Minisci, E.A. and Avanzini, G. and D'Angelo, S. and Dutto, M. (2008) *Multi-objective design of robust flight control systems.* In: ICNPAA 2008, Mathematical Problems in Engineering, Aerospace and Sciences, 25-27th June 2008, Genoa, Italy.

Multi-Objective Design of Robust Flight Control Systems

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A multi–objective evolutionary algorithm is used in the framework of H_∞ control theory to find the controller gains that minimize a weighted combination of the infinite–norm of the sensitivity function (for disturbance attenuation requirements) and complementary sensitivity function (for robust stability requirements). After considering a single operating point for a level flight trim condition of a F-16 fighter aircraft model, two different approaches will then be considered to extend the domain of validity of the control law: 1) the controller is designed for different operating points and gain scheduling is adopted; 2) a single control law is designed for all the considered operating points by multiobjective minimisation. The two approaches are analyzed and compared in terms of effectiveness of the design method and resulting closed loop performance of the system.

Keywords Evolutionary optimization, Robust control, Aircraft control

1 Introduction

In this paper a control synthesis technique in the framework of H_∞ control theory is proposed, based on the application of a modern multi–objective evolutionary optimization algorithm to the associated minimization problem. The objective of the research is not limited to the simple demonstration of the capability of the optimization method in this challenging scenario. Focusing more on the application itself, the work aims at providing a tool that can handle the issue of parameter variation throughout the flight envelope.

In the last two decades, multiple redundant, full authority, fail/safe operational, fly–by–wire control systems have been brought to a very mature state. As a result, many aircrafts, from earlier designs such as the F-16, F-18, and Tornado through the more recent Mirage 2000, European Fighter Aircraft (EFA), Rafale, and advanced demonstrators such as X-29 and X-31, are highly augmented, actively controlled vehicles that possess either a marginal or negative static stability margin without augmentation, for reasons related to improved performances, weight/cost reduction, and/or low observability [1].

Highly augmented and/or superaugmented aircraft require the synthesis of a control system that artificially provides the required level of stability for satisfactory handling qualities, enhancing pilot capability by properly tailoring the aircraft response to the manoeuvre state [9]. At the same time, modern high performance fighter aircraft are characterized by an extended flight envelope in order to allow the pilot to reach unprecedented maneuvering capabilities at high angles of attack [3]. Such a result can be achieved only if the control system maintains adequate performance in presence of considerable variations of the aircraft response characteristics, avoiding instabilities related to the presence of control surface position and rate saturation limits.

Such a result can be obtained by use of robust controllers. H_∞ control theory was developed in this framework [10], in order to provide robustness to the closed–loop system to both external disturbance and model uncertainties of known "size". The controller is synthesized by minimizing the infinite norm of the system, determined as the maximum singular value $\bar{\sigma}$ of the tranfer function matrix $\boldsymbol{G}(s)$ for a multi–input/multi–output (MIMO) system. In more physically meaningful terms $\bar{\sigma}$ represents the maximum gain for a (disturbance) signal in the exptected frequency range: the system is robust to the worst expected disturbance if $\bar{\sigma}$ is less than 1, in which case all the disturbances will be attenuated by the closed–loop system. The cost of robustness is a certain degree of "conservativeness" of the controller, which may reduce closed–loop performance. For this reason the requirement for robust stability may be accompanied by requirements in the time domain (such as raise time, overshoot, and settling time), that can be enforced as inequality constraints to the optimization problem in order to pursue a minimum acceptable level of performance. In aircraft applications these constraints can be easily derived from requirements on the handling qualities.

The synthesis of the controller in the framework of H_∞ control theory is usually carried out by means of Linear Matrix Inequalities (LMI) [6]. In the present work an approach based on evolutionary optimisation is proposed in order to fulfill different (and possibly competing) requirements in different flight conditions. Highly manoeuvrable aircraft control offers a particularly challenging scenario, where on one side a controller synthesized for a single trim condition will unlikely perform well over a sufficiently wide portion of the operating envelope, even by use of robust techniques. In this respect, the classic solution is to use gain scheduled controllers, where the gain are varied as a function of reference parameters for the flight condition (*e.g.* Mach number or dynamic pressure). This classical procedure allows for adapting the system to parameter variation, but still requires a certain degree of robustness when the aircraft is flying off–nominal conditions between the design points where the controllers were synthesized. For this reason a gain scheduled controller for an F–16 fighter aircraft reduced short period model will be derived in three different conditions and gain scheduling used for interpolating the gains (Method 1). This approach will be compared with the synthesis of a single robust controller derived by enforcing simultaneously the requirements in all the considered operating points (Method 2).

The F–16 offers a good test–benchmark for the technique as it features most of the characteristics of a modern jet fighter (instability, high-α flight, etc.) [5]. After the description of aircraft model and control system architecture and a brief review of H_∞ control theory in the next Section, the used optimization method is briefly recalled in Section 3. The synthesis of a controller in the neighbourhood of a single trim condition and a comparison between a gain–scheduled controller and a controller synthesized for different competing merit functions is then carried out and discussed in Section 4. A Section of Conclusions ends the paper.

2 Aircraft dynamic model and control system architecture

2.1 Aircraft model and control system architecture

The longitudinal dynamics of a rigid aircraft can be represented fairly well by a set of 4 linear ordinary differential equations in the form [9]

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & V_0 + Z_q & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + \begin{bmatrix} X_{\delta_E} & X_{\delta_E} \\ Z_{\delta_E} & 0 \\ M_{\delta_E} & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \delta_E \\ \delta_T \end{pmatrix} \tag{1}$$

where, for a level flight reference trim condition, the state variables are perturbations of velocity components, u and w, pitch angular velocity, q, and pitch angle, θ. The control variables are elevetor deflection δ_E and throttle setting δ_T.

The stability derivatives in Eq. (1) depend on the considered flight condition. This means that the response of the aircraft to control action will vary with V_0. In order to deal with a simplified model, it is possible to consider the response to a reduced order short period model, under the assumption that attitude variables (q and $\alpha \approx w/V_0$) respond to control input on a faster time–scale then trajectory ones (namely velocity V and flight–path angle γ, where for longitudinal flight it is $\theta = \alpha + \gamma$), so that V can be considered approximately constant during an attitude manoeuvre. The reduced order model is given by

$$\begin{pmatrix} \dot{\alpha} \\ \dot{q} \end{pmatrix} = \begin{bmatrix} Z_\alpha & 1 + Z_q/V_0 \\ M_\alpha & M_q \end{bmatrix} \begin{pmatrix} \alpha \\ q \end{pmatrix} + \begin{bmatrix} Z_{\delta_E} \\ M_{\delta_E} \end{bmatrix} \delta_E \tag{2}$$

In order to enance model fidelity, a simple first order lag is introduced for representing the response of elevator deflection to pilot or automatic control inputs:

$$\dot{\delta}_E = \left(\delta_{E_{\text{com}}} - \delta_E\right)/\tau_A \tag{3}$$

Both position ($|\delta_E| < \delta_{E_{max}}$) and rate ($|\dot{\delta_E}| < \dot{\delta_{E_{max}}}$) saturation limits are included in the actuator model.

In what follows, an F-16 fighter aircraft model will be considered. The original model, taken from Ref. 9, features a nonlinear aerodynamic model for $-10 \leq \alpha \leq 45$ deg and $|\beta| \leq 30$ deg. Finite differences are used to linearize the aircraft model in the neighbourhood of each trim condition and obtain approximate values for the stability derivatives in Eqs. (1) and (2).

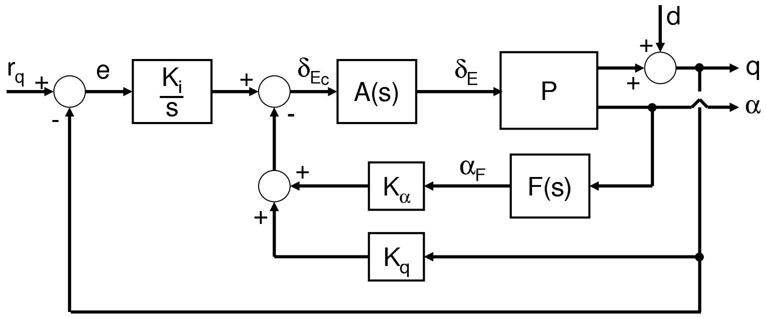

Fig. 1 Control system architecture.

Figure 1 depicts the structure of a longitudinal stability and control augmentation system (SCAS). The blocks P and A represent aircraft and elevator actuator dynamics, respectively. The stability augmentation provides increased pitch damping (q–feedback) and artificial static stability (α feedback). In this latter case a filter is included for reducing α sensor noise, with a cut–off frequency of $\tau_F = 10$ rad/s, $F(s) = \tau_F/(s + \tau_F)$.

The control augmentation system transforms the longitudinal pilot command into a rate command, where the tracked variable is the pitch angular velocity q. In order to provide the system with zero steady–state error an integrator is included in the pitch angular velocity error channel. The resulting open loop dynamics is described by a linear system of ordinary differential equations in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}\ ; \quad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \tag{4}$$

where the state vector is $\boldsymbol{x} = (\alpha, q, \delta_E, \alpha_F, \varepsilon)^T$, ε being the integrator variable, such that $\dot{\varepsilon} = r_q - q$. The only input variable is the pitch velocity reference signal r_q. Provided that the output variables are $\boldsymbol{y} = (\alpha, q, \varepsilon)^T$, the state, control, and output matrices are defined respectively as

$$\boldsymbol{A} = \begin{bmatrix} Z_w & V_0 + Z_q & M_{\delta_E} & 0 & 0 \\ M_w & M_q & M_{\delta_E} & 0 & 0 \\ 0 & 0 & -\tau_A & 0 & 0 \\ 0 & 0 & 0 & -\tau_F & 0 \\ 0 & -\frac{180}{\pi} & 0 & 0 & 0 \end{bmatrix}\ ; \ \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \tau_A \\ 0 \\ 0 \end{bmatrix}\ ; \ \boldsymbol{C} = \begin{bmatrix} \frac{180}{\pi} & 0 & 0 & 0 & 0 \\ 0 & \frac{180}{\pi} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

The optimization algorithm will be exploited in order to find the gains of the stability augmentation system (K_α and K_q) and the integral gain of the control autmentation system (K_i), which minimize the H_∞ norm of the closed-loop system while fullfilling handling quality requirements.

2.2 Robust control

Consider the system depicted in Fig. 2.a, where $\boldsymbol{P}_0(s)$ is the nominal model of a plant with n_i inputs and n_o outputs, $\boldsymbol{C}(s)$ is the controller, $\boldsymbol{r}(s)$ is the reference input signal $\boldsymbol{y}(s)$, $\boldsymbol{d}$ is the noise on the output signal and $\boldsymbol{n}(s)$ is the noise on the sensor channels. Given the definition of the output transfer matrix as $\boldsymbol{L}_o = \boldsymbol{P}_0\boldsymbol{C}$, the sensitivity at the output is defined as the transfer matrix $\boldsymbol{y}/\boldsymbol{d}$, that is

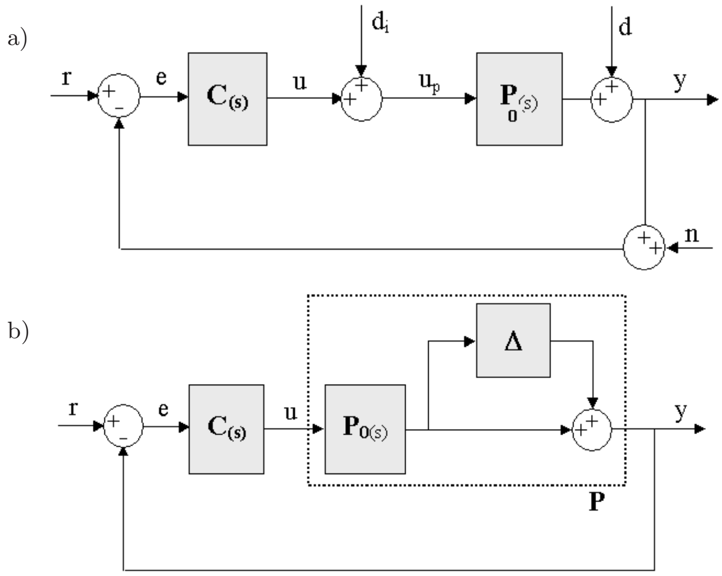

Fig. 2 General feedback configuration (a); feedback configuration with multiplicative uncerntainties of the nominal model (b).

$$\boldsymbol{S}_o = (\boldsymbol{I} + \boldsymbol{L}_o)^{-1}, \quad \boldsymbol{y} = \boldsymbol{S}_o \boldsymbol{d} \tag{6}$$

and the complementary sensitivity function at the output is

$$\boldsymbol{T}_o = \boldsymbol{I} - \boldsymbol{S}_o = \boldsymbol{L}_o(\boldsymbol{I} + \boldsymbol{L}_o)^{-1} \tag{7}$$

From the system represented in Fig. 2.a, it is easy to derive that

$$\boldsymbol{y} = \boldsymbol{T}_o \boldsymbol{r} - \boldsymbol{T}_o \boldsymbol{n} + \boldsymbol{S}_o \boldsymbol{P} \boldsymbol{d}_i + \boldsymbol{S}_o \boldsymbol{d} \tag{8}$$

It is thus clear that in order to reduce as much as possible the effects of noise on the response of the system, it is necessary to operate on $\boldsymbol{T}_o$ and $\boldsymbol{S}_o$.

Moreover, apart from external noise affecting the signals, the system may be characterized by other kind of uncertainties. Usually, the nominal model $\boldsymbol{P}_0$ will only provide an approximation of the response of the actual plant, due to simplifying assumptions and/or linearization. Taking into account a multiplicative uncertainty on the plant model (Fig. 2.b), the following expression for the output is obtained:

$$\boldsymbol{y} = \frac{\boldsymbol{T}_o + \Delta \boldsymbol{T}_o}{\boldsymbol{I} + \Delta \boldsymbol{T}_o} \boldsymbol{r} \tag{9}$$

In order to reduce the effects of the uncertainty it is necessary to tailor the complementary sensitivity function of the uncertainty itself, $\Delta \boldsymbol{T}_o$.

The main idea behind H_∞ control theory and the design process derived in this framework is to find the values of the controller parameters by minimizing the infinite norm of the weighted sensitivity and complementary sensitivity functions. In mathematical terms, the following functions need to be minimized:

$$\|\boldsymbol{W}_1(s)\boldsymbol{S}_o(s)\| = \min \; ; \quad \|\boldsymbol{W}_3(s)\boldsymbol{T}_o(s)\| = \min \tag{10}$$

so that the effects of noise on the output (Eq. 10) and uncertainty of the nominal model $\boldsymbol{P}_0$ are both reduced.

Since the H_∞ norm of a system $\boldsymbol{G}(s)$ is

$$\|\boldsymbol{G}\|_\infty = \sup_{\omega_m < \omega < \omega_M} \{\bar{\sigma}[\boldsymbol{G}(j\omega)]\} \tag{11}$$

where $\bar{\sigma}(\cdot)$ is the maximum singular value, this kind of norm provides the worst gain for a sinuisoidal input over a determined frequency range bounded by ω_m and ω_M, corresponding to the worst energetic gain of the system. The use of weighted functions allows for dealing with different kinds of signals, when MIMO systems are considered. Moreover, and more important, weights allow to focus the optimization process only within prescribed frequency ranges. As an example, in order to reduce low frequency noise a weight function with high gains at low frequency will be used, that is

$$\|\boldsymbol{W}_g(s)\boldsymbol{G}(s)\|_\infty < 1 \; ; \quad \|\boldsymbol{G}(s)\|_\infty < 1/\boldsymbol{W}_g(s) \tag{12}$$

3 The multi–objective optimization process

3.1 Optimization technique

For this work we used a particular type of evolutionary algorithm which belongs to the sub-class of Estimation of Distribution Algorithms (EDAs) [7]. In general terms, these methods try to identify a probabilistic model of the search space from the results for the current populations. Crossover and mutation operators, tipical of classical Genetic Algorithms [8], are replaced with statistical sampling. MOPED (Multi-Objective Parzen based Estimation of Distribution) algorithm is a multi-objective optimisation algorithm for continuous problems that uses the Parzen method to build a probabilistic representation of Pareto solutions, with multivariate dependencies among variables [2,4].

3.2 Statement of the optimization problem

In the framework of Method 1, the optimization process is aimed at minimizing the sum of the sensitivity and complementary sentitivity functions, each one appropriately weighted, in three different trim conditions. The optimal gains are then interpolated and the resulting gain scheduled control law (C1) is tested in off-nominal conditions by choosing two intermediate trim conditions in between the design operating points.

The objective function is

$$F = |\boldsymbol{W}_1(s)\boldsymbol{S}(s)\|_\infty + |\boldsymbol{W}_3(s)\boldsymbol{T}(s)\|_\infty \tag{13}$$

where the shape function $\boldsymbol{W}_1$ is chosen so that the action on the sensitivity function is enfasized in the low frequency zone, where the main disturbance, which can affect aircraft performance, are expected, while $\boldsymbol{W}_3$ is modeled on the basis of assumed uncertainties on the nominal model of the plant. The weight functions chosen for the example are

$$W_1 = \frac{1 + 100s}{100s + 1}; \quad W_3 = \frac{100 + 10s}{s + 1000} \tag{14}$$

Finally, constraints on rise time t_p, settling time t_s and overshoot M_p are considered for each trim condition:

$$t_p \leq 1[sec]; \;\; t_s \leq 3[sec]; \;\; M_p \leq 0.05 \tag{15}$$

The 3-dimensional search domain is bounded by $\boldsymbol{lb} = (-30, -30, -30)^T$ and $\boldsymbol{ub} = (0, 0, 0)^T$.

The second approach is carried out by an optimization process, which simultaneously handles 3 objective functions and 9 constraints related to all the 3 design points. The controller derived by Method 2 (C2) is caracterized by constant gains over the considered portion of the flight envelope. Again, closed–loop system performance are tested for the same intermediate trim conditions employed for C1.

4 Results and discussion

Five trim conditions for the F–16 aircraft model were considered (Tab. 1). Trim condition D1, D2 and D3 were considered for controller gain synthesis while conditions A1 and A2 were used for simulation of the closed–loop behaviour in off–nominal conditions.

Figure 3 shows the results obtained from a simulation of the closed–loop response to a step input on the input channel r_q for C1 (left) and C2 (right) in three different trim conditions. It should be noted how, in all the considered cases, the response of the tracked variable is satisfactory, and it is only marginally affected by the variation of the trim condition. At the same time, the off–nominal response in A1 lies between those for the design points (D1 and D2), thus proving that both the gain scheduling and the global approaches provide the required degree of robustness with respect to model parameter variation. Note that similar results are also obtained when considering D2 and D3 as reference trim conditions for the controller gain synthesis and A2 as the off–nominal condition, cases not reported in the figures for the sake of conciseness.

The global controller is forced to exploit the available control power in order to satisfy the 9 concurrent constraints in the 3 considered situations. This is equivalent to a min–max optimization process, where only some of the inequality constraints are active during the process that forces the worst case below the prescribed threshold. After this first constraint enforcement phase, the algorithm explores a relatively small portion of the search space, where all the 9 constraints are satisfied, looking for

Table 1 Trim conditions

	V [ft/s]	h [ft]	Q [psf]
D1	500	0	297
D2	700	6 000	486
D3	900	12 000	666
A1	600	3 000	391
A2	800	9 000	579

the optimal solutions in terms of minima for the objective functions $\boldsymbol{F}$ in the 3 design points.

The resulting contoller provides a high level of damping, with excellent performance in terms of time response. The price paid is lack of robusteness, as $\|\boldsymbol{W}_3(s)\boldsymbol{T}(s)\|_\infty > 1$ in all the considered trim conditions. Only by relaxing requirements on time response (especially rise time for the third design point) it is possible to achieve the desired level of robustness.

The obtained results demonstrate that the evolutionary approach can be successfully adopted to manage the control synthesis process (Method 1), usually carried out by a trial–and–error technique (e.g. by solving LMI until all the constraints are satisfied). On the other hand, the design of a single control law for a wide range of trim conditions appears impractical, because the controller cannot simulaneously guarantee the required time response and an adequate level of robusteness when large parameter variations are to be dealt with.

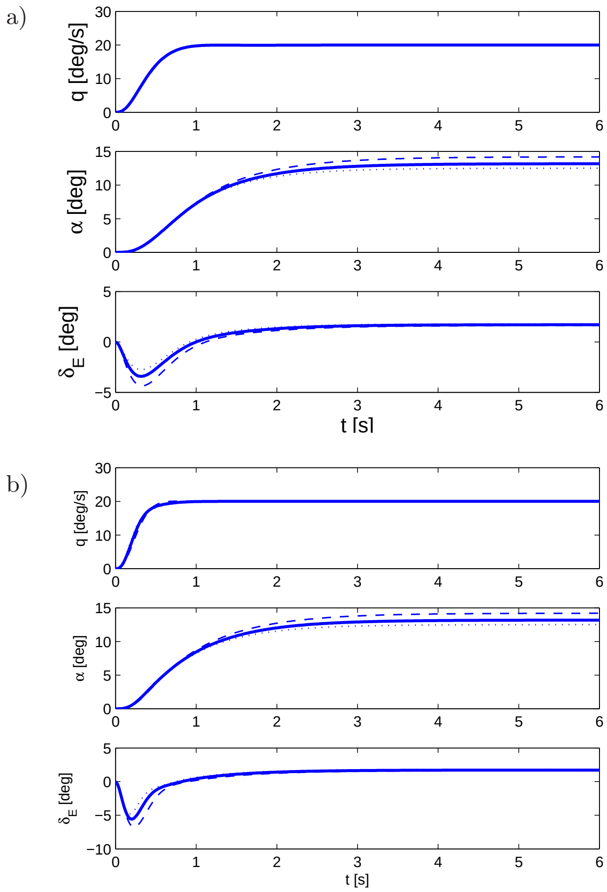

Fig. 3 Step responses of scheduled (a) and global (b) controllers in D1 (dashed line), A1 (thick line), and D2 (dotted line).

4 Conclusions and future work

In this paper an evolutionary optimization technique was demonstrated as a means for control gain synthesis in the framework of H_∞ control problems. Two different techniques were presented: the first one is based on solving three optimization problems at different operating points. Gain scheduling is then used for extending the controller operations to off–nominal conditions and verify controller performance over a wide portion of the flight envelope. In the second framework, a single set of gains was searched for, which satisfies control constraits and performance requirements in the same set of operating points. Satisfactory results were obtained in both cases, although the second one provided a more aggressive controller on one side, at the expenses of some lack of robustness, which can be obtained only by relaxing constraints on time response.

The research will now focus on improving the search of an optimal solution for both techniques (more aggressive controllers in the first case, robust in the whole considered flight envelope for second one). Moreover, a more demanding scenario will also be considered, where simulations are performed by using the fully nonlinear six-degrees-of-freedom model, in order to assess more convincingly the robustness of the control system to both parameter variations and unmodeled dynamics.

Acknowledgements

Giulio Avanzini is grateful to the Department of Aerospace Engineering of the University of Glasgow for inviting him as visiting lecturer in the Fall semester of Academic Year 2007–08. A grant from the Erasmus Office of the Italian National Agency LLP (Lifelong Learning Program) and support from Politecnico di Torino in the framework of "Young Researchers Program" 2007 are also acknowledged.

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