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Pro-rata vs User-centric in the music streaming industry

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A B S T R A C T

An endogenous model, which allows the artists to determine the streaming times strategically, is used to compare two remuneration rules in the music streaming industry: Pro-rata (P rule) and User-centric (U rule). The two judgement criteria are 1. efficiency, in terms of no dominance on quality profile. 2. egalitarian fairness, in terms of the lowest royalty among all artists. Our main result is that P rule always outperforms U rule in efficiency and fairness when the superstar's marginal cost is the lowest. This means that the transition from P rule to U rule can not only enlarge the existing royalty gap but also decrease the efficiency of the music streaming industry.

1. Introduction

In October 2020, the House of Commons Digital, Culture, Media and Sport Committee launched an inquiry into the music streaming industry to deal with the long-lasting complaints on unfair creator remuneration (The Digital, Culture, Media and Sport Committee, 2021). While the methods based on Shapley value from cooperative game theory provide a theoretical foundation for revenue sharing, it requests stand-alone costs, which are not observable or not simple to compute (Shiller and Waldfogel, 2013). There are two widely used payment methods in the music streaming industry. Most industry heavyweights (e.g. Spotify and Apple Music) use the pro-rata rule (P rule). Under P rule, the subscription fees of all subscribers are aggregated at first as a royalty pot and then proportionally divided. In contrast, under the user-centric rule (U rule), which Deezer uses, the subscription fee of each subscriber is first proportionally split and then aggregated (See Example 1).

Example 1. P rule vs. U rule in 2×2 case

There are two songs A, B and two subscribers 1,2. The streaming matrix is in Table 1. In each small cell, the number is the streaming times for a song of a subscriber.

Suppose the subscription fee is £10, which is the same for all subscribers. So, the total royalty pot is £20. Under P rule, the royalty R for each artist are $R_A^P = \frac{20+10}{20+10+10+60} \times £20 = £6$ and $R_B^P = \frac{10+60}{20+10+10+60} \times £20 = £14$. Under U rule, the royalty for each artist are $R_A^U = \frac{20}{20+10} \times £10 + \frac{10}{10+60} \times £10 = £8.1$ and $R_B^U = \frac{10}{20+10} \times £10 + \frac{60}{10+60} \times £10 = £11.9$.

There is a hot debate on how the royalty pot should be allocated and what the properties of different rules are (Page and Safir, 2018a,b). There is an argument from empirical research (Muikku, 2017; Pedersen, 2014; Hesmondhalgh, 2021), Law (Dimont, 2018) and industry news (Dredge, 2021) that U rule can benefit the specialists (unpopular artists with low total streaming times) more than P rule by giving them more royalty (as Example 1 shows). This may be true when the streaming matrix is exogenously given. However, this relation reverses when the streaming matrix is endogenously determined.

In this paper, we construct an endogenous model that allows the artists to strategically change their songs' quality to influence the streaming matrix and maximize their payoff. Preferences and quality profiles jointly determine the consumers' streaming time on each song, and the streaming matrix is endogenously determined in equilibrium. By comparing equilibria, we aim to answer two widely debated questions: 1. which is more efficient? 2. which is fairer? No dominance of quality profile captures efficiency. Egalitarianism captures fairness since, in reality, the specialists complain about their low royalty. Our main theorem is that in the two-artist model, P rule outperforms U rule in efficiency and fairness. We find the theorem still holds when there are one superstar and two identical specialists. Although the cases are special, our endogenous model does give a unique result that contradicts the empirical research, which neglects the artists' strategic behavior.

The structure of the paper is as follows: we discuss other related theoretical papers, and then, Section 2 shows the general setting. Section 3 gives equilibrium and discusses the 2-artist model and 3-artist model with identical specialists. Section 4 concludes the paper with limitations and contributions.

1.1. Literature review

To the best of our knowledge, the only theoretical paper comparing the two rules is Alaei et al. (2020). They use an

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Table 1
Streaming matrix in 2 × 2 case.

		Songs A	Songs B
Subscribers	1	20	10
	2	10	60

extensive-form game to capture the interactions between the platform, the artists and the consumers. They focus on the platform's pricing strategy and how it can sustain a set of artists (participation constraint). We do not consider these two since, in reality, the artists can work part-time. Also, the subscription fee has been fixed for decades due to intensive competition among platforms with highly homogeneous content. We focus on the incentive compatibility constraint that makes the streaming matrix endogenously determined.

For other related stories, Ginsburgh and Zang (2003) considers a problem of sharing the revenue from selling museum passes. Their model can be extended to many real-life cases (e.g., travel cards). Under their setting, the complex Shapley value has a very simple form. However, their model cannot be used in the music streaming industry. In the museum pass game, the consumers' consumption for each museum is binary: 0 for non-visit and 1 for a visit. Here, consumers' consumption of each song in music streaming has intensity. Flores-Szwagrzak and Treibich (2020) considers revenue sharing in teamwork. They use an individual's productivity in his stand-alone project as a proxy for his contribution to teamwork. However, millions of songs are on the platform in the music streaming industry, and it is impossible to find every song's stand-alone price. The last paper (Bergantinos and Moreno-Ternero, 2020) considers the problem of sharing the revenue from selling tickets between football clubs. Again, the story differs from the music streaming industry due to the lack of consumers' consumption intensity.

2. Setting

There are $m+1$ artists on the platform, and each artist publishes one song. The set of the artists is $M = \{0, 1, 2, \dots, m\}$. The artist 0 is a superstar, and all subscribers like him. The artists $1 \sim m$ are specialists and have their fan base. There are n subscribers with private preferences who have paid a subscription fee of p. Let $\pi_0 \in (0, 1)$ denote the probability that a subscriber only likes the song 0. let $\pi_k \in (0, 1)$ denote the probability that a subscriber likes song $k \in \{1, 2, \dots, m\}$ as well as song 0. The distribution of subscribers' types $\pi = \{\pi_0, \pi_1, \pi_2, \dots, \pi_m\}$ is common knowledge. Our assumption that consumers are interested in only one specialist is realistic. For example, we can imagine that artists $1 \sim m$ are the ones who create songs in different styles: classic, jazz, folk, etc. Each of them attracts a small group of consumers with special tastes. While as a superstar, the artist 0 publishes popular music, which is widely accepted. Another explanation is that the superstar publishes songs in English, while the specialists publish songs in their languages like Dutch, Japanese, German, etc.

Given the quality profile $q = \{q_0, q_1, q_2, \dots, q_m\}$. Each consumer determines his streaming times for each song. If a user likes the song k then his utility for streaming song k is $u_k(t_k) = q_k t_k - \frac{1}{2}t_k^2$. Else, his utility for streaming song k is 0. The quadratic and additive utility function allows the simplest linear marginal utility function: $MU(q_k) = q_k - t$. So the streaming time for song k is $t_k = q_k$ for its fans and 0 for the other consumers. So, given any quality profile q, the streaming matrix is given in Table 2. In each cell, there is a corresponding streaming time (equal to quality) for a song of a subscriber.

Table 2
The general streaming matrix.

		Songs/Artists 0	1	2		m
Subscribers	π_0	q_0	0	0	$\cdots$	0
	π_1	q_0	q_1	0	$\cdots$	0
	π_2	q_0	0	q_2	$\cdots$	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	π_m	q_0	0	0	$\cdots$	q_m

The artist k can only control quality q_k while taking other quality q_{-k} as given. So, under P rule, his royalty is:

$$R_k^P(q_k, q_{-k}) = \begin{cases} \frac{q_0}{q_0 + \sum_{j=1}^m \pi_j q_j} np & k = 0 \\ \frac{\pi_k q_k}{q_0 + \sum_{j=1}^m \pi_j q_j} np & k > 0 \end{cases}$$

Under U rule, the artist k's royalty is:

$$R_k^U(q_k, q_{-k}) = \begin{cases} \pi_0 np + \sum_{j=1}^m \frac{q_0}{q_0 + q_j} \pi_j np & k = 0 \\ \frac{q_k}{q_0 + q_k} \pi_k np & k > 0 \end{cases}$$

Taking the first derivative on q_k, we can find first-order condition $MR_k^P(q_k, q_{-k}) = c_k$ under the P rule or $MR_k^U(q_k, q_{-k}) = c_k$ under the U rule. Equilibrium can be found by combining all artists' best response functions. The formal definition of this static game is (M, π, c, n, p). A royalty distribution rule r is a partition of total royalty pot np among the artists in M.

Definition (*Efficiency*)**.** A royalty distribution rule r_1 is more efficient than rule r_2 if r_1 can induce a weakly higher quality profile for all songs and at least one song's quality should be strictly higher:

$$\forall k \in M, q_k^1 \geq q_k^2$$
$$\exists k \in M, q_k^1 > q_k^2$$

Definition (*Fairness*)**.** Fixed the royalty pot, a royalty distribution rule r_1 is egalitarian fairer than rule r_2 if r_1 can induce a higher lowest royalty: let $R^{1*} = min\{R_0^1, R_1^1, \dots, R_m^1\}$ and $R^{2*} = min\{R_0^2, R_1^2, \dots, R_m^2\}$. Rule r_1 is fairer than r_2 if $R^{1*} > R^{2*}$.

3. Equilibrium

To express the equilibrium under P rule, we first define a $m+1 \times m+1$ matrix with marginal costs and consumer's type distribution:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ c_0 & -\frac{c_1}{\pi_1} & c_0 - \frac{c_1}{\pi_1} & c_0 - \frac{c_1}{\pi_1} & \cdots & c_0 - \frac{c_1}{\pi_1} \\ c_0 & c_0 - \frac{c_2}{\pi_2} & -\frac{c_2}{\pi_2} & c_0 - \frac{c_2}{\pi_2} & \cdots & c_0 - \frac{c_2}{\pi_2} \\ c_0 & c_0 - \frac{c_3}{\pi_3} & c_0 - \frac{c_3}{\pi_3} & -\frac{c_3}{\pi_3} & \cdots & c_0 - \frac{c_3}{\pi_3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_0 & c_0 - \frac{c_m}{\pi_m} & c_0 - \frac{c_m}{\pi_m} & c_0 - \frac{c_m}{\pi_m} & \cdots & -\frac{c_m}{\pi_m} \end{bmatrix}$$

The matrix A is derived from the linear system of the FOCs under P rule. The first row is special due to the superstar's popularity in 1. Also we define a group of matrix: A_k, which replace the $(k+1)$'th column of A with a $m+1$-dimension vector: $v = (1, 0, \dots, 0)$. Then by using Cramer's rule, the equilibrium can be found.

Proposition 1. *Under P rule, the equilibrium quality profile and the royalty profile are:*

$$\forall k > 0, q_k^{*P} = \frac{|A_k|}{\pi_k}\frac{np}{c_0}\frac{\sum_{j=1}^m |A_j|}{(\sum_{j=0}^m |A_j|)^2}$$
$$q_0^{*P} = |A_0|\frac{np}{c_0}\frac{\sum_{j=1}^m |A_j|}{(\sum_{j=0}^m |A_j|)^2}$$
$$\forall k, R_k^{*P} = \frac{|A_k|}{\sum_{k=0}^m |A_k|}np$$

Proposition 2. *Under U rule, the equilibrium quality profile and the royalty profile are:*

$$\forall k > 0, q_k^{*U} = np\frac{\sum_{j=1}^m \sqrt{\pi_j c_j}}{\sum_{j=0}^m c_j}\left(\sqrt{\frac{\pi_k}{c_k}} - \frac{\sum_{j=1}^m \sqrt{\pi_j c_j}}{\sum_{j=0}^m c_j}\right)$$
$$q_0^{*U} = \frac{np(\sum_{j=1}^m \sqrt{\pi_j c_j})^2}{(\sum_{j=0}^m c_j)^2}$$
$$\forall k > 0, R_k^{*U} = np(\pi_k - \frac{\sum_{j=1}^m \sqrt{\pi_j c_j}}{\sum_{j=0}^m c_j}\sqrt{\pi_k c_k})$$
$$R_0^{*U} = \pi_0 np + np\frac{(\sum_{j=1}^m \sqrt{\pi_j c_j})^2}{\sum_{j=0}^m c_j}$$

There is no easy way to mathematically compare the quality profile and the royalty gap. We discuss two special cases: the 2-artist and 3-artist models with identical specialists.

Theorem 1. *In the two-artist model, P rule is better than U rule in efficiency and fairness.*

The intuition for better performance of efficiency is from the competition. Under P rule, the artists must compete on the streaming of all subscribers. Under U rule, the artists only compete on the streaming of their fans. The intuition for the dominance of fairness is that the P rule can significantly increase the incentive for the specialists to increase their songs' quality. In contrast, the superstar's incentive increase is not that significant.

Theorem 1 can be easily checked. From Propositions 1 and 2, in this 2-artist model, under P rule, the equilibrium quality profile is $q_0^{*P} = \frac{c_1}{c_0}\frac{\pi_1 np}{c_0(\frac{c_1}{c_0}+\pi_1)^2}$ and $q_1^{*P} = \frac{\pi_1 np}{c_0(\frac{c_1}{c_0}+\pi_1)^2}$. The royalty profile is $R_0^{*P} = \frac{\frac{c_1}{c_0}}{\frac{c_1}{c_0}+\pi_1}np$ and $R_1^{*P} = \frac{\pi_1}{\frac{c_1}{c_0}+\pi_1}np$. Under U rule, the equilibrium quality profile is $q_0^{*U} = \frac{c_1}{c_0}\frac{\pi_1 np}{c_0(\frac{c_1}{c_0}+1)^2}$ and $q_1^{*U} = \frac{\pi_1 np}{c_0(\frac{c_1}{c_0}+1)^2}$. The royalty profile is $R_0^{*U} = \pi_0 np + \pi_1 np\frac{\frac{c_1}{c_0}}{\frac{c_1}{c_0}+1}$ and $R_1^{*U} = \pi_1 np\frac{1}{\frac{c_1}{c_0}+1}$. Given $\pi_1 < 1$, both equilibrium qualities under P rule are higher than that under U rule ($q_0^{*P} > q_0^{*U}$ and $q_1^{*P} > q_1^{*U}$). Also, $R_1^{*P} > R_1^{*U}$, the specialist gets more under P rule.

3.1. Extension: Model with one superstar and two identical specialists

To give a mathematically tractable extension, we assume that there are only two types of artists: one superstar and 2 homogeneous specialists. The superstar 0 has low marginal cost c_L, and the specialists have uniform high marginal cost c_H. The marginal costs satisfy $c_H \geq c_L > 0$. Also, all the specialists share the same popularity. To make the notation coherent, we assume the specialists share uniform popularity $\pi_H(\pi_H \leq \frac{1}{m})$. We find that:

Proposition 3. *In the 3-artist with identical specialists model, Theorem 1 still holds.*

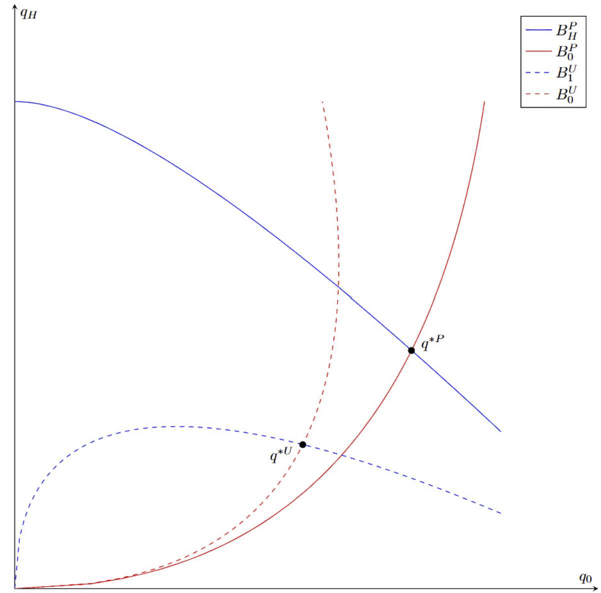

Fig. 1. Best Response functions and Equilibrium Quality Profile.

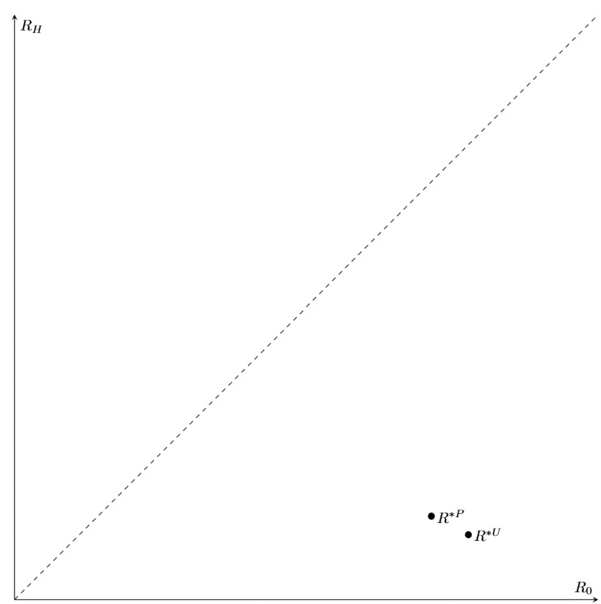

Fig. 2. Royalty profile and egalitarian fairness.

We give a numerical example. Considering the case $\pi_H = \frac{1}{3}$, $c_H = c_L = 10$, $n = 100$, $p = 8$. Let B_0 denote the best response function of the superstar and B_H denote the best response function of the homogeneous specialists. The result is shown in Figs. 1 and 2. Obviously, P rule induces a higher equilibrium quality profile. Also, the royalty profile closer to the 45° line is more egalitarian.

4. Discussion and conclusion

This paper proposes an endogenous model, which shows P rule outperforms U rule in efficiency and fairness. It contradicts the results from other exogenous models. Our result indicates that the current transition from P to U rule can reduce efficiency and fairness in the music streaming industry. So, instead of using U rule, the platforms should increase the exogenous variables (e.g., expanding the market and attracting more subscribers). Although the theorems are compelling, there are some limitations to the model. First, we assume the consumers have uniform marginal utility functions on streaming, which can be heterogeneous. For example, some people are more addicted to music. Second, our analysis can only be limited in some special cases for mathematical tractability.

Data availability

No data was used for the research described in the article.

Appendix A. Proof of propositions

Proof of Proposition 1. The marginal royalty under P rule is:

$$MR_k^P(q_k, q_{-k}) = \begin{cases} \frac{\sum_{j=1}^m \pi_j q_j}{(q_0+\sum_{j=1}^m \pi_j q_j)^2} np & k=0 \\ \frac{\pi_k(q_0+\sum_{j\neq k|j>0}^m \pi_j q_j)}{(q_0+\sum_{j=1}^m \pi_j q_j)^2} np & k>0 \end{cases}$$

To solve the FOC system, let $\forall k>0,\ \pi_k q_k = x_k q_0$ and $x_0 = 1$. The linear system is:

$$\begin{aligned} \forall k>0,\ q_0 &= \frac{\pi_k np}{c_k}\frac{\sum_{j\neq k}^m x_j}{(\sum_{j=0}^m x_j)^2} \\ q_0 &= \frac{np}{c_0}\frac{\sum_{j=1}^m x_j}{(\sum_{j=0}^m x_j)^2} \\ x_0 &= 1 \end{aligned}$$

First, we solve the linear system for $\boldsymbol{x}$ and then the equilibrium quality profile can be found. The linear system for $\boldsymbol{x}$ is:

$$\frac{\sum_{j\neq 0}^m x_j}{c_0} = \frac{\pi_1 \sum_{j\neq 1}^m x_j}{c_1} = \cdots = \frac{\pi_m \sum_{j\neq m}^m x_j}{c_m}$$
$$x_0 = 1$$

In the matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ c_0 & -\frac{c_1}{\pi_1} & c_0-\frac{c_1}{\pi_1} & c_0-\frac{c_1}{\pi_1} & \cdots & c_0-\frac{c_1}{\pi_1} \\ c_0 & c_0-\frac{c_2}{\pi_2} & -\frac{c_2}{\pi_2} & c_0-\frac{c_2}{\pi_2} & \cdots & c_0-\frac{c_2}{\pi_2} \\ c_0 & c_0-\frac{c_3}{\pi_3} & c_0-\frac{c_3}{\pi_3} & -\frac{c_3}{\pi_3} & \cdots & c_0-\frac{c_3}{\pi_3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ c_0 & c_0-\frac{c_m}{\pi_m} & c_0-\frac{c_m}{\pi_m} & c_0-\frac{c_m}{\pi_m} & \cdots & -\frac{c_m}{\pi_m} \end{bmatrix} \times \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

The size of the parameter matrix is $m+1\times m+1$. The solution $\boldsymbol{x}$ can be found using Cramer's rule. □

Proof of Proposition 2. The marginal royalty under U rule is:

$$MR_k^U(q_k, q_{-k}) = \begin{cases} \sum_{j=1}^m \frac{q_j}{(q_0+q_j)^2}\pi_j np & k=0 \\ \frac{q_0}{(q_0+q_k)^2}\pi_k np & k>0 \end{cases}$$

The FOC linear system is:

$$\begin{aligned} \sum_{j=1}^m \frac{\pi_j q_j}{(q_0+q_j)^2} np &= c_0 \\ \frac{\pi_1 q_1}{(q_0+q_1)^2} np &= c_1 \\ &\vdots \\ \frac{\pi_m q_m}{(q_0+q_m)^2} np &= c_m \end{aligned}$$

From the equation $1 \sim m$, there is a relation between $q_k (k > 0)$ and q_0:

$$q_k = \sqrt{\frac{\pi_2}{c_2} npq_0} - q_0$$

Substituting the relation into the equation 0, q_0^{*U} can be found. Then, all the other equilibrium quality q_k^{*U} can be found. □

Proof of Proposition 3. From Propositions 1 and 2, under U rule, the equilibrium quality profile and royalty profile are:

$$\begin{aligned} \forall k>0,\ q_k^{*U} &= np\frac{2\pi_H}{c_L+2c_H}\left(1-\frac{2c_H}{c_L+2c_H}\right) \\ q_0^{*U} &= np\frac{4\pi_H c_H}{(c_L+2c_H)^2} \\ \forall k>0,\ R_k^{*U} &= np\pi_H\left(1-\frac{2c_H}{c_L+2c_H}\right) \\ R_0^{*U} &= np\left((1-2\pi_H)+\pi_H\frac{4c_H}{c_L+2c_H}\right) \end{aligned}$$

In equilibrium, under the P rule, the quality profile and royalty profile are:

$$\begin{aligned} \forall k>0,\ q_k^{*P} &= \frac{c_L^2}{(\frac{2c_Lc_H}{\pi_H}+c_L^2)^2}\frac{2npc_L}{\pi_H} \\ q_0^{*P} &= \frac{\frac{2c_Lc_H}{\pi_H}-c_L^2}{(\frac{2c_Lc_H}{\pi_H}+c_L^2)^2}2npc_L \\ \forall k>0,\ R_k^{*P} &= \frac{c_L}{\frac{2c_H}{\pi_H}+c_L}np \\ R_0^{*P} &= \frac{\frac{2c_H}{\pi_H}-c_L}{\frac{2c_H}{\pi_H}+c_L}np \end{aligned}$$

To check the efficiency property, for the superstar, we have the following:

$$q_0^{*P} - q_0^{*U} = 2npc_L^3\frac{(\frac{8}{\pi_H}-12)c_H^2 - c_L^2 + (\frac{1}{\pi_H}-2-\pi_H)2c_Hc_L}{(c_L+2c_H)^2(\frac{2c_Lc_H}{\pi_H}+c_L^2)^2}$$

The check the sign, the denominator is:

$$\begin{aligned} &\left(\frac{8}{\pi_H}-12\right)c_H^2 - c_L^2 + \left(\frac{1}{\pi_H}-2-\pi_H\right)2c_Hc_L \\ &\geq 4c_H^2 - c_L^2 - c_Hc_L \ \left(\text{Since } \pi_H \leq \frac{1}{2}\right) \\ &\geq 4c_H^2 - c_H^2 - c_H^2 \ (\text{Since } c_L \leq c_H) \\ &= 2c_H^2 \geq 0 \end{aligned}$$

For the specialists:

$$\forall k>0,\ q_k^{*P} - q_k^{*U} = 2npc_L^3\frac{4(\frac{1}{\pi_H}-1)c_Lc_H + (\frac{1}{\pi_H}-\pi_H)c_L^2}{(c_L+2c_H)^2(\frac{2c_Lc_H}{\pi_H}+c_L^2)^2} > 0$$

To check the fairness property, we compare the specialist's royalty under two rules:

$$\forall k > 0, R_k^{*P} - R_k^{*U} = np\frac{(1-\pi_H)c_L^2}{(c_L + 2\frac{c_H}{\pi_H})(c_L + 2c_H)}$$

Since $\pi_H \leq \frac{1}{2}$ and $c_H \geq c_L > 0$, the gap is always positive. □

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