Issues of Fitness for Purpose in Train Simulation Models: a Review

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Abstract. Many simulation models representing the longitudinal dynamics of a train are based on a single point-mass description. This leads to a second-order nonlinear ordinary differential equation, together with algebraic relationships. More complex multi-mass models may be used for models representing long trains involving many separate vehicles. However, in both cases, accuracy is limited by important underlying assumptions, approximations and parametric uncertainties. Another important aspect of train models concerns the direction of information flow. Input variables within conventional train models may represent power or tractive force, with acceleration, speed and distance travelled as output variables. However, inverse simulation methods can also be used, with the required speed or distance as inputs and tractive force, power, or energy as outputs. This allows energy requirements to be established for a given schedule and is useful when investigating fuel or energy economy. Inverse methods can also be used in powertrain design, such as for hybrid hydrogen fuel-cell/battery-electric trains. Issues of fitness for purpose are important in all such applications, both in terms of model uncertainties and in the additional insight offered by inverse simulation methods.

Introduction

In engineering applications, modelling and simulation methods allow early consideration of design trade-offs and system integration issues before any prototypes become available for testing. This is true for railway applications as much as for any other field. Model-based approaches can offer important benefits in terms of cost, safety and timely delivery of final systems in, for example, the development of new train designs and the introduction of improved train operating methods.

Simulation models must always be capable of being used in a convenient and effective fashion for the application at hand. Assessment of the fitness of a given simulation model for a specific application should always include careful testing procedures based on the well-established processes of validation, verification and documentation.

The simulation of longitudinal train motion has attracted new interest recently because of the importance of de-carbonising rail services and the need to reduce energy usage through, for example, the introduction of improved driving strategies. The design of bi-mode traction systems and other hybrid powertrains is one specific area where simulation methods are potentially very important.

In models of longitudinal train motion, the tractive force at any time instant is balanced by an inertial force, plus forces that include aerodynamic and other resistive characteristics of the train, together with route-dependent resistive forces such as gradient resistance, curvature resistance and additional aerodynamic resistance in tunnels.

The objectives of this paper are to review forms of longitudinal train models in current use (e.g. [1]-[7]) and to investigate sources of uncertainty within such models in the context of specific types of application [8]. The use of inverse simulation methods for handling problems involving longitudinal train dynamics is also considered [9].

1 Dynamic Models of Longitudinal Train Motion

The longitudinal motion of a train can be described by a set of nonlinear ordinary differential equations and associated algebraic equations. As in the modelling of other engineering systems, questions of the model structure are linked closely to the intended application. For example, in models used for design of train control systems or the assessment of energy demands, lateral and vertical movements of vehicles are normally neglected and only longitudinal motion is considered.

1.1 General Form of Multi-mass Model

Figure 1 is a schematic diagram of a general lumped-parameter model of the longitudinal motion of a train involving a number of coupled vehicles. Here, the parameters v_1 and k_1 describe viscous and elastic properties of the couplings between the first and second vehicles. Couplings between other vehicles are represented in a similar way.

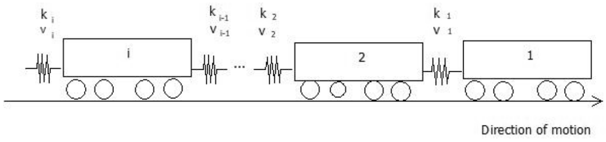

Figure 1: Typical multi-mass representation of a train showing the first second and ith vehicle.

Each powered vehicle is subjected to traction, braking and resistive forces while the unpowered vehicles have no tractive force component. In both cases, train resistance forces involve rolling resistance, air resistance, resistance due to gradients (positive or negative) and resistance due to track curvature.

The equation for the leading vehicle has the form:

$$m_1 \frac{d^2x_1}{dt^2} + v_1\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right) + k_1(x_1 - x_2) + F_{R1} \pm F_{G1} + F_{C1} = F_{T1} - F_{B1} \quad (1)$$

where the variables $x_1(t)$ and $x_2(t)$ are the distances relative to the starting point for the first and second vehicles, and the forces F_{R1}, F_{G1} and F_{C1} are resistive, gravitational and track curvature forces.

The forces F_{T1} and F_{B1} are the tractive and braking forces at the rail, respectively. The effective vehicle mass is represented by the parameter m_1 which is the product of the actual static vehicle mass and a factor $(1 + \phi)$ introduced to account for the inertial effects of rotating parts. The parameter ϕ is typically assigned a value of about 0.1 (see e.g. [5]). This equation can be modified to incorporate more complex nonlinear representations of couplings (see, e.g. [1]).

The vehicle resistive force F_{R1} involves three components, as shown in (2). This is traditionally referred to as the Davis equation (and also, the Leitzmann Formel, von Borries Formel or fonction de Berbier) and is based largely on empirical findings. It involves a constant component a_1, a velocity-dependent component involving b_1 and an aerodynamic component with factor c_1 that depends on the square of the velocity.

$$F_{R1}(t) = a_1 + b_1\frac{dx_1}{dt} + c_1\left(\frac{dx_1}{dt}\right)^2 \quad (2)$$

Equations can be derived in a similar way for the other vehicles, giving an equation for the final vehicle, *n*, of the form shown in (3):

$$m_n \frac{d^2n}{dt^2} + v_{n-1}\left(\frac{dx_n}{dt} - \frac{dx_{n-1}}{dt}\right) + k_{n-1}(x_n - x_{n-1})$$

$$+ F_{Rn} \pm F_{Gn} + F_{Cn} = F_{Tn} - F_{Bn} \quad (3)$$

together with an equation for the resistance F_{Rn}, which is similar in form to (2).

These equations apply to any combination of powered and unpowered vehicles and can therefore be used to describe multiple-unit passenger trains with several powered wheel-sets, or locomotive-hauled passenger or freight trains (e.g. [1], [6], [7]).

This distributed mass model is potentially useful for applications requiring consideration of the kinetic energy in different parts of a train and investigation of associated transient forces that may be exerted at the couplings. Such issues can be important in the modelling and simulation of long trains, especially in the context of train control and braking strategies. This is particularly important for long freight trains, especially on routes with frequent changes of gradient, or many sharp curves and local speed restrictions.

1.2 The Single-mass Model

In many cases where the train length is short compared with features of the route, the model of (1) – (3) may be reduced to involve a single mass (see, e.g. [2]). Comprehensive testing of the validity of this simplified generic model structure has been undertaken for short trains such three-coach or four-coach diesel or electric multiple units, including the use of on-train data (see, e.g. [2], [6], [10]).

Such models have been used widely for investigations of fuel consumption or energy usage.

With this single point-mass approximation, the model of (1) – (3) reduces to:

$$M\frac{d^2x(t)}{dt^2} + R(t) \pm F_G + F_C = F_T(t) - F_B(t) \quad (4)$$

$$\text{where} \quad R(t) = A + B\frac{dx}{dt} + C\left(\frac{dx}{dt}\right)^2 \quad (5)$$

Here the variable $x(t)$ is the position of a chosen reference point on the train (usually the front or the mid-point of the train). The other variables $F_T(t)$, $F_B(t)$ and $R(t)$ are the tractive, braking and train resistance forces acting on the effective mass M_e of the train. This effective train mass is equal to the sum, M, of the static masses of the individual vehicles, multiplied again by an inertial factor $(1+\phi)$. The parameters A, B and C are counterparts of the parameters a_i, b_i, and c_i in the train resistance terms of the distributed-mass model.

1.3 The Route Sub-model

The gravitational terms included in (1) and (3) depend on the gradient profile and this information may be taken from a sub-model providing data for each point on the chosen route.

As shown in Figure 2, the gravitational force acting on each vehicle in the multi-mass model is given by:

$$F_{Gi} = m_i g \sin\alpha \tag{6}$$

where $\sin\alpha = \frac{1}{Y}$ for a gradient of 1 in Y at each point on the route for vehicle i and g is the acceleration due to gravity.

Similarly. for the single-mass model of (4), the angle α gives the gradient at each point on the track and the force acting on the train due to gravity is given by:

$$F_G = Mg \sin\alpha \tag{7}$$

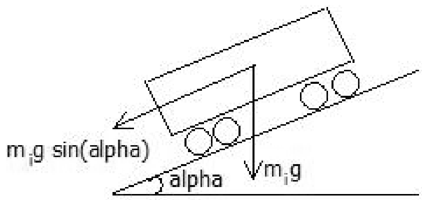

Figure 2: Diagram showing gravitational force acting on the *ith* vehicle of the train.

The route sub-model can also incorporate track curvature data to allow resistance forces $F_{Ci}(x_i(t))$ or $F_C(t)$ to be found.

This is usually based on an empirical approximation and the most widely used relationship involves the product of mass and a factor that depends inversely on curve radius [3] to give:

$$F_{Ci}(x_i(t)) = \frac{k_{cv}}{R(x_i(t))} m_i \tag{8}$$

in the case of the multi-mass model, or

$$F_C(x(t)) = \frac{k_{cv}}{R(x(t))} M \tag{9}$$

for the single-mass approximation. Here $R(x_i(t))$ or $R(x(t))$ represents the radius of curvature (m) at the point $x_i(t)$ or $x(t)$ and k_{cv} is an empirical factor that can vary considerably, depending on environ-mental conditions, state of maintenance of the track and condition of the vehicle.

1.4 Tractive Force, Power and Energy

The sum of the forces $F_T(t)$ and $F_B(t)$ forms a net tractive force variable $T(t)$ which can be positive, zero or negative, depending on the operating condi-tion. Note that, in practical simulation models, the braking force component $F_B(t)$ of the tractive force $T(t)$ can only take a non-zero value if $F_T(t)$ is zero.

In the single-mass model, the power at the rail, $P_R(t)$, is given by:

$$P_R(t) = T(t)\frac{dx}{dt} \tag{10}$$

and a similar equation can be used to determine the power at the rail P_{Ri} of any powered vehicle i within the distributed mass model.

The applied tractive force is limited at low speeds in order to avoid wheel slip. This commonly involves application of a constant tractive force at the rail, T_0, until the speed reaches a value V_{ch} given by:

$$V_{ch} = \frac{P_R}{T_0} \tag{11}$$

where P_R is the value of the applied power at the rail. This condition corresponds to the point on the hyperbolic curve of (10) where the tractive force at the rail is equal to the limiting value T_0, as shown in Figure 3.

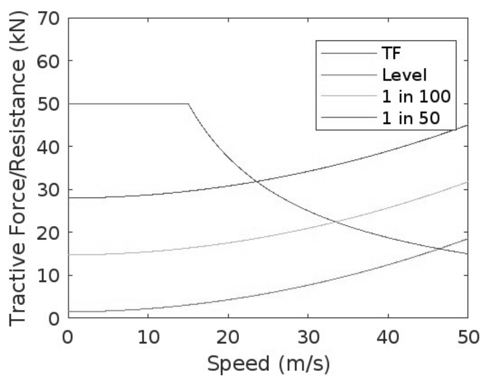

Figure 3: A typical tractive force versus speed curve (TF) together with three train resistance curves for level track (lower curve), a rising gradient of 1 in 100 (middle curve) and a rising gradient of 1 in 50 (upper curve).

It should be noted that this limiting value is mass-dependent and that (10) and (11) apply also to powered vehicles within a multi-mass type of model.

The tractive force and resistance curves of Figure 3 are typical of those for a type of single-mass three-coach multiple unit train used in the United Kingdom. The power level for the hyperbolic part of the curve (defined by (10)) is 750 kW (at the rail). The curves show clearly the variation of resistive forces with speed and also the speed at which the tractive force at the rail and resistance forces balance and how this changes with the gradient.

The energy used for traction is the integral of the power with respect to time, as shown in (12).

$$E = \int P(t)dt \tag{12}$$

Powertrain energy losses and the additional energy for auxiliaries, such as on-board computer systems, heating and air conditioning must also be allowed for.

2 Fitness for Purpose Issues

2.1 Model Structure Issues

For any model, in any field of application, questions concerning model structure are always linked closely to the purpose of the model. A desirable model structure might be thought of as a generic one capable of being used in many different ways, but this approach tends to produce models having a very large number of variables, many of which may not be readily accessed for measurement and also many parameters that are hard to estimate.

For example, the use of a multi-mass description introduces complexities because each vehicle has its own resistance and braking characteristics with inherent uncertainties, and there may also be significant issues about the modelling of inter-vehicle couplings.

Simple linear descriptions of coupling dynamics (such as those used in (1) and (3) above) are unlikely to provide an adequate description of vehicle interactions and more complex nonlinear spring and damper representations may be needed [1]. Often, a single-mass type of structure may be more appropriate since that involves far fewer variables and fewer parametric uncertainties. With the single-mass type of structure the limits within which models can be used with confidence are thus more readily established.

The modelling of train braking systems introduces additional problems and, in many cases, enhancement of the structure of vehicle models becomes essential in order to describe practical systems.

Detailed modelling of braking action is becoming increasingly important due to the current interest in regenerative braking, the development of blended pneumatic/regenerative braking strategies and the introduction of more autonomy within train operations.

In the case of electrified railways, train models may also need to include features of the electrical supply infrastructure to allow investigation of the effects of train dynamics on the overall energy demands and also how supply systems may be affected by regenerative braking. This greatly extends the model boundaries and can add to the complexity of the model. The inclusion of electrical supply system infrastructure within the model also influences the timescales of interest since relatively fast electrical transients may become important.

2.2 Parameter Sensitivity Issues

The structure of the equations (1)-(5) provides useful information about parameter sensitivities. The mass has an obvious and direct effect through the terms involving acceleration and also through the gradient terms as in (6) and (7). The resistance curves of Figure 3 show clearly that the gradient force is the dominant term in the total resistance to motion, except when the train is travelling on level track or on very gentle gradients.

Although the resistance equations are essentially empirical relationships, much work has been done to try to establish a more physical basis for the coefficients *A*, *B* and *C* (see, e.g. [3]-[8]). For example, coefficient *A* is known to depend on the train mass, while *B* is generally believed to depend both on train mass and on the mass of air entering the train for ventilation and cooling. The aerodynamic term *C* depends on the shape of the train. It is directly proportional to the air density, the frontal area of the train and an overall drag coefficient that is the sum of the head and tail drag coefficient plus other drag terms (e.g. [3]-[5]).

In Figure 3 the sensitivity of the resistance to each of the factors *A*, *B* and *C* of (5) may be seen clearly, with the terms involving *B* and *C* becoming more important as the speed increases. The aerodynamic term (i.e. *C*) is clearly most important at high speeds and its components have been investigated in much detail for high-speed train design using both experimental data and the techniques of computational fluid dynamics (CFD). For trains operating in the lower part of the speed range, as shown in Figure 3, the resistance force depends mainly on the train mass through parameter *A*.

The significance of the resistance forces due to the terms involving the *A*, *B* and *C* coefficients compared with the gradient and curvature resistance forces is best seen from a specific example. The data used to generate the resistance curves of Figure 3 demonstrate, for example, that the gradient component of resistance on a rising gradient of 1 in 100 is similar to the sum of all the other resistance forces at 40 m/s for the type of train considered. For the gradient of 1 in 50, the contribution from the gradient term doubles again. In terms of curvature resistance, calculations based on (9) using a typical value for k_{cv} suggest that, for the multiple unit train considered, the curve resistance force is 2751 N for a relatively sharp curve of 200m radius. This is almost twice the resistance force associated with the *A* term of the Davis equation. This resistance is also believed to be much larger (possibly by as much as a factor of two) if the train is moving away from rest while on a curved section of track. Other relevant issues that affect curve resistance include the amount of cant on the curve, the train speed, the rail profile, the design and condition of the vehicles and whether or not rail track lubrication is applied on the curve.

2.3 Uncertainties in Train Parameters

Getting good estimates of the train resistance parameters can present difficulties, even for an existing type of train since these are empirical quantities, which, at present, are usually estimated from full-scale tests using coasting trials [3], [6], [10]. Finding appropriate parameters for individual vehicles in a multi-mass model presents significant difficulties.

Even for the single-mass type of model, accurate estimates are available only for a few specific types of train (see, e.g. [3]-[5]) and values used in practice are often approximate and based on average values for the types of vehicles in question. For example, values often quoted for the resistance parameters for a two-coach Class 156 diesel multiple unit train (a type widely used in the United Kingdom) are A=1500 N, B=6.0 $Nm^{-1}s$ and C=6.7 $Nm^{-2}s^2$ but it is known that these values can vary significantly. For example, with all seats occupied, the value of *A* could, in theory, rise to 1755 and the value of parameter *B* would also be increased.

In the case of new passenger train design, or new freight vehicles, the determination of appropriate values for resistance parameters is clearly difficult in the initial stages of the design and development process.

Once a prototype is available it may be possible to estimate resistance parameters during the initial testing that is often carried out prior to a new train design being approved for service.

2.4 Uncertainties in Route Parameters

Although nominal gradient information is often available, uncertainties may still exist because of the cumulative effects of changes made during track maintenance work and upgrades over long periods of time.

Major changes may have been recorded but the effects of minor adjustments made over many years of track maintenance may not always be known. For example, even changes as small as 25 cm at each end of a 1 km section of track with a nominal gradient of 1 in 100 could lead to an average gradient change to 1 in 95 for that section.

Information about track curvature may also be available but this may be more difficult to incorporate accurately within the route sub-model as, in practice, curved sections of track may involve a transition at the start and finish where the radius changes gradually.

Also, even on a curve having a fixed radius, the parameter k_{cv} can vary between about 500 and 1200 depending on environmental conditions and train speed.

3 Inverse Methods

There are modelling situations in which an inverse approach may allow some issues to be considered directly that would require much repetitive simulation if approached through traditional forward simulation methods. For example, a time history of distance versus time for an existing type of train on a specific route is potentially useful in providing a reference schedule which can be applied as input to an inverse simulation for a new train design.

Estimates can then be obtained directly for the tractive force, power and energy required for that new train operating on that specific route for that schedule. Time-scaling methods allow a reference schedule of this kind to be adjusted to investigate possible performance enhancements [9].

Several methods of inverse simulation are available. One approach that has been used successfully with train models involves a continuous system simulation method based on feedback principles [9].

4 Model Testing

For models developed for system design purposes, no system is available initially to allow rigorous model testing and validation. Only when a prototype system becomes available is it possible to make detailed comparisons between the model are the real system behaviour (e.g. [6], [7], [10]). However, even at an early stage of a new design project, insight may be gained from the use of previously tested models of a broadly similar type and by basing the work on well-established and fully tested models of components. In most design applications repeated testing and refinement of models is essential as the project progresses.

For models of existing trains, data gathered on-board a train (e.g. speed, distance travelled, power or tractive force applied at each time instant over a specific route) can provide quantitative data to allow assessment of predictive accuracy (e.g. [6], [7]). In this context, care in needed in considering use of global positioning system (GPS) data due to inherent errors. Checking of GPS values against independently recorded data based on passing times at key points on the route in question is recommended to detect any inconsistencies.

Some published reports on train performance monitoring suggest that gradients can be estimated from GPS altitude data. However, possible errors in the GPS altitude channel are known to be significantly larger than errors in positional and speed data. Conventional gradient profiles are therefore usually used in train simulations, rather than altitude data.

5 Applications

The choice of model structure depends on the objectives. For example, if the train length is short compared with features of the route, such as the gradient profile and curves, the single-mass type of description may be adequate. However, if the train is longer and especially if it is locomotive-hauled, a multi-mass description may be needed, particularly if the dynamics of different sections of the train are considered important or there is a need to estimate forces at couplings [1]. For example, with regenerative braking alone, the fact that there are no powered axles on trailing vehicles means that forces applied to the locomotive can become unacceptably large during braking actions. However, in conventional pneumatic brake systems, there can be significant delays in the application of the brakes on different vehicles and this can also give rise to undesirable longitudinal dynamic effects.

Thus, investigations of braking strategies and the possible use of blended braking systems involving a combination of pneumatic and regenerative braking, especially with long trains, may require the use of a multi-mass type of model.

In the context of fitness-for-purpose, two specific areas of application have been chosen for discussion because of their current relevance in terms of energy costs and the moves towards de-carbonisation of rail transport. The first of these areas concerns the development of more economical methods of train operation and driving, including lightweight train designs and regenerative braking. The second application area considered involves the design of hybrid trains based on hydrogen fuel cells and batteries for routes where traffic levels do not justify conventional electrification.

5.1 Train Operations Simulation

Dynamic models and simulation methods are already being used to investigate efficiency improvements. Examples of strategies for minimising fuel and energy costs in train operations include the development of improved driving practices and also consideration of possible benefits of infrastructure upgrades on sections of the route where there are severe speed restrictions. As well as reducing travel times, such upgrades could avoid wasting energy through brake applications and subsequent acceleration back to the normal line speed. Other applications of simulation include the investigation of interactions between train operations and the electrical supply system on electrified railways and studies of the use of regenerative braking.

In all such applications, model accuracy is of central importance since inadequate simulation models inevitably lead to incorrect conclusions. For example, optimisation calculations must often be repeated for a range of different model parameter values for different train load conditions.

For passenger services, loads may vary from the minimum (tare weight of the train plus any fuel load and driver) to crush loading where every seat is occupied and there are many standing passengers.

This mass difference is always significant and is particularly large on light rail vehicles. As an example, the CAF Urbos3 trams currently used in Edinburgh have a tare mass of 56000kg with 250 places available. Using an average passenger weight allowance of 80 kg, a fully-loaded tram has a mass of about 76000 kg representing an increase in mass of almost 36% [11].

This is a very significant change that can have a marked effect on longitudinal dynamics and especially performance when accelerating and braking. For a typical UK three-coach diesel multiple-unit (dmu) train, the extra mass contributed by a full load of passengers is smaller, but still significant, at about 15% of the tare value. For crush-loaded conditions, with the maximum possible number of standing and seated passengers, the load increase could be at least 10% greater.

Inverse simulation methods are potentially useful for the investigation of driving strategies and also for the design of braking systems. In the inverse approach, the input variable would be a required profile of distance, speed or acceleration versus time and the simulation model outputs would include the power, tractive force or braking force at the rail.

Again, there are important constraints that must be taken into account in terms of adhesion limits, maximum power levels for motors in generator mode, maximum power levels during the charging of the energy storage system and constraints in terms of the ratings of power-electronic components.

Taking the Edinburgh tram example, it would be interesting to use simulation to investigate whether or not regenerative braking to an on-board supercapacitor system or battery pack could lead to useful operating economies, taking account of weight penalties arising from the additional on-board equipment.

5.2 Hybrid Powertrain Simulations

The current drive to de-carbonise both passenger and freight rail services has generated a strong interest in the development of forms of traction involving combinations of hydrogen fuel cells and electrical storage elements such as batteries, supercapacitors or flywheels. The need for energy storage arises because hydrogen fuel cells tend to be sluggish in their response to changes in demanded power and the stored energy can be used to provide additional tractive effort very rapidly when the train is accelerating or ascending steep gradients. Re-charging takes place when all of the power available from the fuel cell stack is not needed for traction and also during regenerative braking. Further details of hybrid powertrain configurations may be found elsewhere (e.g. [12]-[14])

Supercapacitors (and possibly flywheels) are suitable for light rail vehicles on routes with frequent stops, but batteries are generally considered more appropriate for other applications.

However, batteries introduce significant weight penalties and getting the right balance between the sizes of the fuel-cell stack and battery pack is a complex process. The space required for the storage of hydrogen gas onboard the train is another critically important factor. Simulation methods have a potentially important role in addressing all of these design issues (e.g. [12]-[14]).

The design of hybrid powertrains is also an area in which inverse models are being applied. That approach has been used for automotive powertrain design for some considerable time, but mostly using steady-state or quasi-steady descriptions.

However, in railway applications, the dynamics of the train itself, together with the characteristics of the route, are very important and steady-state descriptions are inappropriate. It is believed that inverse dynamic modelling and simulation can provide important additional insight into the sizing of powertrain components and the optimization of the associated control and energy management systems [12]-[14].

Simulation activities linked to the conversion of a former ScotRail electric multiple-unit train to hydrogen fuel-cell/battery-electric hybrid form are discussed in [13] and further simulation results may be found elsewhere (e.g. [14]). The routes concerned are typical of lines that provide important transport links but involve traffic densities that are too low for a strong business case for conventional electrification. In general, it has been found that hillier routes lead to powertrain configurations involving larger batteries to cope with the frequent changes in demanded power level.

Figures 4(a) – 4(d) show typical inverse simulation results for a specific hybrid train configuration involving hydrogen fuel-cell/battery-electric traction.

This is for a route section approximately 15 km long, which is typical of distances between stations on some rural routes in Scotland. The gradient profile involves an initial section of level track for 1 km where the train accelerates towards the maximum permitted speed of 96 km/hr, a 4 km section with a rising gradient of 1 in 55 and a final section where the train operates on level track, including a coasting phase and a final braking phase. These features of the route are reflected in the speed time-history of Figure 4(b) which corresponds to the distance versus time schedule in Figure 4(a) (generated using a simulation of a conventional diesel multiple unit).

This schedule provides the input to the inverse simulation and Figure 4(c) shows the tractive force that needs to be developed by the hybrid train to match the distance versus time record of Figure 4(a). Negative values of tractive force correspond to braking actions and Figure 4(d) shows the energy usage, including energy recovered through regenerative braking. Such results then provide a basis for decisions regarding the power rating of the fuel-cell stack and battery pack as discussed in [13] and [14]. Results in Figures 4(a) – 4(d) are for a set of parameters corresponding to conditions involving a full load of seated passengers.

This use of the inverse simulation approach, as illustrated in Figures 4(a) – 4(d), allows the necessary total tractive force, total power and total energy to be found for the new train for the set of performance requirements defined using the simulation of the existing dmu. Some of the necessary power is provided by the fuel-cell stack directly and some from energy stored in the battery pack. Results from the inverse simulation provide a starting point for investigation the optimum sizes of these components to ensure that the necessary total power is available at all times, while also satisfying constraints such as limits on the allowable battery state of charge and power ratings of electronic converters. This information then provides a basis for the detailed design of the powertrain control and energy management systems.

Variation of parameters of the train such as *M*, *A*, *B* and *C* or of the route sub-model, such as the gradient profile and curve resistance values, affect the tractive force and energy records in ways that are consistent with results of parameter sensitivity analysis. For example, reducing the mass to correspond to the tare condition with no passengers on board reduces the peak energy value shown in Figure 4(d) by about 10%. Similarly, simulation results suggest that by doubling the passenger load the peak energy use is increased by about 10%.

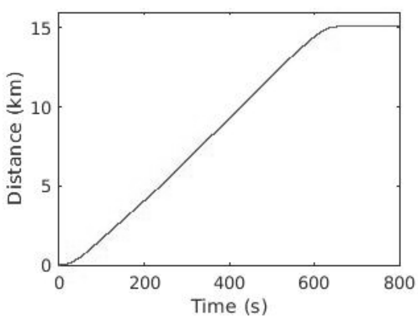

Figure 4(a): Distance versus time reference schedule applied as input to the inverse simulation.

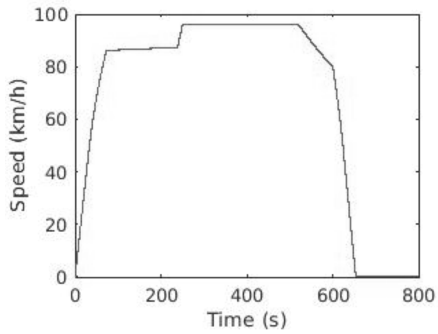

Figure 4(b): Speed versus time record corresponding to the reference schedule of Figure 4(a).

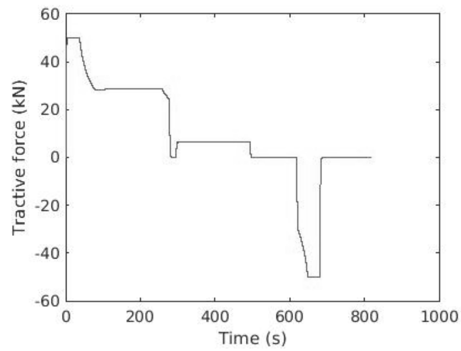

Figure 4(c): Tractive force time-history for a proposed three-coach hybrid multiple-unit train obtained using inverse simulation (for distance versus time schedule of Figure 4(a)).

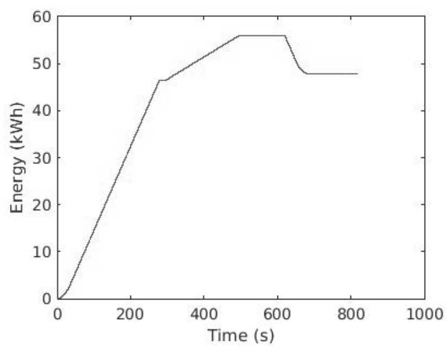

Figure 4(d): Energy record from the inverse simulation for the hybrid train for same conditions as in Figures 4(a), 4(b) and 4(c).

The model used in this work includes a simple representation of driver action in which speed is compared continuously with the speed limit for the current train position. Using that difference, tractive force values at each time step in the simulation are multiplied by a factor representing driver control actions in approaching a limit and adhering to it [9]. Control actions associated with the start and end of coasting and the initiation of braking are introduced through the route model.

6 Discussion

Uncertainties within dynamic models of the longitudinal motion of trains tend to be larger than in many other engineering applications and it is essential to record all inherent assumptions and approximations.

Users must have a good understanding of the range of values possible for each parameter within the chosen model structure and must make full use of this information when applying the model.

Rigorous model testing procedures are essential in ensuring fitness for purpose. The need for repeated testing whenever model changes are made requires robust model management processes, including formal procedures for model version control and the updating of documentation. Ideally, a simulation model needs to be maintained throughout the lifetime of the system that it represents. Not only does this ensure that a model remains fit for its original purpose but it also means that a model developed during one design project may provide a useful starting point for models in future projects. This raises some important issues about the use of simulation models in railway applications that may not, at present, be fully recognised within the industry.

7 Conclusions

Models used to represent the longitudinal dynamics of a single train have many potential applications, including the investigation of more efficient operating strategies and the design of new trains. The fitness for purpose of any model depends on decisions about its boundaries, the extent to which it satisfies accuracy requirements and whether it is to be used in a conventional forward fashion or using an inverse approach.

The model structure, boundaries, parameters and timescales must, therefore, all be tailored to the intended use. Often, a description involving a single-mass approximation is a convenient representation for short multiple-unit passenger trains.

However, multi-mass models may be preferred for longer locomotive-hauled passenger or freight trains, despite the additional complexities. In both the multi-mass and single-mass types of description there are a number of key parameters that have to be chosen to describe the resistance characteristics of the train.

Accurate information must also be available about the gradient profile and track curvature changes for the route in question.

Moves towards de-carbonisation of rail transport and the development of unconventional forms of traction, such as hybrid trains involving fuel-cell/battery-electric powertrains, often lead to complex design problems where the use of dynamic simulation methods can provide valuable insight. However, parametric and structural uncertainties mean that using an envelope of simulation results rather than time histories for a single set of parameter values can be very important.

Dynamic models and computer simulation tools have an increasingly important role in many other railway applications beyond those reviewed in this paper. Fitness-for-purpose is an issue of central importance in all of these and, as in other application areas, all model uncertainties and limitations must be identified, recorded in model documentation, and acknowledged by potential users.

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