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Profit rates: their dispersion and long term determination

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Abstract

This introduces Marx's theory of the determination of profit rates. It contrasts this theory with what happened in the late 19th century to British profit rates with a detailed statistical account. It identifies missing features in the standard presentation and contrasts these with the over-accumulation hypothesis that he presents elsewhere.

A formal mathematical model using the over-accumulation hypothesis is then given and tested against modern empirical data.

1 Marx's account

The concept of a tendency for the rate of profit to fall was a common theme in classical political economy. Smith, Ricardo and Marx all held such a theory. However, their grounds for believing in this tendency were quite various. Smith thought in terms of accumulation leading to an increase in competition between capitals, hence driving down prices and profits. Ricardo dismissed this as a confusion – competition between capitalists influenced the distribution of profit, not its overall amount – and held a theory whose motor lay in the confrontation between rising population and diminishing returns in agriculture. Marx's theory was in a sense more akin to Smith's – at least insofar as it had nothing to do with diminishing returns.

I shall distinguish two elements in Marx's writings that are particularly relevant to our topic. The main argument which Marx set out at length (and which appears in Capital I, his notebooks of the 1860s, and Capital III) to the effect that the rate of profit must tend to fall due to an increase in the organic composition of capital, an increase itself driven by the search for maximum profit on the part of capitalists. The second is what Marx calls "absolute overproduction of capital": this element forms the basis for the revised TRPF theory that we defend below

Marx starts off with a simple numerical example [Marx, 1971, chap. 13]. He assumes that the level of wages and the length of the working day are fixed. Under these circumstances a given sum of money being paid in wages each week can stand as an index for the number of workers employed. Thus if the wage is £1 per week then £100 represents a workforce of 100 people.

He goes on to make the simplifying assumption that the value created by labour will be divided equally between labour and capital so the total value created per week is £200.

In his terminology, the rate of surplus value $s' = \frac{s}{v} = \frac{100}{100} = 100\%$.

This rate of surplus value could however express itself in very different rates of profit depending on the total amount of capital employed. This he designates with a capital C, such that $C = c + v$, the components being variable capital or wages designated by the variable v, with the variable c being what he calls constant capital: raw materials, machinery, buildings etc. He gives the rate of profit p' as the surplus divided by total capital so $p' = \frac{s}{C}$.

He illustrates this with a table of examples. In all cases he assumes $s = 100$.

c	v	C	p'
50	100	150	67%
100	100	200	50%
200	100	300	33%
300	100	400	25%
400	100	500	20%

Note that in real terms these are unrealistically high profit rates.

You might say 'yes 50% is unrealistic, but 20% is within the bounds of possibility'. But Marx has assumed that the £100 represent a week of expenditure on labour power. A profit rate of 20% a week is an astronomical annual profit rate of 1,310,363% [1].

These unrealistic figures are an effect of the ambiguous definitions that Marx uses when discussing profit rates. He is starting out from simple explanatory examples where he tells a story of capitalists advancing constant and variable capital, carrying out production, and then selling the product within a fixed time period. In this sort of example there is a deliberate ambiguity about the time period within which this all takes place. If he had assumed a production period of a year, and variable capital of £1 million, constant capital of one, two or three millions then his example would be more credible. But the ambiguity over the difference between stock and flow measures of capital does pose an obstacle to students of Marx's writings understanding what is actually implied by these examples.

What he has done so far is give a numerical example that illustrates that the rate of profit will be lower if c rises in relation to v, that is to say that the rate of profit will vary inversely with $\frac{c}{v}$.

There is nothing in his example that explicitly depends on time. It could as easily be used to show that industries with a high organic composition of capital must experience a lower rate of profit - which is in fact the case as shown in Figure 1. Marx himself was blocked from making this interpretation of his table because he had, in an earlier chapter [Marx, 1971, chap. 10], hypothesised that profit rates in different industries would equalise. This was the famous 'transformation' argument according to which labour values would be transformed

[1] Since annual profit rate is an exponential of weekly profit rate so $1310363\% = (1.2^{52} - 1) \times 100$.

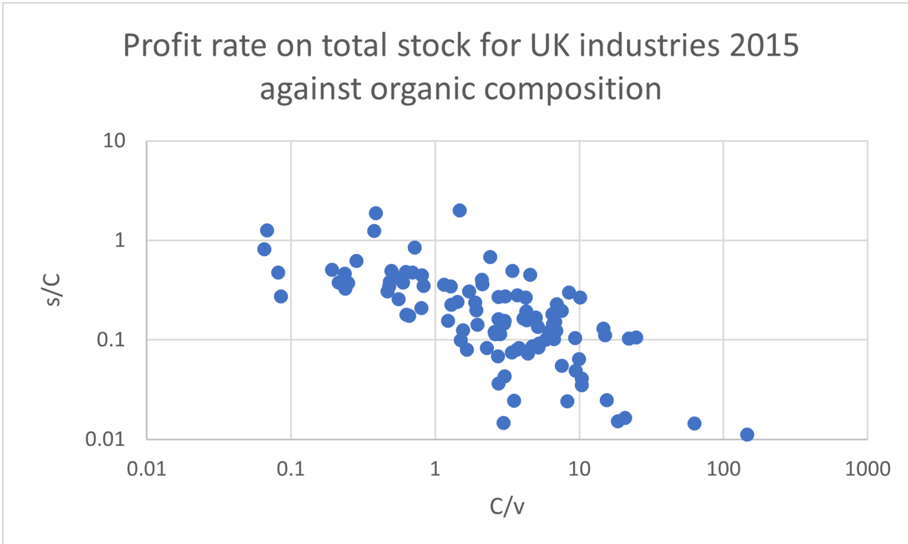

Fig. 1: Profit rates for UK industries show the inverse relationship between capital composition and rate of return that Marx's worked example predicts. Note both axes are given a log scales.

in profit equalising 'production prices'.

It is worth going into both how Figure 1 was computed and the implication of this for Marxian theorisation of profit rates and prices.

Recall that the justification for his transformation process was that it appeared to him *obvious* that profit rates would tend to equalise. If actual profit rates do not equalise it is evident that the production price theory is redundant. It is an attempted answer to an imagined problem. Figure 1 confirms what previous studies for other countries[2] have shown :

- profit rates are significantly dispersed;
- there is a systematic inverse relationship between organic composition of capitals in any given year and their rate of profit.

Studies[3] have shown that for many countries simple labour values are are as good or better than production prices when used to predict the structure of market values of industrial output.

A possible weakness of previous studies lies them carrying over the ambiguity in Marx about what the variable c should actually mean. In Volume I of Capital it is clear that c is being used as a flow quantity. It designates the flow of value

[2] See for example [Cockshott and Cottrell, 1998, Cockshott and Cottrell, 2003, Zachariah, 2006].

[3] See for example [Ochoa, 1989], [Petrovic, 1987], [Fröhlich, 2013] in addition to those cited on the dispersion of profit rates.

into the final product from the means of production. It thus includes both the value of raw materials and the value of wear on the machinery used - what we would now call depreciation. This is fine for a theory of the component parts of output values, but not really for a theory of the rate of profit. The rate of profit has to be calculated on the whole value of the stock of machinery and equipment used, only a small part of which will be depreciated in a given year.

This has practical implications when one comes to calculate what the rates of profit in different industries are. The most common approach in the literature is to work from I/O tables and sum down the columns for each industry, adding up all the different means of production used. This gives a value for c at market prices. To obtain labour values the c at market price is converted to c in labour by first multiplying the columns of the table with a market price to value conversion vector. A similar procedure is used when computing prices of production, except that in this case the initial multiplication is done using a market price to production price conversion vector.

This approach amounts to using c as defined in Volume I, and ignoring fixed capital stocks. But to include fixed capital stocks in a test of production price theory one should not simply take fixed capital stocks at current market price. Instead one should apply a similar pre-multiplication process to that used for constant capital flows. The stocks of industrial equipment should be pre-multiplied by a market price to production price vector to accurately mimic what the supposed profit equalising process would produce[4].

The problem with applying this test empirically is that standard IO tables do not provide capital stock data. For this paper I have attempted to construct dis-aggregated capital stock tables for the UK that allow the pre-multiplication process to be applied to test production price theory. The method of calculation are described in the Appendix.

From the data thus derived it was possible to plot the relationship between s/C in Marx's original terminology and c/v that was shown in Figure 1. As I mentioned earlier, this shows a very strong inverse relationship between organic composition and profitability.

Since this data so strongly contradicts the assumptions of Chapter 10 of Capital III, it is worth doing a detailed calculation of the relative accuracy of labour values and fully transformed prices of production in predicting the market value of different industries.

The results are presented graphically in Figure 2. It is evident to the eye that the labour values are clustered more closely to the diagonal than the prices of production. This is born out by the respective R^2 for the two trend lines. Figure 2 plots the logs of money values of sector outputs against logs of labour values and logs of prices of production. This is done to spread out the plot. Correlations[5] of the logs and Mean Absolute Deviations were found to be:

[4] For example the process described in [Sraffa, 1960].

[5] Correlations are used here rather than a cosine metric since correlation is also numeraire independent and is also more widely used in the sciences. The correlation coefficient is in fact a cosine metric adjusted for shifts of origin.

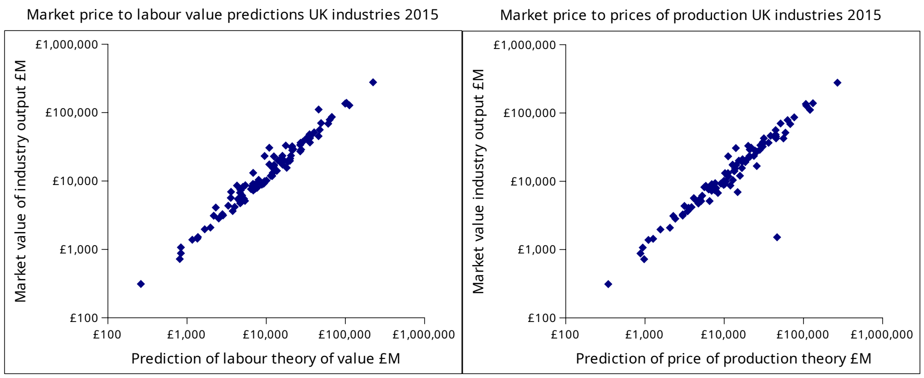

Fig. 2: The relationship between output prices, and two sorts of Marxian prices for the UK industrial sectors 2015.

Correlation log labour value against log market value	98.4%
Correlation price of production against log market value	94.5%
Mean Absolute Deviation market value/labour value	18.7%
Mean Absolute Deviation market value/price of prodution	44.5%

Discussion

Clearly, for the UK in 2015 market prices were better predicted by labour values than by prices of production. The MAD for labour values is only slightly greater than the average value given for the China years in[Cheng and Li, 2020] of 0.168 and well within the range of variations (0.103 to 0.219). The MAD for price of production is much greater than the average given for China (0.086) and outside the range of variation (0.063 to 0.120).

A contributory factor to the poor performance of the prices of production must certainly be the very dispersed rate of profit shown in Figure 1. Correlating profit rate against organic composition reveals an inverse relationship (correlation coeficient -35%) like the data for the USA in [Cockshott and Cottrell, 1998].

It is unclear whether the difference between this result and that obtained by Han Cheng and Minqi Li for China reflects real differences between the two economies or differences in methodology. This could only be determined by repeating their work with a calculation procedure that exluded taxes from their measure of price of prodution.

2 Time trends

Marx, as we mentioned earlier, saw the differences in profit rate caused by varying organic compositions as something that occurred over time, not something that occurred simultaneously in different industries. Marx followed up his examples with saying :

> This is how the same rate of surplus-value would express itself under the same degree of labour exploitation in a falling rate of profit, because the material growth of the constant capital implies also a growth – albeit not in the same proportion – in its value, and consequently in that of the total capital. If it is further assumed that this gradual change in the composition of capital is not confined only to individual spheres of production, but that it occurs more or less in all, or at least in the key spheres of production, so that it involves changes in the average organic composition of the total capital of a certain society, then the gradual growth of constant capital in relation to variable capital must necessarily lead to a gradual fall of the general rate of profit, so long as the rate of surplus-value, or the intensity of exploitation of labour by capital, remain the same. Now we have seen that it is a law of capitalist production that its development is attended by a relative decrease of variable in relation to constant capital, and consequently to the total capital set in motion. This is just another way of saying that owing to the distinctive methods of production developing in the capitalist system the same number of labourers, i.e., the same quantity of labour-power set in motion by a variable capital of a given value, operate, work up and productively consume in the same time span an ever-increasing quantity of means of labour, machinery and fixed capital of all sorts, raw and auxiliary materials-and consequently a constant capital of an ever-increasing value.
>
> [Marx, 1971, Chap. 13]

The argument above rests on the following assumptions:

- A material growth in the means of production implies an increase in the value of the means of production, even if the growth in value is slower.
- The rate of surplus value remains roughly the same.

The first premise is highly questionable, and we will discuss this lower down.

The second premise is fairly safe. Although the rate of surplus value does move, it tends to close to the range assumed by Marx. In 1863 when he was writing this 64% was the British rate of surplus value[6]. One does see situations where the split of income between labour and capital is roughly a 50%/50% split. You do not see situations where the split is 90%/10%. This means that his initial argument to the effect that the total wage bill will act as a rough index of total value created is sound.

On the other hand the fluctuations in constant capital to labour ratios over time and between industries can be much greater. So we can expect profit rates to be negatively correlated with organic composition, as in Marx's argument.

Marx was writing in the 1860s. Figure 3 shows, taking British data from the period 1855 to 1910 that the rate of profit was indeed negatively correlated

[6] Computed from a wage share of 0.61 in table A56 of the *Millenium of Economic Data*.

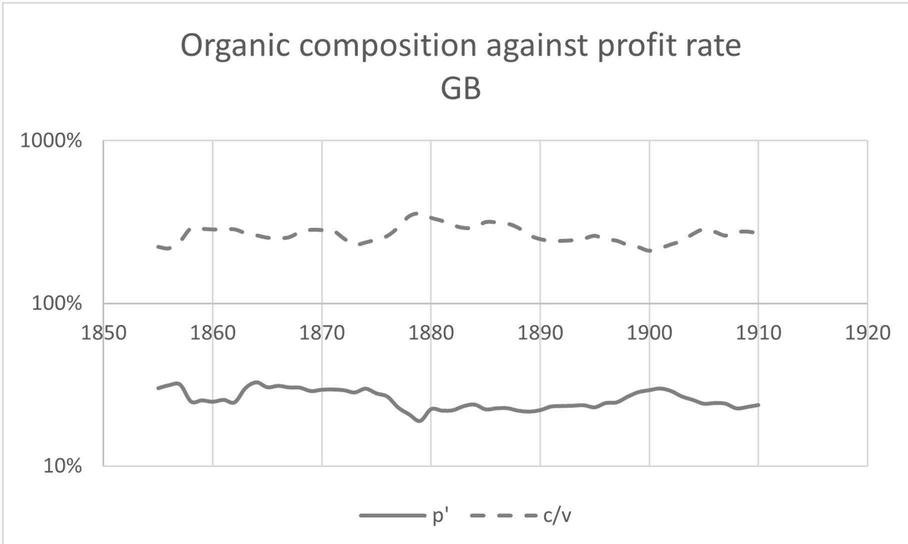

Fig. 3: Actual evolution of the organic composition and rate of profit in Victorian/Edwardian Britain. There is a negative correlation of -61% between organic composition and rate of profit. Data computed from the Bank of England database *A millennium of Economic Data.*.

with the organic composition of capital. His basic argument that the organic composition would be a strong determinant of changes in profit rates is born out by this data.

If we look at the time trend in Figure 3 of the organic composition we see that the expected rise in organic composition was not occurring, and from 1880 to 1900 the trend was downwards.

This must mean that either the physical mass of machinery put in motion by UK workers must have declined, or what Marx called cheapening of the elements of constant capital must have been in operation.

It was certainly the case that the physical mass of basic power machines underwent drastic shrinking during the period in question. The adoption of higher pressure steam first allowed the construction of small high speed triple expansion engines and then of steam turbines (Figure 4). This means that in some areas the actual mass of machinery declined whilst its ability to perform work increased.

In other economic areas the mass of machines certainly did increase. Take shipping as an example, steamships were becoming bigger and more powerful over the period. The issues however when looking at the organic composition of capital would be things like:

1. By how much did the average tonnage of a cargo ship rise.
2. How much did the average crew size change?

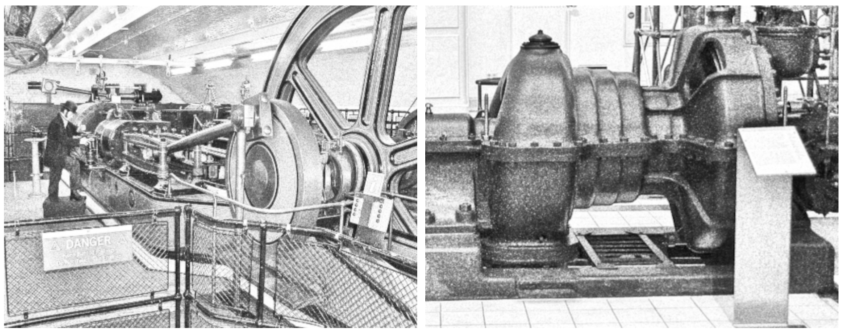

Fig. 4: Left the 150hp engine designed for Tower Bridge by Armstrongs in 1886. Right a 335hp turbine made 1910 by Maschinenbau A.G., Prague. Note how much more compact the more powerful later machine was.

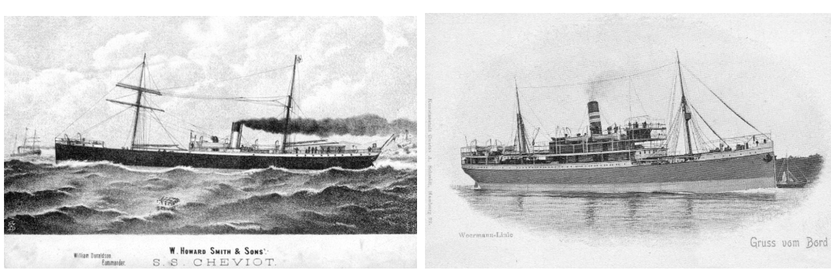

Fig. 5: Size and power of typical cargo ships increased during late 19th Century. Left the Cheviot built on the Tyne in 1870, 1226grt, 120nhp. Right Ernst Woermann built on the Clyde 1900, 4065 grt, 257nhp.

For instance the shift from sailing vessels to steam cargo ships would have reduced the crew requirement per ton displacement. This would correspond to a rise in the technical composition of capital.

3. Most importantly, by how much did the price per ton of displacement change for new ships, whether measured in £s or in labour days.

Could a ship owner have bought a vessel like the Cheviot (Figure 5) in 1870, used her for 30 years, and then with the depreciation money set aside have bought a larger vessel like the Ernst Woermann in 1900?

If productivity in the combined steel and shipbuilding industries had grown by 4% a year he could have done. In fact growth in productivity was somewhat less:

> Between the late 1860's and the early 1890's the price of British iron ships declined about 40 percent. The fall in iron prices and the improvements in building technology were of approximately equal importance in explaining the price decline, while rising wages offset about half of their combined effect.
>
> [Harley, 1970]

This amounts to fall in the cost of constant capital for the shipping industry of about 1.7% a year. Between 1865 and 1895 British organic composition of capital fell at an average rate of 1.4% a year. If the general rise in productivity in what Marx called Department I of the economy, that producing means of production, was the same as in shipbuilding, then it was more than sufficient to account for the decline in organic composition of capital. The actual value of the capital stock per employee could have declined while its physical mass continued to grow at a modest rate.

This possibility was allowed for by Marx:

> Everything said in Part I of this book about factors which raise the rate of profit while the rate of surplus-value remains the same, or regardless of the rate of surplus-value, belongs here. Hence also, with respect to the total capital, that the value of the constant capital does not increase in the same proportion as its material volume. For instance, the quantity of cotton worked up by a single European spinner in a modern factory has grown, tremendously compared to the quantity formerly worked up by a European spinner with a spinning-wheel. Yet the value of the worked-up cotton has not grown in the same proportion as its mass. The same applies to machinery and other fixed capital. In short, the same development which increases the mass of the constant capital in relation to the variable reduces the value of its elements as a result of the increased productivity of labour, and therefore prevents the value of constant capital, although it continually increases, from increasing at the same rate as its material volume, i.e., the material volume of the means of

Year	Capital stock	Average growth rate for period	Employment	Average growth rate for period
1855	994	0.2%	11252000	1.0%
1860	1006	4.1%	11818000	0.8%
1865	1213	2.1%	12262900	0.5%
1870	1338	6.5%	12584300	0.8%
1875	1772	-0.3%	13090500	0.2%
1880	1749	-0.1%	13236600	0.7%
1885	1743	2.0%	13694700	1.8%
1890	1913	0.0%	14935500	0.4%
1895	1909	7.2%	15220300	1.9%
1900	2601	1.3%	16690300	0.3%
1905	2765	1.8%	16982000	1.0%
1910	3011	6.3%	17867000	1.9%

Tab. 1: The growth of British productive capital stock 1855 to 1910. Excludes dwellings. Source Bank of England database *A Millennium of Economic Data*.

> production set in motion by the same amount of labour-power. *In isolated cases the mass of the elements of constant capital may even increase, while its value remains the same, or falls.*[7]
>
> [Marx, 1971, Chap. 14]

Taking into account the proviso shown in italics, the overall account given in Chapters 13 and 14 turn out to be consistent with what was actually happening to the British economy in the latter part of Marx's life.

- He gets right the invese relation between organic composition and profitability.
- He gives an account of how the organic composition of capital can fall and the technical composition rise when, as was the case in the late 19th century, there is a steady rise in labour productivity in Department I.

One is left however, with the feeling that this is not quite what he expected to happen. He seems to have regarded the cheapening of the elements of constant capital as the exception rather than the rule.

3 Accumulation rate

Although the organic composition of capital actually stayed relatively unchanged after 1855, this does not mean that no capital accumulated. Table 1 shows that the nominal value of constant capital stocks rose for 20 years, then stagnated from 1875 to 1885 before growth resumed.

[7] My emphasis.

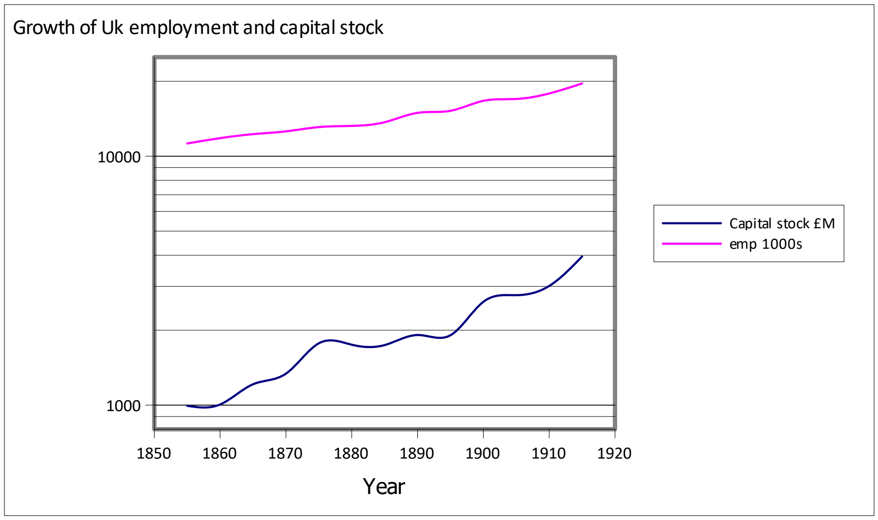

Fig. 6: Growth of capital stock was faster than growth of the workforce.

Capital accumulation can be combined with a falling organic composition, and thus a rising profit rate, if the workforce is increasing. Marx indeed listed relative over-population[8] as one of the offseting factors acting against a fall in the rate of profit in Chapter 14.

If we compare the growth of capital stock to the growth of the workforce in £s in Figure 6 you can see that capital stock was rising faster than the employed workforce. Capital employed per worker was £65 in 1855 and had risen to £155 in 1910. Why then did the organic composition not rise sharply?

It is because Marx posed his argument in terms of real values (labour time) not nominal values in £s. His argument was that given a stable rate of exploitation, then dividing money capital stock through by wages amounts (subject to a constant of scale) to dividing the labour content of the capital stock by the living labour currently activating the stock. The relative constancy of c/v in the face of a rising nominal capital to labour ratio was possible because

[8] "relative over-population becomes so much more apparent in a country, the more the capitalist mode of production is developed in it. This, again, is the reason why, on the one hand, the more or less imperfect subordination of labour to capital continues in many branches of production, and continues longer than seems at first glance compatible with the general stage of development. This is due to the cheapness and abundance of disposable or unemployed wage-labourers, and to the greater resistance, which some branches of production, by their very nature, render to the transformation of manual work into machine production. On the other hand, new lines of production are opened up, especially for the production of luxuries, and it is these that take as their basis this relative over-population, often set free in other lines of production through the increase of their constant capital. These new lines start out predominantly with living labour, and by degrees pass through the same evolution as the other lines of production. In either case the variable capital makes up a considerable portion of the total capital and wages are below the average, so that both the rate and mass of surplus-value in these lines of production are unusually high." [Marx, 1971, Chap. 14.4]

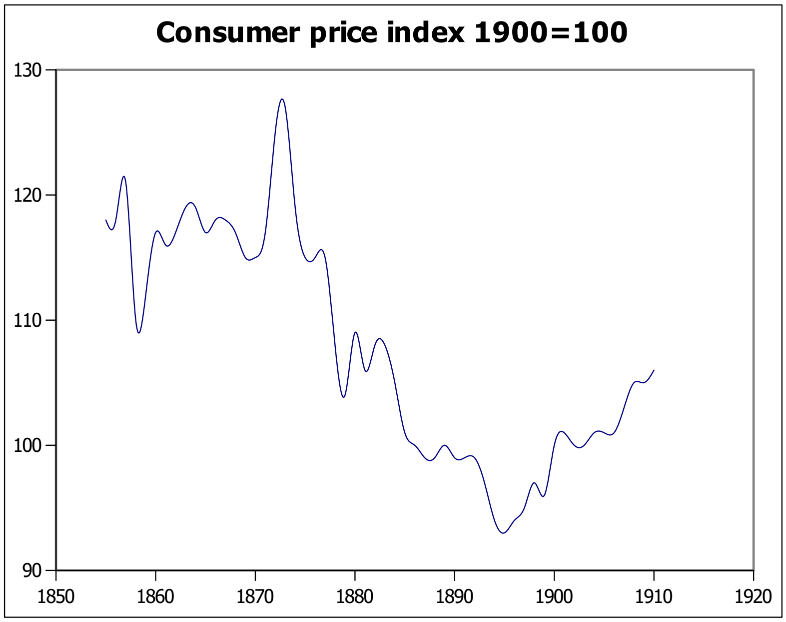

Fig. 7: Consumer price index for late 19th century. Prices were generally falling until the mild inflation caused by rising South African gold production in the early 20th century. Source, table A47 of *A Millennium of Economic Data*.

labour productivity rose. This affected both wages and capital.

- In 1855 average daily wages were 39 (old)pence. In 1910 they had risen to around 81 pence. An index of real, inflation adjusted, earnings per day (taking 1900 as 100) rose from £55 in 1855 to £146 in 1910. The rate of surplus value was 67% in 1855 and 64% in 1910. Thus real wages had risen, even though the share of value added going to labour remained almost unchanged.

- I have not been able to obtain time series for capital goods prices over the period so I will use the CPI shown in Figure 7. Scaling to 1900 prices, this shows that the capital stock per worker in 1855 would have been £75 and in 1910 it would have risen to £159. So in real terms, in terms of productive equipment, it had risen. But what about in terms of labour values?

- Consider Table 2. This computes the Monetary Equivalent of Labour Time (MELT) for the start and end years of our period. It can be seen that the value created by a year's work rose from £61 to £170 over the 55 years. If we use the MELT to convert the constant capital employed per worker given earlier into the number of years of labour needed to reproduce that

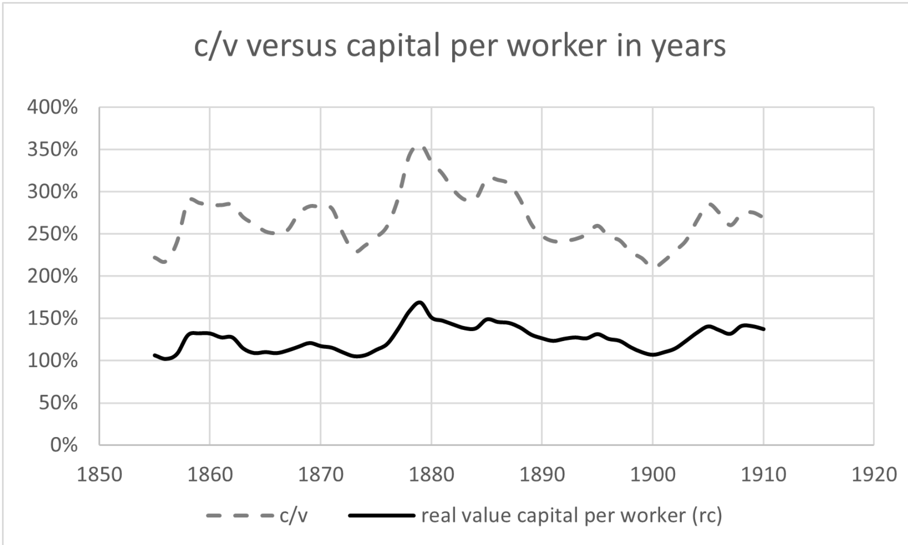

Fig. 8: Marx's index of dead to living labour $\frac{c}{v}$ correlates well (coeff 86%) with the value in person years of the fixed capital operated by each worker.

constant capital, we see that the net capital stock per worker had risen slightly.

If we plot over time the relationship between the organic composition of capital $\frac{c}{v}$ and the ratio of dead to living labour in worker years in Figure 8 we see that they match very well.

It is notable that the broad trend of both is stable. There are short term fluctuations but no long term rising trend. We have accounted for this in terms of a cheapening of constant capital goods. But this is only a partial answer. At a deeper level the cause was the fact that the capitalist class, contrary to Marx's aphorism about accumulation being 'their Moses and their prophets', actually devoted only a minority of their income on accumulation.

4 A model

If you look at Figure 9 it is evident that the upper classes generally consumed more than 80% of surplus value, with accumulation rarely rising above 20%. Comparing the rate of accumulation out of surplus value with the movement of capital stock per worker measured in person years it is clear that the former tended to drive the latter. We can systematise this with a little calculus.

Clearly if the rate of growth of capital stock $\frac{dC}{dt}$ is higher than the rate of growth of the workforce $\frac{d\lambda}{dt}$, then $\frac{c}{v}$ will rise, and $\frac{c}{v}$ will fall if the workforce

year	Gross Domestic product £M	Capital consumption £M	Net domestic product £M	MELT £ per worker year	Capital stock per worker in labour years
1855	715	29	686	61	1.1
1910	22122	108	2014	170	1.4

Tab. 2: Estimating capital stock per worker in labour time for the start and end of the period. The MELT is obtained by dividing the Net Domestic Product by the number of worker years that were required to produce it. Similar methods were used to derive Figure 9.

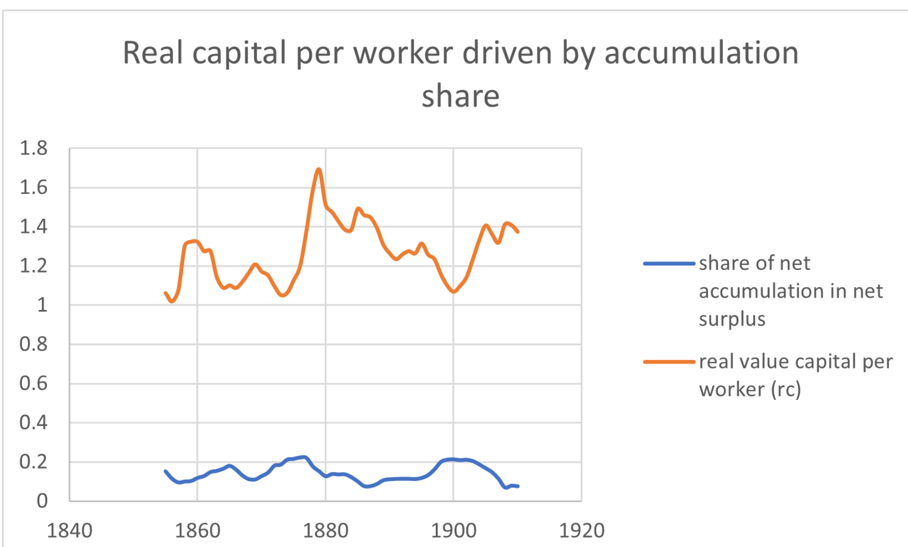

Fig. 9: Long term trends in fixed capital per worker, measured in person years, are driven by the share of surplus being accumulated.

grows faster.

If the workforce and capital stock grow at the same rate then the organic composition and profit rate will stabilise. Let us ignore interest and rent, and assume all surplus value takes the form of profit. We will also initially abstract from depreciation and technical change.

Let $G = \frac{d\lambda}{dt}$ be the rate of growth per year of the workforce.

Let α be the share of net surplus value being accumulated.

Assume the rate of profit has stabilised, and denote this stable rate by p^*.

Clearly we must have relationship

$$p^* = \frac{G}{\alpha} \tag{1}$$

for profit to be stable.

Why?

Consider that the rate of profit is $\frac{s}{C}$ so $\alpha p^* = \frac{\alpha s}{C}$ is the rate of growth per year of the capital stock.

$$\alpha p^* = \frac{\alpha s}{C} = \frac{dC}{dt}$$

But we defined p^* to be the profit rate where $G = \frac{dC}{dt}$, so

$$\alpha p^* = \frac{\alpha s}{C} = \frac{dC}{dt} = G$$

so $p^* = \frac{G}{\alpha}$ Q.E.D.

4.1 From net to gross surplus

Now relax the assumption of no depreciation and not technical change. National income figures give us gross trading surplus and gross capital accumulation along with a separate figure for depreciation or capital consumption.

Let us use S for gross trading surplus, d for the annual depreciation rate as a share of capital, and $P = \frac{S}{C}$ be the gross rate of profit and A denote the share of gross surplus that goes to fund gross accumulation.

We can now modify Equation 1 to obtain a formula for the stable gross profit rate :

$$P^* = \frac{G + d}{A} \tag{2}$$

4.2 Technical change

If labour productivity grows at 5% a year, then clearly the labour value of capital stock at replacement cost also falls by 5% a year. Technical change thus acts on capital stock in the same way as depreciation - Marxist economists call this ‘moral depreciation’.

Let the rate of growth of labour productivity be denoted by t for technical change.

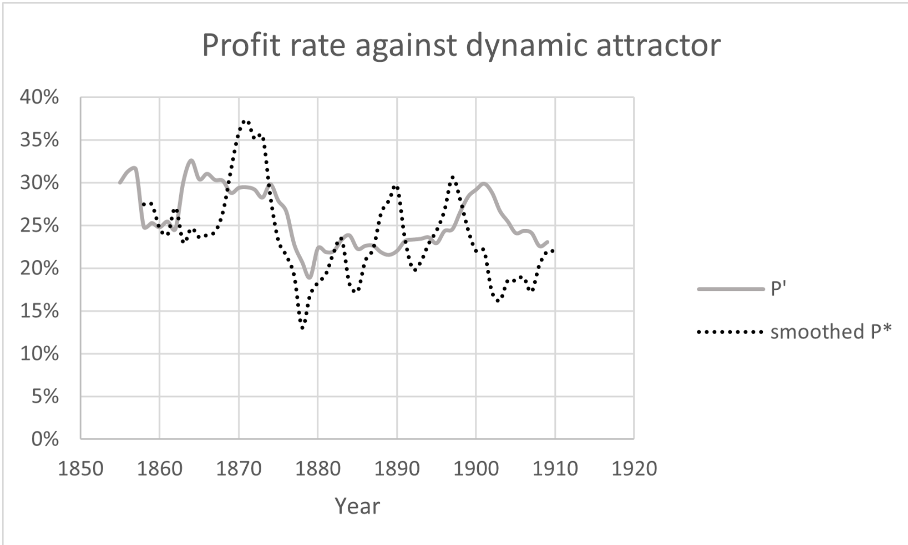

Fig. 10: P' is the gross rate of profit and smoothed P*' is the 4 year average of the dynamic profit rate attractor. All data calculated from *A Millennium of Economic Data*.

A rapid rate of technical change tends to raise the stable rate of profit since it acts to slow down the accumulation process measured in real value terms : that is to say in terms of the worker years represented by the capital stock.

So our final equation is:

$$P^* = \frac{G + d + t}{A} \tag{3}$$

One can consider P^* as the dynamic attractor of the profit rate. If the variables A, G, d, t remain unchanged then the real gross profit rate P' should asymptotically approach P^*.

This basic equation has been derived elsewhere [Cottrell and Cockshott, 2006, Cockshott et al., 2009, Zachariah, 2009] and tested against modern time series. In Figure 10 we show how it can be applied to late Victorian British capitalism.

5 Conclusion

We have shown using old and recent UK data that key elements of Marx's theory of profit rates are valid.

1. In both diachronic and synchronic cases the inverse relationship between organic composition and profit rate predicted by the labour theory of value is observed.

2. As Marx hypothesised $\frac{c}{v}$ in money terms acts as a good proxy for the ratio of dead to living labour.

3. One can, using the assumptions of his theory derive a dynamic model of the rate of profit with non-negligible predictive power.

Figure 10shows how the dynamic attractor of the profit rate, whilst subject to high frequency noise from the business cycle, in absolute scale brackets and in its changes correlates (coeff 42%) to the observed gross profit rate. It shows a tendency to lead the actual profit rate. This is quite striking given that the variables entering into it are all time derivatives - none of the original Marxian variables remains. The actual derivation of the calculus (equations 1 to 3) remains predicated on the assumptions of the labour theory of value. So although the new equation looks unfamiliar, it is axiomatically derived from that given in Capital.

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A Method of calculation

The UK Office of National Statistics(ONS) publish capital stock data annually that give net capital stocks for distinct combinations of 12 asset types and some 90 sectors. Some sectors are given in both aggregate and disaggregated form so the total number of independently specified sectors is slightly less. Whilst the sector names are not identical to those used in the IO table, industrial sector codes are provided so it is relatively easy to translate to the IO table sectors. The data for 2015 was used as this corresponds to the most recent year that a British input output table has been published.

The capital data from ONS are in relational rather than matrix form so an expanded stock matrix was created such that for each column in the original stock matrix for which several sub industries exist in the io table the capital values in the original are spread among the new multiple columns in proportion to their share in the final output of this group of industries. The final output matrix has column names in the same order as the input output table. The resulting intermediate stock matrix has rows with the asset types:

Dwellings
Other buildings and structures
Transport equipment
Computer hardware
Telecommunications equipment
ICT equipment
Cultivated biological resources
Research & development
Computer software and databases
Intellectual property products
Machinery, equipment and weapons systems
Other machinery, equipment and weapons systems

It is again relatively easy to identify the input output table industries producing these categories of goods. The ones that were used in this study are documented in Table 3. Using this and the intermediate capital stock matrix produced by the previous step, software was used to produce an expanded stock matrix with the same layout as the iotable such that

- For each row in the intermediate stock matrix for which several source industries exist in the source index file these are mapped to IO table rows using Table 3.
- Capital values in the original matrix are spread among the new multiple rows in proportion to the flows shown in the corresponding columns in the iotable.
- The underlying assumption for this approach is that the flows shown in the io table are replacement for depreciation and will be proportional to the corresponding capital stocks.

DWELLINGS	CONSTRUCTION						
OTHER BUILDINGS AND STRUCTURES	CONSTRUCTION						
TRANSPORT EQUIPMENT	MANUFACTURE OF MOTOR VEHICLES, TRAILERS AND SEMI-TRAILERS	BUILDING OF SHIPS AND BOATS	MANUFACTURE OF AIR AND SPACECRAFT AND RELATED MACHINERY	MANUFACTURE OF OTHER TRANSPORT EQUIPMENT - 30.2/4/9			
COMPUTER HARDWARE	MANUFACTURE OF COMPUTER, ELECTRONIC AND OPTICAL PRODUCTS						
TELECOMMUNICATIONS EQUIPMENT	MANUFACTURE OF COMPUTER, ELECTRONIC AND OPTICAL PRODUCTS	MANUFACTURE OF ELECTRICAL EQUIPMENT					
ICT EQUIPMENT	MANUFACTURE OF COMPUTER, ELECTRONIC AND OPTICAL PRODUCTS						
CULTIVATED BIOLOGICAL RESOURCES	CROP AND ANIMAL PRODUCTION, HUNTING AND RELATED SERVICE ACTIVITIES	FORESTRY AND LOGGING	FISHING AND AQUACULTURE				
RESEARCH & DEVELOPMENT	SCIENTIFIC RESEARCH AND DEVELOPMENT	OTHER PROFESSIONAL, SCIENTIFIC AND TECHNICAL ACTIVITIES	SCIENTIFIC RESEARCH AND DEVELOPMENT NPISH				
COMPUTER SOFTWARE AND DATABASES	COMPUTER PROGRAMMING, CONSULTANCY AND RELATED ACTIVITIES						
INTELLECTUAL PROPERTY PRODUCTS	MOTION PICTURE, VIDEO & TV PROGRAMME PRODUCTION, SOUND RECORDING & MUSIC PUBLISHING ACTIVITIES & PROGRAMMING AND BROADCASTING ACTIVITIES NON-MARKET	CREATIVE, ARTS AND ENTERTAINMENT ACTIVITIES	SCIENTIFIC RESEARCH AND DEVELOPMENT	ADVERTISING AND MARKET RESEARCH	PUBLISHING ACTIVITIES	MOTION PICTURE, VIDEO & TV PROGRAMME PRODUCTION, SOUND RECORDING & MUSIC PUBLISHING ACTIVITIES & PROGRAMMING AND BROADCASTING ACTIVITIES	CREATIVE, ARTS AND ENTERTAINMENT ACTIVITIES NPISH
MACHINERY, EQUIPMENT AND WEAPONS SYSTEMS	MANUFACTURE OF FABRICATED METAL PRODUCTS, EXCLUDING WEAPONS & AMMUNITION - 25.1-3/5-9	MANUFACTURE OF WEAPONS AND AMMUNITION	MANUFACTURE OF COMPUTER, ELECTRONIC AND OPTICAL PRODUCTS	MANUFACTURE OF ELECTRICAL EQUIPMENT	MANUFACTURE OF MACHINERY AND EQUIPMENT N.E.C.		
OTHER MACHINERY, EQUIPMENT AND WEAPONS SYSTEMS	MANUFACTURE OF FABRICATED METAL PRODUCTS, EXCLUDING WEAPONS & AMMUNITION - 25.1-3/5-9	MANUFACTURE OF WEAPONS AND AMMUNITION	MANUFACTURE OF COMPUTER, ELECTRONIC AND OPTICAL PRODUCTS	MANUFACTURE OF ELECTRICAL EQUIPMENT	MANUFACTURE OF MACHINERY AND EQUIPMENT N.E.C.		

Tab. 3: Mapping from capital stock types to industrial sector in the IO table that produces them.

Handling of tax and imports

The IO table contains rows for taxes on products and production and for the import content of each column listed. Since we are concerned to do an unbiased comparison between prices of production and since Marx's Vol I and Vol III price theories ignore the effect of taxes we do not include them in the calculation. It should however be born in mind that in this and other studies of correspondence between labour values and market prices, the differential impact of taxes of industries will constitute a source of unaccounted for noise in the market prices.

Imports are dealt with by computing the labour content of £1 of exports and imputing the same labour content to each £1 of imports used by an industry.

Values and prices of production

Labour values and prices of production were computed via a Jacobi iterative procedure with 12 iterations. Two temporary vectors $\boldsymbol{v}, \boldsymbol{p}$ are used. The vector $\boldsymbol{v}$ holds the labour content of each £1 of output of the corresponding industry, $\boldsymbol{p}$ holds the production price per £1 of output. Both vectors are initialised to zero.

On each iteration for each industry i the total labour content L_i is computed by adding the direct labour λ_i to $U_i^T.\boldsymbol{v}$, that is to the total obtained by converting the £costs in the ith column of the use matrix U into labour using $\boldsymbol{v}$. Then $\boldsymbol{v}$ is updated by setting $\boldsymbol{v}_i = \frac{L_i}{F_i}$ where F is the final output vector in £.

An analogous procedure is used to update $\boldsymbol{p}$.

In this case the total production cost $P_i$for each industry is computed as

$P_i = \lambda_i + U_i^T.\boldsymbol{p} + r(K_i^T.\boldsymbol{p})$

where K is the capital stock matrix with the same shape as U and r the rate of profit for the whole economy obtained by dividing the Gross Operating Surplus of the economy as a whole by the total capital stock of the economy.

At the end of each iteration P, L are renormalised to ensure that their totals are equal to the total in £of final output F.