharply in the turbulent flow compared to the laminar flow due to the effects of the buoyancy force on the heated plate. Additionally, at a small width of the channel there is not enough space available for the boundary layer to grow fully which

would eventually carry the heat from the heated plate; so as a consequence, the specific heat transfer of the air reaches its maximum temperature early.



Figure 4. The distribution of the velocity (a, c) and the turbulent kinetic energy (b, d) inside the channel at  $Ta = 15^{\circ}$ C and  $Tp = 70^{\circ}$ C.



Figure 5. The critical distance derived from the velocity, kinetic energy and plate heat flux at  $Tp = 70^{\circ}$ C and  $Ta = 15^{\circ}$ C.



Figure 6. Grashof number at the critical distance.

Moreover, Figure 5 shows also that the location of the transition point shifts to a higher distance towards the downstream of the channel when the channel width is increased from b = 0.04 m to 0.08 m. And, particularly, at b = 0.08 m the critical distance reaches its maximum of 1.53 m in the middle of the channel, where the Grashof number is calculated as a function of this critical distance of the velocity to be  $2.8 \times 10^{10}$  (Figure 6). However, as seen, the transition occurs early when the channel width is increased from b = 0.10 m to 0.45 m, and the critical distance at the channel width of 0.45 m becomes similar to that obtained by the one vertical heated plate since the effect from the adiabatic plate at a far distance is negligible. Importantly, the Grashof number, Figure 6, at the critical distance for  $Ta = 15^{\circ}$ C and b = 0.45 m is estimated to be  $2.27 \times 10^{9}$  (1.17×10<sup>9</sup> for the one plate case) which shows a close agreement to ~10<sup>9</sup> obtained by Bejan [1995] in the free convection flow on an isothermal vertical plate.

#### CONCLUSION

The numerical results of the transition inside a channel have been presented in this paper, giving particular attention to the effects of the channel width on the distributions of the velocity, kinetic energy and heat flux plate. The numerical results in the turbulent flow have been compared satisfactorily with the experimental data along with the three different turbulent models, namely the

standard, RNG and Realizable. Importantly, an enhanced wall function was applied to the surfaces to account the developing boundary layers, and the results showed that the Realizable turbulent model gave the least error compared to the other two models.

The distributions of the flow velocity and the turbulent kinetic energy along the vertical channel estimate the critical distance of the developing flow to be very close. We have found that the width of the channel has a major effect on the transition. Importantly, with an increase in the channel width from b = 0.04 m to 0.08 m the transition delays, and particularly, the critical distance reaches its maximum at b = 0.08 m with the Grashof number of  $2.8 \times 10^{10}$ . On the other hand, the transition occurs early when the channel width is increased from 0.08 m, and the critical distance at b = 0.45 m becomes similar to that of the single plate case.

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