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Temptation and self-control for the impure benevolent planner: The case of heterogeneous discounting

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Abstract

This paper presents a "behavioral" model of a normative benevolent social planner, who faces a self-control problem when he/she is in charge of aggregating diverse and conflicting preferences of individuals. The model is presented in the context of aggregating preferences over intertemporal streams of social outcomes, in which Zuber and Jackson and Yariv have shown the impossibility of a time-consistent and Paretian social objective function. Unlike previous studies, our investigation focuses on the compatibility of the Pareto condition with the impure social planner who has a dynamically consistent self-control utility function characterized by Gul and Pesendorfer. Assuming that the social planner is tempted to adopt the majority's opinion when there is a conflict of opinions among individuals, the paper characterizes an aggregation form in which the planner is allowed to depart from dictatorship in ex-post choice under noncommitment.

KEYWORDS

heterogeneous discounting, impure benevolent planner, self-control, temptation

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1 | INTRODUCTION

In social decision making, even if a planner adopts one group's opinion for some normative reason when considering the ranking of social alternatives, it may be difficult for him/her to consistently ignore the opinions of other groups. This difficulty may arise from the social planner's concern about possible backlash from the other groups. As another reason, the social planner may care about his/her own social image. The literature on social preferences (e.g., Dillenberger & Sadowski, 2012; Saito, 2015) has shown that people are subject to social pressure to behave altruistically in public situations that are observed by others when making social allocation decisions.

Such discordance between a social planner representing one group and the other groups can occur in dynamic allocation problems of consumption streams. For example, as shown in Becker (1980), under heterogeneous time preferences among individuals, Pareto efficiency requires that only the most patient individual receives positive consumption in the long-run, and all the others' consumption paths converge to zero. After a long enough time, the majority of less patient individuals may make an objection to stick to this consumption stream. The social planner, influenced by distributive concerns, may lean toward this majority opinion, at which point he/she becomes time-inconsistent.[1]

To capture the internal conflicts and the resulting compromises in dynamic allocation problems, we adopt the dynamic self-control (DSC) model established by Gul and Pesendorfer (2004) for the social planner, who is in charge of aggregating individuals' diverse and conflicting preferences over intertemporal streams of social outcomes. In our setting, the social planner has a normative stationary discounted utility but is tempted by consumption that maximizes a different per-period utility. Such a formulation allows us to consider an "impure" benevolent decision making in which the social planner forms a normative discounted utility that reflects the opinions of one group but is tempted to maximize a utility that reflects the opinions of another group, resulting in choosing from each menu by maximizing a linear combination of the two. The characteristic that distinguishes impure benevolent social planners from purely benevolent social planners is that they use commitment whenever they can avoid temptation from the opinions of other groups.[2]

Our study is also motivated by the impossibility result regarding the existence of a social planner as assumed in the standard dynamic economic analysis. Zuber (2011) and Jackson and Yariv (2015) have shown that when individuals differ in time discounting as well as in tastes over per-period

[1]Formally, this time-inconsistency is implied by nonstationarity and time-invariance of the social planner's preference. To illustrate the point, consider the following example which is common in the macroeconomics literature regarding the existence of a representative agent under heterogeneous time preferences (see Pakhnin (2023) for a survey). There is 1 unit of consumption good at each period, and there are two individuals, A and B. For each $i = A, B$, his/her preference over lifetime consumption streams is represented in the form $\sum_{t=1}^{\infty} \beta_i^{t-1} \ln c_{it}$. Then the solution to maximize a social welfare defined by $\sum_{t=1}^{\infty} \beta_A^{t-1} \ln c_{At} + \sum_{t=1}^{\infty} \beta_B^{t-1} \ln c_{Bt}$ is given as

$$c_{At} = \frac{\beta_A^{t-1}}{\beta_A^{t-1} + \beta_B^{t-1}}, c_{Bt} = \frac{\beta_B^{t-1}}{\beta_A^{t-1} + \beta_B^{t-1}}, \forall\, t = 1, 2, 3, \ldots$$

However, once period 1 passes and we come to period 2, the planner will again optimize the social welfare by maximizing $\sum_{t=2}^{\infty} \beta_A^{t-2} \ln c_{At} + \sum_{t=2}^{\infty} \beta_B^{t-2} \ln c_{Bt}$, which changes the solution to

$$c_{At} = \frac{\beta_A^{t-2}}{\beta_A^{t-2} + \beta_B^{t-2}}, c_{Bt} = \frac{\beta_B^{t-2}}{\beta_A^{t-2} + \beta_B^{t-2}}, \forall\, t = 2, 3, 4, \ldots$$

[2]See Dillenberger and Sadowski (2012) and Saito (2015) for impure altruism.

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social outcomes, only dictatorship can meet the following two requirements: the social planner's objective function satisfies the standard stationary discounted utility theory and the Pareto condition stating that if everybody prefers a stream of social outcomes over another so should the society. It is of interest to investigate the compatibility of Pareto criteria with the DSC representation.

We assume finite individuals having a discounted utility representation over streams of social outcomes. The social planner's preference is defined over recursive menus (such as intertemporal budget sets determined by the current decision on savings or capital accumulation) and represented by a DSC model. Thus, we consider the problem of aggregating individual preferences over *streams* into a social ranking over *menus* of social alternatives.

Three types of axioms are considered for preference aggregation. The first is the Pareto condition applied to commitment streams of social outcomes. The second is that the planner should not be tempted by Pareto-inferior streams, which is natural since we intend to capture temptation arising solely because of disagreements of individuals' opinions. The third is that the social planner should be tempted to accommodate majority's opinion, which illustrates the nature of temptation a benevolent planner should face. The more a smaller minority's opinion is adopted as the social choice, the greater will be the social pressure and the backlash from the other majority that the social planner faces.

In the axiomatic characterization, we first present the most permissive kind of aggregation, which involves no intertemporal trade-offs. When the planner only takes individuals' per-period utility into account, the individuals are divided into two groups: per-period utility functions in one group reflect to the planner's commitment per-period utility function and ones in the other group reflect to its temptation per-period utility.

Next, we present aggregation in which the Pareto condition is imposed for commitment paths. For the same reason as in Zuber (2011) and Jackson and Yariv (2015), the planner's commitment utility function can incorporate only one individual's time preference into account. We call such an individual a “commitment dictator”. Thus, the role of taking care of other individuals is given to maximizing temptation utility. We characterize an aggregation form in which the planner is allowed to depart from dictatorship in ex-post choice because the temptation utility takes care of majority's tastes.

One might be interested in letting temptation utility take care of individuals' time preferences as well. In Section 6, we consider the model of future temptation (FT) by Noor (2007), in which the planner's temptation utility takes the form of stationary discounted utility. We consider the same set of axioms and show that such temptation utility coincides with exactly one individual's discounted utility, and we would call such individual a “temptation dictator.” Thus we show that there is a trade-off between letting temptation utility take care of more individuals' welfare and letting it take care of time preference.

Finally, we give some remarks on other possible settings in which menu preferences are taken into account. As explained above, we consider the problem of aggregating individuals' discounted utilities, defined on consumption streams, to the social planner's preference, defined on menus. If, instead of this setting, individuals can also express opinions about their menu preferences, we may consider an alternative aggregation problem, such as aggregating individuals' preferences over menus into the social planner's menu preference.[3]

[3]For example, Arawatari and Ono (2023) consider that individual voters have time-inconsistency issues due to hyperbolic discounting, while there is heterogeneity in degrees of inconsistency, and the society is to choose a debt rule under difference in desires for a commitment device (smaller menu). Individual voters' time-inconsistency and preference for commitment implied therein are reflected in their preferences over menus. Hence, this setting can be interpreted as aggregating individuals' menu preferences to derive the social planner's menu preference.

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We adopt the current framework for two reasons. One is that each individual cannot evaluate a menu without knowing which one in it is chosen by the society, which depends again on how we aggregate preferences. In single-person cases, an individual without any self-control issue simply evaluates a menu as the best element in it.[4] In a multiperson setting, however, different individuals cannot do this in one society, since they in general have different best elements in the same menu. One may still consider a version of Pareto condition, which states that if everybody prefers a menu over another according to the above-stated sense, then so should the society. Such unanimity may be "spurious" in the sense that different individual have different best ex-post choices in mind, the social ranking may exhibit preference for flexibility which supports a menu based on mutually conflicting reasons (Dekel et al., 2001; Kreps, 1979). In fact, under a reasonable "nondelusional" condition for the social ranking (including temptation-driven preferences such as self-control preferences), only dictatorship can satisfy such Pareto condition (see Proposition 1).

Second is that we aim to emphasize the self-control problem that arises *specifically* at the level of collective decision. More precisely, we are considering here the temptation that the social planner suffers only when there are disagreements of opinions among individuals, such as the social pressure that arises from the other groups with the opposite opinions when only one group's opinion is adopted. It is in the same spirit as in the voting model considered by Lizzeri and Yariv (2017), which arises even when nobody has any self-control issue at an individual level.

The paper is organized as follows. Section 1.1 provides an overview of the existing literature. Section 2 introduces the choice setting, the individuals' utility functions, and the social planner's utility function. Section 3 proposes the consistency between the individuals' preferences and the social planner's preference. We introduce three types of Pareto conditions and axioms that restrict the direction of the social planner's temptation. Section 4 presents characterization theorems on the possible preference aggregation under each set of axioms. Using the optimal growth model as an example, Section 5 discusses the social welfare implications of the optimal growth path when the social planner sequentially optimizes a self-control utility function characterized in the previous section. In Section 6, we explore the possibility of allowing the temptation utility to account for individuals' time preferences. Then Section 7 concludes.

1.1 | Related literature

1.1.1 | Escaping from the impossibility

As shown by Zuber (2011) and Jackson and Yariv (2015), the impossibility theorem states that, assuming a group of individuals with stationary discounted utility, the only social preference that satisfies Pareto efficiency and stationary discounted utility can be dictatorship in the sense that it can only coincide with a particular preference in the group. There have been two main approaches to escape from this dictatorship result. One direction is to relax Pareto efficiency with keeping the stationarity of the social preference, while the other is to relax the stationarity with keeping Pareto efficiency.

Feng and Ke (2018) propose a weaker Pareto axiom that treats successive selves of an identical individual as different individuals, saying that if all such selves prefer a stream over another, so should the society. Their axiom implies that the society must be strictly more patient than every

[4]Such a standard decision making is called strategic rationality and axiomatized in terms of menu preference by Kreps (1979).

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individual, which is interpreted as a kind of paternalism. Hayashi and Lombardi (2021) propose a weaker Pareto axiom in which each individual's lifetime discounted utility is calculated in different ways by using all the individuals' discount factors, not just his/her own discount factor. Their axiom still implies that the society has to adopt exactly one individual's discount factor, but they show that such selection can be done fairly by characterizing the generalized median solution. Billot and Qu (2021) propose an even weaker axiom that limits attention to a restricted class of comparisons involving only two periods of intertemporal trade-offs and obtain a permissive result which allows averaging of discount factors.

As in the above literature, considering a social preference defined on streams implies that the most preferable option in terms of the social preference is eventually chosen among all possible streams that can be chosen in a given choice problem, whether it is under commitment or in a sequential choice environment. Therefore, if the social preference on streams satisfies the Pareto condition, then its maximization solution from any choice problem satisfies Pareto efficiency. In contrast, under self-control preferences, a choice from menus is determined by a compromise between normative and temptation utilities. As explained in detail in Section 3, although we will assume both normative and temptation utilities to respect the Pareto condition in the usual sense (if all individuals prefer one stream to another, so does from the social perspective), this axiom is not strong enough to guarantee that a Pareto-efficient stream is eventually chosen in all choice problems or menus. Thus, as in the above literature, our study can be categorized into a group of studies that avoid the dictatorship by weakening Pareto efficiency.

Giving up or weakening stationary/exponential/geometric discounting at the social level allows us to maintain efficiency. Millner and Heal (2018), Millner (2020) propose time-dependent social welfare function with nonstationary discounting. It is debatable if the planner can commit to the maximization solution for such a nonstationary objective function. Hayashi (2016) argues that commitment to follow the maximization solution for a nonstationary social welfare function requires an extra restriction, which is beyond the mere fact that the solution was already decided in the past period, and proposes a meta axiom stating that if the planner bases welfare judgment at one period on some normative reason the same reason must apply to welfare judgment in the next period as well.

Another possible direction to escape from the dictatorship result is to allow multiple discount factors, either by weakening the completeness axiom at a social level or by allowing a maxmin-type representation for the social objective function (Chambers & Echenique, 2018, 2020).

1.1.2 | Positive analysis of collective choice and time-inconsistency

Positive analyses of time-inconsistency problems arising in aggregating preferences through dynamic voting or bargaining without commitment are closely related.

Formation of household preference through bargaining within a family, as surveyed by Chiappori and Mazzocco (2017), is closely related in particular. In the setting of common time-discounting, in which Pareto-efficiency implies time-invariance of decision powers (welfare weights), they study the time-inconsistency problem arising due to evolution of decision powers under lack of commitment. In contrast, we consider the setting of heterogeneous time-discounting among individuals, in which Pareto-efficiency implies that decision powers of impatient individuals diminish over time exponentially, and study the time-inconsistency problem arising because decision powers are nonetheless time-invariant for political reasons.

Also, there is a huge literature on dynamic voting without commitment. In dynamic macroeconomics/political economy, in which heterogeneity is mostly about asset holdings and

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earnings, lack of commitment to future actions is widely studied to explain attitudes toward income redistribution and the size of government. See Persson and Tabellini (2000), Krusell and Rios-Rull (1999), Azzimonti et al. (2006, 2008), among many.

More closely to the present context, Heal and Millner (2013) show that the planning solution is not time-consistent under lack of commitment, but it is renegotiation-proof. In the setting of accumulation of public capital where agents vote over current savings without commitment to the future, Boylan and McKelvey (1995), Boylan et al. (1996) show that when individuals have common per-period utility function the optimal saving plan for the individual with the median discount factor beats any other saving choice in majority voting at every period (see also Borissov et al., 2017 and Borissov et al., 2019 for related results).

1.1.3 | Self-control preferences

A self-control utility is first axiomatized by Gul and Pesendorfer (2001) as a utility representation of preference over menus. In this model, the rankings over menus are determined through conflicts between the normative utility, corresponding to the commitment utility, and the temptation utility. Gul and Pesendorfer (2004) extend the self-control utility to a recursive domain applicable to infinite-horizon dynamic decision making. Salient features of this model are that (1) the decision maker is tempted only from immediate consumption and (2) the representation is dynamically consistent. This model can explain anomalies such as the preference reversal due to present bias for choices from a menu. To apply the self-control model for addictive behavior, Gul and Pesendorfer (2007) consider a generalization in which the intensity of temptation depends on past consumption history.

Noor (2007) axiomatizes the FT model in which the decision maker is tempted from the entire consumption stream not only from immediate consumption. Here, both normative utility and temptation utility follow a stationary discount model.

Ahn et al. (2020) establish a recursive representation with naiveté. Although their primary concern is a recursive model allowing for naiveté, as a special case, they axiomatize a model, called a sophisticated quasi-hyperbolic discounting representation. As in Noor (2007), the decision maker may be tempted from the entire consumption streams. More specifically, the continuation value function of the temptation utility is the same as that of the normative utility with the current discount factor being specified as in quasi-hyperbolic discounting. Thus, the decision maker exhibits the quasi-hyperbolic discounting only when succumbed to this temptation.

Hayashi and Takeoka Hayashi and Takeoka (2022) are motivated by the projection bias and consider a decision maker who correctly anticipates his/her future preference by considering the effect of habits, while he/she is also tempted to ignore such a habit formation. The decision maker exerts self-control to resist such a self-deception. As in Noor (2007), their decision maker is also tempted to the entire consumption streams.

2 | THE SETTING AND THE DECISION MODEL

While the standard approach in preference aggregation problems is that both individuals' preferences and a social welfare objective belong to the same model of decision making, we consider here that the social planner has a self-control/temptation issue while individuals do not. As detailed in the introduction, we do this for the purpose of isolating a self-control

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problem that arises specifically for a benevolent social planner because of disagreements between individuals.[5]

Time is discrete and infinite. Let C be the set of social outcomes per period, which is assumed to be a compact metric space. Given a compact metric space Y, let $\Delta(Y)$ be the set of lotteries (Borel probability measures) over Y, which is a compact metric space with respect to the Prokhorov metric and is also a convex set of a linear space. Given a compact metric space Y, let $\mathcal{K}(Y)$ be the set of compact subsets of L, which is a compact metric space with respect to the Hausdorff metric.

We consider the domain of infinite-horizon choice problems $\mathcal{Z}$, which satisfies the recursive formula[6]

$$\mathcal{Z} \simeq \mathcal{K}(\Delta(C) \times \mathcal{Z}).$$

A generic element is denoted by $z \in \mathcal{Z}$ and called a menu. Note that $\mathcal{Z}$ includes $\Delta(C)^\infty$, the subdomain of sequences of lotteries over social outcomes. Given $l \in \Delta(C)^\infty$, let $\{l\}$ denote the choice problem in which there is no choice other than committing to $\{l\}$ once and for all.[7]

Let $I = \{1, \ldots, |I|\}$ be the set of individuals. For each individual $i \in I$, let $\succsim_i$ denote his/her preference over $\Delta(C)^\infty$, which is represented in the exponential discounted utility form

$$U_i(l) = \sum_{t=1}^{\infty} u_i(l_t)\beta_i^{t-1},$$

where $u_i : \Delta(C) \to \mathbb{R}$ is a continuous and mixture-linear function and $\beta_i \in (0, 1)$ is a discount factor.

2.1 | DSC preference

We assume that the social planner has a preference $\succsim_0$ over menus $\mathcal{Z}$, which follows the DSC model due to Gul and Pesendorfer (2004): There exist continuous expected utility functions $u_0, v_0 : \Delta(C) \to \mathbb{R}$ (u is nonconstant), a discount factor $\beta_0 \in (0, 1)$, and a parameter $\kappa_0 > 0$ such that $\succsim_0$ is represented by

[5]There are some studies in aggregation problems that take different decision models between individuals and the social planner. Epstein and Segal (1992) consider that the social objective follows the model of quadratic expected utility, which violates the independence axiom, while the individuals' preferences follow the standard expected utility theory. This is for the purpose of isolating desirability of strict randomization at a social level due to fairness concerns. Alon and Gayer (2016) consider that the social objective under subjective uncertainty should exhibit ambiguity aversion even when individuals' beliefs are unambiguous. This is because people disagree in beliefs and we have no clue about who is right, and a cautious social planner should allow for a range of worst-case scenarios.

[6]This domain is smaller than the one established in Gul and Pesendorfer (2004) say $\mathcal{Z}^*$, which satisfies the recursive formula $\mathcal{Z}^* \simeq \mathcal{K}(\Delta(C \times \mathcal{Z}^*))$. It includes the domain of probability trees (Chew & Epstein, 1991; Epstein & Zin, 1989) satisfying the recursive formula $\mathcal{D} \simeq \Delta(C \times \mathcal{D})$. We adopt the current formulation for simplicity of presentation, however, because compound lotteries are not playing any role here. The recursive formula $\mathcal{Z} \simeq \mathcal{K}(\Delta(C) \times \mathcal{Z})$ can be shown directly or by taking a suitable restriction of $\mathcal{Z}^* \simeq \mathcal{K}(\Delta(C \times \mathcal{Z}^*))$.

[7]It is a shorthand and imprecise notation of $\{(l_1, \{(l_2, \{(l_3, \{(l_4, \ldots)\})\})\})\}$, which actually has to involve infinitely many brackets and parentheses.

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$$W_0(z) = \max_{(m,z')\in z} \{U_0(m,z') - \kappa_0(\max_{(n,y')\in z} V_0(n,y') - V_0(m,z'))\}, \quad (1)$$

where

$$U_0(m,z') = u_0(m) + \beta_0 W_0(z'), \quad (2)$$

$$V_0(m,z') = v_0(m). \quad (3)$$

A basic structure of the DSC representation W_0 is the same as the static self-control representation of Gul and Pesendorfer (2001). Two component functions $U_0(m,z')$ and $V_0(m,z')$ are interpreted as normative and temptation utility functions, respectively. From (3), the temptation utility function $V_0(m,z')$ depends only on the current consumption m, which captures the social planner's present bias. The nonnegative term $(\max_{(n,y')\in z} V_0(n,y') - V_0(m,z'))$ is regarded as self-control costs, which are opportunity costs in terms of temptation utilities.

From (1) and (2), W_0 admits a recursive form, and hence, the DSC representation satisfies dynamic consistency. If v_0 is constant, that is, the social planner does not have self-control costs, then W_0 is reduced to a standard stationary discounted utility.

Note also that when we restrict attention to commitment plans $\{l\}$, (1) reduces to the standard discounted expected utility model, that is,

$$W_0(\{l\}) = \sum_{t=1}^{\infty} \beta_0^{t-1} u_0(l_t).$$

The DSC model (1) suggests which element in z is chosen in the ex-post stage. The choice from z should achieve the value function $W_0(z)$, which is equivalent to a maximizer for the problem $\max_{(m,z')\in z}\{u_0(m) + \kappa_0 v_0(m) + \beta_0 W_0(z')\}$. Hence, self-control from temptation will result in a compromise choice between the normative utility and the temptation utility.

The parameter κ_0 captures the intensity of temptation. From (1), we can see that a higher κ_0 implies greater self-control costs, which yields a smaller value of utility representation $W_0(z)$ for all menus z. A behavioral implication is that assuming the other parameters identical, the DSC representation with κ_0^1, denoted by W_0^1, exhibits more desires for commitment than that with $\kappa_0^2 < \kappa_0^1$, denoted by W_0^2. That is, for all $l \in \Delta(C)^\infty$ and all menus z,[8]

$$W_0^2(\{l\}) \geq W_0^2(z) \Rightarrow W_0^1(\{l\}) \geq W_0^1(z).$$

This condition states that whenever the representation with κ_0^2 prefers a commitment to $\{l\}$, the representation with a higher parameter κ_0^1 does so.

Note also that if κ_0 is extremely higher, self-control costs become prohibitively greater. Then, the social planner is likely to yield to the temptation, and a choice from menus tends to be governed by V_0. At the limit, the DSC representation converges to the Strotz representation (Strotz, 1955) as $\kappa_0 \to \infty$.

[8]Since $W_0^1(\{l\}) = W_0^2(\{l\})$ for all $l \in \Delta(C)^\infty$, $W_0^2(\{l\}) \geq W_0^2(z)$ implies $W_0^1(\{l\}) = W_0^2(\{l\}) \geq W_0^2(z) \geq W_0^1(z)$, as desired.

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2.2 | Choice implications of the DSC model

Gul and Pesendorfer (2004) provide an axiomatic foundation for the DSC model defined over a larger domain $\mathcal{Z}^* \simeq \mathcal{K}(\Delta(C \times \mathcal{Z}^*))$. This subsection clarifies choice implications of the DSC model on our parsimonious domain $\mathcal{Z} \subset \mathcal{Z}^*$, where compound lotteries and the correlation between the current consumption and the continuation menus are ignored.[9]

Order: $\succsim_0$ is complete and transitive.
Continuity: $\succsim_0$ is continuous with respect to the Hausdorff metric.
Independence[10]: For all $x, y, z \in \mathcal{Z}$ and $\lambda \in (0, 1)$, $x \succ_0 y$ implies $\lambda x + (1-\lambda)z \succ_0 \lambda y + (1-\lambda)z$.

Order, Continuity and Independence are self-explanatory or technical. Thus, we omit the explanation of them.

Set-Betweenness: For all $x, y \in \mathcal{Z}$, if $x \succsim_0 y$ then $x \succsim_0 x \cup y \succsim_0 y$.

Set-Betweenness states that adding an inferior choice option cannot make the better option even better while the combined menu cannot be worse than the inferior option itself either. In particular, the axiom allows for preference for commitment $x \succ_0 x \cup y$ to avoid temptation from an alternative in menu y. Although the source of temptation has not been specified at this stage, we have in mind the situation, as mentioned in the introduction, where the social planner, when selecting an alternative from a menu, is affected by the conflict among individuals: Despite the social planner wants to choose an alternative that considers some individuals due to normative reasons, the planner may be tempted to choose another that others prefer more. As a result, the planner does not prefer an abundance of choice opportunities, but rather, may strictly prefer to have fewer choice opportunities.

Stationarity: For all $c \in C$ and $x, y \in \mathcal{Z}$, $x \succsim_0 y$ if and only if $\{(c, x)\} \succsim_0 \{(c, y)\}$.

In the statement, $\{(c, x)\} \succsim_0 \{(c, y)\}$ means the ranking between x and y one period ahead. Thus, Stationarity requires that the ranking between menus x and y does not change over time.

Temptation by Immediate Consumption: For all $m, n \in \Delta(C)$ and x, y, z if $\{(m, x)\} \succ_0 \{(m, x), (n, y)\} \succ_0 \{(n, y)\}$ and $\{(m, x)\} \succ_0 \{(m, x), (n, z)\} \succ_0 \{(n, z)\}$ imply $\{(m, x), (n, y)\} \sim_0 \{(m, x), (n, z)\}$.

The presumption of the axiom implies that both (n, y) and (n, z) are more tempting than (m, x), while the social planner exercises self-control. Then, the axiom requires that the social planner is indifferent between $\{(m, x), (n, y)\}$ and $\{(m, x), (n, z)\}$, which suggests that the continuation menus

[9]On our subdomain, Gul and Pesendorfer (2004)'s two axioms, separability and indifference to timing do not play any role. The former requires that the decision maker only cares about the marginal distributions over the current consumption and the continuation menus respectively, and the latter requires that the decision maker does not care about timing of resolution of risk.

[10]For any two menus z, z', and $\alpha \in [0, 1]$, the mixture operation on $\mathcal{Z}^*$ is defined as $\lambda z + (1-\lambda)z' = \{\lambda l + (1-\lambda) l' \mid l \in z, l' \in z'\}$. The mixture operation on $\mathcal{Z}$ should be understood to be its restriction on $\mathcal{Z}$ together with the assumption that randomization over menus, $\lambda \circ x + (1-\lambda) \circ y$, is identified with the menu $\lambda x + (1-\lambda)y$.

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y and z have no impact on the ranking. Hence, the Temptation by Immediate Consumption axiom requires that temptation comes from only immediate consumption. In Section 6, we will consider an alternative model without this assumption.

3 | THE AXIOMS ON AGGREGATION

This section describes the normative axioms that the individuals' discount utility functions and the social planner's self-control utility function should satisfy, as well as the descriptive axioms for characterizing the social planner's temptation.

Throughout, we assume the following Richness Condition. Given $m \in \Delta(C)$, let $m^\infty \in \Delta(C)$ denote the constant sequence of m.

Richness Condition There exists $\underline{m} \in \Delta(C)$ such that for all $i \in I$, there exists $m_i \in \Delta(C)$ with $m_i^\infty \succ_i \underline{m}^\infty$ and $m_i^\infty \sim_h \underline{m}^\infty$ for all $h \neq i$.

Note that the condition implies $\left(\sum_{h \in I} \frac{1}{|I|} m_h\right)^\infty \succ_i \underline{m}^\infty$ holds for all $i \in I$. Richness Condition technically states that the utility possibility set defined over per-period outcomes is full-dimensional and spanned by linearly independent vectors. In particular, this property excludes the possibility that all individuals have the same preference on $\Delta(C)$.

The condition is natural in the context of allocating consumption in the dynamic general equilibrium environment in which each individual has a selfish preference over his/her own private consumption. We can take $\underline{m}$ to be the lottery that is degenerated to the allocation which gives everybody zero consumption. For each $i \in I$, we can take m_i to be the lottery that is degenerated to an allocation that gives positive consumption only to i.

Note that the condition puts no restriction on preferences over private consumption lotteries once we restrict attention to selfish ones, hence it does not exclude the case that everybody has identical taste over private consumption. For example, suppose that c denotes the consumption allocation across n individuals, that is, $c = (c_1, \dots, c_i, \dots, c_n)$, where $c_i \in [0, \bar{a}]$. Then, we may assume that individual i cares only about his/her own consumption c_i, that is, $u_i(c) = u_i(c_i)$. Hence, individuals do not have the same preference over consumption allocations C, but they may well have the same tastes on his/her own consumption, say $u_i(c_i) = \log c_i$.

3.1 | Pareto conditions

First, we present the most permissive Pareto condition in which no intertemporal trade-offs are involved.

Constant Commitment Pareto: For all $m, n \in \Delta(C)$, if $m^\infty \succ_i n^\infty$ for all $i \in I$, then $\{m^\infty\} \succ_0 \{n^\infty\}$.

Since comparisons between m^∞ and n^∞ do not involve intertemporal concerns, the axiom imposes the Pareto condition on static risk preferences.

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The next is the Pareto condition stating that if everybody prefers one stream of social outcomes over another, so should the society. To emphasize that the condition is only about preference over commitment plans, we call it *Commitment Pareto*.

Commitment Pareto: For all $l, l' \in \Delta(C)^\infty$, if $l \succ_i l'$ for all $i \in I$, then $\{l\} \succ_0 \{l'\}$.

When the social planner's decision model is a traditional one in which no self-control issue arises, Commitment Pareto simply implies that the resulting optimization solution is Pareto-efficient in *any* choice problem, since the decision model does not distinguish between commitment problems and noncommitment problems. More formally, for each menu z, say that $l = (l_1, l_2, ...) \in \Delta(C)^\infty$ is feasible in z if there exists a sequence of menus $(z_t)_{t=1}^\infty$ such that $(l_1, z_1) \in z$ and $(l_t, z_t) \in z_{t-1}$ for all $t \geq 2$. A traditional way to evaluate menus, called strategic rationality in Kreps (1979), requires that each menu z admits a feasible stream $l^z \in z$ such that $\{l^z\} \succsim \{l\}$ for all feasible streams $l \in z$ and

$$z \sim_0 \{l^z\}. \tag{4}$$

That is, each menu z is evaluated according to its best element in terms of the commitment ranking among all feasible streams in z. Under this criterion, given any recursive menu $z \in \mathcal{Z}$, an optimal stream in terms of the commitment preferences will be chosen through the sequential decision making. Thus, if the commitment preference respects the Pareto condition, for any recursive menu, a stream eventually chosen through the sequential decision is Pareto efficient.

However, a self-control preference is not consistent with the strategic rationality. Indeed, it is easy to see that condition (4) implies that for all menus z and z',

$$z \succsim_0 z' \;\Rightarrow\; z \sim_0 z \cup z',$$

which is not consistent with either preference for commitment such as $z \succ_0 z \cup z'$ or a self-control preference. Consequently, even under Commitment Pareto, the social planner's choice from a menu may not be Pareto efficient. For example, consider the best stream $l^z = (l_1^z, l_2^z, ...)$ among all feasible steams in a menu z in terms of commitment preference. Let z_1 be a continuation menu after the current outcome l_1^z, which is necessary to choose the stream l^z. If the social planner is tempted from the immediate consumption, for instance, the planner may choose some different $(l_1', z_1') \in z$ over (l_1^z, z_1) when the current outcome l_1' is more tempting than l_1^z. Thus, l^z is not chosen from z.

Although Commitment Pareto appears to be an identical statement to the existing Pareto condition, it is indeed a weaker requirement in the sense that it requires the Pareto condition only on the commitment preference but not on the ex-post choice from menus. The social planner's choice may end up with a Pareto-inferior stream at the end of sequential decision making.

Since Commitment Pareto is a weaker requirement imposed only on the commitment preference, one might think of extending the Pareto condition to the entire domain of choice problems without commitment. Since each individual has a preference $\succsim_i$ over consumption streams, we first extend it to the corresponding preference $\succsim_i^*$ over menus $\mathcal{Z}$ by the procedure

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of strategic rationality. As in (4), for any menu z, let l^z be the best stream in terms of $\succsim_i$ within all feasible streams in z. Then, $\succsim_i^*$ over menus is defined as

$$z \succsim_i^* z' \Leftrightarrow l^z \succsim_i l^{z'}.$$

An attempt to extend a Pareto condition to the whole domain of menus is as follows:

Noncommitment Pareto: For all $z, z' \in \mathcal{Z}$, if $z \succ_i^* z'$ for all $i \in I$, then $z \succ_0 z'$.

By definition, noncommitment Pareto is stronger than Commitment Pareto.

As mentioned in the introduction, the evaluation of menus via strategic rationality may not be valid in terms of the individual level when menus consist of social outcomes. Unlike the evaluation of consumption streams, the evaluation of menus crucially depends on beliefs about in what way options will be chosen from menus. Evaluating a menu of social outcomes via strategic rationality implies that the individual has an optimistic belief that his/her best option in the menu will be always a consequence of social choice. Moreover, such too optimistic beliefs are in general mutually incompatible across individuals.

For an illustration, consider two individuals A and B and three alternatives l, l', l'' such that $l \succ_A l'' \succ_A l'$ and $l' \succ_B l'' \succ_B l$. Under strategic rationality, since $\{l, l'\} \succ_A^* \{l''\}$ and $\{l, l'\} \succ_B^* \{l''\}$, noncommitment Pareto concludes $\{l, l'\} \succ_0 \{l''\}$. This must involve a delusion, since the reason why A prefers $\{l, l'\}$ is that A hopes l is chosen ex-post and the reason why B prefers $\{l, l'\}$ is that B hopes l' is chosen ex-post, and one of them must be wrong eventually.[11]

In fact, the claim below shows that noncommitment Pareto leads to dictatorship unless the planner exhibits preferences for flexibility based on mutually contradicting reasons like the above.

Note that the above discussion, as well as the result leading to dictatorship, is not confined to recursive menus, but also holds for general menus. To present the dictatorship result within a more general setting, we temporarily introduce the following notation and axioms, which will be in use until the end of this subsection.

Let X be a compact metric space in which a mixture operation is defined. Let $\mathcal{K}(X)$ be the set of compact subsets of X endowed with the Hausdorff metric, in which the corresponding set-mixture operation is defined.

Let $\{\succsim_i\}_{i \in I}$ be individual preferences over X which satisfy completeness, transitivity, continuity, and mixture independence: $a \succsim_i b$ implies $\lambda a + (1-\lambda)c \succsim_i \lambda b + (1-\lambda)c$ for all $a, b, c \in X$ and $\lambda \in (0, 1)$.

For each $i \in I$, let $\succsim_i^*$ be the extension of $\succsim_i$ to $\mathcal{K}(X)$ defined as $A \sim_i^* \{a^*\}$ for all $A \in \mathcal{K}(X)$, where $a^* \in A$ is an outcome satisfying $a^* \succsim_i a$ for all $a \in A$.

Let $\succsim_0$ be the social ranking over $\mathcal{K}(X)$ which satisfies completeness, transitivity and continuity. We consider the following three axioms:

Set-Mixture Independence: $A \succsim_0 B$ implies $\lambda A + (1-\lambda)C \succsim_0 \lambda B + (1-\lambda)C$ for all $A, B, C \in \mathcal{K}(X)$ and $\lambda \in (0, 1)$;

[11]This argument against strategic rationality on the social choice environment and noncommitment Pareto is similar to the critique to the Pareto condition in choice under uncertainty, called spurious unanimity. See Mongin (1995) and Gilboa et al. (2004) for the impossibility of Paretian aggregation under heterogeneous beliefs and weakening of the Pareto condition, and also Mongin (2016) for philosophical arguments on the idea of spurious unanimity.

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Weak Set-Betweenness: $A \sim_0 B$ implies $A \cup B \sim_0 A \sim_0 B$ for all $A, B \in \mathcal{K}(X)$.
Weak Set-Pareto: For all $A, B \in \mathcal{K}(X)$, if $A \succ_i^* B$ for all $i \in I$, then $A \succ_0 B$.

Proposition 1. *Assume that there exist $\underline{a} \in X$ such that for all $i \in I$ there exists $a_i \in X$ with $a_i \succ_i \underline{a}$ and $a_i \sim_h \underline{a}$ for all $h \neq i$. Then, $\succsim_0$ satisfies Set-Mixture Independence, Weak Set-Betweenness and Weak Set-Pareto if and only if there exists $i^* \in I$ such that $\succsim_0 = \succsim_{i^*}^*$.*

In the proof (Section A.1), we show that the conditions and axioms except for Weak Set-Betweenness imply a nonnegative additive aggregation of individuals' utility functions over menus having the form of strategic rationality. This welfare function has the same form of representations consistent with preference for flexibility (Dekel et al., 2001; Kreps, 1979). Weak Set-Betweenness is generally incompatible with preference for flexibility and does not allow placing positive weights on multiple individuals. Therefore, by adding this axiom, the weight vector is degenerate on one individual, leading to the dictatorship. Since the self-control preference satisfies an even stronger axiom, that is, Set-Betweenness, Proposition 1 implies that the dictatorship is the only possibility in our setting if noncommitment Pareto is assumed.

3.2 | Disagreement and temptation

We introduce axioms regarding the temptation of the social planner. As we consider the benevolent social planner, the only source of temptation here is disagreement among individuals. Based on such a hypothesis, we introduce two axioms.

The first axiom state that a Pareto-inferior item should not be tempting for the planner. Such a condition is reasonable given the scope of this paper.

No Temptation from Pareto Dominated Outcomes (NTPDO): For all $m, n \in \Delta(C)$, if $m^\infty \succ_i n^\infty$ for all $i \in I$ and $\{m^\infty\} \succ_0 \{n^\infty\}$, then $\{m^\infty\} \sim_0 \{m^\infty, n^\infty\}$.

The presumption of NTPDO states that all individuals prefer a lottery m to n, and so does the social planner (presumably because of the Pareto condition). Since there is no conflict among individuals' opinions, the planner can choose a normatively preferred m^∞ from $\{m^\infty, n^\infty\}$ without any temptation.

The second axiom states that the benevolent social planner should be indeed tempted to accommodate majority when there are disagreements.

Temptation to Accommodate Majority (TAM): For all $i \in I$, for all $m, n \in \Delta(C)$, if $m^\infty \succ_i n^\infty$, $m^\infty \prec_h n^\infty$ for all $h \neq i$ and $\{m^\infty\} \succ_0 \{n^\infty\}$, then $\{m^\infty\} \succ_0 \{m^\infty, n^\infty\}$.

The presumption of the axiom states that all but one individual prefer n^∞ over m^∞ but the planner ranks m^∞ over n^∞ for some reason. The conclusion states that then the planner prefers to avoid an opportunity to choose between m^∞ and n^∞ ex-post, because he/she expects to get tempted to choose n^∞ over m^∞ against his/her commitment preference.

4 | AGGREGATION CHARACTERIZATION

4.1 | Aggregation of per-period-outcome preferences

Let us start with the most permissive type of aggregation, in which no time preference is involved. That is, we restrict attention to preferences over constant streams.

Proposition 2. *The profile $\succsim_0$ and $\{ \succsim_i \}_{i \in I}$ satisfy Constant Commitment Pareto if and only if there exists $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ such that*

$$u_0 = \sum_i \tilde{\alpha}_h u_h.$$

Since Constant Commitment Pareto is interpreted as the Pareto condition on static lotteries, Proposition 2 is the same as Harsanyi's aggregation theorem.

Next, we impose axioms regarding the social planner's temptation and characterize their implications on the temptation risk preference.

Theorem 1. *The profile $\succsim_0$ and $\{\succsim_i\}_{i \in I}$ satisfy Constant Commitment Pareto and NTPDO if and only if there exist $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|}$ such that*

$$u_0 = \sum_h \tilde{\alpha}_h u_h, \text{ and } \quad v_0 = \sum_h \alpha_h u_h.$$

Moreover, $\succsim_0$ and $\{\succsim_i\}_{i \in I}$ satisfy TAM in addition if and only if $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\tilde{\alpha}_h \alpha_h = 0$ for all h.

Theorem 1 shows that an implication of NTPDO is to deliver a nonnegative additive aggregation of individuals' utility functions also for the temptation risk preference. Since NTPDO means that the temptation risk preference respects the Pareto condition, again, by the Harsanyi's aggregation-type argument, it must be written as a nonnegative (possibly zero vector) additive aggregation of individuals' utility functions.

Moreover, under TAM, (α_h) is ensured to be nonzero, which implies that v_0 is not constant. A more important characterization of TAM is the complementarity between the welfare weights associated with normative and temptation utilities. Individuals evaluated with positive weights in the normative utility do not have positive welfare weights in terms of the temptation utility. Thus, under TAM, the population of individuals can be divided into three categories: individuals who are evaluated only through the normative utility, individuals who are evaluated only through the temptation utility, and individuals who are evaluated in neither.

The intuition of the sufficiency of TAM is as follows: If the normative utility is written as a weighted sum of individuals' utilities, we can always find alternatives such that only one individual has the opposite ranking, as in the premise of TAM, but normative utility favors that one individual. Indeed, according to Richness Condition, by choosing, for each individual, two lotteries such that only that individual has a strict ranking and the other individuals are indifferent, and by properly randomizing over them, we can find two alternatives such that only one individual has the opposite ranking from the other and the utility difference between these alternatives for the other individuals

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is infinitely small. Since the improvement of the rest of the individuals can be infinitely close to zero, the opinions of them will have no impact on the normative utility under the additive aggregation. Nevertheless, TAM requires that the social planner is tempted by such an alternative, which leads to the property that the temptation utility is not constant and that individuals with positive weights in the normative utility must have zero weights in the temptation utility.

4.2 | Consideration of time preferences

Let us now see the results incorporating individuals' time preferences. In this subsection, we will state two aggregation results where the planner satisfies Pareto condition not only on constant consumption paths but also on all consumption paths.

First, we consider the case where all individuals agree on time preference.

Proposition 3. *Assume $\beta_i = \beta$ for all i. Then, the profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDO, and TAM if and only if there exist $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|}\setminus\{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|}\setminus\{0\}$ satisfying $\tilde{\alpha}_h\alpha_h = 0$ for all h such that*

$$u_0 = \sum_h \tilde{\alpha}_h u_h, \beta_0 = \beta, \text{ and } \quad v_0 = \sum_h \alpha_h u_h.$$

Since Commitment Pareto implies Constant Commitment Pareto, the aggregation result about risk preferences is obtained as a corollary of Theorem 1. Proposition 3 states that when Commitment Pareto is required, rather than just Constant Commitment Pareto, the discount factor of the benevolent social planner's normative utility function aligns with the one agreed upon among individuals.

Next, we consider the case where individuals disagree about intertemporal trade-offs. The following result states that in this case, the planner's normative utility function must coincide with a specific individual's utility function. Additionally, the planner's temptation utility function aggregates the others' per-period tastes. From now on, we will refer to the individual whose preference coincides with the planner's normative one as the "commitment dictator."

Theorem 2. *Assume $\beta_i \neq \beta_j$ for all i and j. Then, the profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDO, and TAM if and only if there exist $i^* \in I, \tilde{\alpha}_{i^*} > 0$, and $(\alpha_h) \in \mathbb{R}_+^{|I|-1}\setminus\{0\}$ such that*

$$u_0 = \tilde{\alpha}_{i^*} u_{i^*}, \beta_0 = \beta_{i^*}, \text{ and } \quad v_0 = \sum_{h\neq i^*} \alpha_h u_h.$$

The intuition of the result is simple. The existence of the commitment dictator follows from the dictatorship result under Commitment Pareto as in Zuber (2011) and Jackson and Yariv (2015). Then, by Theorem 1, the planner's temptation utility function can aggregate all individuals' per-period tastes except for the commitment dictator.

Our result deviates from the impossibility theorem of Zuber (2011) and Jackson and Yariv (2015). By Theorem 2, we have clarified that the planner following the DSC model, which requires stationarity, can incorporate everyone's per-period tastes via temptation, even though having a dictatorial normative preference.

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However, a social planner's preference still becomes dictatorial if he/she can commit to a specific path. Furthermore, it may lead to an inefficient path due to the behavioral aspect of the DSC model. Using specific resource allocation problems, we will examine in the next section how the planner's value function might lead to such undesired outcomes, as well as how it could take care of distributional equity compared to the conventional value functions imposing efficiency.

5 | IMPLICATIONS TO RESOURCE ALLOCATION

Here we present positive implications of the social welfare functions as characterized in Section 4.2 to resource allocation problems. We look at two dynamic resource allocation problems, one is in which a resource amount is fixed at each period and there is no intertemporal trade-off; the other is an optimal growth problem in which resource is saved and reproduced over time.

In these problems, the standard Pareto efficiency condition has a sharp implication under heterogeneous discounting: only the most patient individual receives positive consumption in the long-run and all the others' consumption paths converge to zero (Becker, 1980).

Even under heterogeneous discounting among individuals, below we obtain solutions to the social planner's problem in which multiple individuals receive positive consumption amounts in the long-run by sacrificing Pareto-efficiency. We see this feature is similar to the one obtained from stationary social welfare functions which violate the Pareto condition (Hayashi & Lombardi, 2021), in the sense that both respond to distributive concerns by sacrificing intertemporal efficiency. The mechanism of doing so is different, however. In the Hayashi-Lombardi model, the distributive property is delivered directly by violating Commitment Pareto. On the other hand, in the current model, impure benevolence allows the planner to deliver the distributive property through ex-post choice under no commitment, even though Commitment Pareto is imposed.

5.1 | Fixed resource

Assume that there are e units of a single good at every time period. This initial resource will be allocated to the consumers in the economy. The set of per-period social outcomes is the set of allocations across consumers and taken as $C = \mathbb{R}^n_+$. Each individual cares only about his/her own consumption, that is, $u_i(c) = u_i(c_i)$. There is no technology to carry over a resource to the next period. Thus, the social planner faces the same menu every period, which is given as

$$z(e) = \left\{ (c, z(e)) \,\middle|\, \sum_{i \in I} c_i = e \right\}.$$

Since the continuation menu for the next period onward is always equal to $z(e)$, the social planner is faced with a static problem such as determining only consumption allocation for each period.

The planner's value function, denoted by $W_0(z)$, is written as

$$W_0(z(e)) = \max_{c \in z(e)} \left\{ \tilde{\alpha}_{i^*} u_{i^*}(c_{i^*}) + \kappa_0 \sum_{h \neq i^*} \alpha_h u_h(c_h) + \beta_{i^*} W_0(z(e)) \right\} - \kappa_0 \max_{c' \in z(e)} \sum_{h \neq i^*} \alpha_h u_h(c'_h).$$

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Since e stays constant over time, $W_0(z(e))$ is constant over time. Since there is effectively no intertemporal choice in this setting, the solution is to provide a time-constant sequence. It is the solution to $\max_{c \in z(e)} \left\{ \tilde{\alpha}_{i^*} u_{i^*}(c_{i^*}) + \kappa_0 \sum_{h \neq i^*} \alpha_h u_h(c_h) \right\}$. Thus, this impure social planner chooses a consumption allocation that maximizes a weighted sum of all individuals' per-period utility functions, and, consequently, behaves like a benevolent social planner.

Such a benevolent allocation is not obtained for free. It is worth emphasizing that a consumption stream chosen by the impure benevolent social planner is generally not Pareto efficient. As suggested in Sections 3.1 and 3.2, the impure benevolent social planner who respects Commitment Pareto and NTPDO will choose a Pareto efficient allocation from menus of streams, but not necessarily from other recursive menus. For an illustration, suppose there are only two consumers, $i = 1, 2$, in the economy. Assume that their per-period utilities are given by $u_1(a) = u_2(a) = \log a$. If consumer 1 is a commitment dictator and welfare of consumer 2 is counted in the temptation utility at each period, the planner chooses

$$\arg\max_{c \in z(e)} \{\tilde{\alpha}_1 \log c_1 + \kappa_0 \alpha_2 \log c_2\} = \left(\frac{\tilde{\alpha}_1}{\tilde{\alpha}_1 + \kappa_0 \alpha_2}, \frac{\kappa_0 \alpha_2}{\tilde{\alpha}_1 + \kappa_0 \alpha_2} \right), \tag{5}$$

and so the constant stream $(c_{1t}, c_{2t}) = \left(\frac{\tilde{\alpha}_1}{\tilde{\alpha}_1 + \kappa_0 \alpha_2}, \frac{\kappa_0 \alpha_2}{\tilde{\alpha}_1 + \kappa_0 \alpha_2} \right)$, $t = 1, 2, \ldots$, is obtained as a social outcome. However, as shown by Becker (1980), if consumers have heterogeneous discount factors, in a Pareto efficient allocation, the most patient consumer will consume all resources in the long run. Hence, if $\beta_1 \neq \beta_2$, a constant stream allocating a positive consumption to all consumers, such as (5), is not Pareto efficient.[12]

5.2 | Optimal growth

Unlike the previous sub-section, which ignores the intertemporal aspect of decision making, this sub-section considers a situation in which capital accumulation is possible. Capital accumulation can be interpreted as the choice of a continuation menu from the next period onward. In choosing from a given menu, benevolent choices are made as a result of a compromise between the normative and temptation utilities, but an impure social planner avoids temptation from the other group's opinion, so the subsequent menu choice tends to be distorted.

We incorporate a production technology into the setting of Section 5.1. There is one good at each period, which can be consumed or used as input for reproduction under strictly concave production function f. Let k_0 be the initial capital. Given the current capital amount k, the social planner faces the constraint

$$\sum_{i \in I} c_i + k' = f(k),$$

[12]Compared to an allocation, where resources are divided in a fixed proportion each period, a Pareto improvement is achieved through a trade such that a more impatient consumer consumes more in earlier periods and a more patient consumer consumes more in later periods.

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where k' denotes the capital amount to be carried over to the next period. Formally, given a current level of capital k, the corresponding menu is written as

$$z(k) = \left\{ (c, z(k')) \middle| \sum_{i \in I} c_i + k' = f(k) \right\}.$$

For notational simplicity, $W_0(z(k))$ is denoted as $W_0(k)$.

For simplicity, consider a symmetric situation that $u_i(c_i) = u(c_i)$ for all $i \in I$ and $\varkappa_0 = n - 1$, $\widetilde{\alpha}_{i^*} = 1$, $\alpha_i = 1/(n-1)$ for all $i \neq i^*$. Assume u is differentiable, strictly increasing, and concave.

Then the social planner's dynamic programming problem is formulated as

$$W_0(k) = \max_{\sum_{i \in I} c_i + k' = f(k)} \left\{ \sum_{i \in I} u(c_i) + \beta_{i^*} W_0(k') \right\} - \max_{\sum_{i \in I} c_i + k' = f(k)} \sum_{h \neq i^*} u(c_h).$$

Let $c_i(k)$ be the consumption function for each i. Then, the first-order condition for the ex-post choice is given by

$$u'(c_i(k)) - \beta_{i^*} W_0' \left(f(k) - \sum_{h=1}^{n} c_h(k) \right) = 0$$

for every $i \in I$. Hence it holds $c_i(k) = c(k)$ for all i, where c is seen as the per-individual consumption function.

On the other hand, since $\max_{\sum_{i \in I} c_i + k' = f(k)} \sum_{h \neq i^*} u(c_h) = (|I| - 1) u\left(\frac{f(k)}{|I| - 1}\right)$, we obtain

$$W_0(k) = |I| u(c(k)) + \beta_{i^*} W_0(f(k) - |I| c(k)) - (|I| - 1) u\left(\frac{f(k)}{|I| - 1}\right).$$

By taking the derivative of both sides of this and combining with the first-order condition, we obtain the envelope condition

$$W_0'(k) = \left[u'(c(k)) - u'\left(\frac{f(k)}{|I| - 1}\right) \right] f'(k).$$

By combining with the first-order condition again, we obtain the Euler equation

$$u'(c(k)) = \beta_{i^*} \left[u'(c(f(k) - |I| c(k))) - u'\left(\frac{f(f(k) - |I| c(k))}{|I| - 1}\right) \right] f'(f(k) - |I| c(k)).$$

Let $k^\star$ be the steady-state capital level for the above Euler equation. Then it must hold $f(k^\star) - |I| c(k^\star) = k^\star$ and

$$u'(c(k^\star)) = \beta_{i^*} \left[u'(c(k^\star)) - u'\left(\frac{f(k^\star)}{|I| - 1}\right) \right] f'(k^\star).$$

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Since $u' > 0$ and $f' > 0$, for the steady-state condition to be met, it is necessary that $c(k^\star) < \frac{f(k^\star)}{|I|-1}$ and $\beta_{i^*} f'(k^\star) > 1$ hold.

Let us compare the performance of the above growth solution with the one obtained from another objective function. Here we consider the objective function as characterized by Hayashi and Lombardi (2021), in which the planner maintains the stationary discounted utility model by adopting one individual's discount factor and taking weighted a sum of individuals' per-period utilities at each period. Again we take the symmetric sum of per-period utilities for simplicity and the “discounting dictator” is the same as the commitment dictator. More precisely, the social planner's dynamic programming problem is

$$\widetilde{W}_0(k) = \max_{\sum_{i\in I} c_i + k' = f(k)} \left\{ \sum_{i\in I} u(c_i) + \beta_{i^*} \widetilde{W}_0(k') \right\}.$$

As shown by Hayashi and Lombardi (2021), this social objective function violates Commitment Pareto, while it satisfies a weaker Pareto condition.

Let $\tilde{c}$ be the consumption function per individual, then the Euler equation is obtained as

$$u'(\tilde{c}(k)) = \beta_{i^*} u'(\tilde{c}(f(k) - |I|\tilde{c}(k))) f'(f(k) - |I|\tilde{c}(k)).$$

and the steady-state capital level $\widetilde{k}$ is given by

$$1 = \beta_{i^*} f'(\widetilde{k}).$$

Since $\beta_{i^*} f'(k^\star) > 1$ and f is strictly concave, we see that $k^\star < \widetilde{k}$. Also, since $(f(k) - k)' = f'(k) - 1 > \frac{1}{\beta_{i^*}} - 1 > 0$ for all $k \in (0, \widetilde{k}]$, we have

$$c(k^\star) = \frac{f(k^\star) - k^\star}{|I|} < \frac{f(\widetilde{k}) - \widetilde{k}}{|I|} = c(\widetilde{k}),$$

meaning that the consumption path converging to $c(k^\star)$ is strictly dominated by the path converging to $c(\widetilde{k})$ in the long-run.

Imposing Commitment Pareto and leaving distributive issues to the planner's impure benevolence results in low growth and low consumption path in the long-run, compared to the one obtained from an objective function that violates Commitment Pareto. This is because carrying over more resource to the future creates a larger self-control cost and the planner prefers to avoid it in the long-run.

Comparing streams of allocations chosen by the above two different objective functions, note that even though one may Pareto-dominate the other, starting from a sufficiently long time period onward, there may be no Pareto-dominance relationship between the two streams when considering the entire period, including the present and the near future. Indeed, the impure benevolent social planner is tempted by the majority's opinion to increase consumption in the present and tends to increase consumption in the short run at the expense of lower steady state in the long run.

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6 | TEMPTATION FROM FUTURE OPTIONS

One might be interested in taking individuals' time preferences into account in the planner's temptation utility as well. To address this interest, in this section, we replace the social planner's decision model from the DSC representation to one where the planner is also tempted by future consumption. We then explore how to aggregate individual preferences to satisfy the same axioms related to the Pareto condition and the cause of temptation as before.

6.1 | FT model

Here, we assume that the social planner has a preference $\succsim_0$ over menus $\mathcal{Z}$, which admits a FT utility representation (Noor, 2007): There exist continuous expected utility functions $u_0, v_0 : \Delta(C) \to \mathbb{R}$ (u is nonconstant), discount factors $\beta_0 \in (0, 1), \gamma_0 \in (0, 1)$, and a parameter $\kappa_0 > 0$, which captures the intensity of temptation, such that $\succsim_0$ is represented by $W_0(z)$ (as given by 1) with $U_0(m, z')$ (as given by 2) and

$$V_0(m, z') = v_0(m) + \gamma_0 \max_{(m', z'') \in z'} V_0(m', z''). \tag{6}$$

Similar to the DSC representation, a basic structure of the FT representation $W_0(z)$ is the same as the self-control representation of Gul and Pesendorfer (2001). From (1) and (2), W_0 admits a recursive form, and hence the FT representation satisfies dynamic consistency. Moreover, from (6), the temptation utility V_0 also has a stationary recursive form. Hence, the social planner is tempted not only by the current consumption, but also by the entire stream of consumption.

At the axiomatic level, the FT model shares common axioms with the DSC model provided in Section 2.2 except Temptation by Immediate Consumption.[13] It is replaced with the following axiom:

Temptation Stationarity: For all $c \in C$ and $x, y \in \mathcal{Z}, x \succ_0 x \cup y$ if and only if $\{(c, x)\} \succ_0 \{(c, x), (c, y)\}$.

The ranking $x \succ_0 x \cup y$ means preference for commitment to x, which implies that y contains an inferior but tempting alternative compared with x. Similarly, $\{(c, x)\} \succ_0 \{(c, x), (c, y)\}$ reveals that (c, y) is inferior but more tempting than (c, x). Since (c, x) and (c, y) are interpreted as menus x and y one period ahead, Temptation Stationarity states that temptation does not change over time. It is worth noting that Temptation Stationarity is logically disjoint to Temptation by Immediate Consumption.

[13]Noor (2007) adopts the domain $\mathcal{Z}^* \simeq \mathcal{K}(\Delta(C \times \mathcal{Z}^*))$ as in Gul and Pesendorfer (2004) and requires one more axiom, named *Indifference to Timing*, to characterize the FT representation. However, in our setting where the domain is smaller than $\mathcal{Z}^*$, this axiom does not play any role.

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6.2 | Aggregation characterization

Before stating the characterization results corresponding to Proposition 3 and Theorem 2, it should be noted that the same claim as Theorem 1 holds here. That is, the profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Constant Commitment Pareto and NTPDO if and only if there exist $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|}\setminus\{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|}$ such that

$$u_0 = \sum_h \widetilde{\alpha}_h u_h, \text{ and } \quad v_0 = \sum_h \alpha_h u_h.$$

Moreover, $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy TAM in addition if and only if $(\alpha_h) \in \mathbb{R}_+^{|I|}\setminus\{0\}$ with $\widetilde{\alpha}_h\alpha_h = 0$ for all h. We will use this fact as well when proving the characterization results described below.

It should also be emphasized that in the two characterization results, we require a stronger axiom than NTPDO, defined as follows.

No Temptation from Pareto Dominated Paths (NTPDP): For all $l, l' \in \Delta(C)^\infty$, if $l \succ_i l'$ for all $i \in I$ and $\{l\} \succ_0 \{l'\}$, then $\{l\} \sim_0 \{l, l'\}$.

Unlike NTPDO, this axiom involves intertemporal trade-offs. NTPDP states that a Pareto-inferior path, not necessarily a constant path, should not be tempting for the social planner. Now, let us state the two results.

First, we consider the case that all individuals agree on discounting.

Proposition 4. *Assume $\beta_i = \beta$ for all i. The profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDP, and TAM if and only if there exist $(\widetilde{\alpha}_h) \in \mathbb{R}_+^{|I|}\setminus\{0\}$ and $(\alpha_h) \in \mathbb{R}_+^{|I|}\setminus\{0\}$ with $\alpha_h\widetilde{\alpha}_h = 0$ for all h such that*

$$u_0 = \sum_h \widetilde{\alpha}_h u_h, v_0 = \sum_h \alpha_h u_h, \text{ and } \quad \beta_0 = \gamma_0 = \beta.$$

Since Commitment Pareto implies Constant Commitment Pareto and NTPDP implies NTPDO, the aggregation result about risk preferences is obtained as a corollary of the claim corresponding to Theorem 1. When there is no disagreement among individuals' time preferences, the benevolent social planner also adopts this common discount factor both as normative and temptation discount factors. In particular, the latter property is ensured by NTPDP. This result can be regarded as a "possibility" result because multiple individuals can have a positive welfare weight either via the normative or the temptation risk preference.

Next, we consider an aggregation problem where individuals disagree about intertemporal trade-offs. The following result states that there exist two kinds of dictators: one determines the commitment utility function, and the other determines the temptation utility function. The former is, as mentioned in Theorem 2, the commitment dictator. The latter, in correspondence, is referred to as the "temptation dictator."

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Theorem 3. *Assume $\beta_i \neq \beta_j$ for all i and j. The profile $\succsim_0$ and $\{\succsim_i\}_{i\in I}$ satisfy Commitment Pareto, NTPDP, and TAM if and only if there exist $i^*, j^* \in I$ with $i^* \neq j^*$ and $\tilde{\alpha}_{i^*} > 0$ and $\alpha_{j^*} > 0$ such that*

$$u_0 = \tilde{\alpha}_{i^*} u_{i^*}, \beta_0 = \beta_{i^*}, v_0 = \alpha_{j^*} u_{j^*}, \text{ and } \quad \gamma_0 = \beta_{j^*}.$$

The intuition about the roles of axioms is simple. The existence of the commitment dictator follows from the dictatorship result under Commitment Pareto as in Zuber (2011) and Jackson and Yariv (2015). Since NTPDP implies that the temptation discounted utility also respects the Pareto condition, a temptation dictator must exist by exactly the same argument as above. Finally, since TAM requires the complementarity between the normative and temptation utilities, the commitment dictator and the temptation dictator must be different.

Under disagreement about time preferences among individuals, Theorem 3 states that at most two individuals can have a positive welfare weight from the planner's viewpoint. Thus, in most economies where the population of individuals is more than three, this theorem is interpreted as an "impossibility".

Moreover, compared with Theorem 2, Theorem 3 reveals a trade-off. Specifically, in Theorem 2 where the social planner follows the DSC model, the planner's preference reflects only individuals' per-period tastes except for the commitment dictator. On the other hand, in Theorem 3 where the social planner follows the FT model, the planner's preference reflects multiple individuals' time preferences as well. However, the number of individuals whose preferences are reflected in the planner's preference is at most two. We need to make a choice between these two aggregation approaches.

7 | CONCLUSION

This paper has investigated how we can aggregate individuals' preferences when the planner faces the potential time-inconsistency problem due to present bias by adopting the decision model of temptation and self-control by Gul and Pesendorfer (2001, 2004).

Although the planner can take only one individual's preferences into account when making commitment choices, he may take other individual's preferences into account in the form of temptation utility (impure benevolence, namely) that affects ex-post choice without commitment. Thus, accommodating with majority's welfare is fulfilled, to some extent, in ex-post choice as a departure from commitment optimum. The DSC model allows such compromise in a recursive and dynamically consistent manner.

We have investigated the positive implications of the models in dynamic resource allocation problems. The solutions allow that all individuals receive positive consumption amounts in the long-run, which contrasts with the implication of the standard Pareto efficiency that only the most patient individual receives a positive consumption amount and all the others' consumption paths converge to zero. Impure benevolence allows the planner to respond to distributive concerns through ex-post choice under noncommitment.

We then investigated how the planner's temptation utility can take care of individuals' time preferences as well by adopting the model of FT by Noor (2007) as the decision model for the planner. Then, we show that there is a trade-off between the coverage of preference parameters and the coverage of individuals that can be taken into account in the form of temptation utility.

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When the planner's temptation utility takes individuals' time preferences into account, it can put positive weight only on one individual.

In the paper, we ruled out the possibility that the planner has a preference for flexibility (Dekel et al., 2001; Kreps, 1979), because it leads to support "spurious" unanimity arising due to different expectations about ex-post choice from a menu.[14] There may be a context, however, in which it is rather appealing to consider that the planner should have preference for flexibility, and this will be worth investigating.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no data sets were generated or analyzed during the current study.

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APPENDIX A: PROOFS

A.1 | Proof of Proposition 1

Necessity is obvious.

To show sufficiency, let $W_0 : \mathcal{K}(X) \to \mathbb{R}$ be a continuous and mixture-linear representation of $\succsim_0$, and let $W_i : \mathcal{K}(X) \to \mathbb{R}$ be a continuous and mixture-linear representation of $\succsim_i^*$, for each $i \in I$, which has the form $W_i(z) = \max_{a \in z} u_i(a)$ with $u_i : X \to \mathbb{R}$ being continuous and mixture-linear. Without loss, assume $u_i(\underline{a}) = 0$ for all $i \in I$.

Because the functions $W_0, W_1, \ldots, W_{|I|}$ are mixture-linear and it holds $W_i\left(\frac{1}{|I|}\sum_{h \in I} a_i\right) > W_i(\underline{a})$ for all $i = 1, \ldots, n$, we can apply a version of Harsanyi theorem (De Meyer & Mongin, 1995; Harsanyi, 1955) so that there is a vector $\alpha \in \mathbb{R}_+^{|I|} \setminus \{\mathbf{0}\}$ such that

$$W_0(z) = \sum_{i \in I} \alpha_i W_i(z)$$

holds for all $z \in \mathcal{Z}$.

We show that there are no two distinct i and j such that $\alpha_i > 0$ and $\alpha_j > 0$. Suppose there are. Without loss of generality, we can take $a_i, a_j \in X$ so that $\alpha_i u_i(a_i) = \alpha_j u_j(a_j) > 0$ and $u_i(a_j) = u_j(a_i) = 0$. Then it holds $W_0(\{a_i\}) = \alpha_i u_i(a_i)$ and $W_0(\{a_j\}) = \alpha_j u_j(a_j)$. On the other hand, it holds

$$W_0(\{a_i, a_j\}) = \alpha_i u_i(a_i) + \alpha_j u_j(a_j).$$

Thus we obtain

$$W_0(\{a_i, a_j\}) > W_0(\{a_i\}) = W_0(\{a_j\}),$$

which is a violation of Weak Set-Betweenness.

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A.2 | Proof of Proposition 2

Only-if part: Take any m and n such that $u_i(m) > u_i(n)$ for all i. This implies $m^\infty \succ_i n^\infty$. By Constant Commitment Pareto, $\{m^\infty\} \succ_0 \{n^\infty\}$. Thus, $W_0(\{m^\infty\}) > W_0(\{n^\infty\})$, or equivalently, $u_0(m) > u_0(n)$. By De Meyer and Mongin (1995, Proposition 2), there exists $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ such that $u_0 = \sum_h \tilde{\alpha}_h u_h$.

If part: Assume $u_0 = \sum_i \tilde{\alpha}_h u_h$ for some $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$. If $m^\infty \succ_i n^\infty$ for all $i \in I$, it implies $u_i(m) > u_i(n)$. Since we have $u_0(m) > u(n)$, $\{m^\infty\} \succ_0 \{n^\infty\}$, as desired.

A.3 | Proof of Theorem 1

Only-if part: By Constant Commitment Pareto and Proposition 2, there exists $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ such that $u_0 = \sum_i \tilde{\alpha}_h u_h$.

Next, we will claim that for all $m, n \in \Delta(C)$, if $u_i(m) \geq u_i(n)$ for all i, then $v_0(m) \geq v_0(n)$. By Richness Condition, for all i, there exists m_i such that $u_i(m_i) > u_i(\underline{m})$ and $u_j(m_i) = u_j(\underline{m})$ for all $j \neq i$. For $\gamma \in (0, 1)$, let $m_\gamma = \gamma \sum_h \frac{1}{|I|} m_i + (1 - \gamma) m$ and $n_\gamma = \gamma \underline{m} + (1 - \gamma) n$. Since $u_i(\sum_h \frac{1}{|I|} m_i) > u_i(\underline{m})$ for all i, $u_i(m_\gamma) > u_i(n_\gamma)$ for all i and γ. Since $u_0 = \sum \tilde{\alpha}_h u_h$, $u_0(m_\gamma) > u_0(n_\gamma)$. Now we have $m_\gamma^\infty \succ_i n_\gamma^\infty$ for all i and $\left\{m_\gamma^\infty\right\} \succ_0 \left\{n_\gamma^\infty\right\}$. By NTPDO, $\left\{m_\gamma^\infty\right\} \sim_0 \left\{m_\gamma^\infty, n_\gamma^\infty\right\}$. From the DSC representation of $\succsim_0$, $v_0(m_\gamma) \geq v_0(n_\gamma)$. Then, $v_0(m) \geq v_0(n)$ as $\gamma \to 0$.

By De Meyer and Mongin (1995, Proposition 1), there exists $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|}$ such that $v_0 = \sum_h \tilde{\alpha}_h u_h$.

Next, suppose that $\succsim_0$ and $\{\succsim_i\}$ satisfy TAM in addition. Since $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$, there exists at least one i with $\tilde{\alpha}_i > 0$. By Richness Condition, there exists m_i such that $u_i(m_i) > u_i(\underline{m})$ and $u_j(m_i) = u_j(\underline{m})$ for all $j \neq i$. For $\gamma \in (0, 1)$, let $m_\gamma = \gamma \sum_h \frac{1}{|I|} m_h + (1 - \gamma) \underline{m}$. Then, for all γ, $u_i(m_\gamma) < u_i(m_i)$ and $u_j(m_\gamma) > u_j(m_i)$ for all $j \neq i$. Since $u_0 = \sum \tilde{\alpha}_h u_h$, for all sufficiently small γ, $u_0(m_i) > u_0(m_\gamma)$. Therefore, for all such γ, we have $m_i^\infty \succ_i m_\gamma^\infty$, $m_\gamma^\infty \succ_j m_i^\infty$ for all $j \neq i$, and $\{m_i^\infty\} \succ_0 \{m_\gamma^\infty\}$. Then, by TAM, $\{m_i^\infty\} \succ_0 \{m_i^\infty, m_\gamma^\infty\}$, which implies $v_0(m_\gamma) > v_0(m_i)$. This means v_0 is not constant. Thus, there exists at least one j such that $\alpha_j > 0$.

Next, we show that if $\tilde{\alpha}_i > 0$, then $\alpha_i = 0$. Seeking a contradiction, suppose there exists i with $\tilde{\alpha}_i > 0$ and $\alpha_i > 0$. By the same argument as above, Richness assumption ensures that for all $\gamma \in (0, 1)$, $u_i(m_\gamma) < u_i(m_i)$ and $u_j(m_\gamma) > u_j(m_i)$ for all $j \neq i$. Moreover, for all sufficiently small γ, $u_0(m_i) > u_0(m_\gamma)$. Therefore, for all such γ, we have $m_i^\infty \succ_i m_\gamma^\infty$, $m_\gamma^\infty \succ_j m_i^\infty$ for all $j \neq i$, and $\{m_i^\infty\} \succ_0 \{m_\gamma^\infty\}$. Then, by TAM, $\{m_i^\infty\} \succ_0 \{m_i^\infty, m_\gamma^\infty\}$, which implies $v_0(m_\gamma) > v_0(m_i)$. Thus, we have $v_0(\underline{m}) \geq v_0(m_i)$ as $\gamma \to 0$. On the other hand, since $v_0 = \sum_h \alpha_h u_h$ with $\alpha_i > 0$,

$$v_0(m_i) = \sum_h \alpha_h u_h(m_i) > \sum_h \alpha_h u_h(\underline{m}) = v_0(\underline{m}),$$

which is a contradiction.

If part: Necessity of Constant Commitment Pareto comes from Proposition 2. For NTPDO, take any $m, n \in \Delta(C)$ such that $m^\infty \succ_i n^\infty$ for all $i \in I$ and $\{m^\infty\} \succ_0 \{n^\infty\}$. Since these conditions imply $u_i(m) > u_i(n)$ for all i, $u_0(m) > u_0(n)$ and $v_0(m) = \sum \alpha_h u_h(m) \geq \sum \alpha_h u_h(n) = v_0(n)$. Then,

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$$
\begin{aligned}
W_0(\{m^\infty, n^\infty\}) &= \max\{u_0(m) + v_0(m) + \beta_0 W_0(\{m^\infty\}), u_0(n) + v_0(n) + \beta_0 W_0(\{n^\infty\})\} \\
&\quad -\max\{v_0(m), v_0(n)\} \\
&= u_0(m) + v_0(m) + \beta_0 W_0(\{m^\infty\}) - v_0(m) \\
&= u_0(m) + \beta_0 W_0(\{m^\infty\}) \\
&= W_0(\{m^\infty\}),
\end{aligned}
$$

as desired.

To show TAM, take any $m, n \in \Delta(C)$ such that $m^\infty \succ_i n^\infty, n^\infty \succ_h m^\infty$ for all $h \neq i$, and $\{m^\infty\} \succ_0 \{n^\infty\}$. These conditions imply $u_i(m) > u_i(n), u_h(n) > u_h(m)$ for all $h \neq i$, and $u_0(m) > u_0(n)$. Since $u_0 = \sum_h \tilde{\alpha}_h u_h$, we must have $\tilde{\alpha}_i > 0$. By the assumption, $\alpha_i = 0$, which implies $v_0(n) = \sum_h \alpha_h u_h(n) > \sum_h \alpha_h u_h(m) = v_0(m)$. From the FT representation for $\succsim_0$, we have $\{m^\infty\} \succ_0 \{m^\infty, n^\infty\}$, as desired.

A.4 | Proof of Proposition 3

Only if part: Since Commitment Pareto implies Constant Commitment Pareto, Theorem 1 implies $u_0 = \sum_h \tilde{\alpha}_h u_h$ for some $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ with $\alpha_h \tilde{\alpha}_h = 0$ for all h.

We want to show $\beta_0 = \beta$. By seeking a contradiction, suppose $\beta_0 \neq \beta$. In particular, assume $\beta_0 > \beta$ (a symmetric argument is applicable when $\beta_0 < \beta$). By Richness Condition, for each i, there exist $m_i, m_i', n_i, n_i' \in \Delta(C)$ such that $u_i(m_i) > u_i(m_i'), u_i(n_i') > u_i(n_i)$, and

$$
\beta_0 > \frac{u_i(m_i) - u_i(m_i')}{u_i(n_i') - u_i(n_i)} > \beta. \tag{A1}
$$

Define $\bar{m} = \sum_i \frac{1}{|I|} m_i, \bar{m}' = \sum_i \frac{1}{|I|} m_i', \bar{n} = \sum_i \frac{1}{|I|} n_i$, and $\bar{n}' = \sum_i \frac{1}{|I|} n_i'$. From (A1),

$$
u_i(\bar{m}) + \beta u_i(\bar{n}) > u_i(\bar{m}') + \beta u_i(\bar{n}'), \text{ and,} \tag{A2}
$$

$$
u_i(\bar{m}) + \beta_0 u_i(\bar{n}) < u_i(\bar{m}') + \beta_0 u_i(\bar{n}') \tag{A3}
$$

for all i. Moreover, since $u_0 = \sum_i \tilde{\alpha}_i u_i$, (A3) implies

$$
u_0(\bar{m}) + \beta_0 u_0(\bar{n}) < u_0(\bar{m}') + \beta_0 u_0(\bar{n}') \tag{A4}
$$

For any $m_0 \in \Delta(C)$, (A2) implies that $(\bar{m}, \bar{n}, m_0, \ldots) \succ_i (\bar{m}', \bar{n}', m_0, \ldots)$. On the other hand, (A4) implies $\{(\bar{m}', \bar{n}', m_0, \ldots)\} \succ_0 \{(\bar{m}, \bar{n}, m_0, \ldots)\}$, which contradicts Commitment Pareto.

If part: NTPDO and TAM follow from Theorem 1. To show Commitment Pareto, suppose $U_i(l) > U_i(l')$ for all i. Then,

$$
\begin{aligned}
W_0(\{l\}) &= \sum_{t=1}^{\infty} \beta^{t-1} \sum_i \tilde{\alpha}_i u_i(l_t) = \sum_i \tilde{\alpha}_i \sum_{t=1}^{\infty} u_i(l_t) \beta^{t-1} \\
&> \sum_i \tilde{\alpha}_i \sum_{t=1}^{\infty} u_i(l_t') \beta^{t-1} = W_0(\{l'\}).
\end{aligned}
$$

A.5 | Proof of Theorem 2

Only if part: Since Commitment Pareto implies Constant Commitment Pareto, together with NTPDO, Theorem 1 implies $u_0 = \sum_h \tilde{\alpha}_h u_h$ for some $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ with $\alpha_h \tilde{\alpha}_h = 0$ for all h.

Observe that when we restrict attention to the subdomain of commitment plans, Commitment Pareto implies the existing dictatorship result (Hayashi & Lombardi, 2021; Jackson & Yariv, 2015; Zuber, 2011). Hence, there exists some $i^* \in I$ such that $\tilde{\alpha}_{i^*} > 0$, $\tilde{\alpha}_h = 0$ for all $h \neq i^*$ and $\beta_0 = \beta_{i^*}$. Since $\alpha_h \tilde{\alpha}_h = 0$ for all h and $\tilde{\alpha}_{i^*} > 0$, $\alpha_{i^*} = 0$. Hence, $v_0 = \sum_{h \neq i^*} \alpha_h u_h$.

If part: Since $u_0 = \tilde{\alpha}_{i^*} u_{i^*}$ and $\beta_0 = \beta_{i^*}$, it is obvious that the profile satisfies Commitment Pareto. NTPDO and TAM follow from Theorem 1.

A.6 | Proof of Proposition 4

Only if part: Since Commitment Pareto implies Constant Commitment Pareto and NTPDP implies NTPDO, Theorem 1 implies $u_0 = \sum_h \tilde{\alpha}_h u_h$ for some $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \backslash \{0\}$ with $\alpha_h \tilde{\alpha}_h = 0$ for all h.

Showing $\beta_0 = \beta$ follows from the proof of Proposition 3. Hence we want to show $\gamma_0 = \beta$. By seeking a contradiction, suppose $\gamma_0 \neq \beta$. In particular, assume $\gamma_0 > \beta$ (a symmetric argument is applicable when $\beta_0 < \beta$). By Richness Condition, for each i, there exist $m_i, m_i', n_i, n_i' \in \Delta(C)$ such that $u_i(m_i) > u_i(m_i')$, $u_i(n_i') > u_i(n_i)$, and

$$\gamma_0 > \frac{u_i(m_i) - u_i(m_i')}{u_i(n_i') - u_i(n_i)} > \beta. \tag{A5}$$

Define $\overline{m} = \sum_i \frac{1}{|I|} m_i$, $\overline{m}' = \sum_i \frac{1}{|I|} m_i'$, $\overline{n} = \sum_i \frac{1}{|I|} n_i$, and $\overline{n}' = \sum_i \frac{1}{|I|} n_i'$. From (A5),

$$u_i(\overline{m}) + \beta u_i(\overline{n}) > u_i(\overline{m}') + \beta u_i(\overline{n}'), \text{ and,} \tag{A6}$$

$$u_i(\overline{m}) + \gamma_0 u_i(\overline{n}) < u_i(\overline{m}') + \gamma_0 u_i(\overline{n}') \tag{A7}$$

for all i. Moreover, since $v_0 = \sum_i \alpha_i u_i$, (A7) implies

$$u_0(\overline{m}) + \gamma_0 u_0(\overline{n}) < u_0(\overline{m}') + \gamma_0 u_0(\overline{n}'). \tag{A8}$$

For any fixed $m_0 \in \Delta(C)$, let $l = (\overline{m}, \overline{n}, m_0, \ldots)$ and $l' = (\overline{m}', \overline{n}', m_0, \ldots)$. Then, (A6) implies that $l \succ_i l'$ for all i. Moreover, by Commitment Pareto, $\{l\} \succ_0 \{l'\}$. Thus, by NTPDP, $\{l\} \sim_0 \{l, l'\}$. On the other hand, (A8) implies $V_0(l') > V_0(l)$. From the FT representation, $\{l\} \succ_0 \{l, l'\}$, which is a contradiction.

If part: TAM follows from Theorem 1. To show Commitment Pareto, suppose $U_i(l) > U_i(l')$ for all i. Then,

$$W_0(\{l\}) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_i \tilde{\alpha}_i u_i(l_t) = \sum_i \tilde{\alpha}_i \sum_{t=1}^{\infty} u_i(l_t) \beta^{t-1}$$
$$> \sum_i \tilde{\alpha}_i \sum_{t=1}^{\infty} u_i(l_t') \beta^{t-1} = W_0(\{l'\}).$$

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Next, to show NTPDP, suppose $U_i(l) > U_i(l')$ for all i and $\{l\} \succ_0 \{l'\}$. Since $\gamma_0 = \beta_i = \beta$,

$$\sum_i \alpha_i U_i(l) = \sum_t \alpha_i \sum_{t=1}^{\infty} \beta_i^{t-1} u_i(l_t) = \sum_{t=1}^{\infty} \beta^{t-1} \sum_t \alpha_i u_i(l_t) = \sum_{t=1}^{\infty} \gamma_0^{t-1} v_0(l_t) = V_0(l).$$

Thus, $U_i(l) > U_i(l')$ implies $V_0(l) > V_0(l')$. By the FT representation, $\{l\} \sim \{l, l'\}$, as desired.

A.7 | Proof of Theorem 3

Only if part: Since Commitment Pareto implies Constant Commitment Pareto and NTPDP implies NTPDO, Theorem 1 implies $u_0 = \sum_h \tilde{\alpha}_h u_h$ for some $(\tilde{\alpha}_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ and $v_0 = \sum_h \alpha_h u_h$ for some $(\alpha_h) \in \mathbb{R}_+^{|I|} \setminus \{0\}$ with $\alpha_h \tilde{\alpha}_h = 0$ for all h.

Observe that when we restrict attention to the subdomain of commitment plans, Commitment Pareto implies the existing dictatorship result (Hayashi & Lombardi, 2021; Jackson & Yariv, 2015; Zuber, 2011). Hence, there exists some $i^* \in I$ such that $\tilde{\alpha}_{i^*} > 0$, $\tilde{\alpha}_h = 0$ for all $h \neq i^*$ and $\beta_0 = \beta_{i^*}$.

Next, we will show $v_0 = \alpha_{j^*} u_{j^*}$ and $\gamma_0 = \beta_{j^*}$ for some fixed $j^* \in I$. Since the profile satisfies TAM, v_0 is nonconstant by the same argument as in Theorem 1. Together with $\gamma_0 > 0$, V_0 is nonconstant.

We will claim that for any $l, l' \in \Delta(C)^\infty$, if $U_i(l) > U_i(l')$ for all i, then $V_0(l) > V_0(l')$. By Commitment Pareto, $\{l\} \succ_0 \{l'\}$. By NTPDP, $\{l\} \sim_0 \{l, l'\}$. From the FT representation of $\succsim_0$, $V_0(l) \geq V_0(l')$. Seeking a contradiction, suppose $V_0(l) = V_0(l')$. Since v_0 is not constant, take any $m, n \in \Delta(C)$ with $v_0(m) > v_0(n)$. For any $\alpha \in (0, 1)$, let $l_\alpha = (\alpha l_1 + (1-\alpha)n, l_2, \ldots)$ and $l'_\alpha = (\alpha l'_1 + (1-\alpha)m, l'_2, \ldots)$. Then, $V_0(l_\alpha) < V_0(l'_\alpha)$. On the other hand, by the continuity of utility functions u_i, for all sufficiently small α, $U_i(l_\alpha) > U_i(l'_\alpha)$. By Commitment Pareto, $\{l_\alpha\} \succ_0 \{l'_\alpha\}$. NTPDP implies $\{l_\alpha\} \sim_0 \{l_\alpha, l'_\alpha\}$, while the FT representation implies $\{l_\alpha\} \succ_0 \{l_\alpha, l'_\alpha\}$, a contradiction.

Since $(U_i)_{i \in I}$ and V_0 are discounted utility functions, the existing dictatorship result (Hayashi & Lombardi, 2021; Jackson & Yariv, 2015; Zuber, 2011) applies. Hence, there exists some $j^* \in I$ such that $\alpha_{j^*} > 0$, $\alpha_h = 0$ for all $h \neq j^*$ and $\gamma_0 = \beta_{j^*}$. And since $\alpha_h \tilde{\alpha}_h = 0$ for all h, it holds $i^* \neq j^*$.

If part: Note that for some $i^* \neq j^*$, $W(\{l\}) = U_{i^*}(l)$ and $V_0(l) = U_{j^*}(l)$. So, it is obvious that the profile satisfies Commitment Pareto.

To show NTPDP, suppose $U_i(l) > U_i(l')$ for all i and $\{l\} \succ_0 \{l'\}$. The latter implies $U_{i^*}(l) > U_{i^*}(l')$ and the former implies $U_{j^*}(l) > U_{j^*}(l')$, or equivalently, $V_0(l) > V_0(l')$. By the FT representation, $\{\tilde{l}\} \sim \{\tilde{l}, \tilde{l}'\}$, as desired.

We show that the profile satisfies TAM. Take any m, n such that $u_i(m) > u_i(n)$ for some i, $u_j(n) > u_j(m)$ for all $j \neq i$, and $\{m^\infty\} \succ_0 \{n^\infty\}$. Since $W(\{l\}) = \tilde{\alpha}_{i^*} U_{i^*}(l)$, we must have $i = i^*$. Since $j^* \neq i^*$, $u_{j^*}(n) > u_{j^*}(m)$, which implies $V_0(n^\infty) = \alpha_{j^*} \frac{u_{j^*}(n)}{1-\beta_{j^*}} > \alpha_{j^*} \frac{u_{j^*}(m)}{1-\beta_{j^*}} = V_0(m^\infty)$. From the FT representation, we have $\{m^\infty\} \succ_0 \{m^\infty, n^\infty\}$, as desired.

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