Eur. Phys. J. Spec. Top.
https://doi.org/10.1140/epjs/s11734-023-00987-2

THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS

Regular Article

Vacuum representations of blackbody radiation

Stephen M. Barnett[a]

School of Physics and Astronomy, University of Glasgow, Glasgow G128QQ, UK

Received 5 April 2023 / Accepted 22 August 2023

Abstract We present and discuss two exact, but perhaps unfamiliar, representations of thermal radiation. The first has the form of a superposition of the quantum vacuum and a stochastic classical field and the second is the pure-state thermofield representation introduced by Takahashi and Umezawa. It is interesting that the former is, essentially, the opposite of Planck's original conception of blackbody radiation.

1 Introduction

The blackbody field has played a crucial role in the development both of quantum theory and thermodynamics [1–3], and it is both natural and timely to return to it in connexion with recent developments in quantum thermodynamics [4, 5]. In this paper, we address the mathematical representation of the blackbody field within quantum theory. That distinct representations exist as a consequence of the fact that mixed states, like the state of each field mode in blackbody radiation, can be thought of as arising from any of numerous equivalent but nevertheless distinct ensembles.

It is the simplest to consider, first, the state of a single discrete electromagnetic field mode in a thermal state with inverse temperature $\beta = 1/k_{\mathrm{B}}T$, where k_{B} is Boltzmann's constant. We can represent this state by the density operator

$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{\mathrm{Tr}\left(e^{-\beta \hat{H}}\right)} = \left(1 - e^{-\beta\hbar\omega}\right) \sum_{n=0}^{\infty} e^{-\beta n \hbar\omega} |n\rangle\langle n|, \tag{1}$$

where ω is the angular frequency of the mode and $|n\rangle$ is the photon-number eigenstate with n photons present. We can consider this state to represent an ensemble of identically prepared modes, with each prepared in a photon number eigenstate with corresponding probabilities $e^{-\beta n\hbar\omega}\left(1 - e^{-\beta\hbar\omega}\right)$. This is often a useful and convenient idea and appears regularly in texts on quantum optics [6–8] and open-system dynamics [9].

Using the density operator formalism for continuum fields has its problems, the most straightforward of which is the fact that it is not possible to normalise the state. Thus in field theory, an alternative approach is favoured. This is based on the similarity between the Boltzmann factor, $e^{-\beta \hat{H}}$, and the time-evolution operator, $e^{-i\hat{H}t/\hbar}$, which suggests identification of the inverse temperature with an imaginary time [10–12]:

$$t = -i\beta\hbar. \tag{2}$$

To determine thermal properties, we can deform an evolution path integral into the complex t plane and apply methods familiar from quantum field theory.

Here we present two alternative representations of the blackbody field. The first is based on idea due to Mollow [13], that there exists a quantum state of the electromagnetic field that corresponds to a superposition of a classical (c-number) field with the quantum (operator) field. The second, thermofield representation [14–18], was primarily designed for use in quantum field theory. It replaces the thermal (mixed) state by an entangled pure state in a doubled state space. Each of these representations provides new insights into the physics of quantum systems interacting with thermal radiation.

2 The vacuum picture

The vacuum picture, introduced by Mollow [13], derives from the fact that a coherent state is related to the vacuum state by means of a unitary transformation. We follow, here, the analysis by Pegg [19] and, for simplicity and brevity, consider just a single discrete field mode, with annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$. The requisite transformation has the form [6–8, 20, 21]

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right)|0\rangle, \tag{3}$$

[a] e-mail: stephen.barnett@glasgow.ac.uk (corresponding author)

where α is a complex amplitude.

To proceed, let us consider the interaction between a single atom and a single (near-resonant) mode of the electromagnetic field. Such a simple situation can be realised in the laboratory using the techniques of cavity quantum electrodynamics [22]. The Hamiltonian for the coupled atom-field system in this simple case has the form

$$\hat{H} = \hat{H}_A + \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) - \hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\mathbf{r}), \tag{4}$$

where $\hat{H}_A$ is the Hamiltonian for the freely evolving atom and ω is the angular frequency of the single cavity mode. The coupling is of the electric–dipole form, with $\hat{\mathbf{d}}$ being the dipole–moment operator for the atom and $\hat{\mathbf{E}}(\mathbf{r})$ being the electric field operator for the cavity mode at the position, $\mathbf{r}$, of the atom:

$$\hat{\mathbf{E}}(\mathbf{r}) = \mathbf{E}_0(\mathbf{r})\hat{a} + \mathbf{E}_0^*(\mathbf{r})\hat{a}^\dagger, \tag{5}$$

where $\mathbf{E}_0(\mathbf{r})$ is a function that depends on the geometry of the enclosing cavity.

To proceed to the vacuum picture, we first transform to an interaction picture [8] in which the time dependence of the field is associated with $\hat{\mathbf{E}}$. This has the effect of removing from the Hamiltonian the term associated with the freely evolving field. We find

$$\hat{H}' = \hat{H}_A - \hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\mathbf{r},\, t), \tag{6}$$

where

$$\hat{\mathbf{E}}(\mathbf{r},\, t) = \mathbf{E}_0(\mathbf{r})\hat{a}e^{-i\omega t} + \mathbf{E}_0^*(\mathbf{r})\hat{a}^\dagger e^{i\omega t}. \tag{7}$$

The description of the coupled atom–field system is completed by the choice of an initial state of the atom and of the field, which we take to have the uncorrelated form $|\psi_A(0)\rangle|\alpha\rangle$. We arrive at the vacuum picture by applying to this initial state the unitary operator $\hat{D}^{-1}(\alpha) = \hat{D}(-\alpha)$, which leaves the initial field state as the vacuum. The transformation is completed by applying the corresponding transformation to the electric field operator:

$$\hat{\mathbf{E}}(\mathbf{r},\, t) \to \hat{D}^{-1}(\alpha)\hat{\mathbf{E}}(\mathbf{r},\, t)\hat{D}(\alpha) = \hat{\mathbf{E}}(\mathbf{r},\, t) + \mathcal{E}(\mathbf{r},\, t), \tag{8}$$

where

$$\mathcal{E}(\mathbf{r},\, t) = \mathbf{E}_0(\mathbf{r})\alpha e^{-i\omega t} + \mathbf{E}_0^*(\mathbf{r})\alpha^* e^{i\omega t} \tag{9}$$

is the c-number electric field. Equation (8) embodies the key idea in the vacuum picture, in that the full electric field is this picture is a *superposition* of a classical field, $\mathcal{E}(\mathbf{r},\, t)$, and a quantum field $\hat{\mathbf{E}}(\mathbf{r},\, t)$, with the corresponding state of the quantum field being the vacuum.

This means that we can write our final Hamiltonian in the form

$$\hat{H}'' = \hat{H}_A - \hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\mathbf{r},\, t) - \hat{\mathbf{d}}\cdot\mathcal{E}(\mathbf{r},\, t), \tag{10}$$

in which there are now two interaction terms: the first induces a coupling between the atom and the surrounding vacuum field and the second couples the atom to the classical driving field.

The representation of a laser field as a c-number field superposed with the full multi-mode vacuum was the key idea behind Mollow's approach [13]. The classical field in this case drives the excitation of the atom and, if it is strong enough, also Rabi oscillations. The coupling between the atom and the quantum vacuum results in spontaneous emission by which the atom fluoresces. The idea has been applied also to cavity quantum electrodynamics to investigate the origin of the quantum collapses and revivals in the evolution of the atomic inversion in the Jaynes–Cummings model [23] in which a single atom interacts with a single resonant cavity mode.

Finally, we note that there is nothing in our analysis that is specific to a single discrete cavity mode and it follows that the same idea, of superposing a classical field with the vacuum, can be applied both to multimode problems and also to continuum fields.

3 The thermal vacuum picture

We can extend, to thermal states, the idea from the preceding section by introducing a distribution of possible coherent state amplitudes. This means treating the state as representing an ensemble of amplitudes, α, for the classical fields with the associated probability distribution

$$P(\alpha) = \frac{1}{\pi\bar{n}}\exp\left(-\frac{|\alpha|^2}{\bar{n}}\right) \tag{11}$$

rather than the single value of α considered in the preceding section. Readers familiar with the rudiments of quantum optics will recognise this as the Glauber P-function for a thermal state of a mode with mean photon number $\bar{n} = (e^{\beta\hbar\omega} - 1)^{-1}$. Indeed we can write the density operator for this state in the form [8, 20]

$$\hat{\rho} = \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|. \tag{12}$$

From this we can calculate the ensemble average of properties of the field mode [8]. For example, the first and the second moments of the photon number are

$$\begin{aligned}\langle\hat{a}^\dagger\hat{a}\rangle &= \int d^2\alpha P(\alpha)|\alpha|^2 \\ &= \bar{n}\end{aligned} \tag{13}$$

$$\begin{aligned}\langle(\hat{a}^\dagger\hat{a})^2\rangle &= \langle\hat{a}^{\dagger 2}\hat{a}^2\rangle + \langle\hat{a}^\dagger\hat{a}\rangle \\ &= \int d^2\alpha P(\alpha)|\alpha|^4 + \bar{n} \\ &= 2\bar{n}^2 + \bar{n}. \end{aligned} \tag{14}$$

Yet the situation in the vacuum picture is different. Here we envisage the quantum state of the mode to be the vacuum state and for the full electric field to be a superposition of the electric field operator for the mode, with a classical field with amplitude α drawn from an ensemble with probability distribution $P(\alpha)$.

It is interesting to ask what insights the thermal vacuum picture might offer, as the results of any calculation must coincide with those found with any other representation of the field. We consider three phenomena for blackbody fields and examine the interpretation of these. The first is Einstein's famous observation that the fluctuations in a blackbody field appear to have both wave and particle contributions [24, 25]. For our single-mode thermal state, this corresponds to the fact that the variance in the observed photon number in the mode is (from Eqs. (13) and (14))

$$\Delta n^2 = \bar{n}^2 + \bar{n}. \tag{15}$$

We can associate the first contribution with wave fluctuations and the second with the shot noise familiar from counting particles, so this variance exhibits both wave and particle characteristics. We can check this in our vacuum picture, as we have precisely a superposition of a classical wave and quantum contributions. One problem needs to be addressed, however, and this is that we do not have a photon description for our classical part of the field. We can address this by considering the fluctuations in the local energy density of our single field mode. We find

$$\begin{aligned}\Delta(E_{\rm Tot}^2(\mathbf{r})^2) &= \Delta(\mathcal{E}^2)^2 + 4\langle\mathcal{E}^2\rangle\langle 0|\hat{E}^2|0\rangle + \Delta(\hat{E}^2)^2 \\ &= 8|E_0|^4\left(\bar{n}^2 + \bar{n} + \frac{1}{4}\right).\end{aligned} \tag{16}$$

The origin of the contributing terms in the vacuum picture is at once clear: the $\bar{n}^2$ term arises solely from the fluctuations in the c-number, classical, part of the field and so may very reasonably be associated with the wave properties of the field. The $\bar{n}$ term arises from both the classical and the quantum parts of the superposition and, indeed, may be thought of as arising from the interference between these. We note that a similar interference occurs in the technique of homodyne detection used to detect optical squeezing [7]. The final term arises from the quantum fluctuations of the vacuum state. Had we analysed the fluctuations in terms of a photodetection experiment, only the $\bar{n}^2$ and $\bar{n}$ parts of this would have appeared [21, 26, 27].

Our second phenomenon is the absorption and emission of radiation by an atom immersed in a blackbody field. Here we identify three fundamental processes: absorption, stimulated emission and spontaneous emission. The rates for the first two of these are governed by the Einstein B coefficients, whilst the last one is determined by his A coefficient. It is common to see absorption and stimulated emission treated within a semiclassical theory, in which the atom or molecule is treated quantum mechanically but the field is described classically. The A coefficient can then be derived later within the fully quantum theory [7]. Our vacuum picture (which is fully quantum) demonstrates why this is possible: the absorption and stimulated emission processes, in this picture, arise from the interaction between the atom and the classical component of the superposed fields. The vacuum component cannot induce absorption and gives rise only to the spontaneous emission, that part of the emission that occurs even in the absence of the blackbody field.

Finally, we turn to consider the question of the coherence of the blackbody field. In the quantum theory of coherence, the observed phenomena depend only on normally ordered moments of the field and it follows necessarily that, in the vacuum picture, the quantum part of the superposed fields does not contribute and we need only consider the classical contributions. These, as we have seen, are determined by a Gaussian random process and we need only evaluate these. For the first two non-trivial moments we find

$$\begin{aligned}\langle\mathcal{E}_i(\mathbf{r}, t)\rangle &= 0 \\ \langle\mathcal{E}_i(\mathbf{r}, t)\mathcal{E}_j(\mathbf{r}, t+\tau)\rangle &= \delta_{ij}\frac{2\hbar}{3\varepsilon_0\pi^2c^3}\left(\pi^4\frac{1+2\cosh^2(\pi\tau/\hbar\beta)}{(\hbar\beta)^4\sinh^4(\pi\tau/\hbar\beta)} - \frac{3}{\tau^4}\right).\end{aligned} \tag{17}$$

In modelling the interaction between a blackbody field and matter, it may be that separating out this fluctuating classical field from the quantum vacuum field may simplify the analysis as well as providing fresh insights into the phenomena arising.

4 Historical interlude

It is interesting to compare the physical picture of blackbody radiation provided by the vacuum picture with that provided in earlier treatments. In particular, there is an interesting comparison to be made with Planck's early thinking on the topic. In his famous 1900 paper, he wrote the following [28].

Let us consider a large number of monochromatically vibrating resonators—N of frequency ν (per second), N' of frequency ν', N'' of frequency ν'',..., with all N large numbers—which are at large distances apart and are enclosed in a diathermic medium with light velocity c and bounded by reflecting walls. Let the system contain a certain amount of energy $E_{\rm t}$ (erg) which is present partly in the medium as travelling radiation and partly in the resonators as vibrational energy. The question is how in a stationary state this energy is distributed over vibrations of the resonators and over the various colours of the radiation present in the medium, and what will be the temperature of the total system.

To answer this question, we first of all consider the vibrations of the resonators and assign to them arbitrarily definite energies, for instance, and energy E to the N resonators ν, E' to the N' resonators ν',.... The sum

$$E + E' + E'' + ... = E_0$$

must, of course, be less than E_t. **The remainder** $E_\mathrm{t} - E_0$ pertains then to the radiation present present in the medium. [My bold.] We must now give the distribution of the energy over the separate resonators of each group, first the distribution of the energy E over the N resonators of frequency ν. If E [the energy of the resonators] is considered to be a continuously divisible quantity, this distribution is possible in infinitely many ways. We consider, however—this is the most essential point of the whole calculation—E to be composed of a very definite number of equal parts and use thereto the constant of nature $h = 6 \cdot 55 \times 10^{-27}$ erg sec. This constant multiplied by the common frequency ν of the resonators gives us the energy element ε in erg, and dividing E by ε we get the number P of energy elements which must be divided over the N resonators. If the ratio is not an integer, we take for P an integer in the neighbourhood.

It is by no means easy to follow, after more than 100 years, precisely the line of Planck's reasoning [3], but it seems to be clear that Planck was, in effect, quantising the energy of the material medium (resonators) in equilibrium with the radiation field and leaving the remaining energy, $E_\mathrm{t} - E_0$, available for the electromagnetic field. Other interpretations are certainly possible, but this at least suggests, in modern terms, something closer to what we refer to, today, as semiclassical theory in which the matter is quantised but the field is treated classically. Our vacuum picture, which is an exact representation, is essentially *the opposite* of this idea: it describes blackbody radiation as a superposition of the, inherently quantum, vacuum state of the field together with a Gaussian random c-number (classical) field, such as would be generated by classical currents in the matter comprising the source of the radiation, where Planck seems to have conceived of quantised matter as a source of a continuously varying (classical) field, we have arrived at a picture that is consistent with classical (noisy) matter superposed with a quantum electromagnetic field.

5 Thermofield representation

A second, but very different, pure-state representation of a thermal state is provided by the thermofield vacuum state, which is a pure state in a doubled state space designed to give rise to thermal statistics in the original space [14–18]. This is possible because the pure state is an entangled state of the original system and the additional system (referred to as a fictitious system) so that the reduced density of operator for the original system is mixed. The idea is most simply conveyed by reference to the specific example of a single field mode, spanned by the number states $\{|n\rangle\}$. To these we add a second fictitious (tilde) oscillator with number states $\{|\tilde{n}\rangle\}$ and write the thermofield vacuum state as

$$|0(\beta)\rangle = \left(1 - e^{\beta\hbar\omega}\right)^{1/2} \sum_{n=0}^{\infty} e^{-n\beta\hbar\omega/2} |n\rangle|\tilde{n}\rangle. \quad (18)$$

The orthogonality of the number states means that the expectation value of any operator acting only on the state space of the original mode will have the same value as that for the thermal density operator, Eq. (1).

The reader may have noticed that the thermofield vacuum state is identical to the two-mode squeezed state generated from the two-mode vacuum state by means of the Bogoliubov unitary transformation [8, 17, 29]:

$$\begin{aligned} \hat{T}(\theta)|0,\,\tilde{0}\rangle &= \exp\left(\theta(\hat{a}^\dagger\hat{\tilde{a}}^\dagger - \hat{\tilde{a}}\hat{a})\right)|0,\,\tilde{0}\rangle \\ &= \mathrm{sech}\theta \sum_{n=0}^{\infty} (\tanh\theta)^n |n,\,\tilde{n}\rangle\,. \end{aligned} \quad (19)$$

The thermofield a vacuum state and the two-mode squeezed state have the same form if we require $\tanh\theta = e^{-\beta\hbar\omega/2}$.

Before discussing the features of the thermofield representation, a short historical digression is in order. Takahashi and Umezawa were not the first to spot the connexion between the pure state, Eq. (19) and single-mode thermal averages [6, 30], but they were, to the best of my knowledge, the first to make systematic use of this fact. The representation of the thermal state by an entangled state in a doubled space can be extended to apply to any mixed state of any quantum system [31]. When this idea was rediscovered in the context of quantum information theory, it became known as purification [32, 33].

The power of the thermofield technique derives from the fact that it is related to the two-mode vacuum state by means of a unitary transformation. This means that thermal expectation values can be reduced to the form of a vacuum expectation value. A simple example may serve to illustrate this point. We may recall that the normal ordered moments for the thermal state have the simple form [8]:

$$\langle \hat{a}^{\dagger m}\hat{a}^k \rangle = \delta_{mk} m!\, \bar{n}^m. \quad (20)$$

We can check this simply using the thermofield vacuum state:

$$\begin{aligned} \langle \hat{a}^{\dagger m}\hat{a}^k \rangle &= \langle 0(\beta)|\hat{a}^{\dagger m}\hat{a}^k|0(\beta)\rangle \\ &= \langle 0,\,\tilde{0}|(\cosh\theta\hat{a}^\dagger + \sinh\theta\hat{\tilde{a}})^m(\cosh\theta\hat{a} + \sinh\theta\hat{\tilde{a}}^\dagger)^k|0,\,\tilde{0}\rangle \\ &= \sinh^{m+k}\langle\tilde{0}|\hat{\tilde{a}}^m\hat{\tilde{a}}^{\dagger k}|\tilde{0}\rangle \end{aligned}$$

$$= \delta_{mk} m!\, \bar{n}^m, \tag{21}$$

where we have used the canonical transformation

$$\hat{T}^\dagger(\theta)\hat{a}\hat{T}(\theta) = \cosh\theta\hat{a} + \sinh\theta\hat{\tilde{a}}^\dagger \tag{22}$$

and its conjugate.

As with the thermal vacuum picture, the thermofield representation provides a distinct physical idea for the form of interactions with the blackbody field. To see this, let us return to the electric-dipole interaction between an atom and a single electromagnetic field mode represented by the Hamiltonian in Eq. (4). The first problem that we need to address is the correct form to take for the free Hamiltonian for the fictitious mode. A natural try might be $\hbar\omega\left(\hat{\tilde{a}}^\dagger\hat{\tilde{a}}\right)$, but this is incorrect; we need take the negative of this so that our Hamiltonian becomes:

$$\hat{H} = \hat{H}_A + \hbar\omega\left(\hat{a}^\dagger\hat{a} - \hat{\tilde{a}}^\dagger\hat{\tilde{a}}\right) - \hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\mathbf{r}). \tag{23}$$

The reason for this is that we require the form of the two-mode free Hamiltonian to be invariant under the action of the unitary operator $\hat{T}(\theta)$:

$$\hat{T}^\dagger(\theta)\left(\hat{a}^\dagger\hat{a} - \hat{\tilde{a}}^\dagger\hat{\tilde{a}}\right)\hat{T}(\theta) = \left(\hat{a}^\dagger\hat{a} - \hat{\tilde{a}}^\dagger\hat{\tilde{a}}\right). \tag{24}$$

This has the important feature that the free evolution of the operator $\cosh\theta\hat{a} + \sinh\theta\hat{\tilde{a}}^\dagger$ corresponds, simply, to multiplication by the phase factor $e^{-i\omega t}$:

$$\hat{T}^\dagger(\theta)\hat{a}(t)\hat{T}(\theta) = \left(\cosh\theta\hat{a} + \sinh\theta\hat{\tilde{a}}^\dagger\right)e^{-i\omega t}. \tag{25}$$

We can transform our coupled atom thermal field system into a system in which the atom is coupled to the vacuum state, $|0, \tilde{0}\rangle$, by first transforming our Hamiltonian into the form given in Eq. (6) and then transforming this using the $\hat{T}(\theta)$ unitary operator:

$$\hat{T}^\dagger(\theta)\hat{H}'\hat{T}(\theta) = \hat{H}_A - \hat{\mathbf{d}}\cdot\left(\cosh\theta\hat{\mathbf{E}}(\mathbf{r}, t) + \sinh\theta\hat{\tilde{\mathbf{E}}}(\mathbf{r}, t)\right), \tag{26}$$

where $\hat{\mathbf{E}}(\mathbf{r}, t)$ is as given in Eq. (7) and the second, tilde, operator has the form

$$\hat{\tilde{\mathbf{E}}}(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r})\hat{\tilde{a}}^\dagger e^{-i\omega t} + \mathbf{E}_0^*(\mathbf{r})\hat{\tilde{a}}e^{i\omega t}. \tag{27}$$

The interpretation of the dynamics associated with this picture differs markedly from from that inferred from models based on the mixed state, thermal density operator or the thermal vacuum picture introduced above. Quanta are provided to excite the atom from the negative-frequency (fictitious) mode and these can then be removed from the atom by exciting the true field mode.

As with the thermal vacuum picture, the dynamics in the thermofield representation are associated with a superposition of two fields. In this case, however, both are quantum in nature, represented by operators. The difference between these is that one has the form of regular harmonic oscillators but the other (fictitious) field is formed from a negative frequency (or inverted) harmonic oscillator. For open systems, such as occurs for an atom immersed in a full blackbody field, the thermofield representation provides, effectively, two reservoirs: the first is a bath of modes in their vacuum state and the second is a bath of inverted oscillators. We have a combination of the first reservoir at temperature 0^+ and the second at the effective temperature 0^-, with the inverse temperature β determined by the relative strengths of the couplings to these two.

As a final thought, we note that that it is often the case that a mixed state corresponds to situations in which correlations with an unobserved system are omitted from consideration. In the thermofield formalism this corresponds to the introduction of the fictitious mode(s). There are physical situations in which such correlated, or entangled, systems are produced. In nonlinear optics, for example, a non-degenerate parametric oscillator produces a two-mode squeezed state of the down-converted mode [17, 30, 34, 35]. At a very different scale, the thermofield representation has been employed to model the Hawking mechanism of black hole evaporation, in which fields trapped within the blackhole become entangled with those escaping to infinity [36, 37].

6 Conclusion

Blackbody radiation occurs for the electromagnetic field in thermal equilibrium with its surroundings at a given temperature. The corresponding mixed state can readily be obtained by maximising the entropy of each field mode subject to a constraint of fixed mean energy [6, 33]. It is an essential feature of all mixed states that they can be decomposed into a great number of possible ensembles, and this suggests that there are many possible ways to decompose the state of the black body field. In this paper, we have presented two such possible representations, each based on reducing the quantum state of each field mode into a vacuum state.

Our first vacuum representation is based on the observation that a field mode prepared in a coherent state may equally well be considered to be a superposition of the vacuum state with a classical, c-number, field. The thermal vacuum representation is obtained by replacing the stable and fixed classical field with a Gaussian random one. We have seen that this representation gives meaning to the idea that absorption and stimulated emission events are semiclassical in origin, whilst spontaneous emission is fully quantum.

Our second vacuum representation is the thermofield picture introduced by Takahashi and Umezawa [14]. This was designed with applications to quantum field theory in mind. The key idea is the use of a doubled state space, with each field mode supplemented by a second "fictitious" mode. The thermal state is then an entangled pure state of the these two modes of the same form as a two-mode squeezed vacuum state [17]. The thermal (mixed state) properties of the original modes arise from this state by virtue of the fact that the fictitious mode is necessarily unobserved. The power of this representation comes, principally, from the fact that the thermofield vacuum state is simply related to the two-mode vacuum state by means of a unitary transformation. This allows us to map thermal averages onto vacuum expectation values and to exploit well-developed techniques from quantum field theory to evaluate thermal averages.

It may be, by virtue of their very different character, that these two vacuum representations may serve to simplify certain models and calculations. Be that as it may, it is certainly true that they provide new physical pictures and insights.

Acknowledgements I wish to express my gratitude to Peter Milonni for the most helpful and encouraging correspondence and to Neel Mackinnon for his suggestions. Work on the thermal vacuum picture was completed and first presented at the meeting on blackbody radiation held at the Erwin Schrödinger Instutute in Vienna. I am grateful to Christoph Dellago and his team for their generous hospitality at the ESI. This work was supported by a Royal Society Research Professorship, grant number RP150122.

Data Availability Statement There is no associated data.

References

1. M. Planck, *The Theory of Heat Radiation* (Dover, New York, 1991)
2. J. Mehra, H. Rechenberg, *The Historical Development of Quantum Theory*, vol. 1 (Springer, New York, 1982)
3. T.S. Kuhn, *Black-body Theory and the Quantum Discontinuity, 1894–1912* (Oxford University Press, Oxford, 1978)
4. J. Gemmer, M. Michel, G. Mahler, *Quantum Thermodynamics* (Springer, Heidelberg, 2004)
5. S. Vinjanampathy, J. Anders, Quantum thermodynamics. Contemp. Phys. **57**, 545–579 (2016)
6. W.H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973)
7. R. Loudon, *The Quantum Theory of Light*, 3rd edn. (Oxford University Press, Oxford, 2000)
8. S.M. Barnett, P.M. Radmore, *Methods in Theoretical Quantum Optics* (Oxford University Press, Oxford, 1997)
9. H.P. Breuer, F. Petrucione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2002)
10. A.A. Abrikosov, L.P. Gorkov, I.E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Dover, New York, 1975)
11. E.M. Lifshitz, L.P. Pitaevskii, *Statistical Physics* (Butterworth-Heinemann, Oxford, 1980)
12. M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, 1996)
13. B.R. Mollow, Pure-state analysis of resonant light scattering: radiative damping, saturation, and multiphoton effects. Phys. Rev. A **12**, 1919–1943 (1975)
14. Y. Takahashi, H. Umezawa, Thermo field dynamics. Collect. Phenom. **2**, 55–80 (1975). (**Reprinted in Int. J. Mod. Phys. 10, 1755–1805 (1996)**)
15. H. Umezawa, H. Matsumoto, M. Tachiki, *Thermo Field Dynamics and Condensed States* (North Holland, Amsterdam, 1982)
16. H. Umezawa, *Advanced Field Theory* (American Institute of Physics, New York, 1993)
17. S.M. Barnett, P.L. Knight, Thermofield analysis of squeezing and statistical mixtures in quantum optics. J. Opt. Soc. Am. B **2**, 467–479 (1985)
18. B.J. Dalton, J. Jeffers, S.M. Barnett, *Phase Space Methods for Degenerate Quantum Gases* (Oxford University Press, Oxford, 2015)
19. D.T. Pegg, Atomic spectroscopy: interaction of atoms with coherent fields, in *Laser Physics*. ed. by D.F. Walls, J.D. Harvey (Academic Press, Sydney, 1980)
20. R.J. Glauber, Coherent and incoherent states of the radiation field. Phys. Rev. **131**, 2766–2788 (1963)
21. R.J. Glauber, *Quantum Theory of Optical Coherence* (Wiley, Weinheim, 2007)
22. S. Haroche, J.-M. Raimond, *Exploring the Quantum* (Oxford University Press, Oxford, 2006)
23. P.L. Knight, P.M. Radmore, Quantum origin of dephasing and revivals in the coherent-state Jaynes–Cummings model. Phys. Rev. A **26**, 676–679 (1982)
24. A. Einstein, Zum gegnwärtigen Stand des Strahlungsproblems *Phys. Zeits.* **10**, 185–193 (1909); republished in English translation as "On the present status of the radiation problem" in The collected papers of Albert Einstein, vol. 2 (Princeton University Press, Princeton, 1989)
25. P.W. Milonni, *An Introduction to Quantum Optics and Quantum Fluctuations* (Oxford University Press, Oxford, 2019)
26. R.J. Glauber, The quantum theory of optical coherence. Phys. Rev. **130**, 2529–2539 (1963)
27. L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995)

28. M. Planck, Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum Verhandlungen der Deutschen Physikalischen Gesellschaft Berlin **2**, 237–245 (1900); Published in English translation as "On the theory of the energy distribution law of the normal spectrum", in *The old quantum theory* D. ter Haar ed. (Pergamon, Oxford, 1967) p. 82–90
29. G.J. Milburn, Multimode minimum uncertainty squeezed states. J. Phys. A Math. Gen. **17**, 737–745 (1984)
30. B.R. Mollow, R.J. Glauber, Quantum theory of parametric amplification II. Phys. Rev. **160**, 1097–1108 (1967)
31. S.M. Barnett, B.J. Dalton, Liouville space description of thermofields and their generalisations. J. Phys. A Math. Gen. **20**, 411–418 (1987)
32. M.A. Nielsen, I. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000)
33. S.M. Barnett, *Quantum Information* (Oxford University Press, Oxford, 2009)
34. W.H. Louisell, A. Yariv, A.E. Siegmann, Quantum fluctuations and noise in parametric processes. Phys. Rev. **124**, 1646–1654 (1961)
35. B.R. Mollow, R.J. Glauber, Quantum theory of parametric amplification I. Phys. Rev. **160**, 1067–1096 (1967)
36. W. Israel, Thermo-field dynamics of black holes. Phys. Lett. A **57**, 107–110 (1976)
37. L. Parker, D. Toms, *Quantum Field Theory in Curved Spacetime* (Cambridge University Press, Cambridge, 2009)