A comparison of UK equity and property duration

by

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Dedication

This paper is dedicated to Nanda Nanthakumaran who died before it was published. He was, not only a dedicated teacher and researcher of international renown, but also a dear friend, sadly missed by everyone who knew him. We hope that the alterations we have made subsequent to his death are in keeping with the high standards he always set.

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Abstract

This paper considers the duration of property and equity. A general formula for duration of asset classes is derived. It is shown that calculations which assume, usually implicitly, that the flow-through of inflation to cash flow is zero, produce misleadingly high durations for property and equities. These are typically in the range 15 to 25 years. Simulations using the formulae show that property has some bond-like characteristics. The results indicate that, for realistic flow-through rates, equities have a higher duration than property. The flow-through rate is the most important variable in the estimation of equities. Using historical data, equity duration is estimated at 8.65 years and property's at 3.15 years. These are substantially lower than those commonly cited. If these values can be substantiated, and if higher values are used in practice, portfolio immunisation strategies may need to be reconsidered.

Keywords: Liabilities risk, immunisation, Macaulay Duration, equities, property

1. Introduction

The role of commercial property in multi asset portfolios has been examined from two main perspectives. First, the allocation to property in asset allocation models has been considered extensively (for example, Fogler, 1984; Firstenberg *et al.*, 1988; Hamelink and Hoesli, 1996; MacGregor and Nanthakumaran, 1992; Ross and Zisler, 1991; Webb *et al.*, 1988). The general conclusion is that property offers diversification benefits although there are substantial concerns about the appraisal based data used for property. Second, property has been considered as a hedge against inflation (see, for example, Barber *et al.*, 1997; Brown, 1991; Brueggeman *et al.*, 1984; Hartzell *et al.*, 1987; Hoesli, *et al.*, 1997; Limmack and Ward, 1988; Liu *et al.*, 1997). These studies on inflation hedging have found varying degrees of support for property as a hedge against both expected inflation and unexpected inflation.

Such findings are important considerations for institutional investors. In contrast, surprisingly little work has been done on the ability of property to contribute to asset/liability matching of institutional investors through measures of its duration. A prime consideration for pension funds and insurance companies is their ability to meet their contractual liabilities as and when they become due. In this context, the institutional objective is to maximise the surplus (of the value of assets over the value of liabilities) subject to an acceptable level of volatility of that surplus.

Investors attempt to achieve the objective of asset/liability matching through the principle of immunisation that ensures assets and liabilities are matched and the risk of any shortfall is thereby avoided. Redington (1952) introduced the use of duration to immunisation. Duration measures the sensitivity of an asset's value to changes in interest rates. The concept of duration was earlier developed by Hicks (1939) and Macaulay (1938). Redington pointed out that, in order to avoid any risk of a shortfall, the mean duration of assets must be equal to the mean duration of liabilities. Therefore, duration matching is seen as a risk minimising strategy in pension fund management.

Extensive work has been done on the duration of bonds but extensions to property and equity are somewhat limited, although the principles are applicable to both asset classes. The present paper considers the duration of both property and equities as investment classes, although its focus is on the former. More specifically, it has the following objectives:

- to derive formulae for the duration of property and to examine the variation in duration with lease structure;
- to derive formulae for the duration of equities and to compare equity and property durations; and
- to derive empirical estimates for the duration of property and equities.

The paper is structured as follows. In section 2, the literature on the duration of property is reviewed. Section 3, develops duration formulae and analyses the variation in duration with lease structure, discount rates and growth rates. In section 4, the empirical results are presented and finally, in section 5, some conclusions are given.

2. Literature review

Macaulay (1938) developed the concept of duration in his seminal analysis of interest rates and bond prices in order to find a more meaningful measure of life or longness of a bond than its term to maturity. He recommended a weighted average of the time to each bond payment where the weights were the present values of each of the payments as a percent of the price of the bond. Duration is, thus, measured in units of time.

In 1939, Hicks showed that duration is equal to the elasticity of the value of a stream of payments with respect to the discount factor $1/(1+r)$. This study, and later work, which showed that the duration and the elasticity are equal, implicitly assume that the income receivable is unaffected by changes in the interest rate. This assumption is valid only for bonds where the income is fixed. Hicks also noted that the relative prices of two income streams are invariant to changes in interest rates if the two have the same elasticity.

Samuelson (1945) in analysing the effects of interest rate changes on capital values of financial institutions came to a similar conclusion. He used the term weighted average distribution for duration and concluded that increased interest rates would help any organisation whose (weighted) average time period of disbursements is greater than the average time of its receipts. In a study of life offices, Redington (1952) computed first derivatives of the values of inflows and outflows with respect to interest rates and called the derivative ‘mean term’. He concluded that the existing

business is immune to a change in interest rates when the mean term of assets equals the mean term of the liabilities.

Most work on duration has focused on bonds because of their sensitivity to changes in interest rates (Bierwag *et al.*, 1990). It has been shown that duration is a measure for the elasticity of price with respect to interest rates. For most bonds, duration varies directly with maturity. Duration and coupon vary inversely. (For a complete review of the application of duration analysis to bonds see Bierwag *et al.*, 1983; and Bierwag *et al.*, 1988.).

Equity duration has been derived mainly from dividend discount models (DDM) that assume constant growth in dividends into perpetuity. In most studies, the assumption is made that changes in interest rates have no effect on the dividend income (for example, see Casabona *et al.*, 1984; Ludvik, 1990). Ludvik estimates duration of 22.2 and 16.7 years, respectively, for equities and property. These estimates are simply the reciprocals of the income yield of 4.5% and 6% for equities and property.

In a discounted cash flow model, the discount rate may change due to changes in the real rate of interest, expected inflation or the risk premium. The estimates which ignore the impact of inflation flow-through on the cash flows produce very high durations (see Diermeier, 1990; Feldstein, 1980; Hoesli *et al*., 1997; Modigliani and Cohn, 1979 for a discussion of the impact of inflation on cashflows). However, if the cash flows are adjusted to allow for increased expectations of inflation, the effect on the capital value will be much lower. According to Leibowitz *et al*. (1989, 30-31):

> *'The early DDM duration calculations typically led to values ranging from 20 to 50 years, with growth companies exhibiting the longest durations. An alternative form of analysis using straightforward regression techniques has been used to estimate empirically actual stock price sensitivity to interest rate changes. This has led to "empirical*

duration" values that range between two and six years - significantly less than the duration estimates derived from the DDM approach.'

Leibowitz (1986) estimates duration of 2.19 for the US equity market. A similar approach, which allows for inflation flow-through, has been adopted for the study of the duration of property. In the UK, Ward (1988) derived formulae for the duration of property based on a discounted cash flow model. He simulated a range of values for property duration assuming sensitivity of rental growth to interest rates of 0, 0.5 and 1. These rates are similar to inflation flow-through rates ($\partial g/\partial r$). The estimates for freehold property ranged from 9.33 to 36.05. Interestingly, the extreme high values were obtained for an inflation-flow through rate of zero. Ward claims that, where positive growth is expected, the duration of property is longer than the equivalent bond. However, this observation appears to be at odds with *a priori* expectations since bond income is fixed whereas property income can vary with changes in expectations of inflation.

Hartzell *et al.* (1988) employed a cash flow valuation model to study the duration of US commercial real estate. They concluded that real estate investors have some control over the duration of the asset through the lease contracting process. Furthermore, they showed, through simulation, that the duration of real estate can vary from 0 to 6 depending on the type of lease contract. They also estimate implied inflation flow-through rates that vary between 0.51 and 1.00 with varying lease terms.

Adams *et al.* (1993), in a study which examined the theoretical volatility of UK commercial property values, assumed no links between changes in interest rates and growth in rental values, that is a zero inflation flow-through rate. Their results must be viewed with this restriction in mind, since evidence shows inflation flow-through rates to be positive and above 0.50 in value in most instances. Adams *et al.* (1999) extend the work using the same basic framework.

In summary, very few studies have been undertaken on the duration of commercial property while there has been relatively more research into the application of duration to equities. These studies show that the values of both asset classes are affected by changes in the interest rates arising from changes in expectation of inflation. However, price volatility is often dampened due to the effect of increased inflationary expectations on the cash flows of the assets. In addition, the duration of property is affected by the nature of the lease contract.

3. The duration of commercial property

In the UK, commercial property is usually let on a long lease (15 - 25 years) with upward-only rent reviews at intervals of five years. When the rent paid is equal to the estimated rental value, the property is said to be 'fully let'. However, between rent review dates, the rent payable will normally be less than the estimated full rental value and this type of property is known as a 'freehold reversion' or as an 'under-rented' property.

In the early nineties, the rental values of commercial and industrial property in the UK fell due to the oversupply of space created by overbuilding in the late eighties. This was a rare phenomenon that occurred after years of growth in nominal rental values. Since lease terms in the UK allow for the revision of rents only in an upward direction, the tenants who had already entered into tenancy agreements were unable to take advantage of declining rents at reviews and were required to pay the originally contracted rent. This led to many properties being rented at values higher than the prevailing market rentals. Such properties are described as 'over-rented'. These properties will remain over-rented until market rental values catch up eventually with rents paid through rental growth or until the lease comes to an end. Under these conditions, uncertainty surrounds both expected future rental growth and the markets discount rate. The greater dependence on current income and the relatively lower certainty with respect to future returns will result in the duration of over-rented properties being lower than the duration for comparable fully-rent properties.

Ward (1988) derives formulae for the duration of *fully let* freehold property and for *leasehold* property. In the present paper, the formula for a freehold *reversion* is derived, both because reversions are more commonly found than fully let investments and because a fully let freehold property is a special case of this general formula which can also be used for the derivation of over-rented property. The formula for a fully let freehold can be derived from that of a reversion in a straightforward manner. Furthermore, the duration of an equity or a bond can readily be obtained from the duration formula of a reversion. The duration formula for a reversion also allows an investigation into the scope for the management of duration through differential leasing.

The main interest of this paper is asset price volatility as measured by the elasticity of price or capital value to changes in the discount rate. The relationship between elasticity (*e*) and duration (*D*) is given by:

$$\mathrm{D} = -\frac{\partial \mathrm{v}}{\partial \mathrm{r}} \cdot \left(\frac{1+\mathrm{r}}{\mathrm{r}} \right) = \boldsymbol{e} \qquad \textbf{(1)}$$

The general formula expressed in equation (1) can be used to derive the duration of a freehold reversion. This equation represents the average sensitivity of an asset's value to changes in the discount rate over the asset's holding period. The capital value *V*, of a freehold reversion can be expressed by:

$$V = R\left[\frac{1-(1+r)^{-t}}{r}\right] + F\left(\frac{1+g}{1+r}\right)^{t} \cdot \frac{\left[(1+r)^{n}-1\right]}{\left[(1+r)^{n}-(1+g)^{n}\right]} \cdot \frac{1}{r} \qquad \textbf{(2)}$$

where: *R* is the rent paid for the property; *F* is the current estimated rental value, *R*< *F*; *t* is the length of the term (the numbers of years before the next rent review or

rental change); g is the growth rate in rental values (assumed constant); n is the rent review period; and r is the risk adjusted discount rate.

The above equation has two parts, comprising the value of the *term* (for which the rent is fixed) and the value of the reversionary part, which is the capital value of an infinite stream of payments which are revisable to the open market rent at each rent review. Differentiating the term and the reversionary part with respect to r and using (1), the duration of the term is given by (see the appendix for full derivations):

$$D_T = \frac{(1+r)}{r} - \frac{t}{(1+r)^t - 1} \qquad \textbf{(3)}$$

The duration of the reversionary part is given by:

$$D_R = \frac{(1+r)}{r} - \frac{n(1+r)^n}{(1+r)^n - 1} + \frac{n(1+r)}{(1+r)^n - (1+g)^n} \cdot \left[(1+r)^{n-1} - (1+g)^{n-1} \frac{\partial g}{\partial r} \right] + t \left[1 - \left(\frac{1+r}{1+g} \right) \frac{\partial g}{\partial r} \right] \qquad \textbf{(4)}$$

From the durations of the constituent parts of equation (2), the duration of a freehold reversion can be derived from:

$$D = w_T \cdot D_T + w_R \cdot D_R \qquad \textbf{(5)}$$

where:

$$w_T = \frac{R\left[\frac{1-(1+r)^{-t}}{r} \right]}{V} \qquad \textbf{(6)}$$

and:

$$w_R = \frac{\left[F\left(\frac{1+g}{1+r} \right)^t \cdot \frac{\left[(1+r)^n - 1 \right]}{\left[(1+r)^n - (1+g)^n \right]} \cdot \frac{1}{r} \right]}{V} \qquad \textbf{(7)}$$

Note that the duration of a fully let freehold property can be obtained by putting $t = 0$ in equation (4). The duration of such an interest is:

$$D_p = \frac{(1+r)}{r} - \frac{n(1+r)^n}{(1+r)^n - 1} + \frac{n(1+r)}{(1+r)^n - (1+g)^n} \cdot \left[(1+r)^{n-1} - (1+g)^{n-1} \cdot \frac{\partial g}{\partial r} \right]. \quad \textbf{(8)}$$

The above equation represents the Macaulay duration. The modified duration which is a more direct measure of volatility, can be calculated by dividing the Macaulay duration by $(1+r)$. In equation (8), $\delta g/\delta r$ is the inflation flow-through (or the growth sensitivity to discount rates). The inflation flow-through parameter gives an indication of the responsiveness of the cash flows of the investment over the holding period to small changes in the discount rate.

Formula (8) is identical to the one derived by Ward (1988).

For equity investments, the duration can be obtained by substituting $n = 1$, since dividend income can be varied at each payment period. Equity duration is therefore:

$$D_E = \frac{(1+r)}{(r-g)} \cdot \left[1 - \frac{\partial g}{\partial r} \right] \quad \textbf{(9)}$$

If the simplifying assumption of a zero inflation flow-through ($\partial g/\partial r = 0$) is made, the familiar duration formula for equities is obtained:

$$D_E = \frac{(1+r)}{(r-g)} \quad \textbf{(10)}$$

4. Results of simulations

4.1 *Freehold reversions*

As explained at the beginning of section 3, an over-rented property is one where the rent passing is greater than the full rental value. In order to study the variation in duration of freehold reversions with lease terms, the duration of a reversionary property was calculated for different lengths of term (t) and for different levels of rental uplifts measured by the ratio R/F. The term of the reversion was made to vary from one year to 30 years. The rent passing (R) for the property was taken as 100 and the full rental value (F) was allowed to take the values 120, 150 and 180. In calculating the duration, the discount rate r and the rental growth rate g were kept at 10% and 4% respectively. In line with standard UK market practice, a value of five was assumed for the rent review period n.

The results of the simulations are shown in Table 1 for different values of the inflation flow-through rate ($\partial g/\partial r$). The table shows that freehold reversions exhibit many bond-like features. These may be summarised as follows:

- duration increases with the term to reversion (t);
- duration is higher, the lower is the R/F ratio; and
- duration is higher, the lower is the inflation flow-through ($\partial g/\partial r$).

The first and second effects are similar to the well-known maturity and coupon effects found in bonds. The cash flows during the term of a freehold reversion are fixed and exhibit bond-like characteristics while the reversion displays equity-like characteristics. Although duration increases with the term, the increase within the normal rent review cycle of five years is not great. The term of the freehold reversion must exceed 20 years to have an appreciable increase over the duration of a four year term. Such terms under modern lease regimes are extremely rare.

Table 1: Duration of freehold reversion when under-rented
(*Rent* = 100, *r* = 10% and *g* = 4%)

$\partial g/\partial r$		*F*		
0	*t*	120	150	180
	1	18.36	18.53	18.65
	2	18.48	18.81	19.03
	3	18.62	19.09	19.41
	4	18.76	19.36	19.78
	10	19.63	20.81	21.70
	20	20.29	21.98	23.38
	30	19.66	21.46	23.05

$\partial g/\partial r$		*F*		
0.25	*t*	120	150	180
	1	14.22	14.35	14.44
	2	14.30	14.55	14.72
	3	14.41	14.76	15.00
	4	14.52	14.97	15.28
	10	15.37	16.21	16.85
	20	16.39	17.57	18.54
	30	16.44	17.68	18.78

$\partial g/\partial r$		*F*		
0.5	*t*	120	150	180
	1	10.09	10.18	10.24
	2	10.13	10.29	10.41
	3	10.19	10.43	10.59
	4	10.29	10.57	10.78
	10	11.11	11.61	11.99
	20	12.50	13.16	13.70
	30	13.21	13.91	14.52

$\partial g/\partial r$		*F*		
0.75	*t*	120	150	180
	1	5.95	6.00	6.04
	2	5.95	6.04	6.10
	3	5.98	6.10	6.18
	4	6.05	6.18	6.28
	10	6.84	7.01	7.14
	20	8.60	8.75	8.87
	30	9.99	10.13	10.25

$\partial g/\partial r$		*F*		
1.00	*t*	120	150	180
	1	1.82	1.83	1.83
	2	1.77	1.78	1.78
	3	1.77	1.77	1.76
	4	1.81	1.79	1.77
	10	2.58	2.41	2.28
	20	4.70	4.33	4.03
	30	6.76	6.35	5.99

4.2 Over-rented Property

The duration of over-rented property was calculated using the same set of equations used in the calculation of under-rented property. The only difference in the application being the term (t) for which the passing rent will continue to be paid had to be calculated given the ratio between the passing rent R and the full rental value F (Note: for over-rented property $F<R$). To facilitate comparison with under-rented property (see Table 1) the same growth and discount rates (g = 4%, r = 10%) are used.

Table 2 considers the duration in 1998 of an over-rented property with rent passing R = 100 and full rented value F = 60. It is assumed that the property was let at the peak of the property boom in 1988 on a 25 year lease that provides for rental changes in an upward-only direction at intervals of five years. The property is currently over-rented by 66.7% ([100-60]/60). Assuming a rental growth rate of 4%, it will take 13 years (year 2011) for the market rental to catch up with the rent paid. Since the lease provides for rental changes only at intervals of five years from 1988, no change to the rent can be effected until the year 2013 which, by coincidence, is also the year the 25 year lease will come to an end. This would mean the passing rent of 100 would continue for another 15 years. In Table 2, the duration is, therefore, given for a term of 15 years with flow-through rates $\partial g/\partial r$ varying from 0 to 1 in steps of 0.25.

Table 2: Duration of freehold reversion when over-rented
(*Rent passing (R)* = 100, *Full rental value (F)* = 60, *r* = 10% and *g* = 4%)

$\partial g/\partial r$	t	Duration
0	15	15.59
0.25	15	12.95
0.5	15	10.32
0.75	15	7.69
1	15	5.06

The figures in Table 2 show that, for the over-rented property, the durations vary appreciably as the flow-through rate varies from 0 to 1. These variations are more marked than those in Table 1 for under-rented property. Table 3 provides durations of fully let property ($R = F$). An examination of the figures in Tables 1-3 suggest that the divergence in duration is greater for over-rented and fully let property than it is for under-rented and fully let property (see Tables for r = 10% and g = 4%). This result indicates that over-rented property offers better scope for duration management. However, over-renting is a market phenomenon and the owner has no control over the degree of over-renting.

The implication is that there is very little scope for the owner to vary duration by varying the terms of the lease. Under modern leasing arrangements where the norm is

for rent reviews at intervals of five years, terms of more than five years will be rare. Table 1 clearly shows that durations do not vary significantly for terms less than five years. However, there will be opportunities for varying the duration of a portfolio of properties by having a mix of low and high yielding properties. Examples of low and high yielding properties can be found respectively in the retail and industrial property sectors.

4.3 Fully let freehold property

A fully let property is one which is let at its full rental value, subject to rent reviews at regular intervals of time. In the UK, this period is five years. Equation (8) gives the duration of a fully let freehold property.[1] Table 3 shows simulated values of duration for the following values of the discount rate (r), rental growth rate (g) and inflation flow-through rate ($\partial g/\partial r$):

- discount rates: 9% to 13% in steps of 1%;
- rental growth rates: 2% to 6% in steps of 1%;
- rent review period: 5 years; and
- inflation flow-through: 0, 0.25, 0.5, 0.75 and 1.

A number of interesting observations can be made from the results:

- high growth or low yielding property tend to have high durations (except when $\partial g/\partial r = 1$);

[1] These calculations assume that the property is let on a long lease with upward only rent reviews. However, University of Aberdeen and IPD (1997) and DETR (2000) have shown that the average lease length in the UK is shortening. Evidence suggests that leases which have 10 or less years to expire will have up to a 2% rise on the all-risks yield [Crosby, Baum and Murdoch, 1996]. This rise in discount rate will cause a decrease in property durations. Another effect of the changing structure of leases is the shortening of the rent review period. In the event that this occurs, calculations with shorter rent review periods imply that duration decreases with the rent review periods.

In addition, the UK government is currently contemplating legislating against upward only rent reviews. The removal of the assumption of an upward only rent review clause may have two effects. Firstly, it may reduce the potential for rental growth. Secondly, it would also impact on the risk premium attached to property investments, pushing up discount rates. Lower rental growth and higher discount rates would combine to create lower durations.

- the lower the inflation flow-through ($\partial g/\partial r$), the higher is the duration; and
- duration varies inversely with the discount rate.

In Table 3, the highest duration of 36.2 is found when the discount rate is 9%, the growth rate is 6% and $\partial g/\partial r$ is 0. In reality it is difficult to find property with such a low yield (about 3.17%) and no inflation flow-through. An income yield of around 6.5% - 7% is more likely (which may be considered as the approximate difference between r and g), and thus the duration of property is more likely to fall within the range of 5 and 12 (for r = 12%; g = 5% and $\partial g / \partial r$ = 0.25 - 0.75). In section 4.6 an empirical estimate is made based on historical values of the inputs.

Table 3: Duration as a function of *g* and *r* for various levels of $\partial g/\partial r$ and *n*.

	n	*r*				
$\delta g/\delta r$	5	9%	10%	11%	12%	13%
0	2%	15.53	13.71	12.29	11.16	10.23
	3%	18.11	15.66	13.82	12.39	11.24
	4%	21.72	18.26	15.78	13.92	12.48
	5%	27.15	21.9	18.4	15.9	14.03
	6%	36.22	27.38	22.08	18.55	16.03

	n	*r*				
$\delta g/\delta r$	5	9%	10%	11%	12%	13%
0.25	2%	12.14	10.77	9.709	8.859	8.163
	3%	14.07	12.23	10.85	9.773	8.915
	4%	16.77	14.17	12.31	10.92	9.838
	5%	20.84	16.9	14.28	12.4	11
	6%	27.63	21.01	17.03	14.38	12.49

0.5	5	9%	10%	11%	12%	13%
	2%	8.744	7.833	7.124	6.557	6.092
	3%	10.02	8.796	7.876	7.16	6.588
	4%	11.82	10.09	8.849	7.92	7.197
	5%	14.53	11.9	10.15	8.901	7.963
	6%	19.05	14.63	11.98	10.22	8.954

0.75	5	9%	10%	11%	12%	13%
	2%	5.35	4.894	4.539	4.255	4.022
	3%	5.981	5.367	4.906	4.548	4.26
	4%	6.87	6.003	5.383	4.918	4.556
	5%	8.214	6.901	6.026	5.4	4.931
	6%	10.47	8.258	6.933	6.049	5.417

1	5	9%	10%	11%	12%	13%
	2%	1.956	1.955	1.954	1.952	1.951
	3%	1.938	1.937	1.936	1.935	1.933
	4%	1.92	1.919	1.918	1.917	1.916
	5%	1.902	1.901	1.9	1.899	1.898
	6%	1.88	1.883	1.882	1.882	1.881

Calculations were also undertaken for rent review periods of 3 and 7 years. These showed that duration increases with the rent review period but the magnitude of the variation is not very significant. For example, for a discount rate, r = 10%, growth

rate, g = 4% and inflation flow-through, $\partial g/\partial r$ = 0.5 the duration is 9.7, 10.1 and 10.5, for rent review periods of 3, 5 and 7 years, respectively.

4.4 Equity duration

The general formula for the duration of equity is given by equation (9). Equation (10) gives the formula when the assumption of independence between discount rate changes and dividend growth changes are made. The formula in equation (10) has been widely used to calculate the duration of equity in the region of 20. However, evidence shows that changes in expectations of inflation and changes in interest rates affect profits and dividends. The empirical estimates of equity show a much lower value (for example, see Leibowitz *et al.*, 1989). The simulated values of equity durations are shown in Table 4. Clearly, the high values of equity duration are found when $\partial g/\partial r$ = 0. The following observations can be made from Table 4:

- high growth (low yield) shares tend to have high durations, and
- duration of equities vary inversely with the value of $\partial g/\partial r$.

4.5 A comparison of property and equity duration

Table 4 compares the duration of property with that of equities. The figures express the ratio of equity duration to property duration. The figures for property duration are for a rent review period of 5 years. The figures show that for given values of r and g, property duration is marginally greater than equity duration except when $\partial g/\partial r$ = 0. This could be explained by the lagged response in the cash flows in the case of property due to the rent review period not being annual. In the calculations, it has been assumed that income from property can only be varied at intervals of 5 years. However, this comparison has limitations since the same values cannot be assumed for the discount rate, r, growth rate, g, and the flow-through rate, $\partial g/\partial r$ for the two asset classes. A comparison of data on equity and property discount and growth rates reveal that both the discount rate and growth rate tends to be greater for equities (see Table 5). In addition to this empirical evidence, Hoesli and MacGregor (2000)

demonstrate that the discount rates are greater for equities. A comparison of data on equity and property discount and growth rates reveal that both the discount rate and growth rate tends to be greater for equities (see Table 5). In addition to this empirical evidence, Hoesli and MacGregor (2000) demonstrate that discount rates are greater for equities.

Table 4: Equity and property duration compared

Equity Duration

$\delta g \delta/r$	n		r				
0	1		9%	10%	11%	12%	13%
		2%	15.570	13.750	12.330	11.200	10.270
		3%	18.170	15.710	13.880	12.440	11.300
		4%	21.800	18.330	15.860	14.000	12.560
		5%	27.250	22.000	18.500	16.000	14.130
		6%	36.330	27.500	22.200	18.670	16.140
$\delta g \delta/r$			9%	10%	11%	12%	13%
0.25	1	2%	11.68	10.31	9.35	8.40	7.71
		3%	13.63	11.79	10.41	9.33	8.48
		4%	16.35	13.75	11.89	10.50	9.42
		5%	20.44	16.50	13.88	12.00	10.59
		6%	27.25	20.63	16.65	14.00	12.11
$\delta g \delta/r$			9%	10%	11%	12%	13%
0.5	1	2%	7.786	6.875	6.167	5.600	5.136
		3%	9.083	7.875	6.938	6.222	5.650
		4%	10.900	9.167	7.929	7.000	6.278
		5%	13.630	11.000	9.250	8.000	7.063
		6%	18.170	13.750	11.100	9.333	8.071
$\delta g \delta/r$			9%	10%	11%	12%	13%
0.75	1	2%	3.893	3.438	3.083	2.800	2.568
		3%	4.542	3.929	3.469	3.111	2.825
		4%	5.450	4.583	3.964	3.500	3.139
		5%	6.813	5.500	4.625	4.000	3.531
		6%	9.083	6.875	5.550	4.667	4.036

Equity Duration as a percentage of property duration

$\delta g \delta/r$	n		r				
0	5		9%	10%	11%	12%	13%
		2%	1.003	1.003	1.003	1.003	1.004
		3%	1.003	1.004	1.004	1.005	1.005
		4%	1.004	1.004	1.005	1.006	1.006
		5%	1.004	1.004	1.005	1.006	1.007
		6%	1.003	1.004	1.005	1.006	1.007
0.25	5		9%	10%	11%	12%	13%
		2%	0.962	0.957	0.953	0.948	0.944
		3%	0.969	0.964	0.959	0.955	0.951
		4%	0.975	0.970	0.966	0.961	0.957
		5%	0.981	0.976	0.972	0.968	0.963
		6%	0.986	0.982	0.977	0.973	0.969
0.5	5		9%	10%	11%	12%	13%
		2%	0.890	0.878	0.866	0.854	0.843
		3%	0.906	0.893	0.881	0.869	0.858
		4%	0.922	0.909	0.896	0.884	0.872
		5%	0.938	0.924	0.911	0.899	0.887
		6%	0.954	0.940	0.926	0.914	0.901
0.75	5		9%	10%	11%	12%	13%
		2%	0.728	0.702	0.679	0.658	0.639
		3%	0.759	0.732	0.707	0.684	0.663
		4%	0.793	0.763	0.736	0.712	0.689
		5%	0.829	0.797	0.768	0.741	0.716
		6%	0.868	0.833	0.801	0.771	0.745

4.6 Empirical estimates of duration

In this sub-section an attempt is made to estimate the duration of equity and property from historical data. From equations (8) and (9) it is evident that estimates are required for the discount rate, r, the growth rate, g, and the flow-through rate, $\partial g/\partial r$. In order to undertake this estimation, data for equities are drawn from the BZW Equity-Gilts study while property data come from the Investment Property Databank (IPD). The data cover a 27-year period from 1970-96.

The discount rates, r, for equities and property are taken as the redemption yield on a 20-year Government bond plus a risk premium of 3% and 2%, respectively[2]. For equities, the difference between the discount rate and the dividend yield is calculated for the 27-year period. This gives the implied long-term constant growth rate in dividends, g. Similarly for property, the average yields and the discount rates are used to calculate the implied long-term constant growth rates for the 27-year period.[3] The mean values of r and g are used in the estimation of duration for the two asset.

The inflation flow-through parameter ($\partial g/\partial r$) measures the effect of inflation on the growth in share and rental income. Its value is difficult to estimate. Following similar analyses, it can be proxied by the cross correlation between r and g [for example, Hartzell *et al*., (1988)], however the estimates will depend on how the lead-lag structure is modelled. Alternatively, and arguable more technically correct, the relationship between g and r can be measured as the cross correlation between the change in r and change in g. Thus, the two versions of the inflation flow-through proxy are included in the duration calculations.

Table 5: Empirical estimates of duration

[2] The 2% risk premium amounts to the 'conventional wisdom' of the property market. The delivered risk premium attached to shares has been about 2.5% although in recent years it has been much higher. A share premium of 3% is taken as an illustrative example, however the figure may be higher in practice [Hoesli and MacGregor, 2000].

[3] For property, the estimate of g is net of depreciation. Further, it has been assumed that the risk premium and the rate of depreciation are constant.

	r	g	ρ using r to g	Duration	ρ using Δr to Δg	Duration
Equities	13.82%	8.82%	0.62	8.65	-0.09	24.81
Property	12.82%	5.76%	0.91	3.15	0.88	3.57

The values used in the calculations are shown in Table 5. Substituting these values in equations (9) and (8), gives the duration of equities as 8.65 and of property as 3.15 when the cross correlation between r and g are applied. Using the cross correlation between the change in the discount and growth rates increases the duration of equities to 24.81 while the duration of property rises only slightly to 3.57.

The substantial difference in the duration of equities arises because the inflation flow-through parameter dramatically falls when r and g are differenced. This results in a negative value for the inflation flow-through. It can be shown that any increase in r arising from an increase in expected inflation will lead to an increase in the net cash flows received from an investment. This implies that the value for $\partial g/\partial r$ should be positive [Leibowitz *et al*., 1989] and questions the accuracy of this inflation flow-through measure.

Figure 1 plots the variation in the duration for different values of $\partial g/\partial r$. The duration of equities was calculated using the mean values of r and g used in the above calculation but the value of $\partial g/\partial r$ was allowed to vary from 0 to 1 in steps of 0.1. The same procedure was repeated for property. The Figure shows that for the values of r and g used, the duration of the two asset classes is equal, at a value of 4.92 when the flow-through rate is 0.78. Equity duration is greater than property duration up to a flow-through rate of 0.78. At a flow-through rate of 1, equity duration is 0 whereas property duration has a value of 1.88. At a flow-through rate of 0, equity and property have durations of 22.76 and 15.91, respectively.

As a check, OLS regression of equity prices (BZW, 1996) and discount rates (both expressed in logarithms) was undertaken. The slope of the equation gives a measure

of the elasticity. Since Hicks (1939) and later work, proved that elasticity and duration are equal (see equation 1), the estimation of elasticity using historical data will provide an appropriate measure of duration.

A similar equation was estimated for property and the results are summarised in Table 6. Since a price based index is not available for property, the IPD capital value index was used. The regressions produced estimates of 4.82 for equity and 1.65 for property. The IPD capital value index is derived from valuations and is subject to a degree of smoothing. This series as, therefore, 'de-smoothed'[4] and the regression equation re-estimated which produced an elasticity of 3.04. The regression results together with the results from Table 5, produce a range of 4 – 25 for equity and 1 – 4 for property.

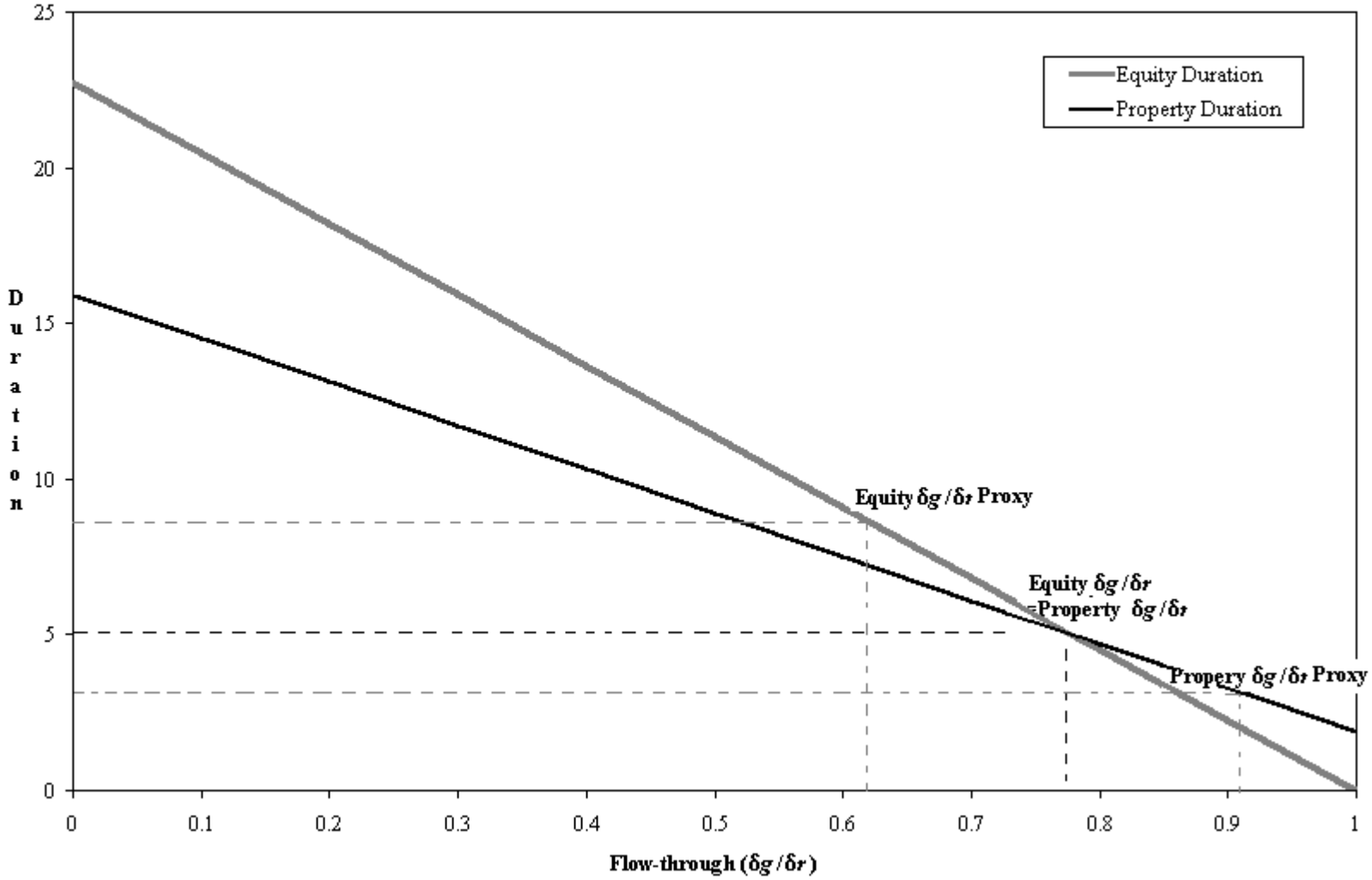
Table 6: Regression elasticity measures for equities and property.

	Equities	**Unadjusted Property**	**De-smoothed Property**
Constant	18.45*	9.918*	14.21*
Elasticity	-4.822*	-1.648*	-3.036**
Adjusted R^2	0.532	0.207	0.093
F-statistic F(1,25)	30.58*	7.792*	3.569**
Serial correlation χ^2 (1)	16.571 [0.000]	17.69 [0.000]	2.399 [0.121]
Functional form χ^2 (1)	0.122 [0.727]	0.291 [0.589]	3.139 [0.076]
Normality χ^2 (2)	7.11 [0.029]	9.353 [0.009]	1.12 [0.571]
Heteroscedasticity χ^2 (1)	1.311 [0.252]	1.349 [0.245]	2.116 [0.146]

*Significant at the 1% confidence level.
** Significant at the 10% confidence level.

[4] The series was de-smoothed assuming that the capital values follow an autoregressive (AR1) process. For details see Blundell and Ward, 1987.

Figure 1: Equity duration and property duration versus flow-through



The empirical estimates using the cross correlation between the discount and growth rates and the regression results are broadly in agreement with the findings of US studies based on regression which suggest a range of 2 – 6 for the equity market and about 1 – 4 for property (Leibowitz *et al*., 1989 and Hartzell *et al.*, 1988). It is also important to note that these estimates are much lower than those based on duration formulae that assume perfect independence between discount rates and dividends of shares or rents of property. However, while a useful check, the regression results must be treated with a degree of caution. The results from the bivariate regressions are likely to suffer from the 'omitted variables' problem since prices and capital values are likely to be influenced by a series of other variables in addition to long-term interest rates (see Chau *et al*., 1998). Also, the summary diagnostic tests suggest that the regressions for equities and unadjusted property reject the assumptions of normality and serial independence while the model for de-smoothed property appears to have a heteroscedasticity problem.

5. Summary and conclusion

This paper has considered the durations of equities and property and, it is believed, this is the first attempt to compare the two assets. The formulae for the duration of the asset classes were derived. It was shown that calculations, which assumed, usually implicitly, that the flow-through of inflation to cash flow was zero, produced misleadingly high durations for property and equities. These were typically in the range 15 to 25 years.

Simulations using the formulae also showed that property has some bond-like characteristics. Attempts at empirical estimation produced results which indicated that, for realistic flow-through rates, equities tend to have a higher duration than property. However, the flow-through rate emerged as the most important variable in the estimation of equity duration. This necessitates further work to estimate robust values for the crucial flow-through parameters.

Using historical data, equities' duration was estimated at 8.65 years and property's at 3.15 years. These are substantially lower than those commonly cited[5]. If these values can be substantiated, and if higher values are used in practice, immunisation strategies may need to be reconsidered.

Finally, it should be noted that duration measures the responsiveness of values/prices for only very small changes in the interest rates. A more complete measure for volatility should combine duration with convexity. This would allow a finite variation in the interest rates to be studied.[6]

[5] There may be an equivalent effect on the income streams of liabilities which means durations are still matched.

[6] For small changes in interest rate, duration gives a close approximation to the actual change in capital value, however a better approximation is obtained if a convexity term is included. Convexity measures the rate of change of changes in value with respect to changes in the interest rate. It is a second-order measure of the sensitivity of capital value to changes in the interest rate, and is an indicator of the sensitivity of capital value to changes in interest rate. The improvement in the accuracy of the results when a convexity term is included can be seen in Nanthakumaran and Orr (1998)

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Appendix: Derivation of formula for the duration of a freehold

The value (V) of a freehold reversion is given by:

$$V = R\left[\frac{1-(1+r)^{-t}}{r}\right] + F\left(\frac{1+g}{1+r}\right)^{t} \cdot \frac{\left[(1+r)^{n}-1\right]}{\left[(1+r)^{n}-(1+g)^{n}\right]} \cdot \frac{1}{r} \qquad \textbf{(A1)}$$

where: R is the rent paid for the property; F is the current estimated rental value, $R < F$; t is the length of the term (the numbers of years before the next rent review); g is the growth rate in rental values (assumed constant); n is the rent review period; and r is the risk adjusted discount rate.

The rent R will be paid for a period of t years ($t < n$), at the end of which period the owner will be able to revise the rent to the prevailing full market rental. The rent of the property is reviewed at intervals of n years. Usually, n equals 5 for UK commercial leases.

Equation (A1) may be rewritten in the following form:

$$V = R \cdot y + F \cdot u \cdot v \cdot w \qquad \textbf{(A2)}$$

where: $R \cdot y$ is the value of the term (V_T) and $F \cdot u \cdot v \cdot w$ is the value of the reversion (V_R).

The duration of the reversion can be derived by differentiating the Equation (A1) with respect to the discount rate (r).

Thus, for the term:

$$y = \frac{1-(1+r)^{-t}}{r}$$

$$\frac{\partial y}{\partial r} = \frac{r\left[t(1+r)^{-t-1}\right] - \left[1-(1+r)^{-t}\right]}{r^2}$$

$$= \frac{t}{r(1+r)^{t+1}} - \frac{\left(1-(1+r)^{-t}\right)}{r^2} \qquad \textbf{(A3)}$$

For the reversion:

$$u = \frac{(1+g)^t}{(1+r)^t}$$

$$\frac{\partial u}{\partial r} = \frac{t(1+r)^t \cdot (1+g)^{t-1} \cdot \frac{\partial g}{\partial r} - t(1+g)^t (1+r)^{t-1}}{(1+r)^{2t}}$$

$$= \frac{t(1+g)^{t-1} \cdot (1+r)^{t-1}\left[(1+r)\frac{\partial g}{\partial r} - (1+g)\right]}{(1+r)^{2t}}$$

$$= \frac{t(1+g)^{t-1}\left[(1+r)\frac{\partial g}{\partial r} - (1+g)\right]}{(1+r)^{t+1}} \qquad \textbf{(A4)}$$

and:

$$v = \frac{(1+r)^n - 1}{r}$$

$$\frac{\partial v}{\partial r} = \frac{\left[rn(1+r)^{n-1} - (1+r)^n + 1\right]}{r^2} \qquad \textbf{(A5)}$$

$$w = \frac{1}{(1+r)^n - (1+g)^n}$$

$$\frac{\partial w}{\partial r} = -\frac{1}{\left[(1+r)^n - (1+g)\right]^2} \cdot \left[n(1+r)^{n-1} - n(1+g)^{n-1} \cdot \frac{\partial g}{\partial r} \right]$$

$$= \frac{n\left[(1+g)^{n-1} \cdot \frac{\partial g}{\partial r} - (1+r)^{n-1} \right]}{\left[(1+r)^n - (1+g)^n \right]^2} \qquad \textbf{(A6)}$$

Now:

$$V_R = F \cdot u \cdot v \cdot w$$

$$\frac{\partial V_R}{\partial r} = F \cdot v \cdot w \cdot \frac{\partial u}{\partial r} + F \cdot u \cdot w \cdot \frac{\partial v}{\partial r} + F \cdot u \cdot v \cdot \frac{\partial w}{\partial r}$$

$$\frac{\frac{\partial V_R}{\partial r}}{V_R} = \frac{\frac{\partial u}{\partial r}}{u} + \frac{\frac{\partial v}{\partial r}}{v} + \frac{\frac{\partial w}{\partial r}}{w} \qquad \textbf{(A7)}$$

Therefore, Duration (D_R) is given by:

$$D_R = -\frac{\partial V_R}{\partial r} \cdot \left(\frac{1+r}{v} \right) = -\frac{\partial V_R}{\partial r} \cdot \left(\frac{1+r}{V_R} \right)$$

(A8)

Substituting Equation (A8) for $\frac{\partial V_R}{\partial r} \cdot \frac{1}{V_R}$ and Equations (A4), (A5) and (A6) for the derivatives of *u*, *v* and *w*, the duration of the reversion (D_R) is given by:

$$D_R = \frac{1+r}{r} - \frac{n(1+r)^n}{(1+r)^{n-1}} - \frac{n(1+r)}{(1+r)^n - (1+g)^n} \left\{ (1+g)^{n-1} \frac{\partial g}{\partial r} - (1+r)^{n-1} \right\} - t\left[\left(\frac{1+r}{1+g} \right) \cdot \frac{\partial g}{\partial r} - 1 \right]$$

(A9)

This gives the duration of a freehold that commences after *t* years.

Using similar reasoning the duration of the term (V_T) is derived by using the derivative given in Equation (A3). Therefore:

$$D_T = -\frac{\partial V_T}{\partial r} \cdot \frac{1+r}{V}$$

$$D_T = \frac{1+r}{r} - \frac{t}{(1+r)^t - 1} \qquad \textbf{(A10)}$$

The duration of a freehold reversion given by Equation (A1) can now be obtained by using the values of D_T and D_R.

$$D = w_T \cdot D_T + w_R \cdot D_R \qquad \textbf{(A11)}$$

where: w_T and w_R are the proportions of the value of the term and reversion to the total value of the freehold reversion.