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1 **Simulating spatial and temporal evolution of multiple wing cracks around faults in**  
2 **crystalline basement rocks**

3

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10 **Running title:** Wing crack evolution around faults

11 **Keywords:** mechanical modeling, faults, splay fractures, damage zone, dynamic

12

12 **Abstract**

13 Faults zones are structurally highly spatially heterogeneous and hence extremely  
14 complex. Observations of fluid flow through fault zones over several scales show that  
15 this structural complexity is reflected in the hydrogeological properties of faults.  
16 Information on faults at depth is scarce, hence, it is highly valuable to understand the  
17 controls on spatial and temporal fault zone development. In this paper we increase our  
18 understanding of fault damage zone development in crystalline rocks by dynamically  
19 simulating the growth of single and multiple splay fractures produced from failure on a  
20 pre-existing fault. We present a new simulation model, MOPEDZ, that simulates fault  
21 evolution through solution of Navier's equation with a combined Mohr-Coulomb and  
22 tensile failure criteria. Simulations suggest that location, frequency, mode of failure and  
23 orientation of splay fractures are significantly affected both by the orientation of the fault  
24 with respect to the maximum principal compressive stress and the conditions of  
25 differential stress. Model predictions compare well with published field outcrop data,  
26 confirming that this model produces realistic damage zone geometries.

27

28

## 29 **Introduction**

30 Faults are structurally highly spatially heterogeneous and hence extremely complex  
31 [Aydin, 2000; Caine *et al.*, 1996; do Nascimento *et al.*, 2005; Fairley and Hinds, 2004;  
32 Galli *et al.*, 2004; Wibberley and Shimamoto, 2003]. Observations of spatially  
33 heterogeneous fluid flow through fault zones over several scales e.g. [do Nascimento *et*  
34 *al.*, 2005; Fairley and Hinds, 2004] show that this structural complexity is reflected in the  
35 hydrogeological properties of faults. Within crystalline basement rocks, permeable faults  
36 are a dominant feature of subsurface flow systems. The dominant deformation structures  
37 in these systems are fractures that may be open or filled with minerals or gouge. Recent  
38 research at the European Union's Soultz-sous-Forêt Hot Dry Rock test site [Evans *et al.*,  
39 2005a], underlines the importance of characterising fault zones. Data taken from low  
40 pressure injection tests on the open borehole at Soultz show that almost all flow occurs  
41 within a single fault zone located at 3490 m depth, and that just 10 major open fractures  
42 account for 95% of the flow.

43

44 Fault zone structure can also vary temporally due to continued movement on the fault  
45 and/or changing stress conditions. This can result in the creation of new fractures and the  
46 reopening of sealed existing fractures, both of which may lead to increased fault zone  
47 permeability. Observations during the high pressure injection tests at the HDR Soultz-  
48 sous-Forêt site confirm this link between fault damage zone evolution and increased flow  
49 to the borehole [Evans, 2005; Evans *et al.*, 2005b; Evans *et al.*, 2005c]). Temporal  
50 changes in fault hydraulic properties have also been observed in the hydrocarbon industry  
51 [Anderson *et al.*, 1994; Losh, 1998; Losh, 2006].

52

53 The inaccessibility of subsurface faults, in combination with their spatial and temporal  
54 complexity, makes it hard either to assess fault architectural structure or to predict fault  
55 permeability. Data from boreholes and seismic surveys are available, however neither  
56 provide the information necessary to constrain values of fault permeability. Given the  
57 scarcity of information on faults at depth, it is advantageous to gain as much knowledge  
58 as possible on the processes that govern spatial and temporal fault zone development.  
59 Ultimately, if these processes can be numerically simulated, then predicted fault zone  
60 architectures can be up-scaled to provide statistical estimates of bulk permeability fields.  
61 The aim of our research is to predict damage zone features on small (subseismic) faults  
62 with geometries that are geologically realistic. It is particularly important for estimation  
63 of bulk fault permeability in crystalline rocks to accurately simulate the orientation and  
64 connected nature of evolving fractures. In this paper, we present the first stage in  
65 achieving this aim; simulating the dynamic growth of single and multiple splay fractures  
66 produced from failure on pre-existing faults or joints.

67

68 *Previous simulation studies of fault damage zone development*

69 A number of researchers have employed numerical simulation methods to predict the  
70 evolution of damage zone features surrounding faults. The modeling studies of  
71 [Burgmann *et al.*, 1994; Du and Aydin, 1993; 1995] employ Linear Elastic Fracture  
72 Mechanics, first presented in [Pollard and Segall, 1987], to simulate the growth of splay  
73 fractures from original faults or joints. [Du and Aydin, 1995] calculate the distribution of  
74 strain energy around pre-existing features such as isolated fault tips, echelon fault steps

75 and fault bends, and infer the directions of propagating fractures from the direction of  
76 maximum distortional strain energy. [Shen and Stephansson, 1993] simulate the  
77 propagation of shear fractures using the F-criteria based on combining the Maximum  
78 Principal Stress and the Maximum Strain Energy Release Rate criteria, which they  
79 propose for simulation of Mode I and Mode II fractures. Using the F-criteria, they  
80 simulate damage at the fault tips whereby an initial single high angle tension fracture  
81 evolves followed by a single shear fracture that propagates in the same direction as the  
82 original fault.

83

84 More recently, research has focused on simulation of fault zone damage from dynamic  
85 fault rupture in seismically active faults to improve estimates of energy losses during  
86 earthquakes. [Yamashita, 2000] combines laboratory experiments with numerical  
87 simulation to predict the generation of microcracks during macroscopic shear rupture. He  
88 simulates the temporal evolution of maximum tensile stress at a rupture tip to infer the  
89 location and orientation of associated microcracks in the surrounding rock. He concludes  
90 that dynamically propagating earthquake faults generate a large number of tensile  
91 microcracks in the surrounding fault zone. [Dalguer et al., 2003] develop and apply a 3D  
92 Discrete Element Model to simulate crack propagation during the seismic rupture of a  
93 pre-existing fault. They simulate the generation of new cracks within the surrounding  
94 fault zone caused by progression of a single rupture patch on the fault with a tensile  
95 failure criterion: as the rupture front progresses, tensile cracks expand and new cracks are  
96 generated at the tip of the dynamic rupture patch.

97

98 In this paper we extend fault zone modeling of fault growth, such as that by [Burgmann *et*  
99 *al.*, 1994; Du and Aydin, 1993; 1995; Shen and Stephansson, 1993], to include spatial and  
100 temporal evolution of single and multiple wing cracks within a fault damage zone in  
101 crystalline basement. The model presented here introduces several novel processes that  
102 have not been previously accounted for: cracks are propagated dynamically based on the  
103 temporally evolving stress field; simulations investigate progressive microscopic-to-  
104 macroscopic failure; the model uses a combined Mohr Coulomb and tensile failure  
105 criterion that allows for both shear and tensile failure of the rock.

106

### 107 **Formulation of a Numerical Model for the Evolution of Fault Damage Zones**

108 This paper presents a new model for the simulation of fault damage zone evolution. The  
109 code is a two-dimensional coupled hydro-mechanical model, MOPEDZ (Modelling Of  
110 Permeability Evolution in the Damage Zone surrounding faults), and is based on a finite  
111 element approach to solving Navier's equation with a combined Mohr Coulomb-tensile  
112 failure criterion, coupled to the groundwater flow equation [Willson *et al.*, 2005]. We  
113 present the results of simulations using the mechanical modeling component of MOPEDZ  
114 in order to investigate the controls on the evolution of wing crack generation around a  
115 single pre-existing fault.

116

117 Navier's equation is well known as describing the displacement of a body subject to  
118 external forcing. The steady-state form of the equation can be written as

$$119 \quad \nabla \cdot c \nabla \mathbf{u} = \mathbf{F} \quad (1)$$

120 Where  $\mathbf{F}$  is the vector of external forces,  $\mathbf{u}$  is a vector describing displacement and  $c$  is a  
121 matrix where each entry is a function of the first and second Lamé constants. The failure  
122 criteria employed to simulate rock fracturing in MOPEDZ are the Mohr-Coulomb and  
123 tensile failure criteria, generally written as

$$124 \quad \sigma_1 \leq C_0 + \left[ (\mu^2 + 1)^{1/2} + \mu \right] \sigma_3 \text{ and } \sigma_3 \leq -T_0 \quad (2)$$

125 respectively, where  $\sigma_1$  is the maximum principal compressive stress,  $\sigma_3$  is the minimum  
126 principal compressive stress,  $C_0$  is the uniaxial compressive strength,  $\mu$  is the coefficient  
127 of friction and  $T_0$  is the tensile strength.

128

129 MOPEDZ is built using the commercially available finite element software, FEMLAB  
130 [COMSOL, 2004]. FEMLAB is used to provide finite element subroutines that are called  
131 from the MOPEDZ code, which has been developed, and is executed, within MATLAB.  
132 MOPEDZ models the propagation of fractures by solution of Navier's equation. Failure  
133 is predicted by a combined Mohr-Coulomb and tensile failure criterion, which results in  
134 changes to the material parameters at failed locations. Elements that contain fractures (as  
135 opposed to intact host rock only) are represented by a reduction in the values of Young's  
136 Modulus, Poisson's ratio and the material strength. This approach is similar to that of  
137 [Tang, 1997] which has been successfully applied to simulate laboratory experiments and  
138 reproduce fracture patterns around tunnels and boreholes.

139

140 The aim of MOPEDZ is to reproduce the change in material properties (Young's  
141 Modulus, Poisson's ratio and the material strength) of a rock as it fails i.e. the first failure  
142 is triggered by a deformation of the boundaries, but subsequent failures can occur



143 spontaneously because of stress redistribution around previous failures. These subsequent  
144 failures can be adjacent to previous failures, i.e. the extension of a fracture, or they can  
145 occur in locations that are disconnected from any previous failure, but fail because of the  
146 redistribution of stress. MOPEDZ solves Navier's equation as a progressive series of  
147 steady-states. Initially, the top and bottom boundaries of the model domain are displaced  
148 inwards by a small increment. Navier's equation is then solved and the number of  
149 predicted failures examined. The displacement increment is then adjusted, and Navier's  
150 equation re-solved, such that a pre-defined small number of elements are predicted to fail,  
151 resulting in a reduction in material properties for those elements. This approach of  
152 numerically representing fracturing of an element of rock by reducing its material  
153 properties was first developed and validated in rock mechanics using laboratory data  
154 [*Tang, 1997*]. After the material properties have been reduced within MOPEDZ, Navier's  
155 equation is resolved (with no further boundary displacement). If elements are still  
156 predicted to fail, the same process is repeated until no more failure is predicted i.e. a  
157 steady-state is reached. Once a steady-state solution has been achieved for a given  
158 boundary displacement, the boundaries are once more displaced to produce further shear  
159 damage and the whole solution process is repeated.

160

161 The restriction of only a few elements for failure within each iterative solution ensures  
162 stability of the model solution and a temporal propagation of cracks upon failure. For all  
163 the simulations presented here, the number of failures for a single boundary displacement  
164 was 10. In general, the higher the displacement, the more rapidly the model runs. Ten

165 failures were selected as it produced results almost identical to those allowing only a  
166 single failure with each iteration.

167

## 168 **Results**

169 Figure 1 shows the temporal evolution of damage for a pre-existing fault with an  
170 orientation of 30° to the maximum principal stress. In all simulations presented in this  
171 paper, the maximum principal stress is orientated along the y-axis (i.e. top-to-bottom).  
172 The accompanying simulation parameters (Table 1) are based on available laboratory  
173 data for granite. In this simulation,  $\sigma_3=0$  on the lateral boundaries of the model and the  
174 top and bottom boundaries are gradually displaced inwards as described above. Note that  
175 the position of the pre-existing fault is non-central, with the fault being located slightly to  
176 the right hand side of the domain. Figure 1 shows progressive temporal development of a  
177 single splay fracture, formed under tension, propagating from each end of the fault at an  
178 angle of 70° measured anticlockwise from the plane of the fault. Due to the asymmetry of  
179 the fault in the domain, damage progresses more rapidly at the upper fault tip.  
180 Simulations (not shown here) with a central fault predict the evolution of symmetric  
181 damage zone structures. The results in Figure 1 are in keeping with LEFM, with the final  
182 structure being similar to that in Figure 7a of [Burgmann *et al.*, 1994] where tensile wing  
183 cracks are predicted (using a steady-state model) to initiate at 70° with a boundary  
184 condition of a prescribed shear force applied to the pre-existing fault.

185

186 Figure 1 demonstrates that MOPEDZ can predict the evolution of fault zone structures  
187 similar to those of other authors. In the following sections, we use the novel aspects of

188 MOPEDZ to explore *temporal* damage zone evolution under differing conceptual  
189 scenarios. We investigate for the first time, the effects on temporal and spatial damage  
190 zone evolution of: progressive breakdown of the rock; the magnitude and orientation of  
191 the confining stress; the host rock heterogeneity.

192

### 193 *How Does Damage to Rocks Progressively Occur?*

194 The method by which the material properties of a finite element should be changed as it  
195 becomes progressively more fractured is dependent on the how the process of fracturing  
196 is conceptualized i.e. the micro-scale processes that represent different bulk weakening  
197 behaviors. There are two possible scenarios for the failure process within the elements.  
198 The first scenario is that upon failure, a single fracture occurs that spans the whole  
199 element, and subsequent failure produces either further smaller fractures that propagate  
200 off this fracture or increases the aperture on the original fracture, resulting in progressive  
201 weakening of the element (Figure 2a). The second scenario is that when the confining  
202 stress is sufficiently high, many locations fail within the element, producing a large  
203 number of microfractures. Further stress increases cause the microfractures to eventually  
204 coalesce and a single main fracture dominates and spans the whole of the element (Figure  
205 2b). Scenario 1 is equivalent to a large drop in strength followed by progressive  
206 continued strength breakdown (infilling with weak minerals [*Niemeijer and Spiers, 2005*]  
207 or absorption of water into fault gouge [*Morrow et al., 2000*]). Scenario 2 would  
208 correspond to progressive strength breakdown followed by a large drop in strength. This  
209 would be equivalent to the process zone model for fracture growth [*Lockner et al., 1992*].  
210 Both of these scenarios ultimately result in a similar reduction of bulk material properties

211 for an element that represents both microscopic damage and a through-going fracture  
212 (See final frames of Figure 2).

213

214 To reproduce the above scenarios, MOPEDZ simulations were conducted, again using the  
215 parameter values in Table 1, but allowing for a progressive reduction in Young's  
216 modulus on a failed element. The first scenario above assumes that most damage occurs  
217 in the first failure followed by smaller subsequent failures. This can be expressed as a  
218 geometric series, relating the value of the material parameter of interest,  $D_n$ , after failure,  
219 to the previous value of that material parameter,  $D_{n-1}$

$$220 \quad D_n = \left( \frac{M_f}{M_{hr}} \right)^{\frac{1}{m}} D_{n-1} \quad \text{where } n = 1, 2, \dots, m \quad (2)$$

221 where  $M_f$  is a constant representing the lowest possible value of the material parameter  
222 for a completely fractured element,  $M_{hr}$  is the value of the material parameter  
223 representing intact host rock,  $m$  is the number of levels of progressive damage, and  $D_0$  is  
224 equal to  $M_{hr}$ .

225

226 In the second scenario, microfracturing occurs first, followed by macroscopic failure. In  
227 this case, the relative reductions in the above series were reversed to produce small initial  
228 reductions in material properties followed by progressively larger ones. Results for these  
229 two scenarios allowing a total of three progressive failures of a single element (i.e.  $m=3$ )  
230 are shown in Figure 3(a) and (b). In the case of initial microfracturing followed by  
231 macroscopic failure (i.e. the second scenario above) a single tensile fracture evolves at  
232 each fault tip (Figure 3a). By contrast, for immediate macroscopic fracturing of an

233 element, multiple parallel tensile fractures are observed (Figure 3b). The first fracture to  
234 evolve is the longest fracture, at the tip, with subsequent fractures evolving progressively  
235 toward the centre of the fault. Fracture length decreases linearly toward the centre of the  
236 fault.

237

238 To investigate the role of mesh refinement on the development and location of the  
239 multiple splay fractures predicted in Figure 3(b), a further simulation was conducted  
240 using a finer mesh. Figure 3(c) shows results from a simulation identical to that in Figure  
241 3(b) but with a  $160 \times 160$  mesh i.e. four times the number of elements (this was the limit  
242 of the virtual memory available within MATLAB). The preexisting fault for the finer  
243 mesh has the same physical thickness as that in the  $80 \times 80$  case, although its diagonal  
244 representation within the square mesh results in a smoother pixelisation of the fault  
245 surface. A comparison of Figures 3(b) and (c) shows the results to very similar, both  
246 simulations produce parallel splay fractures that decrease in length toward the centre of  
247 the fault. In Figure 3c, the finer mesh allows these splays to appear slightly earlier in the  
248 simulation and to be closer together, hence they are more concentrated toward the fault  
249 tip. Further simulations not shown here, using an even finer adaptive triangular mesh to  
250 represent a completely smooth initial fault surface, with an increased length to width ratio  
251 (a thinner fault), also produced results similar to those in Figure 3(c): the cracks  
252 concentrate at the fault tip and are slightly shorter relative to the length of the original  
253 fault. In summary, Figures 3(b) and (c) show that the basic geometry of the results  
254 remains very similar as the mesh is refined but that the multiple splays are closer together

255 and more concentrated at the fault tip, this is consistent with observations of multiple  
256 splay fractures in the field (see discussion and Figure 8a).

257

258 In all subsequent simulations within this paper, the first scenario of immediate  
259 macroscopic fracturing, followed by progressive but decreasing weakening, is adopted to  
260 investigate the evolution of *multiple* splay fractures in fault damage zones. This allows  
261 for investigation of differing mechanical phenomena to those presented by previous  
262 authors such as [Du and Aydin, 1995; Shen and Stephansson, 1993]. For computational  
263 feasibility, the mesh resolution in the following sections is  $80 \times 80$  which enabled multiple  
264 simulations to be performed within a reasonable time scale.

265

#### 266 *How are Fault Damage Zone Structures Influenced by the Confining Stress?*

267 To investigate the role of confining stress in influencing the damage surrounding faults,  
268 four different cases were investigated. The final structures obtained for each of these  
269 cases are presented in Figure 4. In Figure 4(a)  $\sigma_3 = 0$ . Figure 4(b) shows results for the  
270 same simulation but with  $\sigma_1/\sigma_3 = 5$ . In this simulation multiple parallel tension cracks are  
271 again produced, but here the longest splay fracture is not now associated with the fault  
272 tip. In Figure 4(c), the final damage zone structure is shown for  $\sigma_1/\sigma_3 = 2.5$ . Now a  
273 different pattern of damage is observed: wing cracks evolve due to shear failure at the  
274 fault tips and propagate at a much lower angle to the original fault plane. In addition to  
275 these shear cracks, small tensional cracks form at a higher orientation to the fault and  
276 away from the fault tip. In the final simulation (Figure 4d) the boundaries are completely  
277 rigid. Here, low angle shear fractures form at the fault tips, and subsequent small

278 perpendicular tertiary fractures then propagate from these shear fractures. The same  
279 structures as those in Figure 4(d) were produced as  $\sigma_1/\sigma_3$  tended to a value of one (i.e.  
280  $\sigma_1 \rightarrow \sigma_3$ ). The results in Figure 4 imply that for large differential stress (i.e.  $\sigma_1 \gg \sigma_3$ )  
281 multiple parallel tension fractures are formed, whereas for small differential stress ( $\sigma_1$   
282 close to  $\sigma_3$ ) single shear fractures form at the fault tip followed by small higher angle  
283 tension fractures.

284

285 The effect of varying the orientation of the maximum principal stress with respect to the  
286 pre-existing fault is examined in Figure 5. Simulations were conducted for faults at  
287 angles of  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  and  $75^\circ$  to  $\sigma_1$  for the two extreme cases in Figure 4 of  $\sigma_3 = 0$  and of  
288 rigid lateral boundaries, from here termed high and low differential stress respectively.  
289 For high differential stress (Figure 5a) all wing cracks form in tension and ultimately  
290 propagate in the direction of the maximum principal stress. Figure 5a shows that faults at  
291 low angles to the maximum principal stress ( $30^\circ$  and  $45^\circ$ ) produce single curved splay  
292 fractures. As the angle between the fault and  $\sigma_1$  increases, multiple parallel fractures  
293 evolve that decrease in length toward the centre of the fault. Once the fault is oriented at  
294  $75^\circ$  to  $\sigma_1$  the geometry of the fracturing becomes more erratic, splay fractures evolve on  
295 both sides of the fault at the same tip, and there is no clear pattern to fracture length. In  
296 the case of low differential stress (Figure 5b) the predicted damage zone structures are  
297 very different. For a fault at an angle of  $30^\circ$  to  $\sigma_1$ , wing cracks form in shear and  
298 propagate back into the compressive quadrant. As the angle increases, shear fractures  
299 begin to propagate at the fault tip at an angle of around  $35^\circ$  to the original fault. For large

300 angles, short tension fractures also form away from the fault tip at an angle of 85° to the  
301 original fault.

302

303 *Does Host Rock Heterogeneity Affect Damage Zone Formation?*

304 To explore the effect of host rock heterogeneity, simulations were conducted using both  
305 purely random and spatially correlated fields for host rock material properties. Six  
306 realizations were simulated for each statistical material property distribution, in  
307 conditions of high differential stress. Final damage zone structures for two of the  
308 realizations of the host rock material properties in each case are shown in Figure 6.  
309 Figure 6 (row 1) shows final damage zone structures for an uncorrelated, purely random  
310 field where the mean Young's modulus of the host rock is 60 GPa (as on previous  
311 simulations) with a standard deviation of 1 GPa (i.e. 95% of the field lies between 58GPa  
312 and 62 GPa). For comparison, a completely fractured element is represented by a  
313 Young's modulus of 1.2 GPa, so the original fault remains quite distinct from any  
314 underlying variations in the host rock. Visual inspection of the simulations in Figure 6  
315 row 1 shows that both realizations produce multiple parallel splay fractures, but in the  
316 first realization a splay fracture forms beyond the fault tip.

317

318 Figure 6, rows 2 to 4, show the results of simulations identical to those in row 1 but with  
319 a host rock Young's modulus that is described by a spatially correlated random field. The  
320 covariance structure of this field is described by an exponential function of the form

321 
$$C(h) = \sigma^2 - \exp(-h/\lambda) \tag{4}$$



322 where  $\lambda$  is termed the correlation length and effectively governs the size of the  
323 ‘patches’ of high or low Young’s modulus in the host rock,  $\sigma^2$  is the variance and  $h$  is the  
324 distance between pairs of points in the field. The results for the final damage zone  
325 structure for 3 different correlation lengths are shown in Figure 6 (rows 2 to 4) two  
326 realizations are shown for each different correlation length. Again, multiple parallel splay  
327 fractures are produced with some of these forming beyond the fault tip. Realizations with  
328 high numbers of parallel fractures correspond to those with patches of low host rock  
329 Young’s modulus adjacent to the fault. For the simulation with the largest correlation  
330 length, a shear fracture can be seen propagating from the left hand fault tip. It is clear that  
331 whilst the macroscopic damage zone pattern remains very similar in these simulations,  
332 the exact locations and frequencies of evolving splay fractures are heavily influenced by  
333 the heterogeneous material properties of the host rock.

334

335

335 **Discussion**

336 The simulation results presented here have investigated the formation of fractures around  
337 a single pre-existing feature (described here as a fault, but which could represent other  
338 pre-existing features such as joints and dikes [*d'Alessio and Martel, 2005*]). We have  
339 investigated varying the effect of: the conceptual scenario of progressive fracture  
340 damage; the orientation of the initial fault with respect to the maximum principal  
341 compressive stress; the confining boundary conditions; and the heterogeneity of the host  
342 rock material properties. The range of resulting damage zone structural styles are  
343 summarized in Figure 7. Typical structures are a) single tensile wing cracks, b) multiple  
344 tensile wing cracks, c) unconnected tensile fractures, d) low angle wing cracks that have  
345 formed in shear e) low angle shear cracks with tertiary fractures subsequently  
346 propagating off their sides, and f) high angle shear fractures that propagate at angles  
347 between  $180^\circ$  and  $270^\circ$  (measured anticlockwise from the original fault plane).  
348 Interestingly, one key form of damage zone evolution that is not predicted here is an  
349 extension of the initial fault in its own plane. Fault plane extension has been previously  
350 simulated by [*Du and Aydin, 1993; 1995*], however, in their conceptual model of  
351 mechanical failure, tensile failure is suppressed.

352

353 *Comparison with previous theory and outcrop measurements.*

354 This research has produced tensile wing cracks that vary in angle from  $30^\circ$  to  $90^\circ$  and  
355 synthetic shear wing cracks that vary from  $23^\circ$  to  $40^\circ$ . This range is far broader than the  
356 range predicted by Linear Elastic Fracture Mechanics [*Pollard and Segall, 1987*] which  
357 predicts all fractures should initially propagate at  $70^\circ$ .

358

359 Wing crack measurements on outcrops are generally low, but can range from 15° to 70°.  
360 The angles produced by MOPEDZ compare well with field observation of wing crack  
361 angles (Table 2). In our simulations, the most influential factor is the orientation of the  
362 fault relative to the maximum principal stress direction. Since faults in crystalline rocks  
363 are generally thought to evolve from the linking of pre-existing joints [*Martel, 1990*] or  
364 dikes [*d'Alessio and Martel, 2004*], the maximum principal stress direction must have  
365 originally been aligned with their pre-existing structures. It seems reasonable to presume  
366 that smaller rotations of the stress field will be more common than larger ones, and this  
367 would result in wing cracks frequently occurring at angles at the lower end of the possible  
368 range.

369

370 For a comparison of MOPEDZ with field observations, Figure 8a shows a 14m long fault  
371 developed in granodiorite in the Kip Camp area of the Sierra Nevada, California mapped  
372 by [*Lim, 1998*]. The length of the wing cracks in Figure 8a is related to distance from the  
373 fault tip, with a general decrease in length of the wing cracks as the distance from the  
374 fault tip increases. The adjacent MOPEDZ simulation shown compares well with the type  
375 of damage shown in [*Lim, 1998*]. The simulation has high differential stress (i.e.  $\sigma_1 \gg \sigma_3$ )  
376 assumes immediate macroscopic damage of the host rock on initial failure, and has a fault  
377 inclined at 60° to the maximum principal compressive stress. The simulation produces  
378 wing cracks that propagate at around 60°, that are distributed along the length of the joint  
379 and that decrease in length as the distance from the tip increases. This simulation suggests  
380 that there was high differential stress when fracturing around the fault developed and that

381 the direction of the maximum principal stress for the fault in Figure 8a was  
382 approximately north-south.

383

384 Figure 8b shows the Alligerville fault, New York state, mapped by *Vermilye & Scholz*  
385 [1998]. The 40m long vertical fault is in quartzite [*Vermilye & Scholz* 1998]. The fault  
386 has shear wing cracks that propagate in the compressional quadrants at angles of around  
387 225°, which are distributed along the length of the fault, and decrease in length as the  
388 distance from the fault tip increases. The fault also shows pressure solution cleavage in  
389 the compressional quadrant, striking at approximately -60°. The adjacent MOPEDZ  
390 simulation shown in Figure 8b compares well with the Alligerville fault trace. This  
391 simulation has confined boundary conditions, assumes immediate macroscopic damage  
392 of the host rock on initial failure (Figure 3a) and has an initial fault orientation of 30° to  
393 the maximum principal compressive stress. The simulation produces shear wing cracks  
394 that propagate from the fault tips into the compressional quadrant at around 242°. In the  
395 Alligerville fault, multiple fractures are formed which are not reproduced in the  
396 MOPEDZ simulations, this may be due to the heterogeneity of the host rock, or due to the  
397 clear deviations from a planar fault surface (which would concentrate stress) that are  
398 apparent on the Alligerville fault. Based on our simulations, we suggest that the direction  
399 of the maximum principal stress around the Alligerville fault was approximately north-  
400 west/south-east when fracturing developed, and that the differential stress was low.

401

402

402 *Implications for the temporal sequence of multiple wing cracks.*

403 A number of conceptual temporal sequences for the evolution of multiple splay fractures  
404 have been proposed e.g. [Martel and Pollard, 1989]. Four conceptual temporal sequences  
405 (S1 to S4) are possible (Figure 9). In S1, wing cracks form behind (i.e. away from) the  
406 fault tip, and then further wing cracks form progressively nearer to the fault tip (Figure  
407 9a); this is essentially the sequence proposed in [Martel and Pollard, 1989] and simulated  
408 for seismic rupture in [Dalguer et al., 2003]. In S2, wing cracks form initially at the tips  
409 of the fault and then further wing cracks form at locations that are progressively closer to  
410 the middle of the joint (Figure 9b). In S3, wing cracks form at the tips, subsequently the  
411 fault extends in its original plane, and then further wing cracks form at the new tips  
412 (Figure 9c). Finally in S4, wing cracks form at the tips of the fault, parallel wing cracks  
413 form that are away from and unconnected to the fault tip, the tip then extends and  
414 connects to these parallel cracks (Figure 9d) .

415

416 Based on the simulations presented here, the most likely sequence for temporal evolution  
417 is that of S2 (Figure 9b). Almost all MOPEDZ simulations predict the first wing crack to  
418 occur at the fault tip. In general, further wing cracks then form progressively further  
419 away from the tip toward the centre of the fault. This temporal sequence matches that of  
420 S2 in Figure 9b. However, in simulations with low differential stress and an angle of at  
421 least  $75^\circ$  to the maximum principal compressive stress the first wing cracks are predicted  
422 to initiate away from the fault tip (S1, Figure 9a). For these high angle faults, the  
423 formation of wing cracks is predicted to be highly irregular in length, with a few cracks  
424 also appearing in the compressional quadrant.

425 Finally, in a few of the realizations incorporating a fault within a heterogeneous host  
426 rock, fractures occur beyond the tip of the fault without any extension of the original fault  
427 in its own plane. Once these fractures have formed it is possible that subsequent  
428 fracturing could link these off-tip fractures (sequence S4, Figure 9d) but this final linking  
429 stage has not been produced here.

430

#### 431 *Implications for fluid flow*

432 The future application of physically-based simulation models to improve estimates of  
433 fault hydraulic properties in the subsurface, depends critically on being able to accurately  
434 predict the orientation, frequency and connected nature of evolving fractures in the  
435 damage zone. In this paper, we demonstrate that it is possible to simulate the spatial and  
436 temporal evolution of multiple splay fractures from a single pre-existing fault.  
437 Simulations are compared with field observation data and the orientation of the predicted  
438 fractures compares well with observed fault zone geometries. The model can now  
439 confidently be applied to investigate more complex fault zone geometries, such as fault  
440 zone development from linkage of pre-existing joints [Martel, 1990], and combined with  
441 fluid flow simulation [Willson *et al.*, 2005] for bulk permeability estimation.

442

#### 443 **Conclusions**

444 We present a new model, MOPEDZ, for dynamic simulation of multiple and single splay  
445 fractures originating from a single feature such as a fault, joint or dike. The model is  
446 based on a finite element solution of Navier's equation using a combined Mohr-Coulomb  
447 and tensile failure criterion. Simulations compare well with field observations of fault

448 zone geometry. Investigations of damage zone evolution under differing boundary  
449 conditions and material properties predict that:

- 450 • Changing the conceptual sequence of microscopic versus macroscopic fracturing  
451 of an element results in important differences in the structure of the damage zone  
452 around slipping faults. Simulations show that a conceptual sequence of  
453 microscopic damage followed by macroscopic failure tends to produce single  
454 splay fractures at the fault tip, whereas immediate macroscopic failure produces  
455 multiple parallel splay fractures that decrease in length away from the fault tip.
- 456 • Due to the combined Mohr Coulomb and tensile failure criteria, which allows  
457 both shear and tensile failure, the orientations of damage zone features  
458 surrounding the fault are heavily influenced by the differential stress. For low  
459 differential stress, simulations predict single shear fractures at the fault tip that  
460 propagate at a low angle to the fault. For high differential stress, fractures form in  
461 tension at the fault tips followed by multiple parallel fractures that are  
462 progressively closer to the fault centre and shorter in length.
- 463 • Faults that are reactivated at a high angle to the maximum principal compressive  
464 stress form multiple parallel splay fractures. By comparison, faults at a low angle  
465 to the maximum principal stress, form single fractures at the fault tips. For  
466 simulations where the differential stress is low, these single fractures propagate  
467 into the compressional quadrant.
- 468 • Host rock heterogeneity effects the locations of evolving splay fractures  
469 sometimes causing them to originate beyond the tip of the original fault.

- 470
- In the case of multiple splay fractures, simulations generally support a temporal
- 471 evolution in which the initial fracture occurs at the fault tip. This is then followed
- 472 by the evolution of other parallel features that are progressively both shorter in
- 473 length and further from the tip.



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**Figure 1** Temporal development of wing cracks at the tips of a single fault, using the parameters listed in Table 1 and  $\sigma_3=0$ . The maximum principal stress ( $\sigma_1$ ) is oriented top to bottom in these simulations. Each frame shows the structure that has developed by increasing the boundary displacement,  $\Delta y$ , to the amount indicated. The grey scale represents material Young's modulus; grey is unfractured (60GPa) and black is fractured (1.2GPa). The angle between the fault and the wing crack is measured anticlockwise from the plane of the fault

**Figure 2** Sequences of progressive fracturing for a single finite element **a)** development of fracturing assumes that the major fracture happens first, and subsequent fractures are smaller (shown for three steps), **b)** development of fracturing assumes that microfractures occur first, and larger fractures develop through coalescence of these fractures.

**Figure 3** Temporal damage zone progression **a)** assuming initial macroscopic failure with an  $80 \times 80$  mesh (see Figure 2a) **b)** initial microscopic failure with an  $80 \times 80$  mesh (see Figure 2b) **c)** assuming initial macroscopic failure with an  $160 \times 160$  mesh (see Figure 2a) **d)** assuming initial microscopic failure with an  $160 \times 160$  mesh (see Figure 2b). The maximum principal stress ( $\sigma_1$ ) is oriented top to bottom.

**Figure 4** Comparison of simulations with different ratios of  $\sigma_1$  to  $\sigma_3$ . The maximum principal stress ( $\sigma_1$ ) is oriented top to bottom. **a)**  $\sigma_3 = 0$  (this is a reproduction of the final damage structure in Figure 3a), **b)** a ratio of  $\sigma_1 / \sigma_3 = 5$ , **c)** a ratio of  $\sigma_1 / \sigma_3 = 2.5$ , **d)** rigid left and right boundaries.

**Figure 5** Resulting structures when varying fault orientation for a boundary displacement of approximately 6.8mm. The orientation between the fault and  $\sigma_1$  (oriented top to bottom) is  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$  and  $75^\circ$  for **a)** high differential stress and **b)** low differential stress.

**Figure 6** Resulting structures using a heterogeneous Young's modulus field for a boundary displacement of approximately 3mm. Each simulation starts with a different heterogeneous Young's modulus matrix. The maximum principal stress ( $\sigma_1$ ) is oriented top to bottom. There are 2 realisations each of a purely random field and of three different correlation lengths of 0.15m, 0.375m and 0.45m. Colour scale is logarithmic and indicates Young's modulus.

**Figure 7** Six fault tip structural styles that have been produced with MOPEDZ.

**Figure 8 a)** A fault in granodiorite showing multiple wing cracks distributed along its entire length [Lim, 1998] alongside a structure produced by MOPEDZ with high differential stress starting with a fault inclined at  $60^\circ$  to the maximum principal stress. The mapped fault strikes at  $57^\circ$  and dips  $81^\circ$  to the south, the grey rectangle on the map indicates ground cover, and it is presumed the fault is continuous [Lim, 1998]. **b)** Outcrop map of a fault after Vermilye & Scholz [1998] alongside a simulation produced by MOPEDZ with low differential stress, starting from a fault that is inclined at  $30^\circ$  to the maximum principal compressive stress

**Figure 9** Four possible sequences for the formation of multiple wing cracks. **a)** cracks form behind the fault tip and further fractures occur towards the tip, **b)** cracks form at the fault tip and further fractures occur towards the centre, **c)** cracks always form at the tip, but the fault extends in length, **d)** cracks form at the tip, further fractures form beyond the fault tip which are subsequently linked by extension of the fault

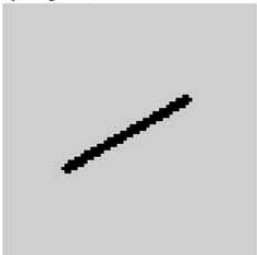
Domain length	3m	
Domain width	3m	
Matrix rows	80	
Matrix columns	80	
Fracture length	1.6875m	
Fracture rotation	60°	
Host rock Young's Modulus	60 GPa	[ <i>Martin, 1997</i> ] [ <i>Turcotte and Schubert, 1982</i> ]
Host rock Poisson's ratio	0.2	[ <i>Turcotte and Schubert, 1982</i> ]
Young's Modulus of fractured element	1.2 GPa	[ <i>Segall and Pollard, 1983</i> ]
Poisson's ratio of fractured element	0.02	
C <sub>0</sub> (Shear strength)	130 MPa	[ <i>Martin, 1997</i> ]
$\mu$ (Coefficient of friction)	0.6	[ <i>Byerlee, 1967</i> ]
T <sub>0</sub> (Tensile strength)	10 MPa	[ <i>Martin, 1997</i> ]
Failed locations Limit	10	
Young's modulus of the platen	120 GPa	

**Table 1 MOPEDZ simulation parameters, where relevant the right hand column contains the reference from which the value of the mechanical property for granite was derived**

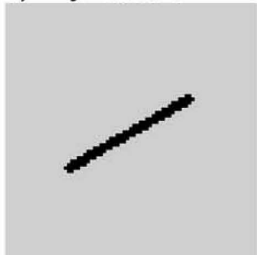
Author	Range of wing crack angles	Rock type and location
[ <i>Lim, 1998</i> ]	20° to 70°	Granite, Sierra Nevada, California
[ <i>Segall and Pollard, 1983</i> ]	15° to 35°	Granite, Sierra Nevada, California
[ <i>Cruikshank et al., 1991</i> ]	35° to 50°	Sandstone, Arches National Park, Utah

**Table 2 Wing crack angles measured in the field**

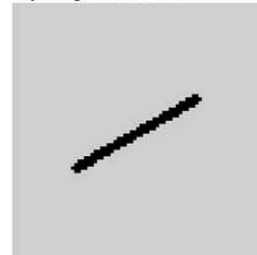
i)  $\Delta y = 0$



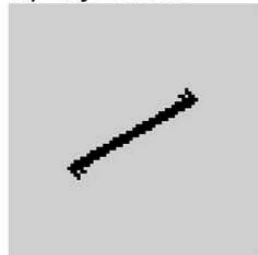
ii)  $\Delta y = 0.0002$



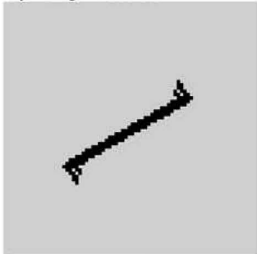
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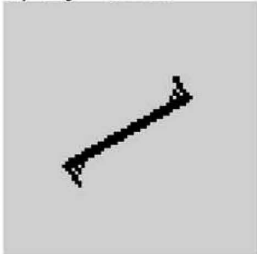
iv)  $\Delta y = 0.0008$



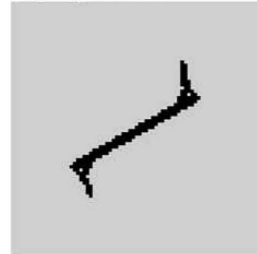
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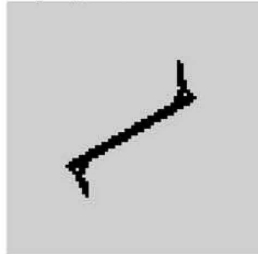
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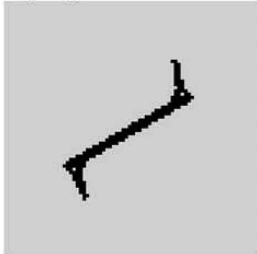
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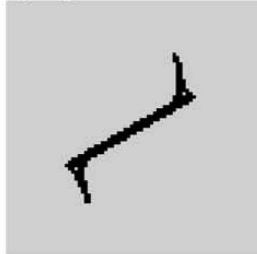
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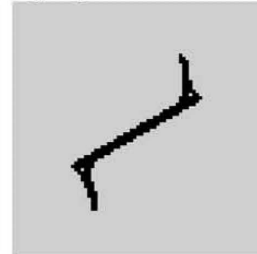
ix)  $\Delta y = 0.0018$



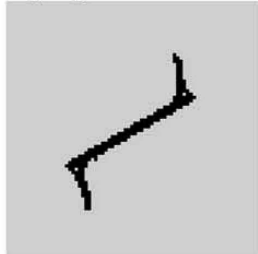
x)  $\Delta y = 0.002$



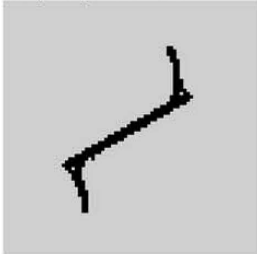
xi)  $\Delta y = 0.0022$



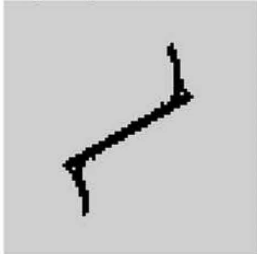
xii)  $\Delta y = 0.0024$



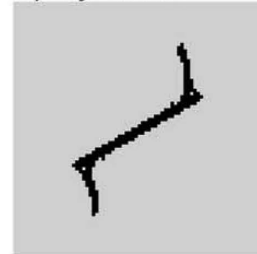
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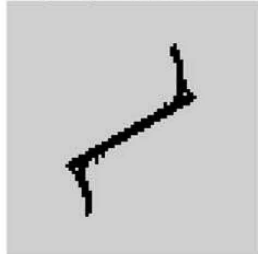
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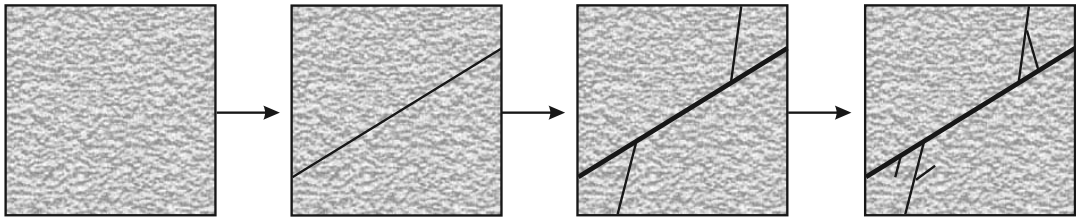
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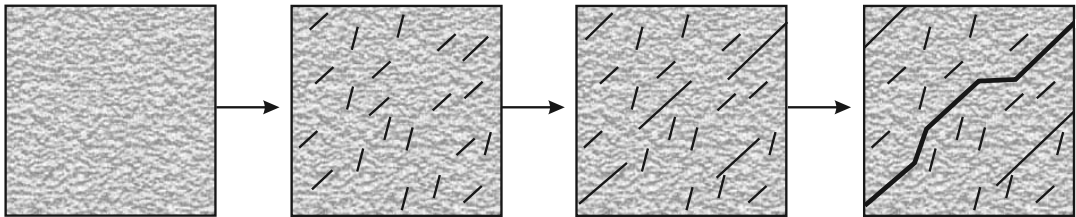
xvi)  $\Delta y = 0.00295$



a)



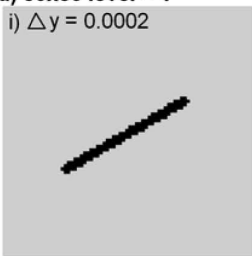
b)



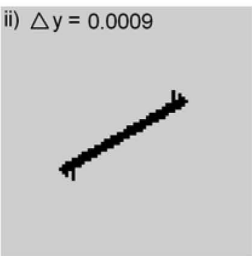


a) 80x80 level = 1

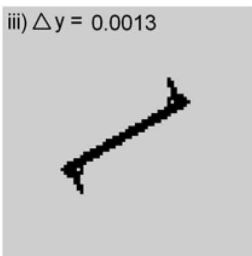
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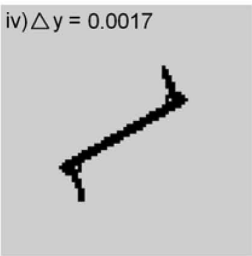
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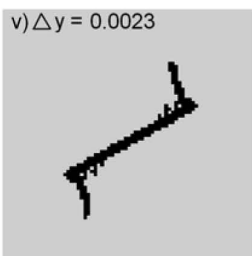
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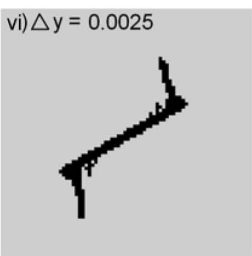
iv)  $\Delta y = 0.0017$



v)  $\Delta y = 0.0023$

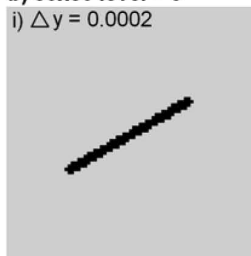


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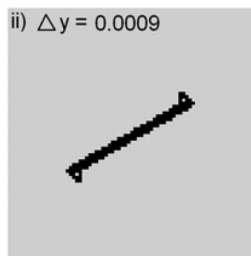


b) 80x80 level = 3

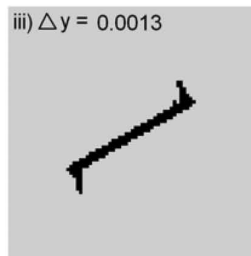
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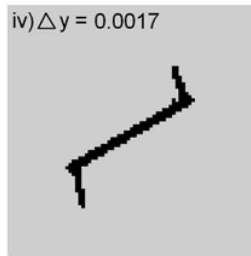
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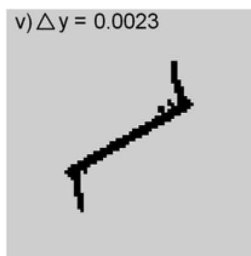
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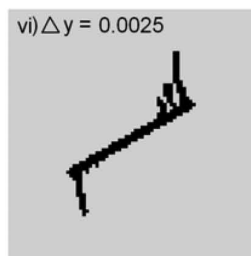
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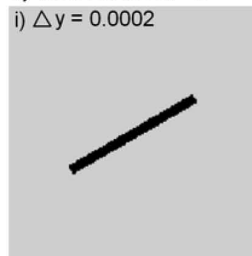


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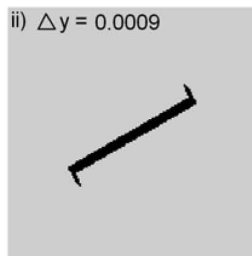


c) 160x160 level = 3

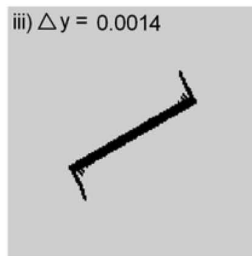
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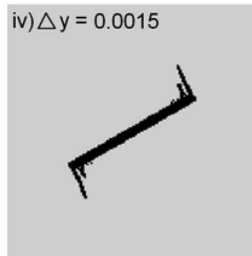
ii)  $\Delta y = 0.0009$



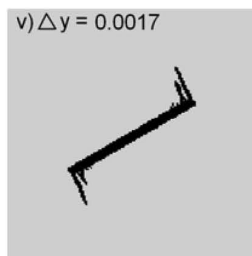
iii)  $\Delta y = 0.0014$



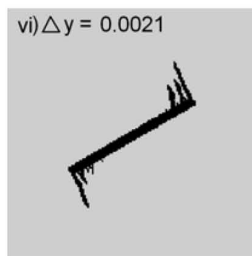
iv)  $\Delta y = 0.0015$



v)  $\Delta y = 0.0017$



vi)  $\Delta y = 0.0021$



a)  $\sigma_3 = 0$



b)  $\sigma_1 / \sigma_3 = 5$



c)  $\sigma_1 / \sigma_3 = 2.5$



Shear fracture

d) Rigid lateral boundaries



**a) High differential stress**

Rotation = 30 deg.  
 $\Delta y = 0.003$



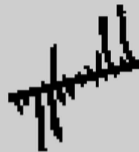
Rotation = 45 deg.  
 $\Delta y = 0.003$



Rotation = 60 deg.  
 $\Delta y = 0.003$

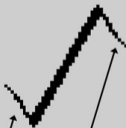


Rotation = 75 deg.  
 $\Delta y = 0.002725$



**b) Low differential stress**

Rotation = 30 deg.  
 $\Delta y = 0.0067$

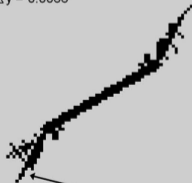


Shear fracture

Rotation = 45 deg.  
 $\Delta y = 0.007$

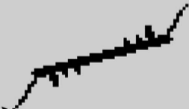


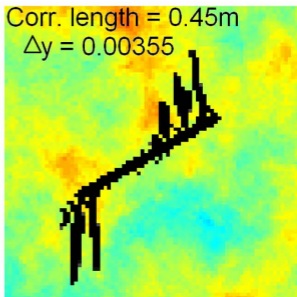
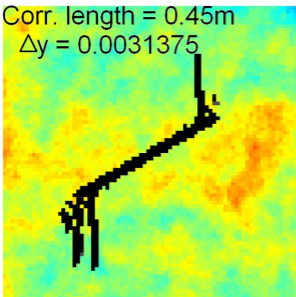
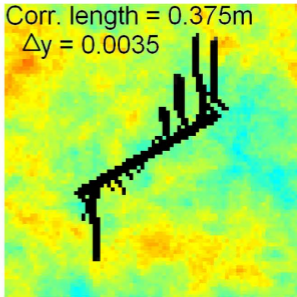
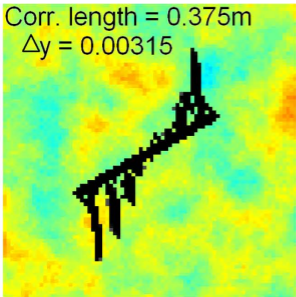
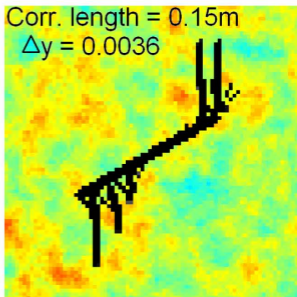
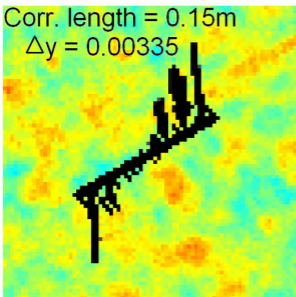
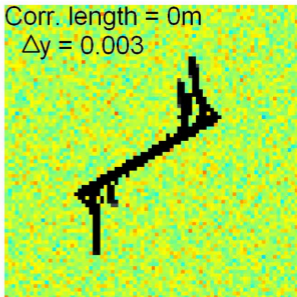
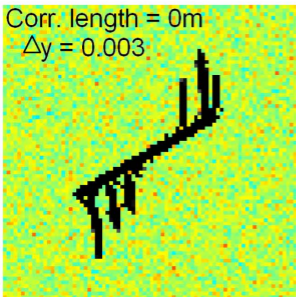
Rotation = 60 deg.  
 $\Delta y = 0.0066$



Shear fracture

Rotation = 75 deg.  
 $\Delta y = 0.006325$

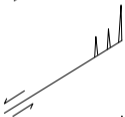




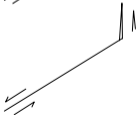
(1) Single tensile wing cracks



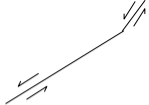
(2) Multiple tensile wing cracks



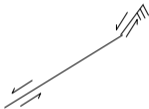
(3) Fractures beyond the joint tip



(4) Shear wing cracks



(5) Shear wing cracks with tertiary antithetic fractures



(6) Shear fractures propagating in the compressional quadrant



