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Teaching Place Value Conceptually – Part II

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Place value understanding is a basis for school mathematics (Moeller et al., 2011). Place value is needed for success in operations including their standard algorithms (van de Walle et al., 2019). However, children do not acquire a profound understanding of place value (Fuson et al., 1997) and suffer the consequences of this during their entire schooling (Chan et al., 2014). Instead of dealing with the root problem, the regular practice in schools is to teach place value and standard algorithms in a procedural way (Kamii, 2004). Our experience also suggest that teachers mostly treat the concept of place value as social knowledge, not logico-mathematical knowledge in primary schools. Piaget (1971, cited in Kamii, 2004) identified three different kinds of knowledge: social, physical and logico-mathematical knowledge. Social knowledge is knowledge of conventions, rules, habits, or agreed-upon information. Physical knowledge is the knowledge of observables or experiments, in other words, noticeable characteristics of an object or event. Logico-mathematical knowledge is the knowledge of relations between at least two objects established by individuals in their minds.

Place value grids (Th H T U) are heavily used in schools and pupils attach labels to the place a digit occupies. For example, it is not uncommon for pupils to describe a given number, say 1034, as consisting of "one thousand, no hundreds, three tens and four ones", dismissing the fact that the number can also be described as having 10 hundreds, possibly because pupils consider "0" as a label or place holder in the hundreds column without considering its connection to the value of "1034". In other words, the face value of a digit overshadows its place value within the number. Therefore, teaching place value as labels of digits or placeholders is the same as treating a mathematical relation as a label (social) without being aware of the relation it involves (logico-mathematical). Mechanical manipulation of base-ten blocks and representing numbers with blocks (physical) without paying attention to the embedded relationships (logico-mathematical) will also fail to build appropriate connections.

In addition to the problem of treating the concept of place value as social knowledge, two other problems may hinder pupils' understanding of place value: teachers' over-reliance on manipulatives (Kamii et al., 2001) such as base-ten blocks in physical, pictorial or virtual form, and limiting the application of place value to the base-ten system only. In our first article (Zembat et al., 2022), we explicated on the concept of place value on a theoretical basis and provided a sample lesson developed by the first author that has a real potential to improve pupils' understanding of place value as logico-mathematical knowledge by paying close attention to these problems. Here, we analyse the design principles of this novel lesson and explain how it fosters pupil understanding.

Design Principles of Place Value Lesson and How it Fosters Pupil Understanding

The lesson design involves two major activities with a set of tasks. Activity 1 involves tasks representing a certain number of items (eight, thirty four, forty one, fifty one, sixty one, seventy nine) in groups of 3, 5, 7 and 10 using sticks. Activity 2 consists of grouping the same set of items in groups of 4, 6 and 10 using interlocking cubes. In each activity, pupils are asked about the minimum number of items for a

group/bundle, the maximum number of items for a group/bundle and the group/bundle growth rate.

The purpose of these two activities is to help pupils conceptualise the logico-mathematical knowledge of place value: the relation between the magnitude of a number and the symbolic representation of that number. For example, a number like 153 consists of three partitions or columns each of which has two meanings involved in it - the column/partition that involves "5" means that we have 5 bundles but it also means that each bundle has 10 items in it. Understanding the relation between the magnitude of a number and the symbolic representation of that number in this sense requires an understanding of the coordination among three key pieces of information:

- Size of the groups/bundles (*n*)
- Quantity of the groups/bundles, including the minimum (0) and the maximum (*n*-1) number of groups/bundles of each type (order)
- Growth rate of consecutive types (order) of groups (1:*n* ratio)

Therefore, the lesson focuses pupils' attention on these three key pieces in each given task.

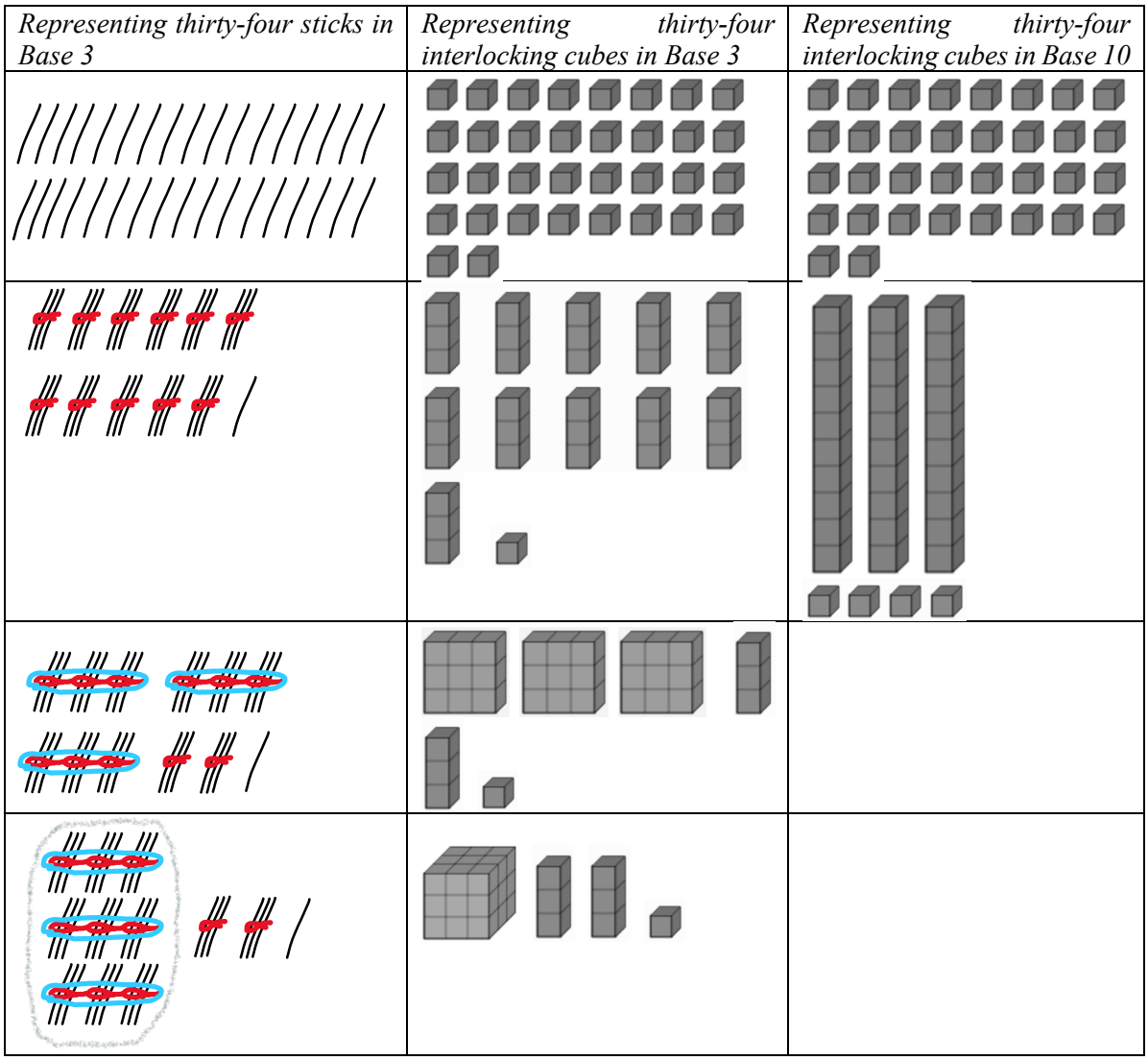

Figure 1. Grouping/Bundling 34 using sticks, interlocking cubes and base-ten blocks

The prior knowledge required to go through this lesson is:

1. to be able to represent the magnitude of a given quantity with manipulatives (e.g., drawing a picture of 'thirty-four' sticks or putting 'thirty-four' sticks on the table to represent 'thirty-four')
2. to know what it means to group using a certain group size (e.g., grouping by 3)
3. to know the symbols for numerals (0, 1, 2, …, 9).

The activities given in the lesson are structured and sequenced to give pupils the opportunity to think about organising a given number of items in groups, groups of groups, and so on in a hierarchical manner using different-size groups or bases. For example, pupils put 34 sticks on table (using prior knowledge #1), group 34 in groups of 3 (using prior knowledge #2) and represent it as a combination of 1 'individual' stick, 2 groups of '3-stick' and 1 group of '3-group-of-3-stick' as illustrated in Figure 1 and finally fill in the given table using symbols, 1, 2, 1 (using prior knowledge #3). They go through the same physical or mental action sequence for each given task. Repeating the same tasks for different bases encourages pupils to reflect on the role of the group "size and the quantity of the bundle it represents" (Herzog et al., 2019) before they start thinking about the base-ten system.

Instead of restricting the tasks to base 10, the lesson explores place value in different bases to allow pupils to question constructs that they may or may not already be familiar with. Through experiencing and comparing several bases, pupils will have the opportunity to reflect on and abstract the commonality of the three key pieces of information above. In addition, pupils who already have experience with base 10 and teachers who have been teaching place value in the same way for years might take base 10 for granted or as arising naturally and might ignore the deeper, more fundamental, aspect of it. Working with different bases better allows pupils to understand the relation between the symbolic representation of a number and its magnitude. Therefore, expanding to other bases is a crucial component of this lesson as highlighted by Thanheiser (2015) when working with teacher candidates.

Another difference between this lesson and more traditional ones is the initial use of sticks, before moving on to interlocking cubes and finally to base-ten blocks. The initial activities with sticks focus the pupils' attention on the action of grouping and organising these groups in a hierarchical manner. Pupils have the choice to bundle or organise the sticks in groups in any way they want. Sticks focus pupils' attention on the nature of the grouping while cubes focus their attention on the organisation of the physical objects (Figure 1). Continuing the instruction with sticks might limit students' understanding of the nature of grouping whereas moving to interlocking cubes will pave the way to understanding the work with base-ten blocks. Note that the interlocking-cube part of the activity can be completed using an online applet developed by the last author (Kabaca, 2020) and the sticks part of the activity can be done using an online source E-concrete manipulatives (2022).

One danger of using manipulatives is that the pupils do not see beyond the particular manipulative being used and that their understanding of the concept is limited (e.g., physical knowledge) or hidden by the concrete nature of their actions. Letting pupils experience grouping with different manipulatives tries to overcome this problem as pupils might be able to abstract the idea of grouping when they apply their actions with different concrete materials (logico-mathematical knowledge). Therefore, in this design, the activities are to be completed initially with sticks and then with interlocking cubes to teach pupils that place value refers to a particular type of grouping regardless of the manipulative used. It is important for pupils to construct the groups and reflect on the nature of them, instead of using pre-assembled 'tens', 'hundreds', 'thousands' in base-10 blocks. When using pre-made base-10 materials, pupils might simply attach labels (i.e., using social knowledge) to those groups and this may overpower the meaning 'behind' the manipulatives, what the manipulatives represent. Hence, using sticks and interlocking cubes gives no danger of 'familiarity' with these manipulatives in this context.

Depending on the age and level of the pupils, the teacher might start using vocabulary that pupils will encounter in secondary school, such as 'first order', 'second order' groups. The teacher can use the language of 'groups of first order', 'groups of second order', to orient pupils' thinking toward the idea that there is a change in the nature and size of the groups as one moves from the rightmost column (leftovers) to the left. This language is useful in preparing children for later years when they learn to represent multiples of 10 as powers of 10 for place value (e.g., considering "hundreds" as 10^2). However, the teacher might want not to focus too much on this terminology and use it together with the more "child-friendly" "groups of groups".

The activities described above would take 2-3 weeks to be applied to give pupils enough time and space to think and develop an understanding of the meaning of place value. The lesson can be adapted to different grade levels, either as a first introduction to place value or for revisiting the concept. We feel that the time dedicated to these activities will be rewarded when the pupils can construct the meaning of number operations and algorithms on a solid foundation of place value understanding.

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