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LARGE ANGLE REORIENTATION MANOEUVRE OF SPACECRAFT USING ROBUST BACKSTEPPING CONTROL

Imran Ali
University of Glasgow, Glasgow, United Kingdom
i.ali@aero.gla.ac.uk

Jongrae Kim
University of Glasgow, Glasgow, United Kingdom
jkim@aero.gla.ac.uk

Gianmarco Radice
University of Glasgow, Glasgow, United Kingdom
gradice@aero.gla.ac.uk

ABSTRACT

The nonlinear control design problem for large angle reorientation manoeuvre of spacecraft has a proper structure for the direct application of backstepping design as its dynamics and kinematics are naturally in a cascade form. In this paper, the robustness of the backstepping control against the uncertainties in the moment of inertia matrix is investigated and a sufficient condition for the robust stability is derived. Numerical simulations show the validity of the condition.

1. INTRODUCTION

BACKSTEPPING is a systematic Lyapunov-based control design method for nonlinear systems, especially those in a cascade form [1]. The basic idea is to use a part of the system states as virtual controls to control the other states. The name backstepping refers to the recursive nature of the control design procedure where a control law as well as a control Lyapunov function is recursively constructed to guarantee the stability. Generating a family of globally asymptotically stabilizing control laws is the main advantage of this method and it can be exploited for addressing robustness issues and solving adaptive problems.

Large angle reorientation manoeuvre problem of spacecraft, whose dynamics and kinematics are naturally in a cascade form, is a good candidate for this technique [2,3]. However, the control actuators used for spacecraft attitude manoeuvre problems, e.g. reaction wheels, control moment gyros or thrusters, have an upper bound on the magnitude of torque they can exert onto the system and the simple or conventional backstepping control method may result in excessive control input beyond that saturation bound of the actuators.

A family of augmented Lyapunov functions proposed in [4] introduces a constant gain that is helpful to lower the required control torque bound. For this approach, the analytical bound for the control torque was derived in [5]. Lowering of the control torque bound may cause settling time performance degradation and [5] has also employed the nonlinear function based tracking function in [3] to reduce the settling time. The same conventional backstepping control law as considered by [5] is being analysed in the present paper for robustness against the uncertainties of the spacecraft moments of inertia.

The paper is organized as follows: Firstly, the kinematics and dynamics of rigid spacecraft attitude motion are summarised. Secondly, the analytical estimates of the moment of inertia matrix bounds in terms of eigenvalues and the sufficient condition of robust stability are presented. Finally, numerical simulation shows the validity of the robustness condition and the summary is presented.

2. RIGID SPACECRAFT ATTITUDE MOTION

Spacecraft is assumed to be a rigid body with actuators that provide torques about three mutually perpendicular axes that define a body-fixed frame with origin at the centre of mass of spacecraft. The equations of rotational motion of spacecraft are given by [6]

$$\dot{\mathbf{q}}_{13} = \frac{1}{2}(q_4\boldsymbol{\omega} - \boldsymbol{\omega}\times\mathbf{q}_{13}), \quad \dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^T\mathbf{q}_{13} \qquad (1)$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times[\mathbf{J}\boldsymbol{\omega}] = \mathbf{T} \qquad (2)$$

where $\mathbf{q}_{13} \in \Re^3$ and $q_4 \in \Re$ satisfy $\mathbf{q}_{13}^T\mathbf{q}_{13} + q_4^2 = 1$, $\mathbf{q} = [\mathbf{q}_{13}^T,\ q_4]^T$ denotes the unit quaternion that represents the orientation of spacecraft with respect to an inertial frame, $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ denotes the angular velocity of the spacecraft with respect to the inertial frame expressed in the body frame, $\mathbf{J} = \mathbf{J}^T$ denotes the body frame referenced positive definite inertia matrix of spacecraft, $\mathbf{T} = [T_1, T_2, T_3]^T \in \Re^3$ denotes the control torque with components in the body frame. We define the three subscripts *i*, *j* and *k* as $(i, j, k) \in \{(1,2,3),\ (2,3,1),\ (3,1,2)\}$ and Eq. (1) can be written as

$$\dot{q}_i = \tfrac{1}{2}(q_4\omega_i - q_k\omega_j + q_j\omega_k) \qquad (3)$$

and choosing $\mathbf{J} = diag(J_1, J_2, J_3)$ Eq. (2) becomes

$$\dot{\omega}_i = p_i\omega_j\omega_k + u_i \qquad (4)$$

where $p_i = (J_j - J_k)/J_i$ and $u_i = T_i / J_i$.

3. ROBUSTNESS ANALYSIS

In this section, the backstepping controller design procedure is summarised and the sufficient condition for the robustness with respect to the moment inertia uncertainties is derived.

The candidate Lyapunov function for the kinematics subsystem stabilization is

$$V = \tfrac{1}{2}\left[\|\mathbf{q}_{13}\|^2 + (1-q_4)^2\right] \qquad (5)$$

The pseudo control law for the kinematics subsystem stabilization $\boldsymbol{\omega}_s$ is written as

$$\boldsymbol{\omega}_s = -s\,\mathrm{sgn}(q_4)\boldsymbol{\phi}(\mathbf{q}_{13}) \qquad (6)$$

where s is a positive constant and $\boldsymbol{\phi} = \boldsymbol{\phi}(\mathbf{q}_{13}) = \left[\phi(q_1) \quad \phi(q_2) \quad \phi(q_3)\right]^T$ with the nonlinear tracking function $\phi(q_i)$ as given in [3]

$$\phi(q_i) = \alpha \tan^{-1}(\beta q_i) \qquad (7)$$

where α and β are positive constants. The function sgn(.) denotes the sign function defined by [3]

$$\mathrm{sgn}(x) = \begin{cases} -1, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

or (8)

$$\mathrm{sgn}(x) = \begin{cases} -1, & x \leq 0 \\ 1, & x > 0 \end{cases}$$

The Lyapunov function in Eq. (5) is augmented for the overall system as follows [4]:

$$U = \tfrac{1}{2}\left[\left\|\mathbf{q}_{13}\right\|^2 + (1-q_4)^2\right] + \tfrac{1}{2}\left\|\boldsymbol{\Omega} - \boldsymbol{\Omega}_s\right\|^2 \qquad (9)$$

where $\boldsymbol{\Omega} = \boldsymbol{\Omega}(\boldsymbol{\omega}) = \left[\Omega(\omega_1) \quad \Omega(\omega_2) \quad \Omega(\omega_3)\right]^T$, $\boldsymbol{\Omega}_s = \boldsymbol{\Omega}(\boldsymbol{\omega}_s) = \left[\Omega(\omega_{s1}) \quad \Omega(\omega_{s2}) \quad \Omega(\omega_{s3})\right]^T$, and $\Omega(\cdot)$ is the class κ_∞ function, i.e. it is zero at zero, strictly increasing and becomes unbounded as the argument is unbounded [4]. The time derivative of Eq. (9) becomes

$$\begin{aligned}\dot{U} &= \tfrac{1}{2}\mathrm{sgn}(q_4)\boldsymbol{\omega}^T\mathbf{q}_{13} + \left(\boldsymbol{\Omega} - \boldsymbol{\Omega}_s\right)^T\left(\dot{\boldsymbol{\Omega}} - \dot{\boldsymbol{\Omega}}_s\right) \\ &= \tfrac{1}{2}\mathrm{sgn}(q_4)\boldsymbol{\omega}_s^T\mathbf{q}_{13} + \tfrac{1}{2}\mathrm{sgn}(q_4)(\boldsymbol{\omega} - \boldsymbol{\omega}_s)^T\mathbf{q}_{13} + \\ &\quad \left(\boldsymbol{\Omega} - \boldsymbol{\Omega}_s\right)^T\left(\mathrm{diag}(\boldsymbol{\Omega}')\dot{\boldsymbol{\omega}} - \mathrm{diag}(\boldsymbol{\Omega}_s')\dot{\boldsymbol{\omega}}_s\right) \\ &= -\tfrac{1}{2}s\boldsymbol{\Phi}(\mathbf{q}_{13})^T\mathbf{q}_{13} + \\ &\quad (\boldsymbol{\omega} - \boldsymbol{\omega}_s)^T\Big[\tfrac{1}{2}\mathrm{sgn}(q_4)\mathbf{q}_{13} + \\ &\quad \mathrm{diag}(\boldsymbol{\omega} - \boldsymbol{\omega}_s)^{-1}\mathrm{diag}\left(\boldsymbol{\Omega} - \boldsymbol{\Omega}_s\right)\left(\mathrm{diag}(\boldsymbol{\Omega}')\dot{\boldsymbol{\omega}} - \mathrm{diag}(\boldsymbol{\Omega}_s')\dot{\boldsymbol{\omega}}_s\right)\Big]\end{aligned} \qquad (10)$$

where $\boldsymbol{\Omega}' = \boldsymbol{\Omega}'(\boldsymbol{\omega}) = \left[\Omega'(\omega_1) \quad \Omega'(\omega_2) \quad \Omega'(\omega_3)\right]^T$, $\boldsymbol{\Omega}_s' = \boldsymbol{\Omega}'(\boldsymbol{\omega}_s) = \left[\Omega'(\omega_{s1}) \quad \Omega'(\omega_{s2}) \quad \Omega'(\omega_{s3})\right]^T$, and $\Omega'(x)$ defines the derivative of $\Omega(x)$ with respect to x. Moreover, for a vector $\mathbf{a} = \left[a_1, a_2, a_3\right]^T$, $\mathrm{diag}(\mathbf{a})$ denotes the diagonal matrix $\mathrm{diag}(a_1, a_2, a_3)$. In order to make the time derivation of the Lyapunov function be negative definite, the control law is chosen as follows:

$$\begin{aligned}\mathbf{u} = -\mathrm{diag}(\boldsymbol{\Omega}')^{-1}\Big[&\mathrm{diag}(\boldsymbol{\Omega} - \boldsymbol{\Omega}_s)^{-1}\mathrm{diag}(\boldsymbol{\omega} - \boldsymbol{\omega}_s)\left(\tfrac{1}{2}\mathrm{sgn}(q_4)\mathbf{q}_{13} + \right. \\ &\left. g(\boldsymbol{\omega} - \boldsymbol{\omega}_s)\right) - \mathrm{diag}(\boldsymbol{\Omega}_s')\dot{\boldsymbol{\omega}}_s\Big] + \mathbf{J}^{-1}\boldsymbol{\omega}^{\times}\mathbf{J}\boldsymbol{\omega}\end{aligned} \qquad (11)$$

Substituting the above backstepping controller in Eq. (10) we get

$$\dot{U} = -\tfrac{1}{2}s\boldsymbol{\phi}(\mathbf{q}_{13})^T\mathbf{q}_{13} - g(\boldsymbol{\omega} - \boldsymbol{\omega}_s)^T(\boldsymbol{\omega} - \boldsymbol{\omega}_s) \qquad (12)$$

where g is a positive constant. Hence, the control law of Eq. (11) is globally asymptotically stable. Let

$\mathbf{J}_a$ denote the actual spacecraft inertia matrix which is different from $\mathbf{J}$, the one assumed for the design of the control law of Eq. (11). Substituting the control input of Eq. (11) in conjunction with the actual inertia matrix $\mathbf{J}_a$, Eq. (10) becomes

$$\dot{U}=-\tfrac{1}{2}s\boldsymbol{\phi}(\mathbf{q}_{13})^T\mathbf{q}_{13}-g(\boldsymbol{\omega}-\boldsymbol{\omega}_s)^T(\boldsymbol{\omega}-\boldsymbol{\omega}_s)-(\boldsymbol{\omega}-\boldsymbol{\omega}_s)^T\operatorname{diag}(\boldsymbol{\omega}-\boldsymbol{\omega}_s)^{-1}\operatorname{diag}\left(\boldsymbol{\Omega}-\boldsymbol{\Omega}_s\right)\operatorname{diag}(\boldsymbol{\Omega}')\left[\mathbf{J}_a^{-1}\boldsymbol{\omega}^{\times}\mathbf{J}_a-\mathbf{J}^{-1}\boldsymbol{\omega}^{\times}\mathbf{J}\right]\boldsymbol{\omega} \tag{13}$$

Considering the simple case of $\Omega(\omega_i)=\eta\omega_i$ with $\eta>1$ and defining $\mathbf{e}=\boldsymbol{\omega}-\boldsymbol{\omega}_s$, $\mathbf{J}_a=\mathbf{J}+\delta\mathbf{J}_1$ and $\mathbf{J}_a^{-1}=\mathbf{J}^{-1}+\delta\mathbf{J}_2$ where $\delta\mathbf{J}_2=-\left(\mathbf{I}_3+\delta\mathbf{J}_1^{-1}\mathbf{J}\right)\mathbf{J}^{-1}$, [7], $\|\mathbf{J}\|\le\overline{\lambda}_J$, $\|\mathbf{J}^{-1}\|\le 1/\underline{\lambda}_J$, $\|\delta\mathbf{J}_1\|\le\gamma_1$ and $\|\delta\mathbf{J}_2\|\le\gamma_2$ the above equation can be written as

$$\begin{aligned}\dot{U}&=-\tfrac{1}{2}s\boldsymbol{\phi}(\mathbf{q}_{13})^T\mathbf{q}_{13}-g\|\mathbf{e}\|^2-\eta^2\mathbf{e}^T\left(\mathbf{J}^{-1}\boldsymbol{\omega}^{\times}\delta\mathbf{J}_1+\delta\mathbf{J}_2\boldsymbol{\omega}^{\times}\mathbf{J}+\delta\mathbf{J}_2\boldsymbol{\omega}^{\times}\delta\mathbf{J}_1\right)\boldsymbol{\omega}\\&\le-\tfrac{1}{2}s\boldsymbol{\phi}(\mathbf{q}_{13})^T\mathbf{q}_{13}-\|\mathbf{e}\|\Big[g\|\mathbf{e}\|-\eta^2\left(\|\mathbf{J}^{-1}\|\|\delta\mathbf{J}_1\|+\|\delta\mathbf{J}_2\|\|\mathbf{J}\|+\|\delta\mathbf{J}_2\|\|\delta\mathbf{J}_1\|\right)\|\boldsymbol{\omega}\|^2\Big]\\&\le-\tfrac{1}{2}s\boldsymbol{\phi}(\mathbf{q}_{13})^T\mathbf{q}_{13}-\|\mathbf{e}\|\Big[g\|\mathbf{e}\|-\eta^2\left(\gamma_1/\underline{\lambda}_J+\gamma_2\overline{\lambda}_J+\gamma_1\gamma_2\right)\|\boldsymbol{\omega}\|^2\Big]\end{aligned} \tag{14}$$

As a result, the negative definiteness of $\dot{U}$ is guaranteed if the following inequality is satisfied:

$$g\|\mathbf{e}\|\ge\eta^2\left(\gamma_1/\underline{\lambda}_J+\gamma_2\overline{\lambda}_J+\gamma_1\gamma_2\right)\|\boldsymbol{\omega}\|^2 \tag{15}$$

and the whole system is robustly stable with respect to the moment of inertia uncertainties.

4. NUMERICAL SIMULATION

The usefulness of the condition given by Eq. (15) is demonstrated through the numerical simulation of a benchmark rest-to-rest slew manoeuvre [2,3,5]. The simulation scenario is as follows:

$$\mathbf{J}=\operatorname{diag}(J_1,J_2,J_3)=\operatorname{diag}(10,15,20)\qquad(\text{kg m}^2)$$

$$\boldsymbol{q}(t_0)=[0.4646\ \ 0.1928\ \ 0.8047\ \ 0.3153]^T$$

$$\boldsymbol{q}(t_f)=[0\ \ 0\ \ 0\ \ 1]^T$$

where t_0 and t_f are the starting and the final times, respectively. The values for the control gain g and α and β for the nonlinear tracking function $\phi(q_i)$ are chosen the same as given in [3], i.e., 10, 0.75 and 8.0, respectively. Also, the same true moment of inertia matrix in [3], $\mathbf{J}_a$, is adopted as follows:

$$\mathbf{J}_a=\operatorname{diag}(8,16.5,24)\qquad(\text{kg m}^2)$$

For the given $\mathbf{J}$ and $\mathbf{J}_a$, the parameters for the bound, Eq. (15), are $\overline{\lambda}_J=20$, $\underline{\lambda}_J=10$, $\gamma_1=4$ and $\gamma_2=0.025$. Finally, selecting the control gains η and s equal to 6 and 10, respectively, the sufficient condition for the robust stability given by Eq. (15) becomes $\|\mathbf{e}\|\ge 10\|\boldsymbol{\omega}\|^2$.

Fig. (1)–(5) show the simulation results for the specified manoeuvre. The whole scenario is the same as considered in [5] except the control gains η and s have been retuned in order to ensure the robustness against the uncertainties in the moments of inertia. The peak control torque from Fig. 3 is similar to the one reported in the simulation results of [5] but the performance in terms of the settling time has been sacrificed because of the effort of meeting the robustness condition, Eq. (15). A considerate retuning of all of control gains may rectify this issue of the sluggish motion response.

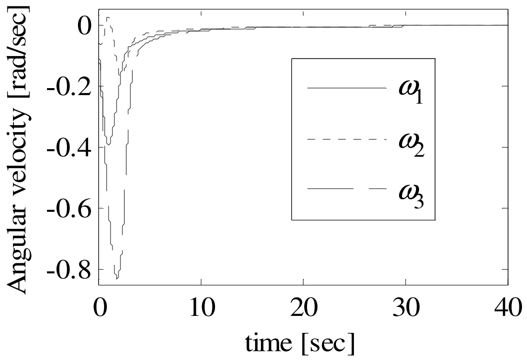

Fig. 1. Angular velocity $\boldsymbol{\omega}$ history

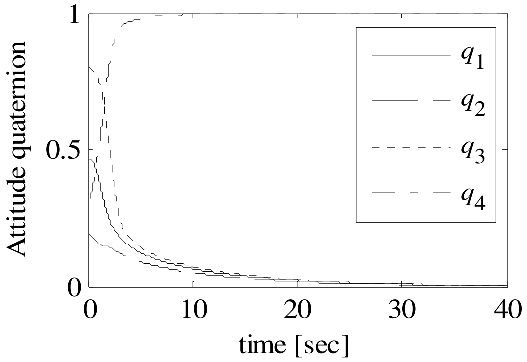

Fig. 2. Unit attitude quaternion $\mathbf{q}$ history.

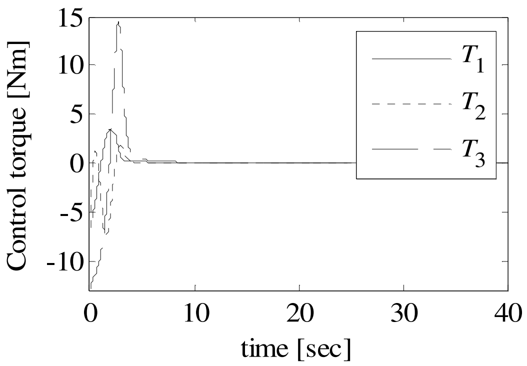

Fig. 3. Control torque $\mathbf{T}$ history.

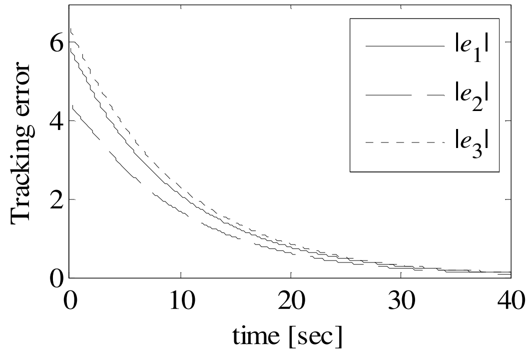

Fig. 4. Tracking error $\mathbf{e}$ history.

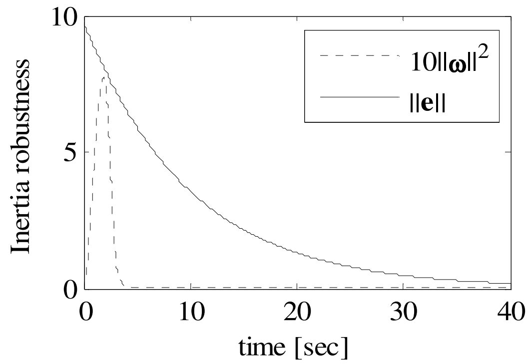

Fig. 5. Inertia robustness condition

5. CONCLUSION

The conventional backstepping controller for the spacecraft large angle reorientation manoeuvre problem has been analysed in the context of robustness against the uncertain variations in the moments of inertia and a sufficient condition has been derived that ensures the robust stability to the prescribed bounded uncertainties.

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