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Joint Population and cosmological properties inference with gravitational waves standard sirens and galaxy surveys

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Gravitational wave (GW) sources at cosmological distances can be used to probe the expansion rate of the Universe. GWs directly provide a distance estimation of the source but no direct information on its redshift. The optimal scenario to obtain a redshift is through the direct identification of an electromagnetic (EM) counterpart and its host galaxy. With almost 100 GW sources detected without EM counterparts (dark sirens), it is becoming crucial to have statistical techniques able to perform cosmological studies in the absence of EM emission. Currently, only two techniques for dark sirens are used on GW observations: the spectral siren method, which is based on the source-frame mass distribution to estimate conjointly cosmology and the source's merger rate, and the galaxy survey method, which uses galaxy surveys to assign a probabilistic redshift to the source while fitting cosmology. It has been recognized, however, that these two methods are two sides of the same coin. In this paper, we present a novel approach to unify these two methods. We apply this approach to several observed GW events using the GLADE+ galaxy catalog discussing limiting cases. We provide estimates of the Hubble constant, modified gravity propagation effects, and population properties for binary black holes. We also estimate the binary black hole merger rate per galaxy to be $10^{-6} - 10^{-5} \text{yr}^{-1}$ depending on the galaxy catalog hypotheses.

I. INTRODUCTION

Compact binary coalescences (CBCs) detected via gravitational waves (GWs) are rapidly becoming a central tool to study cosmology. GWs directly provide the luminosity distance of the source, but they are unable to provide the cosmological redshift estimation that is required to measure the cosmic expansion. An estimation of the source redshift could be obtained from the observation of an electromagnetic counterpart (EM) as occurred in the case of the binary neutron star merger GW170817 [1, 2] observed by the LIGO [3] and Virgo [4] GW detectors. However, most of the sources currently detected are binary black holes (BBHs) [5] which are not accompanied by an EM counterpart. In order to exploit this population of “dark sirens” for cosmological studies several methods have been proposed. In this study, we focus on two methods which, however, as we will see, are not independent: the galaxy catalog method and the spectral siren method. Additional methods to assign an implicit redshift from the GW alone include the “cross-correlation method”, which explores the spatial clustering between GW sources and galaxies [6–10], and the “Equation of State method”, which uses the measurement of tidal deformability of neutron stars and knowledge of their Equation of State [11, 12]. For a review on these methods, see [13, 14].

The *galaxy catalog* method, also referred to as the “statistical method”, consists in using galaxy catalogs to statistically assign a host galaxy, hence a redshift, to the GW event [15–17]. This method has been applied in recent studies [18–23] using various galaxy catalogs. One of the main bottlenecks of this method is the fact that at high redshift galaxy catalogs are not complete and therefore could not bring any relevant information on the source redshift. The galaxy catalog method also makes assumptions about the source frame mass distribution that we comment on more in detail later. Another method used in the literature is the *spectral siren* method [24–33] which aims at obtaining a redshift estimate for the GW sources from the relation between source-frame masses and detector-frame masses. In fact, it is possible to obtain a constraint on cosmology by jointly fitting for the population distribution of source-frame masses. The spectral siren method comes with the price of assuming a functional form and some priors for the source-frame mass distribution. In particular, wrong modeling of the population distribution of dark sirens (in particular source-frame masses) can introduce a bias in the estimation of the cosmological parameters [28]. In analogy to the mass method, information on the possible redshift of GW events can also come from the CBC merger rate as a function of redshift. If the CBC merger rate presents local overdensities, such as a global merger rate redshift peak, then one can perform cosmological studies even in the absence of electromagnetic counterparts [34, 35].

As discussed in [28], the galaxy catalog method is in fact a particular extension of the spectral siren method in which

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additional information from galaxy surveys is encoded. The galaxy catalog method requires the definition of a source-frame mass distribution, or more generally a population model for the CBCs. Assumptions about the population of sources typically enter the galaxy catalog method in two ways: (i) they provide a systematic preference among host candidates at different redshift and (ii) they provide a redshift preference for the detected source through the *completeness correction*. The completeness correction accounts for the missing galaxies in the catalog. In fact, in the case that the galaxy catalog is 0% complete (it contains no observed galaxy), the catalog method reduces to the spectral siren method. The tight connection between the spectral siren and the galaxy catalog methods has strong implications in interpreting results from current GW detections. For instance, in [36] it is shown that the constraint on the Hubble constant (H_0) from 42 BBHs reported in the third GW transient catalog (GWTC-3) is dominated by the population assumptions for the distribution of BBHs even when a galaxy catalog is employed.

In this paper, we present a new methodology, which we refer to as the “*galaxy density method*”, that is able to unify the galaxy catalog *and* the spectral siren methods. As a result, it is possible to marginalize over all the uncertainties describing the population model while taking into account information from a galaxy catalog. The core of this new method is to link the CBC merger rate (spectral siren) to the galaxy number density (galaxy catalog).

The paper is organized as follows. In Sec. II we review the basics of hierarchical Bayesian inference in presence of selection biases in the context of GW detections. In Sec. III we introduce a parametrization for the CBC merger rate in terms of galaxy number density, and we discuss limiting cases, including how this new approach collapses to the standard spectral siren method. In Sec. IV, we show some tests and discuss the behavior of the method with the two best localized standard sirens to date, GW170817 and GW190814. In Sec. V we perform a full population and galaxy catalog-based analysis of the 42 BBH detections from GWTC-3 analyzed in [36] and we discuss the obtained results. Finally, in Sec. VI we draw our conclusions.

II. THE HIERARCHICAL LIKELIHOOD

The GW detection of N_{GW} events in a given observing time T_{obs} , collectively represented by some data $\{x\}$, can be described in terms of an inhomogeneous Poissonian process [37, 38] characterised by some parameters Λ by using the following hierarchical likelihood:

$$\mathcal{L}(\{x\}|\Lambda) \propto e^{-N_{\text{exp}}(\Lambda)} \prod_i^{N_{\text{GW}}} T_{\text{obs}} \int d\theta dz \mathcal{L}_{\text{GW}}(x_i|\theta, z, \Lambda_c) \times \frac{1}{1+z} \frac{dN_{\text{CBC}}}{d\theta dz dt_s}(\Lambda). \quad (1)$$

Here N_{exp} is the number of expected CBC detections in T_{obs} , while z and θ indicate the redshift and a set of parameters characteristic of each GW event (such as spins and luminosity dis-

tance), respectively. The central quantity of the hierarchical inference is the CBC merger rate in source-frame time t_s :

$$\frac{dN_{\text{CBC}}}{d\theta dz dt_s}(\Lambda), \quad (2)$$

where $\Lambda = \{\Lambda_p, \Lambda_c\}$ contains population-level parameters Λ_p as well as cosmological parameters Λ_c such as H_0 and Ω_m . The information about how precisely we are able to measure the GW parameters θ (such as spins and source masses) from the data is given by the GW likelihood $\mathcal{L}_{\text{GW}}(x_i|\theta, z, \Lambda_c)$. The expected number of GW events accounts for the GW selection biases:

$$N_{\text{exp}}(\Lambda) = T_{\text{obs}} \int d\theta dz P_{\text{det}}(z, \theta, \Lambda_c) \frac{dN_{\text{CBC}}}{dz d\theta dt_s}(\Lambda) \frac{1}{1+z}, \quad (3)$$

where the detection probability $P_{\text{det}}(z, \theta, \Lambda)$ is defined as:

$$P_{\text{det}}(z, \theta, \Lambda_c) = \int_{x \in \text{detectable}} dx \mathcal{L}_{\text{GW}}(x_i|\theta, z, \Lambda_c). \quad (4)$$

Eq. 1 is formally equivalent to:

$$\mathcal{L}(\{x\}|\Lambda) \propto e^{-N_{\text{exp}}(\Lambda)} [N_{\text{exp}}(\Lambda)]^{N_{\text{GW}}} \times \prod_i^{N_{\text{GW}}} \frac{\int d\theta dz \mathcal{L}_{\text{GW}}(x_i|\theta, z, \Lambda_c) \frac{1}{1+z} \frac{dN_{\text{CBC}}}{d\theta dz dt_s}(\Lambda)}{\int d\theta dz P_{\text{det}}(\theta, z, \Lambda_c) \frac{1}{1+z} \frac{dN_{\text{CBC}}}{d\theta dz dt_s}(\Lambda)}, \quad (5)$$

that can be rewritten as:

$$\mathcal{L}(\{x\}|\Lambda) \propto e^{-N_{\text{exp}}(\Lambda)} [N_{\text{exp}}(\Lambda)]^{N_{\text{GW}}} \times \prod_i^{N_{\text{GW}}} \frac{\int d\theta dz \mathcal{L}_{\text{GW}}(x_i|\theta, z, \Lambda_c) \pi_{\text{pop}}(\theta, z|\Lambda)}{\int d\theta dz P_{\text{det}}(\theta, z, \Lambda_c) \pi_{\text{pop}}(\theta, z|\Lambda)}, \quad (6)$$

by defining the population parameter probability as:

$$\pi_{\text{pop}}(\theta, z|\Lambda) = \frac{1}{C(\Lambda)} \frac{1}{1+z} \frac{dN_{\text{CBC}}}{d\theta dz dt_s}(\Lambda), \quad (7)$$

where $C(\Lambda)$ is a normalization constant. Eq. 6 is the form of the hierarchical likelihood typically reported in [28, 39, 40].

However, from a numerical point of view, Eq. 1 has some advantages in comparison to Eq. 6 [41]: (i) it is more connected to a physical quantity, the CBC merger rate and (ii) it involves the numerical computation of two integrals instead of three. In order to obtain Eq. 6 from Eq. 1, recall that the observing time is given by:

$$T_{\text{obs}} = \frac{N_{\text{exp}}(\Lambda)}{\int d\theta dz P_{\text{det}}(z, \theta, \Lambda_c) \frac{dN_{\text{CBC}}}{dz d\theta dt_s}(\Lambda) \frac{1}{1+z}}, \quad (8)$$

which, after substitution into Eq. 1, gives Eq. 6.

The two forms of the hierarchical likelihood in Eq. 1 and Eq. 6 can be written in a “scale-free” version that does not depend on N_{exp} (but still accounts for GW selection biases). By choosing a prior¹ $\pi(N_{\text{exp}}) \propto 1/N_{\text{exp}}$, it is possible to marginalize analytically over N_{exp} and obtain [42]

$$\mathcal{L}(\{x\}|\Lambda) \propto \prod_i^{N_{\text{GW}}} \frac{\int d\theta dz \mathcal{L}(x_i|\theta, z, \Lambda_c) \frac{dN_{\text{CBC}}(\Lambda)}{dz d\theta dt_s} \frac{1}{1+z}}{\int d\theta dz P_{\text{det}}(\theta, z, \Lambda_c) \frac{dN_{\text{CBC}}(\Lambda)}{dz d\theta dt_s} \frac{1}{1+z}}. \quad (9)$$

¹ Note that a prior $\pi(N_{\text{exp}}) \propto 1/N_{\text{exp}}$ would correspond to a prior $\pi(R_0) \propto 1/R_0$, where R_0 is the CBC merger rate per $\text{Gpc}^{-3}\text{yr}^{-1}$.

The Poissonian process described by the hierarchical likelihoods in Eqs. 1-6-9 can be used to jointly measure the CBC merger rate and cosmological parameters. The fundamental aspect that opens up this possibility is the fact that GW detectors directly measure the luminosity distance d_L to the source and the detector frame masses \vec{m}_d . We recall that, for a flat Λ CDM model, the luminosity distance is:

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}, \quad (10)$$

where $E(z) = \sqrt{\Omega_m(1+z)^3 + (1-\Omega_m)}$. Furthermore, detector masses are linked to source masses through $\vec{m}_d = \vec{m}_s(1+z)$. In [24–29, 31–33] it was shown that by modeling the CBC merger rate in terms of source-frame masses, it is possible to obtain a joint measurement of both the CBC merger rate and the cosmological parameters. Similarly, in [34, 35] it was demonstrated that if the CBC merger rate shows some features in redshift, such as peaks, this might help to measure cosmological parameters even without any information about the source-frame masses. In this work, we will apply Eq. 6 and Eq. 9 either to measure the Hubble constant only, or to jointly measure H_0 and the population parameters related to the mass and redshift models used (see App. A). In the rest of this paper, we assume a flat Λ CDM cosmology model with free H_0 and fixed $\Omega_m = 0.308$, with $\Omega_\Lambda = 1 - \Omega_m$.

III. CBC MERGER RATE PARAMETRIZATIONS

In Sec. II we have seen that the relevant GW parameters which are crucial for GW cosmology are the source-frame masses \vec{m}_s and the redshift z . However, when working with galaxy catalogs also the sky position, which we label by Ω , is a central quantity since it can help us to identify possible host galaxies along a given line-of-sight. Therefore, in the remainder of this paper, we will work with a CBC merger rate written as:

$$\frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s}(\Lambda). \quad (11)$$

Note that Eq. 11 is a CBC source-frame merger rate per source-frame mass, per GW parameters θ , per sky position Ω , per redshift z . To simplify the notation, we will refer to this quantity as the ‘‘CBC merger rate’’, unless we want to discuss an explicit parametrization of it. For brevity, we will also drop the Λ dependence.

A. The CBC spectral siren parametrization

In standard cosmological-population analyses such as [28, 41], which do not make explicit use of galaxy surveys, the CBC merger rate is always factorized in terms of comoving volume rather than the redshift, since it is easier to model and also easier to motivate physically. The CBC merger rate thus

becomes:

$$\begin{aligned} \frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s} &= \frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta dV_c dt_s} \frac{dV_c}{dz d\Omega}(\Lambda_c) \\ &= R_0 \psi(z; \Lambda) p_{\text{pop}}(\vec{m}_s, \theta | z, \Lambda) \frac{dV_c}{dz d\Omega}, \end{aligned} \quad (12)$$

where R_0 is the CBC merger rate per comoving volume at redshift $z = 0$, $\psi(z; \Lambda)$ is a phenomenological parametrization for the CBC merger rate as a function of redshift such that $\psi(z = 0; \Lambda) = 1$, p_{pop} is a probability density function describing the distribution of GW sources in masses and other parameters θ , and

$$\frac{dV_c}{dz d\Omega} = \left[\frac{c}{H_0} \right]^3 \left[\int_0^z \frac{dz'}{E(z')} \right]^2 \quad (13)$$

is the differential of the comoving volume. Typically, it is assumed that p_{pop} in Eq. 12 is independent of redshift, *i.e.* the distribution of masses and GW population parameters does not evolve with redshift. This is indeed a simplifying assumption as source mass distributions might depend on redshift [43, 44] as well as spin parameters [45, 46]. This assumption, if wrong, could introduce a bias in the estimation of the cosmological parameters. However, it has been shown in [32] that for a mild evolution one should still be able to obtain unbiased estimation of H_0 .

B. The CBC merger rate informed by galaxy surveys

We now proceed to derive the CBC merger rate encoding information from the galaxy catalog. To construct this CBC merger rate, we will make two assumptions: (*i*) mergers can only occur inside galaxies and (*ii*) mergers can be more or less frequent according to the absolute magnitude M (or luminosity) and redshift of the galaxy. In this paper, we will mostly use galaxies observed in the K-band, but also other colors could be used, in particular if they trace the binary merger rate in redshift. We can write the overall CBC merger rate as the integral over the absolute magnitude of the CBC rate per galaxy absolute magnitude, namely:

$$\frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s} = \int dM \frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s dM}. \quad (14)$$

Similar to the spectral siren case (see Eq. 12), we parametrize the CBC rate per galaxy (rather than comoving volume), namely:

$$\frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s dM} = \frac{dN_{\text{CBC}}}{dN_{\text{gal}} d\theta d\vec{m}_s dt_s} \frac{dN_{\text{gal}}}{dz d\Omega dM}. \quad (15)$$

In order to write Eq. 15, we have further assumed that the number of galaxies only depends on sky position, absolute magnitude, and redshift. The first term in Eq. 15 is the CBC rate per galaxy which, similarly to Eq. 12, we parametrize as:

$$\frac{dN_{\text{CBC}}}{dN_{\text{gal}} d\vec{m}_s d\theta dt_s} = R_{\text{gal},0}^* \Psi_L(z, M; \Lambda_p) p_{\text{pop}}(\vec{m}_s, \theta | z, M, \Lambda_p), \quad (16)$$

where $R_{\text{gal},0}^*$ is the CBC rate per year at a reference galaxy luminosity (or absolute magnitude) $L_* = L(M_*)$, while $\Psi_L(z, M; \Lambda_p)$ is an evolution function for the CBC merger rate per galaxy such that $\Psi_L(z = 0, M = M_*; \Lambda_p) = 1$. In Eq. 16 p_{pop} has been expressed in a general form assuming that the probability of having some GW parameters might be dependent on the galaxy redshift and absolute magnitude. For the analysis presented in this paper, we will not consider this possibility and we will choose $p_{\text{pop}}(\vec{m}_s, \theta|z, M, \Lambda_p) = p_{\text{pop}}(\vec{m}_s, \theta|\Lambda_p)$. Regarding the rate function $\Psi_L(z, M; \Lambda_p)$ we use the following parametrization:

$$\Psi_L(z, L(M); \Lambda_p) = \psi(z; \Lambda_p) \left\| \frac{L}{L_*} \right\|^\epsilon = \psi(z; \Lambda_p) 10^{0.4\epsilon(M_* - M)}, \quad (17)$$

i.e. we factorize the rate evolution function into a redshift-dependent part and a factor that depends on the luminosity of the galaxy. Similarly to Eq. 12, the redshift part $\psi(z; \Lambda_p)$ is modeled to track the merger rate as a function of redshift, while the luminosity part models the fact that the merger rate might depend on the luminosity of the galaxy (which could

be a tracer of the Star Formation Rate [47]). As an example, the parameter $\epsilon > 0$ indicates that more luminous galaxies are more likely to host CBCs, while $\epsilon = 0$ indicates that there is no such preference. In principle, one could also treat ϵ as a population parameter and jointly fit also for it. However, considering ϵ as a free parameter poses some numerical challenges because when trying different values of ϵ , the galaxy catalog must be handled multiple times.

A good choice for the reference luminosity L_* is the “knee” luminosity of the Schechter function. The Schechter function describes the number density of galaxies in the comoving volume per luminosity (or absolute magnitude). In terms of absolute magnitude, it is identified by five parameters: a galaxy number density ϕ_* per comoving volume, a knee absolute magnitude M_* , a faint end M_{max} , a bright end M_{min} , and a slope parameter α . Note that in practice the bright end choice is not important as the Schechter function quickly drops to 0 after $M_*(H_0)$. However, here we write it explicitly for a pedagogic derivation. We can thus write the Schechter function as:

$$\text{Sch}(M; \phi_*, M_*, M_{\text{max}}, M_{\text{min}}, \alpha) = \begin{cases} 0.4 \ln(10) \phi_* 10^{0.4(\alpha+1)(M_* - M)} \exp[-10^{0.4(M_* - M)}], & M_{\text{min}} \leq M \leq M_{\text{max}}, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Usually, Schechter parameters are provided for $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $h = 1$, so one typically needs to rescale them. The scaling relations are given in terms of absolute magnitude by:

$$\begin{aligned} M_*(h) &= M_*(h = 1) + 5 \log_{10} h, & (19) \\ \phi_*(h) &= \phi_*(h = 1) h^3. & (20) \end{aligned}$$

Note that in writing down the Schechter function, we have implicitly assumed that it does not evolve in redshift. This is a simplification as we can measure for some colors an evolution of the Schechter function in redshift (see *e.g.* [48]). As we will see later, the modeling of the Schechter function is crucial in the case that the galaxy catalog is not complete, as it enters the completeness correction. A wrong modelization of the Schechter function evolution might result in a wrong modelization of the CBC merger rate as a function of redshift (as mergers are expected to track galaxy density). This will translate to bias on H_0 . In order to understand the magnitude of the bias on H_0 , and how this interacts with other population parameters, we would require a dedicated mock data challenge which is beyond the scope of this paper.

We now discuss how to build the number density of galaxies in Eq. 15. In the limit in which we are provided with a galaxy survey containing all the galaxies in the Universe, this term can be constructed as a sum of delta functions at the redshift of each galaxy, sky position, and absolute magnitude. However, galaxy catalogs are not complete and therefore the galaxy density must be corrected to include the missing galaxies. In other words, the overall galaxy density can be calcu-

lated as the sum of a galaxy density detected by the catalog and a completeness correction, *i.e.*

$$\frac{dN_{\text{gal}}}{dz d\Omega dM} = \frac{dN_{\text{gal,cat}}}{dz d\Omega dM} + \frac{dN_{\text{gal,out}}}{dz d\Omega dM}. \quad (21)$$

The overall CBC merger rate will be then given by plugging the galaxy density distribution, Eq. 21, and the model of the CBC rate per galaxy, Eq. 17, into Eq. 16, thus obtaining:

$$\begin{aligned} \frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s dM} &= R_{\text{gal},0}^* p_{\text{pop}}(\vec{m}_s, \theta|\Lambda_p) \psi(z; \Lambda_p) \times \\ &\times 10^{0.4\epsilon(M_* - M)} \left[\frac{dN_{\text{gal,cat}}}{dz d\Omega dM} + \frac{dN_{\text{gal,out}}}{dz d\Omega dM} \right]. \end{aligned} \quad (22)$$

Note that in order to compute the hierarchical likelihood in Eq. 1 we need to marginalize out the absolute magnitude from Eq. 22, as GW detectors are not able to measure this quantity. In fact, we will need to calculate:

$$\begin{aligned} \frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s} &= R_{\text{gal},0}^* p_{\text{pop}}(\vec{m}_s, \theta|\Lambda_p) \psi(z; \Lambda_p) \times \\ &\times \int_{M_{\text{min}}}^{M_{\text{max}}} dM 10^{0.4\epsilon(M_* - M)} \left[\frac{dN_{\text{gal,cat}}}{dz d\Omega dM} + \frac{dN_{\text{gal,out}}}{dz d\Omega dM} \right]. \end{aligned} \quad (23)$$

1. The completeness correction

The second term in Eq. 21, which is the completeness correction, or “out-of-catalog” term [21, 22], can be written as

the background galaxy density times the probability p_{miss} to miss a galaxy at redshift z , absolute magnitude M , and sky position Ω , namely:

$$\frac{dN_{\text{gal,out}}}{dzd\Omega dM} = \frac{dN_{\text{gal,bg}}}{dzd\Omega dM} p_{\text{miss}}(z, \Omega, M). \quad (24)$$

The background galaxy density can be defined in terms of Schechter function with slope α as:

$$\frac{dN_{\text{gal,bg}}}{dzd\Omega dM} = \text{Sch}(M; \alpha) \frac{dV_c}{dzd\Omega}, \quad (25)$$

where for brevity we just write $\text{Sch}(M; \alpha)$ to indicate the Schechter function. Moving to p_{miss} , we recall that usually galaxy catalogs are flux-limited, *i.e.* a galaxy is detected if its apparent magnitude m is brighter than a given threshold m_{thr} . Note that m_{thr} could be sky-angle dependent to indicate that the galaxy survey is not able to cover in the same way all the sky, *e.g.* a blind region would be identified by $m_{\text{thr}} = -\infty$. Therefore, we can model p_{miss} as an Heaviside step function, $p_{\text{miss}} = \Theta[(M > M_{\text{thr}}(z, m_{\text{thr}}(\Omega), H_0))]$.

We can now use the out-of-catalog part to calculate its contribution to the CBC merger rate in Eq. 23. In doing so we first note that the integral of the Schechter function multiplied by the luminosity weight can be written as:

$$\begin{aligned} & \int_{M_{\text{min}}}^{M_{\text{max}}} dM \text{Sch}(M; \alpha) 10^{0.4\epsilon(M_*(H_0)-M)} = \\ & = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \text{Sch}(M; \alpha + \epsilon) \\ & = \phi_*(H_0) \Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{min}}, x_{\text{max}}), \end{aligned} \quad (26)$$

where Γ_{inc} is the incomplete gamma function, while in the last step of the above equation, we have changed variable to $x = 10^{0.4(M_*(H_0)-M)}$ and $x_{\text{min/max}} = 10^{0.4(M_*(H_0)-M_{\text{min/max}})}$. Bearing in mind the result obtained in Eq. 26 and using Eq. 24 we can define an “effective” galaxy density per steradian per redshift which are CBC emitters:

$$\begin{aligned} \frac{dN_{\text{gal,out}}^{\text{eff}}}{dzd\Omega} &= \int_{M_{\text{min}}(H_0)}^{M_{\text{max}}(H_0)} dM 10^{0.4\epsilon(M_*(H_0)-M)} \frac{dN_{\text{gal,out}}}{dzd\Omega dM} \\ &= \int_{M_{\text{thr}}(H_0, m_{\text{thr}}(\Omega), z)}^{M_{\text{max}}(H_0)} dM \text{Sch}(M; \alpha + \epsilon) \frac{dV_c}{dzd\Omega} \\ &= \phi_*(H_0) \Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}, x_{\text{thr}}) \frac{dV_c}{dzd\Omega}, \end{aligned} \quad (27)$$

where $x_{\text{thr}} = 10^{0.4(M_*(H_0)-M_{\text{thr}}(H_0))}$. The bright end of the integration is given by the absolute magnitude threshold, which is computed from the sky-dependent apparent magnitude threshold as a function of redshift and H_0 . Before explaining the physical sense of the above result, let us note that Eq. 27 is not actually dependent on H_0 . On one hand, $\phi_*(H_0) \propto H_0^3$, while $\frac{dV_c}{dzd\Omega} \propto H_0^{-3}$, therefore the H_0 -dependence will cancel. On the other hand, the incomplete gamma function depends on the slope parameters α and ϵ (which are cosmology-independent) and on the parameter x . The parameter x is

function of $M_*(H_0) - M_{\text{max/thr}}(H_0)$, and this difference is H_0 -independent as the two absolute magnitudes will scale in the same way with H_0 .

Eq. 27 can be thought of as an “effective” density of galaxy per steradian per redshift, which are the CBC emitters that are missing from the catalog. As an example, if the catalog is 0% complete (no galaxies are present and $M_{\text{thr}} = -\infty$) and no luminosity weight is applied ($\epsilon = 0$), then Eq. 27 will simply reduce to the integral of the Schechter function times the differential of the comoving volume, which is by definition the number density of galaxies per redshift per steradian. If the galaxy catalog is 100% complete (all the galaxies are present and $M_{\text{thr}} = \infty$), then the integral in Eq. 27 will evaluate to 0 as by definition the Schechter function is null for M fainter than M_{max} .

Using Eq. 27 we can also give a more formal definition of “incompleteness”. This can be defined as the fraction of galaxies that are CBC emitters that we think are not present in the catalog part. More formally, we can define the incompleteness $\mathcal{I}_{\text{CBC}}(z, \Omega)$ as:

$$\begin{aligned} \mathcal{I}_{\text{CBC}}(z, \Omega) &= \frac{\int_{M_{\text{thr}}(H_0, m_{\text{thr}}(\Omega), z)}^{M_{\text{max}}(H_0)} dM \text{Sch}(M; \alpha + \epsilon) \frac{dV_c}{dzd\Omega}}{\int_{M_{\text{min}}}^{M_{\text{max}}(H_0)} dM \text{Sch}(M; \alpha + \epsilon) \frac{dV_c}{dzd\Omega}} \\ &= \frac{\Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}, x_{\text{thr}}(\Omega, z))}{\Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}, x_{\text{min}})}. \end{aligned} \quad (28)$$

In analogy with the discussion in the previous paragraph, we note that the incompleteness is also a quantity that does not depend on H_0 .

2. The catalog part

We now calculate the “in-catalog” part of Eq. 23, which is built starting from the galaxies present in the catalog. The discussion in this section follows the same logic as presented in [49]. In analogy to what we discussed in the previous section, even for the catalog part, we can define an effective number of CBC emitters:

$$\begin{aligned} \frac{dN_{\text{gal,cat}}^{\text{eff}}}{dzd\Omega} &= \\ &= \int_{M_{\text{min}}}^{M_{\text{thr}}(z, m_{\text{thr}}(\Omega), H_0)} dM 10^{0.4\epsilon(M_*(H_0)-M)} \frac{dN_{\text{gal,cat}}}{dzd\Omega dM}. \end{aligned} \quad (29)$$

The integral in Eq. 29 is limited up to a faint end identified by $M_{\text{thr}}(z, m_{\text{thr}}(\Omega), H_0)$, since by definition we can not detect galaxies that are fainter than a given threshold value (which are accounted for by the completeness correction). Since galaxy surveys usually provide lists of galaxy redshifts, sky locations, and apparent magnitudes, it is more convenient to express the above integral in terms of apparent magnitudes m as:

$$\begin{aligned} \frac{dN_{\text{gal,cat}}^{\text{eff}}}{dzd\Omega} &= \\ &= \int_{m_{\text{min}}}^{m_{\text{thr}}(\Omega)} dm 10^{0.4\epsilon(M_*(H_0)-M(H_0, m, z))} \frac{dN_{\text{gal,cat}}}{dzd\Omega dm}. \end{aligned} \quad (30)$$

The galaxy number density can be built from the observed list of galaxies as the product of the total number of galaxies observed in the catalog and the probability of finding a galaxy at redshift z , sky position Ω , and apparent magnitude m , given our observation of the catalog (labeling by C the observed quantities):

$$\frac{dN_{\text{gal,cat}}}{dzd\Omega dm} = N_{\text{gal}} p_{\text{gal}}(z, \Omega, m|C). \quad (31)$$

If the galaxy catalog provides us with perfectly measured redshift, sky position, and apparent magnitudes, $p_{\text{gal}}(z, \Omega, m|C)$ would simply be a sum of delta functions centered at the galaxy redshift, sky position, and apparent magnitude. However, we are not usually able to measure perfectly some of these quantities, in particular redshift, so we need to account for this.

Following [49], the probability of finding a galaxy at redshift z , sky position Ω , and apparent magnitude m given the observed survey can be written as:

$$\begin{aligned} p_{\text{gal}}(z, \Omega, m|C) &= \int d\{z\}_{\text{gal}} d\{\Omega\}_{\text{gal}} d\{m\}_{\text{gal}} \times \\ &\times p_{\text{loc}}(z, \Omega, m|\{z\}_{\text{gal}}, \{\Omega\}_{\text{gal}}, \{m\}_{\text{gal}}, C) \times \\ &\times p_{\text{cat}}(\{z\}_{\text{gal}}, \{\Omega\}_{\text{gal}}, \{m\}_{\text{gal}}|C). \end{aligned} \quad (32)$$

In Eq. 32 $\{z_{\text{gal}}\}$, $\{\Omega_{\text{gal}}\}$, and $\{m_{\text{gal}}\}$ indicate a set of *true* redshifts, sky positions, and apparent magnitudes for all the N_{gal} galaxies reported in the survey. The term p_{loc} is the probability of having a galaxy at z, Ω, m once we know the true $z_{\text{gal}}, \Omega_{\text{gal}},$ and m_{gal} of all the other galaxies. This probability does not actually depend on the galaxy catalog and is simply:

$$\begin{aligned} p_{\text{loc}}(z, \Omega, m|\{z\}_{\text{gal}}, \{\Omega\}_{\text{gal}}, \{m\}_{\text{gal}}) &= \\ &= \frac{1}{N_{\text{gal}}} \sum_{j=1}^{N_{\text{gal}}} \delta(z - z_{\text{gal}}^j) \delta(\Omega - \Omega_{\text{gal}}^j) \delta(m - m_{\text{gal}}^j). \end{aligned} \quad (33)$$

The term p_{cat} represents the probability that the galaxies have their true values of redshift, magnitude, and sky location of the galaxies $\{z_{\text{gal}}\}$, $\{\Omega_{\text{gal}}\}$, and $\{m_{\text{gal}}\}$, given the *observed* galaxy survey. Generally, the galaxy catalog is made of a collection of galaxies, each of them observed *independently* and reported with *observed* apparent magnitude, redshift, and sky location $\{m\}_{\text{obs}}, \{\Omega\}_{\text{obs}}, \{z\}_{\text{obs}}$ and typical uncertainties $\{\sigma_m\}_{\text{obs}}, \{\sigma_\Omega\}_{\text{obs}}, \{\sigma_z\}_{\text{obs}}$. Given the *observed* quantities C , the probability that a given galaxy j has a *true* redshift z_{gal}^j , sky location Ω_{gal}^j , and apparent magnitude m_{gal}^j can be written as:

$$\begin{aligned} p(z_{\text{gal}}^j, \Omega_{\text{gal}}^j, m_{\text{gal}}^j|C) &= p(z_{\text{gal}}^j|z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j) \times \\ &\times p(\Omega_{\text{gal}}^j|\Omega_{\text{obs}}^j, \sigma_{\Omega,\text{obs}}^j) \times \\ &\times p(m_{\text{gal}}^j|m_{\text{obs}}^j, \sigma_{m,\text{obs}}^j), \end{aligned} \quad (34)$$

and since galaxies are observed independently from each other, we can write:

$$\begin{aligned} p_{\text{cat}}(\{z\}_{\text{gal}}, \{\Omega\}_{\text{gal}}, \{m\}_{\text{gal}}|C) &= \prod_{j=0}^{N_{\text{gal}}} p(z_{\text{gal}}^j|z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j) \times \\ &\times p(\Omega_{\text{gal}}^j|\Omega_{\text{obs}}^j, \sigma_{\Omega,\text{obs}}^j) p(m_{\text{gal}}^j|m_{\text{obs}}^j, \sigma_{m,\text{obs}}^j). \end{aligned} \quad (35)$$

By substituting Eqs. 33 and 35 into Eq. 32, we find that the reconstructed galaxy number density is:

$$\begin{aligned} \frac{dN_{\text{gal,cat}}}{dzd\Omega dm} &= \sum_j^{N_{\text{gal}}(\Omega)} p(z|z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j) p(\Omega|\Omega_{\text{obs}}^j, \sigma_{\Omega,\text{obs}}^j) \times \\ &\times p(m|m_{\text{obs}}^j, \sigma_{m,\text{obs}}^j). \end{aligned} \quad (36)$$

We now use Eq. 36 to calculate the effective CBC emitters hosts in Eq. 30. If we now assume that the apparent magnitude and sky position of each galaxy are measured perfectly (this is a reasonable assumption for galaxy surveys), and that the sky is divided in equal-size pixels of area $\Delta\Omega$, we can write the effective CBC emitters hosts in a given sky direction as:

$$\begin{aligned} \frac{dN_{\text{gal,cat}}^{\text{eff}}}{dzd\Omega} &\approx \frac{1}{\Delta\Omega} \sum_j^{N_{\text{gal}}(\Omega)} 10^{0.4\epsilon(M_*(H_0) - M(H_0, m_j, z))} \times \\ &\times p(z|z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j), \end{aligned} \quad (37)$$

where the sum is only on galaxies in a given sky direction. Eq. 37 is the relation needed to calculate the effective number of CBC hosts from the galaxy survey. The quantity $p(z|z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j)$ is the distribution of what true redshift we expect the galaxy to have, namely

$$p(z|z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j) \propto \mathcal{N}(z_{\text{obs}}^j, \sigma_{z,\text{obs}}^j) \frac{dV_c}{dz}. \quad (38)$$

where \mathcal{N} is a Gaussian. In Eq.38, we have assumed that the galaxy redshift is measured from the catalog with gaussian uncertainty and that we apply a prior proportional to $\frac{dV_c}{dz}$ [49].

We now make the following remarks. Eq. 37 does not depend on H_0 for the same motivations explained for the out-of-catalog part, see the discussion below Eq. 27. Therefore, Eq. 37 can be computed only once for each observed galaxy catalog, thus decreasing significantly the computational cost of the analysis. Strictly speaking, as discussed in [49], in principle, the *true* redshift of each galaxy should be treated as a ‘‘population’’ parameter for the hierarchical inference. In other words, in the limit that the galaxy redshifts are not observed perfectly, the effective density of CBC hosts can not be reconstructed as a sum of redshift probability density distributions centered around the redshift values provided by the galaxy catalog. However, in the limit that the number density of observed CBC events is much lower than the number density of CBC hosts, Eq. 37 should still be valid to perform the hierarchical inference. Finally, we adopt also the same color corrections used in [36] to calculate the absolute magnitude for the apparent magnitude.

IV. TESTS WITH WELL-LOCALIZED DARK SIRENS

In this section, we provide some working examples of the methodology discussed in the previous sections using the well-localized standard sirens GW170817 [1] and GW190814 [50]. We use the GLADE+ galaxy catalog [51] and the galaxy luminosities reported for the infrared K-band. GLADE+ already

reports redshift values which are corrected for galaxies peculiar motion. Additionally, we consider the peculiar motion uncertainties addition by summing it in quadrature to the reported spectroscopic or photometric redshift uncertainty. In this section we apply Eq. 9 to infer H_0 only, while we fix all the other population parameters (see App. A for more details).

To compute the completeness correction in Eq. 27, we adopt a Schechter function specified by the parameters $M_{\min} = -27.85$, $M_{\max} = -19.84$, $\alpha = -1.09$, $\phi_* = 0.03 \text{ Mpc}^{-3}$ (for $H_0 = 67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$). To calculate the effective hosts' number from the galaxy catalog in Eq. 37, we subdivide the sky into 49152 pixels² and a choice of equal area of 0.8 deg^2 . The choice of the pixel size is driven by the fact that it should be smaller than the localization areas of GW190814, which is 18.5 deg^2 [50]. While for GW170817, since we will fix the sky position, we should take a pixel size large enough to contain the galaxies' localization errors and angular size.

To calculate the apparent magnitude threshold, we follow the same procedure as in [36]. The apparent magnitude threshold is computed all over the sky using equal-sized pixels of 3.35 deg^2 . On one hand, the pixel size for the calculation of the apparent magnitude threshold should be small enough to appreciate the different characteristics of the surveys composing GLADE+. On the other hand, the pixel size should be high enough to contain a reasonable number of galaxies to calculate the apparent magnitude threshold. The apparent magnitude threshold is defined as the median of the apparent magnitude of the galaxies reported in each pixel. This is an over-conservative threshold that is applied to avoid any systematic that could be potentially introduced by galaxy catalogs selection biases. We find that in a pixel of 3.35 deg^2 the median number of galaxies present is 80, which is enough to calculate an apparent magnitude threshold. All the galaxies fainter than the apparent magnitude threshold are removed from the calculation of the CBC host redshift density.

A. GW170817 as quasi-dark siren: the pencil beam approach

So far, GW170817 is the only GW source for which it was possible to detect an associated electromagnetic counterpart and identify its host galaxy: NGC4993 [52]. GW170817 is also the only standard siren observed with EM counterpart. Using the GW luminosity distance estimation and the redshift of NGC4993 it was possible to estimate $H_0 = 70_{-8}^{+19} \text{ km s}^{-1} \text{ Mpc}^{-1}$ [53, 54]. An analysis of GW170817 as “dark siren” was also performed in [55] with the GLADE [56] catalog using B-band luminosities, obtaining $H_0 = 77_{-18}^{+37} \text{ km s}^{-1} \text{ Mpc}^{-1}$ [55].

Here, we employ GW170817 as a “quasi-dark siren”, namely, we assume the sky position to be perfectly known but the host galaxy not identified. This is type of methodology is also referred to as the “pencil beam” approach [19].

We made this choice in order to show the connection between the framework presented in Sec. III and the various

TABLE I. Galaxies with K-band luminosities from the GLADE+ galaxy catalog reported in the 0.8 deg^2 pixel of GW170817. From left to right columns, the reported values are observed redshift, redshift uncertainty (including peculiar motion), apparent magnitude in the K-band, and absolute magnitude calculated for a Planck15 [58] cosmology. The first entry is NGC4993, the host of GW170817. The apparent magnitude threshold for this particular pixel is $m_{\text{thr}} = 13.68$.

z_{obs}	$\sigma_{z,\text{obs}}$	m_K	M_K
0.0092	0.0005	9.3	-23.7
0.0301	0.0148	13.0	-22.6
0.0423	0.0014	12.9	-23.4
0.0613	0.0153	13.5	-23.7
0.0709	0.0152	13.0	-24.4
0.1207	0.0151	13.4	-25.1
0.1505	0.0151	13.4	-25.6

physical quantities involved. For this study, we use the parameter estimation (PE) samples from the IMRPHENOMPv2NRT waveform model with low spin priors³ released for GWTC-1 [57]. The PE samples are generated by fixing GW170817 sky location to the one its EM counterpart. We take all the galaxies falling within the 0.8 deg^2 pixel of GW170817's counterpart, for which we obtain an apparent magnitude threshold of $m_{\text{thr}} = 13.68$ (defined as the median of the apparent magnitude threshold of all the galaxies reported in the pixel). After removing all the galaxies fainter than m_{thr} from GLADE+, we obtain only 7 galaxies with redshift spanning from 0.0092 to 0.15 in the pixel of GW170817. The observed redshifts and luminosities are reported in Tab. I, where the first row of the table is NGC4993, the host of GW170817. We can immediately notice that farther galaxies display a lower (brighter) absolute magnitude. This is due to the presence of a selection bias, for which at high redshift we can observe only the brightest galaxies, and which is accounted using the completeness correction.

Using the galaxies reported in Tab. I and the apparent magnitude threshold previously computed, we calculate the effective CBC hosts density present in the catalog (Eq. 37) and their completeness correction (Eq. 27). The top panels of Fig. 1 show a comparison between the effective number density of CBC hosts “in” and “out” of the galaxy catalog. The figure displays two cases: one where we assume that the CBC merger rate does not depend on the intrinsic galaxy luminosity ($\epsilon = 0$, left column), and one where we assume the CBC merger rate is proportional to the galaxy luminosity ($\epsilon = 1$, right column). Importantly, we note that in the $\epsilon = 1$ case, the effective number of CBC hosts in the catalog is lower. This is because in this model galaxies fainter than M_* , which are the majority of galaxies according to the Schechter function, host fewer GW sources. We also note that in the $\epsilon = 1$ model, the CBC completeness of the galaxy catalog (second row in Fig. 1) is higher since the brightest galaxies included in the catalog are the majority fraction of the CBC emitters. Finally, from Fig. 1 we notice that from the in-catalog part, only two

² calculated with healpy and a choice of nside=64

³ <https://dcc.ligo.org/LIGO-P1800370/public>

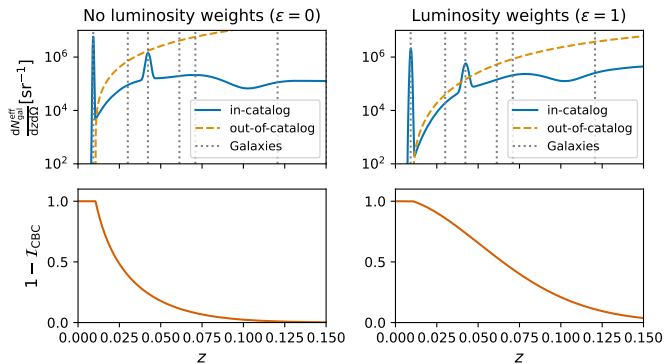


FIG. 1. *Top row*: Effective number of in-catalog (solid lines) and out-of-catalog (dashed line) CBC emitters per redshift per steradian for GW170817 as a quasi-dark siren. The blue solid line is the contribution from the galaxy catalog, while the orange dashed line is the completeness correction. The vertical dotted lines indicate the observed redshift values reported in Tab. I. *Bottom row*: CBC completeness $1 - I_{\text{CBC}}$, with I_{CBC} given by Eq. 28. The luminosity weights are assigned using a probability $p(L|\epsilon) \propto L^\epsilon$.

The first column is generated using a CBC emission rate independent of galaxy luminosity ($\epsilon = 0$), while the second is generated using a rate linearly proportional to the galaxy luminosity ($\epsilon = 1$).

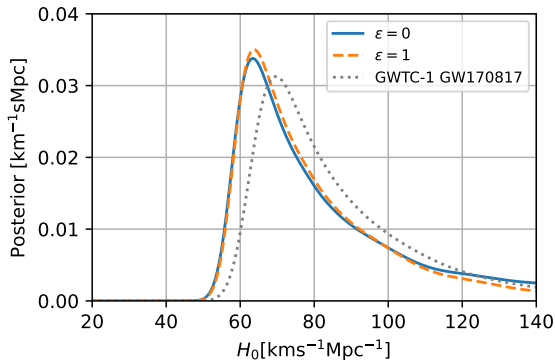


FIG. 2. Hubble constant posterior distribution obtained for GW170817 as quasi-dark siren. The blue solid and orange dashed lines indicate the posterior obtained using a CBC merger rate independent of the galaxies’ luminosity and proportional to it, respectively. The luminosity weights are assigned using a probability $p(L|\epsilon) \propto L^\epsilon$.

The gray dotted line is the posterior of GW170817 obtained assuming the identification of the host galaxy with $z = 0.0100 \pm 0.0005$ [59].

galaxies can be well localized in redshift and provide a redshift scale (the first peak is NGC4993). The other galaxies have large redshift uncertainties (cf. Tab. I) and will not significantly contribute to the measure.

We use the CBC host densities shown in Fig. 1 to calculate the CBC rate in Eq. 23, which is then used to evaluate the scale-free hierarchical likelihood in Eq. 9 and obtain a posterior distribution on the value of H_0 . We consider a single

population model for CBC rates in terms of masses and redshift. In App. A we provide more details about the CBC rate models for masses and redshift used, as well as the corrections of selection biases. Fig. 2 shows the H_0 posterior obtained for the different cases for which we computed the in-catalog and out-of-catalog CBC hosts densities. We observe that, for GW170817, the H_0 measure is nearly independent of the choice of using or not luminosity weights. This is a consequence of the fact that the only structure in the CBC host density contributing to the H_0 estimation is the one associated with NGC4993. Compared to the posterior obtained by the LIGO-Virgo Collaboration using GW posterior samples from GWTC-1 [59], we obtain an H_0 posterior slightly narrower with a maximum a posteriori shifted to smaller values. This is due to the fact that in GLADE+, NGC4993 redshift is slightly lower ($z = 0.0092 \pm 0.0005$) than the value of $z = 0.0100 \pm 0.0005$ used for the analysis in [59]. Moreover, in the pencil-beam approach, the H_0 inference gets some contribution from galaxies not well localized at high redshifts.

Moreover, in our analysis, we also have the contribution from galaxies at higher redshifts which are not well localized (see Fig. 1): this introduces a slight preference for lower values of H_0 .

B. GW190814 as a typical dark siren

GW190814 [50] is one of the best localized GW events so far, second only to GW170817. The 90% credible interval sky area of GW190814 covers about 20 deg^2 and its localization volume (assuming the Planck15 cosmology [58]) is about 900 Mpc^3 [36]. Previous H_0 studies for this event using the galaxy catalog method with the GLADE (B-band) and the Dark Energy Survey [60] found a posterior with a maximum a posteriori in the Hubble constant tension region. However, it was not possible to constrain the H_0 value in the prior range of $[20, 140] \text{ km s}^{-1} \text{ Mpc}^{-1}$. One of the peculiar aspects of this event is that the mass of the lighter object falls between the expected gap between neutron stars and black hole masses.

In our test case, we classify GW190814 as a massive neutron star black hole merger. Note that, when considering cosmology unknown we are intrinsically changing the source frame mass of the secondary object, so one should use a population model that includes neutron stars and black holes altogether. This is due to the fact that the secondary object might end up in the neutron star or black hole mass region depending on the cosmological model. However, we do not expect results for this event to strongly depend on the population model of the neutron star masses. For the prior extremes of H_0 assumed, this event would change its source frame mass from a median value of $2.6 M_\odot$ (for Planck cosmology),⁴ to a range of $2.46 M_\odot - 2.67 M_\odot$ (depending on which H_0 extreme we are using). This range is well between the “massive” neutron star region and it is also smaller than the typical statistical uncertainties.

⁴ value taken from <https://gwosc.org/eventapi/html/all/events/>

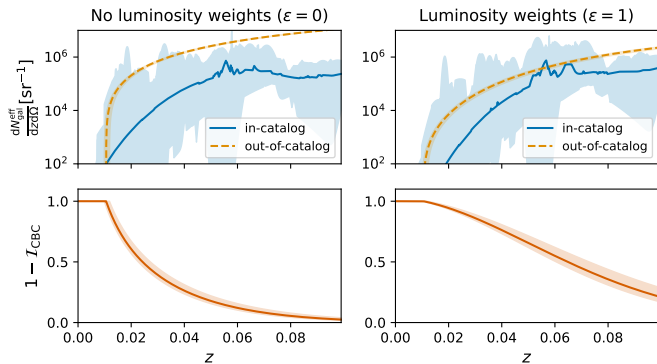


FIG. 3. *Top row*: Effective number of in-catalog (solid lines) and out-of-catalog (dashed line) CBC emitters per redshift per steradian in the 90% credible interval sky area of GW190814. *Bottom row*: CBC completeness $1 - I_{\text{CBC}}$, with I_{CBC} given by Eq. 28. The shaded areas indicate the contours identified by the 90% credible sky area of GW190814, while the lines correspond to the median values. The luminosity weights are assigned using a probability $p(L|\epsilon) \propto L^\epsilon$.

The first column is generated using a CBC emission rate independent of galaxy luminosity ($\epsilon = 0$), while the second is generated using a rate linearly proportional to the galaxy luminosity ($\epsilon = 1$).

The population model we fix for neutron star black hole mergers is described in App. A. For the GW190814 PE samples, we use the ‘‘Publication Samples’’⁵ that are a combination of the samples obtained with the two IMRPHENOM and EOB waveform families released with GWTC-2 [61].

Consistently with [36], we find that GW190814’s 90% credible sky localization contains 90 galaxies with K-band luminosities reported in GLADE+, while we find a median apparent magnitude threshold equal to $m_{\text{thr}} = 13.55$. Fig. 3 displays the effective number density of in-catalog and out-of-catalog CBC hosts calculated for different models. Given the larger sky area covered by GW190814, in Fig. 3 we show the 90% credible intervals of the different quantities. We observe that there is an overdensity of galaxies in the redshift region from 0.05 to 0.06, which is the redshift expected for GW190814 if one assumes a cosmological model with H_0 priors in the Hubble tension region.

In Fig. 4 we show the posteriors obtained for H_0 using different prescriptions for the galaxy catalog. We note that when we assume a CBC rate proportional to the galaxies’ intrinsic luminosity ($\epsilon = 1$), the Hubble constant posterior displays a local maximum in the H_0 -tension region. The local maximum is still present, but less pronounced when a constant CBC rate with respect to the galaxies’ luminosity is assumed ($\epsilon = 0$), the reason being that the CBC completeness is lower for the uniform rate, so that the out-of-catalog term washes out the impact of redshift features introduced by the in-catalog term.

Regarding the comparison with the GWTC-2 estimate, the analysis in [36] also uses a sky pixel of 0.8 deg^2 , but it makes

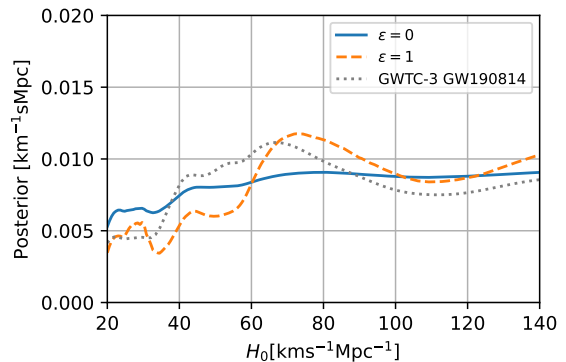


FIG. 4. Hubble constant posterior distribution obtained for the dark siren GW190814. The blue solid and orange dashed lines indicate the posterior obtained using a CBC merger rate independent of the galaxies’ luminosity and proportional to it. The luminosity weights are assigned using a probability $p(L|\epsilon) \propto L^\epsilon$.

The gray dotted line is the posterior of GW190814 obtained by the LVK collaboration in [36].

the choice of considering pixels with less than 10 galaxies as ‘‘empty’’. This choice translates to a higher incompleteness of the galaxy catalog for GW190814, resulting in a less informative H_0 posterior.

V. APPLICATION TO BINARY BLACK HOLES IN GWTC-3

We now apply our framework to probe the current expansion rate of the Universe using BBHs reported in GWTC-3. In [36, 41], it has been shown that BBHs can be used to probe cosmology either by using the standard galaxy catalog approach, or knowledge from the source-frame mass distribution. Although in [36, 41] these two aspects are considered separately, it is important to consider them conjointly [28], in order to obtain an unbiased estimate of the cosmological parameters. For the analyses presented in this section, and consistently with [36], we use galaxies with K-band luminosities reported in the GLADE+ catalog. To calculate the apparent magnitude threshold, we divide the sky into pixels of equal area⁶ of 13 deg^2 . This resolution is suitable for the best localized BBHs detections, which are usually constrained in a sky region larger than $50 - 100 \text{ deg}^2$ [36]. Smaller pixel areas will not affect the results also because, as we will see later, most of the H_0 information will actually come from the completeness correction. In this section, we apply Eq. 6 to infer H_0 jointly with the population parameters.

⁵ <https://dcc.ligo.org/LIGO-P2000223/public>

⁶ calculated with `healpy` and `nside=16`.

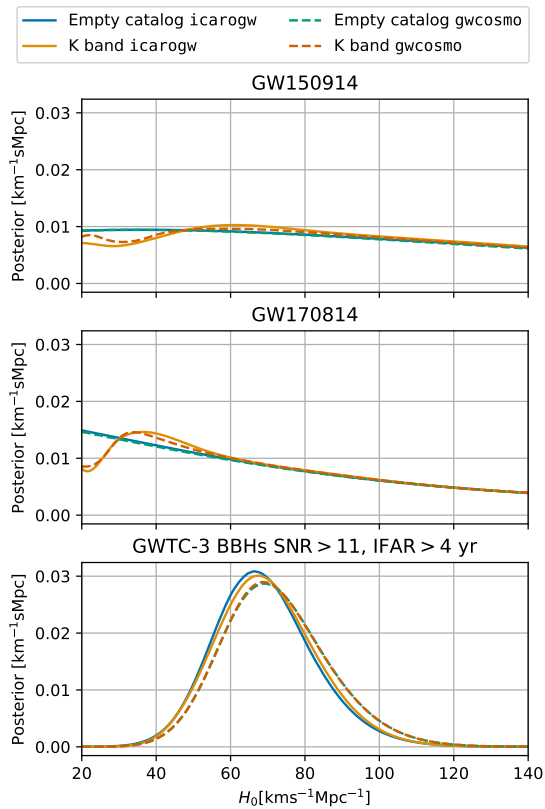


FIG. 5. Hubble constant posteriors distributions for an analysis of the 42 BBHs from GWTC-3 with SNR above 11 and IFAR higher than 4, fixing the BBH population model in the inference. The dashed lines are the H_0 posterior obtained by gwcosmo in [36], while the solid lines are the ones obtained in this work. The “empty catalog” assumes a 0% complete galaxy catalog, where the inference is driven mostly by BBH population assumptions. *Top panel:* H_0 posterior for GW150914. *Middle panel:* H_0 posterior for GW170814. *Bottom panel:* H_0 posterior for the combined 42 BBHs considered in the analysis.

A. Catalog-only benchmark analysis

We first perform a benchmark analysis. We fix the BBH population model (source mass spectrum) to the one used by the galaxy catalog analysis in [36] (see App. A for the population models employed). We select 42 BBHs from GWTC-3 with a signal-to-noise ratio (SNR) above 11 and inverse false alarm rate (IFAR) higher than 4 yr. In Fig. 5 we show the H_0 posterior obtained in this analysis for two events (GW150914 and GW170814) and the combination of all 42 dark sirens. In general, we find a good agreement between the results in [36] generated by gwcosmo and our approach. The plots show two test cases: the “empty catalog” case and the “full catalog” case. The former is generated by artificially assuming that the galaxy catalog contains no galaxies, while the latter uses the entire galaxy catalog. The empty catalog case is used to quantify the impact of population assumptions as the redshift information is dominated by the completeness correction. Compar-

ing the empty and full catalog curves in Fig. 5, we see that, for the combined posterior, the H_0 inference is completely dominated by the BBH population prescription, while on a single-event level, adding redshift information from the galaxy catalog we are able to disfavor lower values of H_0 . This is explained by the fact that for low H_0 values, the redshift of the GW events is lower and is located in a region of the catalog that is more complete.

B. Full catalog and BBH population analysis

Next, we perform a joint fit of H_0 and BBH population properties, this time using information also from the galaxy catalog. Following [36], for the BBH population model, we adopt a POWER LAW + PEAK phenomenological mass model and a Madau-Dickinson [62] model $\psi(z; \Lambda_p) = \psi_{\text{MD}}(z; \gamma, k, z_p)$ for the CBC merger rate as a function of redshift. Also for this analysis, we selected 42 BBHs from GWTC-3 with SNR above 11 and IFAR higher than 4 yr. More details about the redshift-related parameters as well as the BBH population models with their prior ranges can be found in App. A. In this analysis, we model the CBC merger rate per galaxy as:

$$\frac{dN_{\text{CBC}}}{dN_{\text{gal}}dt_s} = R_{\text{gal},0}^* \left(\frac{L}{L_*} \right)^\epsilon \psi_{\text{MD}}(z; \gamma, k, z_p). \quad (39)$$

We consider three scenarios characterized by different assumptions on the luminosity weight: $\epsilon = 1$, $\epsilon = 0$, and $\epsilon = 1$ but with the further assumption that the galaxy catalog is 0% complete (empty catalog case). note that for the “empty catalog” case, changing ϵ would only result in a different $R_{\text{gal},0}^*$, the GW population parameters are only determined by the completeness correction.

In all the cases, we find that the population parameters related to the mass distribution do not strongly depend on the prescription used for the galaxy catalog (see Tab. II). This is a consequence of the fact that our CBC mass rate model in Eq. 16 does not explicitly depend on galaxies’ redshift and absolute magnitude.

We also note that the parameters related to the shape of CBC rate as a function of redshift ψ_{MD} do not strongly depend on the catalog prescription (see Tab. III). Redshift-related population parameters are independent of the catalog prescription since (i) GW events are in general not precisely localised in the redshift space, thus they are associated to thousands of galaxies and (ii) the galaxy catalog is highly incomplete at $z > 0.1$, and the CBC redshift rate contribution is mostly calculated with the completeness correction (dependent only on the comoving volume).

Let us now focus on the estimation of $R_{\text{gal},0}^*$, the CBC merger rate per galaxy with a reference luminosity L_* , jointly with H_0 . In Fig. 6 we report the joint posterior distribution for the 42 BBHs selected in the analysis and the various galaxy catalog and rate prescriptions. Firstly, we find that the value of $R_{\text{gal},0}^*$ is strongly dependent on the prescription used for the CBC merger rate as a function of the galaxy luminosity (ϵ). In particular, the CBC merger rate per galaxy should be

TABLE II. Maximum a posteriori and symmetric 68.3% credible intervals of the marginal distributions for the mass-related population parameters using 3 galaxy catalog prescriptions used in the analysis of the 42 BBHs from GWTC-3 having SNR above 12 and IFAR higher than 4 yr. The reported population parameters are used to build the BBH mass spectrum, see App. A for more details. Parameters with no entry are not constrained within their prior ranges.

Model	α	β	m_{\min}	m_{\max}	$\delta_m [M_\odot]$	$\mu_g [M_\odot]$	$\sigma_g [M_\odot]$	λ
$\epsilon = 1$	$3.85^{+0.50}_{-0.52}$	$0.7^{+1.3}_{-1.1}$	$5.17^{+0.69}_{-0.95}$	-	4.8 ± 2.4	$32.3^{+2.8}_{-4.3}$	$1.9^{+3.4}_{-1.5}$	$0.019^{+0.030}_{-0.017}$
$\epsilon = 0$	$3.81^{+0.49}_{-0.48}$	0.7 ± 1.2	$5.16^{+0.66}_{-0.96}$	-	$4.7^{+2.9}_{-1.8}$	$33.2^{+2.8}_{-4.3}$	$2.8^{+3.0}_{-2.3}$	$0.018^{+0.029}_{-0.016}$
Empty ($\epsilon = 1$)	$3.82^{+0.46}_{-0.51}$	$0.56^{+1.47}_{-0.97}$	$4.99^{+0.95}_{-0.68}$	-	$5.0^{+2.4}_{-2.2}$	$33.0^{+3.0}_{-4.0}$	$2.8^{+3.0}_{-2.4}$	$0.019^{+0.030}_{-0.017}$

TABLE III. Maximum a posteriori and symmetric 68.3% credible intervals of the marginal distributions of the cosmology and redshift-related parameters for the 3 galaxy catalog prescriptions used in the analysis of the 42 BBHs from GWTC-3 having SNR above 12 and IFAR higher than 4 yr. The reported population parameters are used to build the BBH rate as function of redshift, see App. A for more details. Parameters with no entry are not constrained within their prior ranges.

Model	H_0 [km s ⁻¹ Mpc ⁻¹]	γ	k	z_p	$\log_{10} \left[\frac{R_{\text{gal},0}^*}{\text{yr}^{-1}} \right]$
$\epsilon = 1$	71^{+35}_{-30}	$4.5^{+2.0}_{-1.7}$	-	-	$-5.67^{+0.63}_{-0.50}$
$\epsilon = 0$	43^{+48}_{-18}	$4.9^{+2.2}_{-2.1}$	-	-	$-6.10^{+0.81}_{-0.51}$
Empty ($\epsilon = 1$)	52^{+36}_{-25}	$4.9^{+2.3}_{-2.1}$	-	-	$-5.48^{+0.80}_{-0.50}$

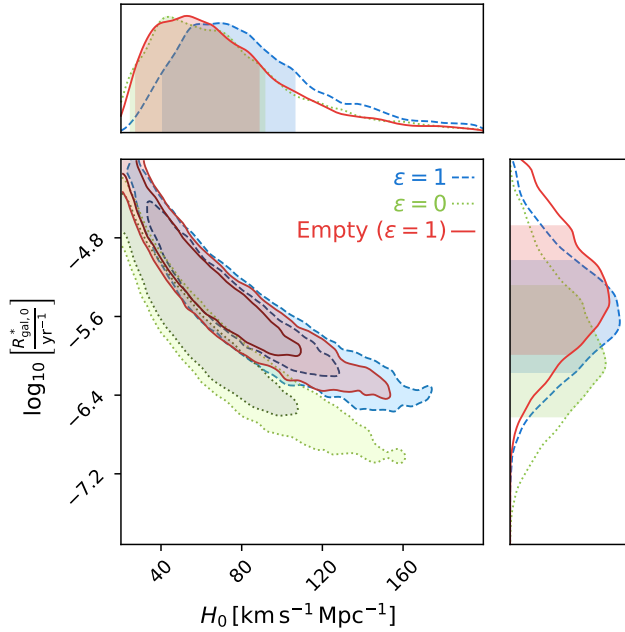


FIG. 6. Two dimensional and one-dimensional marginal distributions on the logarithm of the CBC merger rate per galaxy today $R_{\text{gal},0}^*$ and H_0 . The inner contour of the two-dimensional posterior indicates the 68.3% credible intervals while the outer contour is the 90%. The luminosity weights are assigned using a probability $p(L|\epsilon) \propto L^\epsilon$.

The marginal 1-dimensional distribution reports the 68.3% credible intervals. Different colors correspond to the three galaxy catalog prescriptions.

higher when we assume that brighter galaxies are more likely to host GW events. This is expected as brighter galaxies are significantly lower in number than fainter galaxies, therefore to keep constant the expected number of CBC detections per year $R_{\text{gal},0}^*$ must significantly increase. Secondly, we find that there is a strong anti-correlation between $R_{\text{gal},0}^*$ and H_0 . Cor-

relations are introduced when the variation of a certain combination of population-level parameters keeps the hierarchical likelihood in Eq. 1 (or Eq. 6) constant. For the particular case of $R_{\text{gal},0}^*$ and H_0 , the two parameters are degenerate for the calculation of the expected number of CBC detections N_{exp} . The expected number of CBC detections N_{exp} is linearly dependent on $R_{\text{gal},0}^*$ and is also dependent on H_0 , as for lower values of H_0 the GW detection range can include galaxies at higher redshift. In other words:

$$N_{\text{exp}} \approx R_{\text{gal},0}^* T_{\text{obs}} N_{\text{gal}}(z < z(d_L^H, H_0)), \quad (40)$$

where d_L^H is a typical luminosity distance horizon for GW detection and $N_{\text{gal}}(z < z(d_L^H, H_0))$ the number of galaxies enclosed in a redshift sphere with radius $z(d_L^H, H_0)$. The number of galaxies enclosed in the GW detection horizon will roughly scale as $N_{\text{gal}} \propto [d_L^H H_0]^3$. This implies that, to keep constant the value of N_{exp} we can either increase $R_{\text{gal},0}^*$ and decrease H_0 or viceversa. Note that this type of correlation is not usually observed between H_0 and the CBC merger rate per comoving volume R_0 , which is the parametrization typically used in population studies. This is due to the fact that the R_0 -parameterization depends on the differential of the comoving volume (which scales as H_0^{-3}):

$$N_{\text{exp}} \approx R_0 T_{\text{obs}} V_c(z < z(d_L^H, H_0)), \quad (41)$$

where $V_c(z < z(d_L^H, H_0))$ is the comoving volume within a redshift shell with radius $z(d_L^H, H_0)$. As the comoving volume scales as $V_c \propto [z(d_L^H, H_0)]^3 / H_0^3$ and $z(d_L^H, H_0) \propto d_L^H H_0$, then V_c will not strongly scale with H_0 . As a consequence R_0 is not correlated with H_0 during the inference. In App. B we discuss in more detail how R_0 and $R_{\text{gal},0}^*$ are related to each other.

Let us finally comment about H_0 . On one hand, the marginal posterior for H_0 is almost identical in the empty catalog and $\epsilon = 0$ case. In fact, as we have shown in Sec. IV, assuming $\epsilon = 0$ corresponds to a galaxy catalog that is significantly less complete from the CBC hosts' point of view

(see Fig. 3 as an example). On the other hand, assuming $\epsilon = 1$ with the galaxy catalog we obtain a marginal H_0 posterior which disfavors low values of H_0 . This is due to the fact that for small H_0 values, GW events are located at lower redshifts, where the galaxy catalog is more complete (especially for $\epsilon = 1$). Galaxy catalogs, if complete, can provide additional constraints to lower values of H_0 , therefore it is crucial to use them to improve current constraints.

C. GWTC-3 and modified gravity

Proposed modifications to general relativity (GR) that include a friction term for the GW propagation can result in different expressions for the GW luminosity distance d_L^{GW} and the classical (EM) luminosity distance d_L^{EM} . Different parametrizations of d_L^{GW} have been proposed. Here we focus on three models: the Ξ_0 -model [63], the extra-dimensions model [64], and the running Planck mass model [65]. The Ξ_0 -model adds the two phenomenological parameters Ξ_0 and n_{Ξ_0} to describe the distance relation. The extra-dimensions model adds three parameters: the number of spacetime dimensions D , the screening distance R_c , and a scaling factor n_D . The running Planck mass adds only one extra parameter c_M . These models parametrize the relation between d_L^{GW} and d_L^{EM} as:

$$d_L^{\text{GW}} = d_L^{\text{EM}} \left[\Xi_0 + \frac{1 - \Xi_0}{(1+z)^{n_{\Xi_0}}} \right], \quad (42)$$

$$d_L^{\text{GW}} = d_L^{\text{EM}} \left[1 + \left(\frac{d_L^{\text{EM}}}{(1+z)R_c} \right)^n \right]^{\frac{D-4}{2n_D}}, \quad (43)$$

$$d_L^{\text{GW}} = d_L^{\text{EM}} \exp \left[\frac{c_M}{2} \int_0^z \frac{1}{(1+z')E^2(z')} dz' \right]. \quad (44)$$

Bright sirens are often used to put constraints on these models (see [31] and references therein). However, even the galaxy catalog method [66] and the spectral siren method [30, 31, 41, 67] can be used.

Here we repeat the full spectral siren and spectral+catalog analyses for these three models. To do so, we use the same selection of events used in Sec. V A and Sec. V B. This time, we fix the value of H_0 to $67.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ because the determination of H_0 and the modified gravity parameters is strongly degenerate and with the current number of events we are unable to jointly constrain both. For the Ξ_0 -model, we use a uniform prior in $[0.1, 50]$ for Ξ_0 and a uniform prior in $[1, 10]$ for n_{Ξ_0} . For the extra dimension model, we use a uniform prior in $[3.6, 8]$ for D , a log uniform prior in $[0.1, 100]$ for n_D , and a log uniform prior in $[10, 10^5] \text{ Mpc}$ for R_c . For the running Planck mass model, we use a uniform prior in $[-10, 50]$. The rest of the priors assumed for the population model are equal to the ones used in Sec. V B.

The marginal posterior distributions that we obtain on the modified gravity parameters are displayed in Figs. 7-8-9. As we can see from the marginal posterior distributions, the constraints on the modified gravity parameters are equal when only using source mass information (spectral) and the galaxy catalog information. This is due to the fact that the galaxy

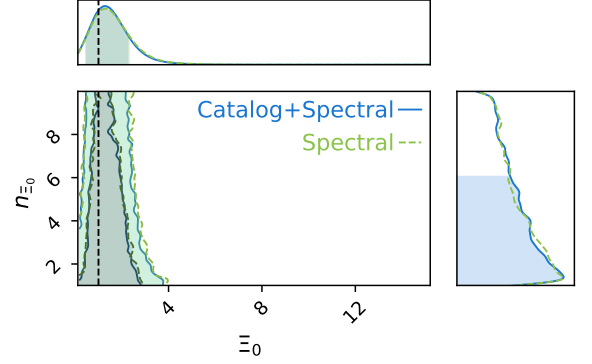


FIG. 7. Marginal posterior distribution for the parameters of the Ξ_0 model for the catalog+spectral (blue solid line) and spectral (green dashed line) siren analyses. The black dashed line is the GR value ($\Xi_0 = 0$). The plots report the 68.3% and 90% credible intervals. The marginal 1-dimensional distribution reports the 68.3% credible intervals.

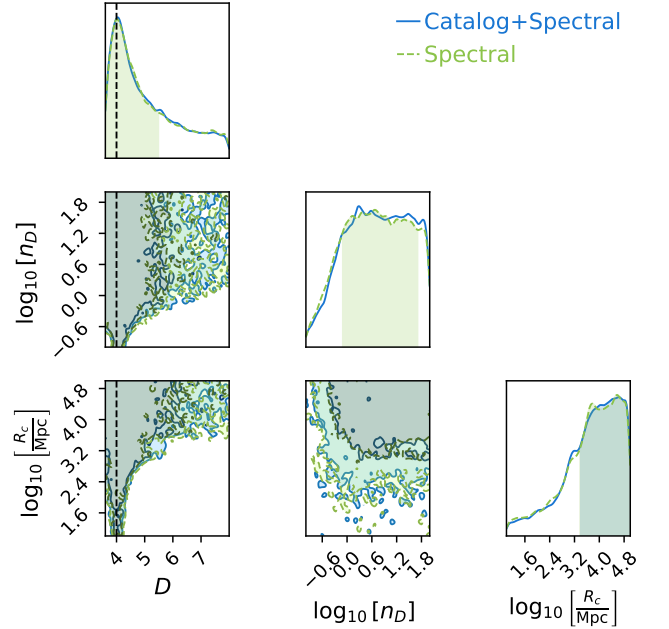


FIG. 8. Marginal posterior distribution for the parameters of the extra-dimensions model for the catalog+spectral (blue solid line) and spectral (green dashed line) siren analyses. The black dashed line is the GR value ($D = 4$). The plots report the 68.3% and 90% credible intervals. The marginal 1-dimensional distribution reports the 68.3% credible intervals.

catalog is strongly incomplete and all the information on the redshift distribution is obtained from the source frame mass distribution. In all the cases, we obtain marginal posterior distributions consistent with no deviations from GR.

For the running Planck mass model, we obtain a constraint of $1.0^{+2.6}_{-3.4}$ and $1.6^{+2.2}_{-4.0}$ on c_M for the catalog+spectral and spectral siren analyses respectively (credible intervals reported at

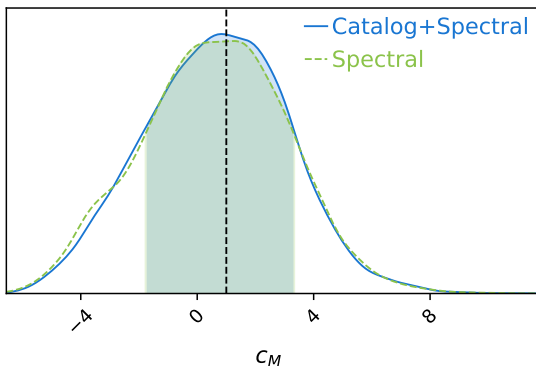


FIG. 9. Marginal posterior distribution for the parameter of the running Planck mass model for the catalog+spectral (blue solid line) and spectral (green dashed line) siren analyses. The black dashed line is the GR value ($c_M = 1$). The plots report the 68.3% and 90% credible intervals. The marginal 1-dimensional distribution reports the 68.3% credible intervals.

68.3% credible interval unless stated otherwise). For the Ξ_0 model we obtain a bound of $1.44^{+1.17}_{-0.93}$ and $1.37^{+1.36}_{-0.90}$ on Ξ_0 for the catalog+spectral and spectral siren cases, but we are unable to constrain n_{Ξ_0} . For the extra dimensions model, we are not able to place strong bounds on any of the parameters as the number of space time dimensions and screening distance are strongly degenerate. If the screening distance is smaller than ~ 100 Mpc, then the number of spacetime dimensions is strongly constrained to 4. The constraint on the rest of the population parameters is reported in Tab. IV

VI. CONCLUSION

In this paper, we have presented a new methodology to jointly estimate the CBC merger rate and cosmological parameters using GW data and galaxies surveys.

In Sec. III, we presented a new parametrization for the CBC merger rate as a function of galaxies' number density. We have discussed how the galaxies' number density can be constructed from a real survey also accounting for a completeness correction and errors in the redshift evaluation of each galaxy. The proposed CBC merger rate parameterization introduces new astrophysical quantities such as $R_{\text{gal},0}^*$, the CBC merger rate per galaxy. The parametrization allows us to fit the number density of galaxies once for the entire analysis, thus making the study computationally feasible, while at the same time linking CBC physics to galaxy physics.

In Sec. IV and Sec. V, we have shown how this method could be applied to well-localized dark sirens and to a population of GW events, inferring both cosmology and population properties. We have discussed extensively the relation between H_0 and the CBC merger rate per galaxy $R_{\text{gal},0}^*$. We constrained the value of $\log_{10} R_{\text{gal},0}^*$ to $-5.67^{+0.63}_{-0.50}$ and $-6.10^{+0.81}_{-0.51}$ at 90% CI in the two scenarios where more luminous galaxies are more likely to emit GW events and where there is no such

dependency, respectively. We have also shown that, if more luminous galaxies are more likely to host GW sources, then we can exclude lower values of H_0 just by using the 42 BBHs used in the LVK analysis in [36].

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TABLE IV. Constraints on the population parameters for the spectral and catalog+spectral analyses performed with 42 BBHs and various modified gravity models. The values are reported as the median with symmetric 68.3% credible intervals.

Model	α	β	m_{\min}	m_{\max}	$\delta_m [M_\odot]$	$\mu_g [M_\odot]$	$\sigma_g [M_\odot]$	λ	γ	k	z_p
Ξ_0 (Catalog+Spectral)	$3.74^{+0.49}_{-0.41}$	$0.8^{+1.2}_{-1.1}$	$4.98^{+0.95}_{-0.64}$	-	$5.0^{+2.6}_{-2.2}$	$32.9^{+3.2}_{-2.2}$	-	$0.019^{+0.032}_{-0.018}$	-	-	-
Extra dimensions (Catalog+Spectral)	$3.80^{+0.45}_{-0.48}$	$0.6^{+1.2}_{-1.1}$	$5.37^{+0.55}_{-1.02}$	-	$4.6^{+2.7}_{-2.0}$	$33.1^{+2.1}_{-2.4}$	$2.3^{+3.0}_{-1.9}$	$0.017^{+0.031}_{-0.015}$	-	-	-
c_M (Catalog+Spectral)	$3.72^{+0.50}_{-0.45}$	$0.41^{+1.49}_{-0.82}$	$5.08^{+0.78}_{-0.81}$	-	-	$32.9^{+2.7}_{-2.8}$	-	$0.018^{+0.031}_{-0.017}$	-	-	-
Ξ_0 (Spectral)	$3.71^{+0.49}_{-0.40}$	$0.7^{+1.4}_{-1.1}$	$5.14^{+0.77}_{-0.79}$	-	5.2 ± 2.4	$33.3^{+2.7}_{-3.1}$	$3.7^{+3.2}_{-2.5}$	$0.018^{+0.035}_{-0.015}$	-	-	-
Extra dimensions (Spectral)	$3.86^{+0.42}_{-0.50}$	$0.7^{+1.2}_{-1.1}$	$5.13^{+0.84}_{-0.70}$	-	$4.9^{+2.5}_{-2.1}$	33.0 ± 2.3	$2.4^{+3.0}_{-2.0}$	$0.016^{+0.032}_{-0.014}$	-	-	-
c_M (Spectral)	$3.79^{+0.46}_{-0.50}$	$0.7^{+1.2}_{-1.1}$	$5.22^{+0.64}_{-0.90}$	-	$5.3^{+2.3}_{-2.4}$	$32.7^{+2.6}_{-2.7}$	$3.0^{+2.4}_{-2.6}$	$0.017^{+0.032}_{-0.015}$	-	-	-

Appendix A: CBC mass and redshift models

1. Redshift model

In this paper, we employ a CBC rate model as a function of redshift following the Madau-Dickinson star formation rate [62]. We parametrize the CBC merger rate evolution $\psi(z; \Lambda_p)$ as:

$$\psi_{\text{MD}}(z; \gamma, k, z_p) = [1 + (1 + z_p)^{-\gamma-k}] \frac{(1+z)^\gamma}{1 + \left(\frac{1+z}{1+z_p}\right)^{\gamma+k}}. \quad (\text{A1})$$

For the analysis with GW170817 in Sec. IV A we fix $\gamma = 0, k = 0, z_p = 2.47$ a uniform in comoving volume rate. For the analysis with G190814 in Sec. IV B and the catalog analysis in Sec. V A we fix $\gamma = 4.59, k = 2.86, z_p = 2.47$ [36], while for the full catalog and population analysis presented in Sec. V B we assume priors equal to the ones used in [36], see Tab. V.

Parameter	Description	Prior
R_0	BBH merger rate today in $\text{Gpc}^{-3} \text{yr}^{-1}$	$\mathcal{U}(0.0, 100.0)$
γ	Slope of the power law regime for the rate evolution before the point z_p	$\mathcal{U}(0.0, 12.0)$
k	Slope of the power law regime for the rate evolution after the point z_p	$\mathcal{U}(0.0, 6.0)$
z_p	Redshift turning point between the power law regimes with γ and k	$\mathcal{U}(0.0, 4.0)$

TABLE V. Summary of the prior hyper-parameters used for the merger rate evolution models adopted in this paper.

2. Mass models

The source-frame mass models we use are composed of two statistical distributions: a truncated power law distribution:

$$\mathcal{P}(x|x_{\min}, x_{\max}, \alpha) \propto \begin{cases} x^\alpha, & (x_{\min} \leq x \leq x_{\max}), \\ 0, & \text{otherwise,} \end{cases} \quad (\text{A2})$$

and a Gaussian distribution with mean μ and standard deviation σ :

$$\mathcal{G}(x|\mu, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]. \quad (\text{A3})$$

The distribution of the source frame masses m_1, m_2 is factorized as:

$$\pi(m_1, m_2|\Phi_m) = \pi(m_1|\Lambda_p)\pi(m_2|m_1, \Lambda_p), \quad (\text{A4})$$

where $\pi(m_1|\Phi_m)$ is the primary mass distribution (different for the various cases of the paper, as specified below). The secondary mass distribution $\pi(m_2|m_1, \Phi_m)$ is a truncated power law distribution. The secondary mass is conditioned to the primary mass as $m_2 < m_1$:

$$\pi(m_2|m_1, m_{\min}, \alpha) = \mathcal{P}(m_2|m_{\min}, m_1, \beta). \quad (\text{A5})$$

Below we list the primary mass models and priors used in this paper:

- Sec. IV A (GW170817 as a quasi-dark siren): The primary mass model is a truncated power law with $\alpha = 0, m_{\max} = 3M_\odot, m_{\min} = 1M_\odot$; the secondary mass model is a truncated power law with $\beta = 0.81, m_{\max} = m_1, m_{\min} = 1M_\odot$ [36].
- Sec. IV B (GW190814 as dark siren): For the primary mass we use the POWER LAW+PEAK model from [36] to describe the source frame distribution. The POWER LAW+PEAK distribution is:

$$\pi(m_1|m_{\min}, m_{\max}, \alpha, \lambda_g, \mu_g, \sigma_g) = (1 - \lambda_g)\mathcal{P}(m_1|m_{\min}, m_{\max}, -\alpha) + \lambda_g\mathcal{G}(m_1|\mu_g, \sigma_g), \quad (\text{A6})$$

where the power law part has slope $-\alpha$ between m_{\min} and m_{\max} , while the Gaussian component has mean μ_g and standard deviation σ_g and accounts λ_g of the total fraction of the distribution. We also apply an additional smoothing at the lower edge of the distribution:

$$\pi(m_1, m_2|\Phi_m) = [\pi(m_1|\Phi_m)S(m_1|\delta_m, m_{\min})][\pi(m_2|m_1, \Phi_m)S(m_2|\delta_m, m_{\min})], \quad (\text{A7})$$

where S is a sigmoid-like window function as described in [39]. The population parameters fixed for the analysis are $\alpha = 3.78, \beta = 0.81, m_{\max} = 112.5M_{\odot}, m_{\min} = 4.98M_{\odot}, \delta_m = 4.8M_{\odot}, \sigma_g = 3.88M_{\odot}, \mu_g = 342.27M_{\odot}, \lambda_g = 0.03$. For the secondary mass (the neutron star) we use a truncated power law with $m_{\min, \text{NS}} = 1M_{\odot}, m_{\max, \text{NS}} = 3M_{\odot}$ [36].

- Sec. V A (catalog-only BBH analysis): For the primary mass we use the same POWER LAW + PEAK population model used in Sec. IV B. For the secondary mass, we use a truncated power law with $\beta = 0.81$ and $m_{\min} = 4.88M_{\odot}, m_{\max} = m_1$.
- Sec. V B (full catalog and population BBH analysis): the baseline model of the primary mass is always the POWER LAW + PEAK, but this time the population parameters of the model are also sampled using priors equal to the ones used in [36].

Parameter	Description	Prior
α	Spectral index for the PL of the primary mass distribution.	$\mathcal{U}(1.5, 12.0)$
β	Spectral index for the PL of the mass ratio distribution.	$\mathcal{U}(-4.0, 12.0)$
m_{\min}	Minimum mass of the PL component of the primary mass distribution.	$\mathcal{U}(2.0M_{\odot}, 10.0M_{\odot})$
m_{\max}	Maximum mass of the PL component of the primary mass distribution.	$\mathcal{U}(50.0M_{\odot}, 200.0M_{\odot})$
λ_g	Fraction of the model in the Gaussian component.	$\mathcal{U}(0.0, 1.0)$
μ_g	Mean of the Gaussian component in the primary mass distribution.	$\mathcal{U}(20.0M_{\odot}, 50.0M_{\odot})$
σ_g	Width of the Gaussian component in the primary mass distribution.	$\mathcal{U}(0.4M_{\odot}, 10.0M_{\odot})$
δ_m	Range of mass tapering at the lower end of the mass distribution.	$\mathcal{U}(0.0M_{\odot}, 10.0M_{\odot})$

TABLE VI. Summary of the priors used for the population hyper-parameters for the three phenomenological mass models.

3. Correction of selection biases

To correct the selection biases, we need to calculate the expected number of detections

$$N_{\text{exp}}(\Lambda) = T_{\text{obs}} \int d\theta dz P_{\text{det}}(z, \theta, \Lambda_c) \frac{dN_{\text{CBC}}}{dzd\theta dt_s}(\Lambda) \frac{1}{1+z}. \quad (\text{A8})$$

The integral above can be approximated as a Monte Carlo sum over a set of simulated injections

$$N_{\text{exp}} \approx \frac{T_{\text{obs}}}{N_{\text{gen}}} \sum_{j=1}^{N_{\text{det}}} \frac{1}{\pi_{\text{inj}}(z_j, \theta_j)} \frac{1}{1+z_j} \frac{dN_{\text{CBC}}}{dzd\theta dt_s} \Big|_j \equiv \frac{T_{\text{obs}}}{N_{\text{gen}}} \sum_{j=1}^{N_{\text{det}}} s_j, \quad (\text{A9})$$

where T_{obs} is the observing time, N_{gen} the number of simulated signals (also the non-detected ones), N_{det} the number of detected signals, and $\pi_{\text{inj}}(z_j, \theta_j)$ the prior used to generate the injections. Following Farr [68], we also define a numerical stability estimator for the selection bias. We require that the effective number of injections used to evaluate the selection bias $N_{\text{eff, inj}}$ is 4 times higher than the observed number of GW signals. The effective number of injections is given by

$$N_{\text{eff, inj}} = \frac{\left[\sum_j^{N_{\text{det}}} s_j \right]^2}{\left[\sum_j^{N_{\text{det}}} s_j^2 - N_{\text{gen}}^{-1} \left(\sum_j^{N_{\text{det}}} s_j \right)^2 \right]}. \quad (\text{A10})$$

In this work, we used different injection sets. For the GW170817 analysis, we used the BNSs and NSBHs injections released after O3⁷. For the BBH analysis we use the injection set released with [36].

Appendix B: The relation between R_0 and $R_{\text{gal},0}^*$ for the empty catalog case

In this Appendix we show how the CBC merger rate based on galaxy number density in Sec. III B relates to the spectral siren parameterization in Sec. III A. The CBC merger rate written in terms of galaxy number density can be written as

$$\frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s} = R_{\text{gal},0}^* P_{\text{pop}}(\vec{m}_s, \theta | \Lambda_p) \psi(z; \Lambda_p) \int dM 10^{0.4\epsilon(M_s - M)} \left[\frac{dN_{\text{gal,cat}}}{dz d\Omega dM} + \frac{dN_{\text{gal,out}}}{dz d\Omega dM} \right]. \quad (\text{B1})$$

⁷ <https://zenodo.org/record/5546676>

Let us assume that the catalog is completely empty and our rate is determined by the completeness correction; then we have:

$$\frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s} = R_{\text{gal},0}^* p_{\text{pop}}(\vec{m}_s, \theta | \Lambda_p) \psi(z; \Lambda_p) \phi_*(H_0) \Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}, x_{\text{min}}) \frac{dV_c}{dz d\Omega}, \quad (\text{B2})$$

where $\phi_*(H_0) \Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}, x_{\text{min}})$ is simply the overall integral of the entire Schechter function (with the addition of the luminosity weight). The spectral siren rate parameterization is:

$$\frac{dN_{\text{CBC}}}{d\vec{m}_s d\theta d\Omega dz dt_s} = R_0 p_{\text{pop}}(\vec{m}_s, \theta | \Lambda_p) \psi(z; \Lambda_p) \frac{dV_c}{dz d\Omega}. \quad (\text{B3})$$

By equating Eq. B2 and Eq. B3, we obtain the relation between the spectral siren and galaxy rate, namely:

$$R_0 = R_{\text{gal},0}^* \phi_*(H_0) \Gamma_{\text{inc}}(\alpha + \epsilon + 1, x_{\text{max}}, x_{\text{min}}) \propto R_{\text{gal},0}^* H_0^3. \quad (\text{B4})$$

The above relation also shows why in the full population parameter estimation of Sec. V B $R_{\text{gal},0}^*$ and H_0 are correlated.

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