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ANALYSIS OF ARCHING AROUND SHALLOW TUNNELS USING THE BOUNDARY ELEMENT METHOD

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ABSTRACT : This paper describes a study of the effect of soil arching around shallow tunnels using the Boundary Element Method. Due to the different stiffnesses of the soil and the underground structure, then, under surface loading, the differential displacements generate shearing (arching) forces which increase or decrease the load on the tunnel. Under elastic conditions, the soil parameters (elastic modulus (E) and Poisson's ratio (υ)) both affect these arching forces. The Boundary Element Method is an effective numerical technique to achieve accurate results as it deals directly with the tractions at the tunnel/soil interface. Moreover, it is more efficient than the Finite Element Method in this case because no elements are needed within the soil itself.

KEYWORDS : *Arching, Thrust, Rigid lined Tunnel, Tunnel stability, Boundary element method.*

1. INTRODUCTION

Accurate calculation of tunnel loads and settlements due to surface loads above tunnels has to take into account the change in stress distribution attributable to the tunnel/ground interaction. Kolymbas (2008)[6] discusses the significance of this phenomenon, termed arching. Tien (1996)[9], in a wide-ranging review, explains that arching arises when differential settlements arising in heterogenous materials gives rise to shearing stresses. The phenomenon can be exploited (e.g., GeoTech (1999)[4], to reduce loads on conduits and thus reduce the cost of construction.

This study focuses on the analysis of the arching phenomenon around circular shallow-depth tunnels subjected to various types of surface loading, using the boundary element method. The ground is assumed to deform elastically while the tunnel lining is assumed to be perfectly rigid. Although the model assumptions constitute a significant simplification of reality, the results are expected to yield useful insights into the major factors which control the degree of arching around tunnels.

2. ARCHING

2.1 Positive arching and negative arching

Arching involves the transmission of shear stress between the deforming soil mass and the adjoining buried structure (Terzaghi, 1943)[6]. Tien (1996)[9], reviews this early work which identifies arching of two types, namely "positive arching" and "negative arching". Positive arching ("simple arching") occurs when the buried structure is more compressible than the surrounding soil. When the ground surface is loaded, the relatively large deformation of the structure induces shearing in the surrounding soil, which consequently bears a greater proportion of the loading.

Negative arching occurs when the buried structure is less compressible than the surrounding soil. Now the soil tends to deform more than the buried structure and the structure bears a greater proportion of the surface loading. Further, the additional stresses on the structure tend to be concentrated at the structure's extremities.

Of course, it is possible that the stiffness of the structure and the surrounding ground is perfectly matched, so that a condition of neutral arching arises. This is difficult to arrange in practice.

2.2 Arching ratio

It is convenient to define an 'arching ratio' which describes the degree of arching, defined as the ratio of the total thrust developed in the tunnel walls to the product of the nominal vertical pressure at tunnel level and the tunnel span. Thus,

$$R = \frac{F}{p'D} \qquad (1)$$

where
F = total vertical thrust in tunnel walls,
D = diameter (span) of tunnel
p' = nominal vertical pressure at the level of the tunnel soffit. [With reference to **Figure 1,** p' = pL/(L+S)]

Accordingly,
R = 1 Neutral arching
R < 1 (Positive) arching
R > 1 Negative arching

3. NUMERICAL MODELLING

3.1 Model description

For simplicity, the problem was analyzed assuming that plane-strain conditions are applicable, which reduces the problem to two dimensions.

The extent of the soil domain was defined by a lower boundary at a depth H and lateral boundaries at a distance W. The consequence of assuming a finite domain was investigated by comparing results obtained using a wide range of H/D and W/D ratios (where D is the tunnel diameter).

Figures 1 and **2** define the geometric quantities used in this study, which are used later to form the dimensionless parameters S/D, L/D and I/D.

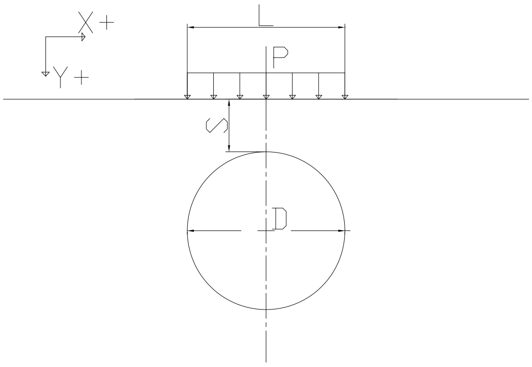

Figure 1: Definition of (symmetric) problem geometry

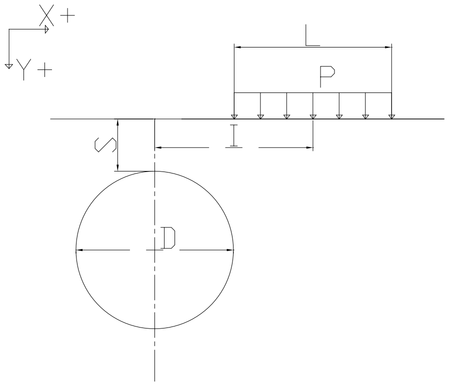

Figure 2: Definition of (asymmetric) problem geometry.

The boundary element method [Brebbia (1984)[1], Crouch & Starfield (1997)[2] , Gao & Davies (2002)[3]] can implicitly incorporate infinite boundaries but this possibility has not been exploited here because, as indicated below, the deformation of a half-plane is theoretically unbounded. We therefore adopt a finite boundary Γ_e , which includes the ground surface, the artificially truncated lower and lateral boundaries and the tunnel/soil interface, as shown in **Figure 3**.

Figure 3: Subdivision of the boundary Γ_e into elements

The boundary Γ_e is divided into N_e quadratic boundary elements, defined by three nodes. Numerical integration of the Kelvin singular fundamental solution over each element leads to a set of linear equations of order ($4xN_e$). This set can be solved by substitution of the known boundary conditions at the ground surface and the assumed boundary conditions at the truncated boundaries.

The boundary element model, **Figure 4**, shows a typical configuration of the far-field boundaries, around a tunnel located at the top-centre of the figure. In this figure, S = 2, D = 2.

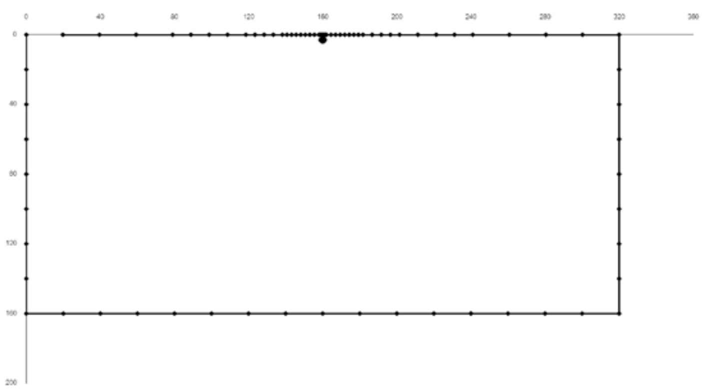

Figure 4: A typical far-field boundary around a tunnel

If the tunnel lining is rigid, then all nodes displace equally. However, this is not a prescribed boundary condition because the magnitude and direction of the displacement is not known apriori but is the consequence of the ground surface loading. But this rigid body motion can be determined (iteratively) by making use of the fact that the net force on the tunnel must be zero.

For the soil domain, the boundaries were truncated, in the first instance, by setting H/D = 40 and W/D = 40. Initially, just 30 boundary elements were used along these boundaries, with a further four elements on the soil/tunnel interface. Roller boundary conditions were assumed on the truncated boundaries, while the ground surface was assumed to be traction-free, except in that area subjected to surcharge.

3.2 Tunnel rigid body displacement

The displacement of the tunnel can be obtained iteratively as follows:

1. Apply an arbitrary identical displacement to every node on the tunnel/soil interface.
2. Calculate the resultant tractions on the interface, using the boundary element method.
3. Calculate the resultant force on the tunnel. (e.g., $F_y = \int t_y ds$)
4. Check whether the resultant force is sufficiently small. If so, go to 5. Otherwise, apply an improved estimate (by trial and error or otherwise) of the arbitrary displacement and go to 2 (See **Figure 5**).
5. This yields the rigid body displacement. The thrust in the tunnel liner can then be computed from the resultant of the tractions over the upper (or lower) half of the tunnel/soil interface.

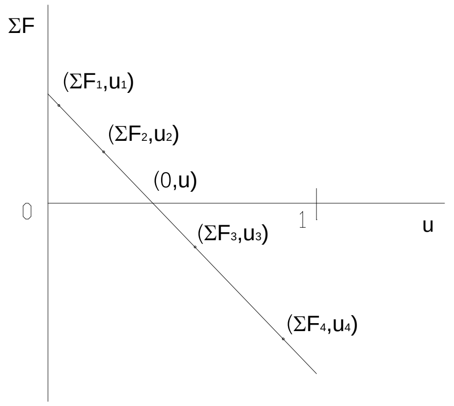

Figure 5: Resultant force on tunnel due to prescribed displacement (root at ΣF=0)

3.3 Boundary and Mesh convergence

Convergence analysis is necessary to establish the reliability of the model. However, it is also necessary to optimize the cost of increasing the model resolution (e.g., number of elements, location of truncated boundaries) against the accuracy obtained.

To establish the effect of boundary truncation, results obtained by setting H/D = 80, 160, 320 and W/D = 80, 160, 320 respectively are shown in **Figure 6** and **Figure 7.** **Figure 8** shows the effect of increasing the number of elements at the tunnel/soil interface

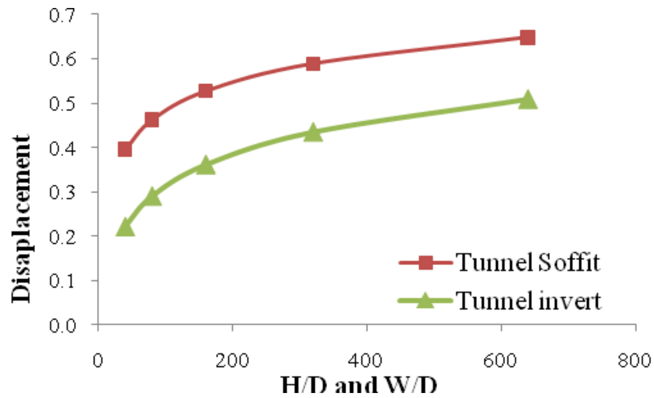

Figure 6: Effect of boundary truncation on tunnel displacements.

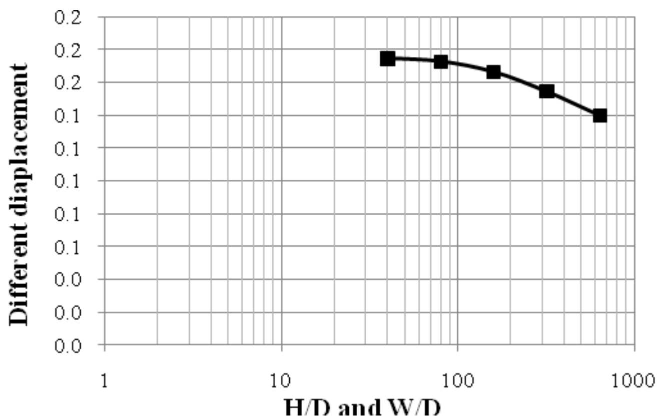

Figure 7: Effect of boundary truncation on differential tunnel displacements.

Clearly, tunnel displacements increase as the boundary becomes more remote. Plotted on a semi-logarithmic scale (**Figure 7**) it is evident that the displacements increase logarithmically, once any local effects are excluded. It appears that if the boundaries are fixed beyond a certain distance (approximately 50 to 100 diameters), then the local effects are eliminated: the logarithmic increase in displacements is simply a consequence of the logarithmic unboundedness of the half-plane problem. For the purposes of this paper, it is assumed that the boundaries are fixed at eighty diameters (H/D=80, W/D=80).

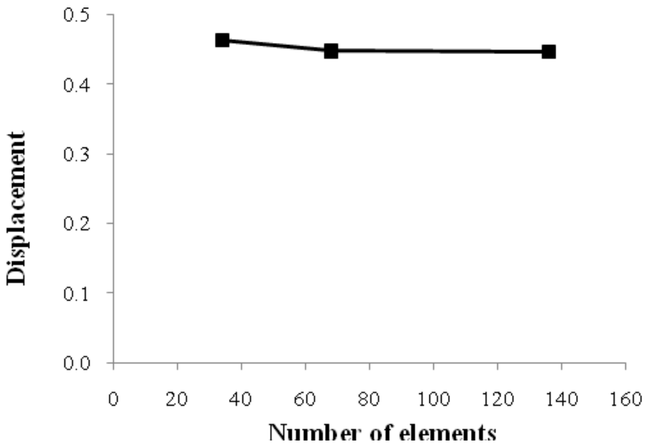

Figure 8: Convergence of soffit displacement

Figure 8 shows how accurate results for tunnel soffit displacements can be obtained with very few elements. These data were used to establish the mesh geometry – sufficiently accurate results can be obtained by using no more than 80 elements in total, including ten elements around the tunnel/soil interface itself.

4. RESULTS

Figure 10 shows results obtained while investigating the effect of tunnel depth on arching. In this case, the surface loading is symmetric and extends over the plan area of the tunnel (L/D = 1). It can be seen that arching is effective at shallow depths, but negative arching occurs around deeper tunnels (i.e., these tunnels carry greater load than would be expected if the stresses decayed as expected in a homogenous medium). The effect of Poisson's ratio is significant only for shallow tunnels.

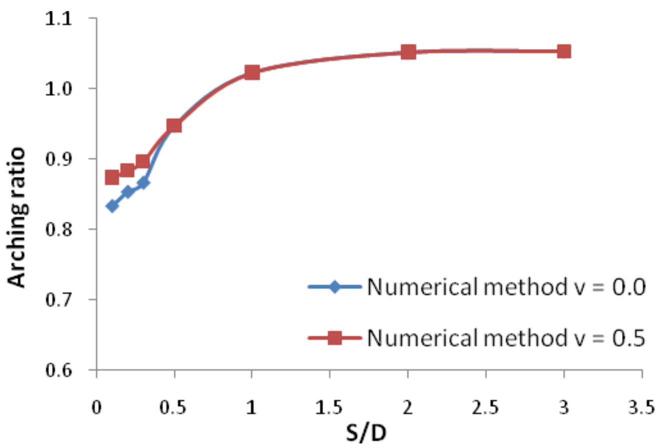

Figure 10: The effect of tunnel depth S on arching.

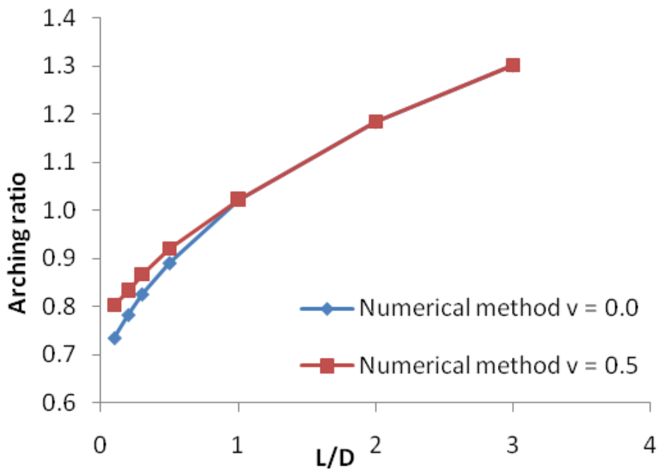

Figure 11: The effect of loaded area on arching (S/D = 1)

Figure 11 shows the effect of increasing the loaded area (characterized by the length L) on arching. The results are relevant to a tunnel buried at moderate (S/D = 1). Provided that the extent of the loaded area is small, then (positive) arching occurs. However, if the surface loading is extensive, then as might be expected, the rigid tunnel carries a relatively high proportion of the load (negative arching). Arching is reduced (higher arching ratio) in incompressible soils but this effect is only significant if the loaded area is less than the plan area of the tunnel.

Figure 12 and **Figure 13** show the effect of increasing the asymmetry of the surface loading on vertical and horizontal (normalized) displacements, respectively. These data refer to a moderately shallow tunnel (S/D=1). These figures show the very significant effect of Poisson's ratio: an increase in Poisson's ratio reduces vertical displacements considerably, while lateral displacements may be reversed. In particular, in incompressible soils (ν=0.5), vertical asymmetric loading results in outward translation of the tunnel away from the loading. More generally, the results show that shallow tunnels are affected to a significant degree by surface loading well outside their plan area.

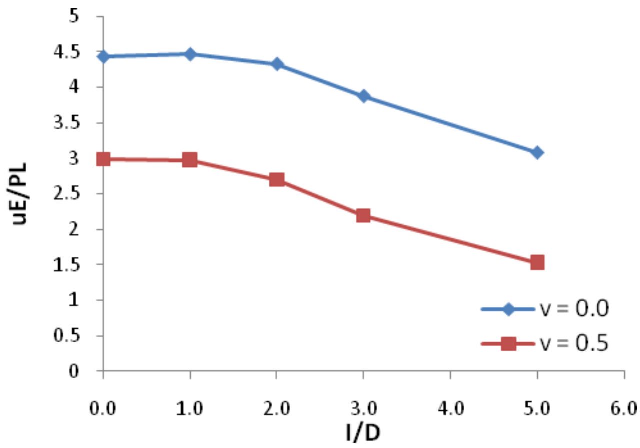

Figure 12: Vertical tunnel displacement under asymmetric surface loading

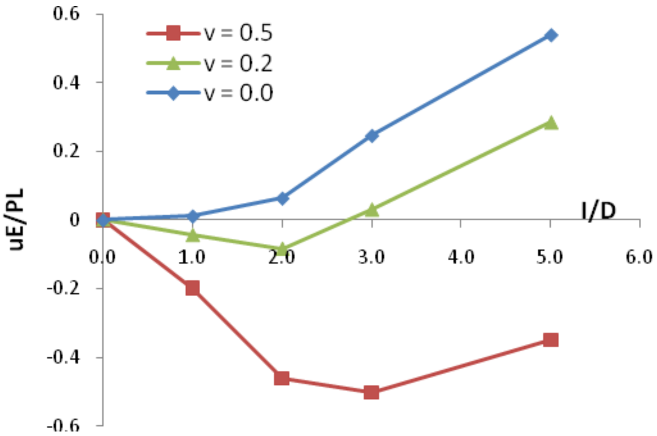

Figure 13: Horizontal tunnel displacement under asymmetric surface loading

5. CONCLUSION

The Boundary Element Method (BEM) is well-suited to analysis of this problem because it requires boundary-only discretisation and also because it provides results where they are needed (i.e., at the tunnel/soil interface) rather than the interior. Convergence studies show that accurate results can be obtained with very few elements. They also verify the theoretical result that the displacement problem in the half-plane is unbounded. Some sample results show that positive or negative arching can occur around rigid tunnels, depending on the depth of the tunnel. Moreover, tunnels may undergo significant displacements under surface loads which have been applied well outside the tunnel plan area.

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