# Online Supplement to "Bayesian dynamic variable selection in high-dimensions"


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## Contents




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# A Settings used in forecasting exercise

While all technical details regarding our methodology are provided in detail in the paper, we have skipped details for the numerous competing algorithms used in the Monte Carlo and empirical exercises.

- **DSS algorithm, Rockova and McAlinn (2021)**: We followed the authors and tried the various settings they suggest in their Section 7: Synthetic high-dimensional data. For our DGP the best performance was achieved with $\phi_0 = 0$, $\phi_1 = 0.98$, $\lambda_1 = 10 \times (1 - \phi_1^2)$, $\lambda_0 = 0.9$ and $\Theta = 0.92$ (note that for $p = 50$ the authors suggest $\Theta = 0.98$, but we found that a lower value does better as $p$ gets larger, while it doesn't deteriorate performance for $p = 50$).

- **MCMC algorithm, Chan and Jeliazkov (2009)**: This is the standard time-varying parameter regression model used in economics, see for example Cogley and Sargent (2005). It consists of equations (9) and (10) in the main text, but with the addition that the measurement error variance follows a geometric random walk. As with VBDVS, the crucial setting that affects the amount of time-variation in regression coefficients is the prior on the state variances, which is of the form $w_j^{-1} \sim Gamma(v_1, v_2)$. We set the conservative choice $v_1 = 3$ and $v_2 = 20$, which implies that $w_j$ has prior mean around 0.016. In order to estimate this model efficiently, we use the Gibbs sampler algorithm of Chan and Jeliazkov (2009).

- **Dynamic Model Averaging, Koop and Korobilis (2012):** We use standard settings described in Koop and Korobilis (2012) with $\alpha = 0.99$, $\lambda = 0.99$ and $\kappa = 0.96$.

- **Bagging, Breiman (1996):** With the bagging algorithm we first resample our data $B$ times with replacement blocks of size $m$. For each pseudo-generated dataset we estimate with ordinary least squares using the Newey and West estimator of the covariance with lag truncation parameter $int\left\{T^{1/4}\right\}$. We select the optimal model using only those predictors that have t-statistics larger than a threshold $c^*$ in absolute value. We forecast with the optimal model, and the bagging forecast is obtained as the average of all forecasts over the $B$ Bootstrap replications. We set $B = 1000$, $m = 1$ and $c^* = 2.807$.

- **Elastic Net, Zou and Hastie (2005):** We use the MATLAB function "lasso" that is available in the Statistics and Machine Learning Toolbox. We use 10-fold cross validation for selecting the optimal $\lambda$ parameter, and we fix $\alpha = 0.75$.

- **Gaussian Process Regression:** Gaussian Process Regression (GPR) is a very powerful machine learning method that allows flexible nonparametric estimation targeted towards prediction. We use the MATLAB function "fitrgp" that is available in the Statistics and Machine Learning Toolbox. This is estimated using the following settings:
  `fitrgp(X,y,'Basis','linear','Optimizer','QuasiNewton','verbose',1,`
  `'FitMethod','exact','PredictMethod','exact')`

- **Partial Least Squares:** Partial Least Squares (PLS) is a method that originated in chemometrics. It allows to estimate factors that are extracted with reference to the variable to be predicted (target variable). Principal components instead maximize only the variance explained by the large dataset, and may not be optimal for prediction of the target variable. While more elegant methods have been proposed recently, such as the three-pass regression filter, the PLS is undeniably a good benchmark for assessing whether we can improve on the information content of simple principal component estimates. We use again the MATLAB function "plsregress" available in the Statistics and Machine Learning Toolbox, and we extract five factors from our dataset.

- **Structural breaks model, Koop and Potter (2007):** The Koop and Potter (2007) specification is a structural breaks model that builds on the more general time-varying parameter (TVP) specification but doesn't allow breaks to occur necessarily in each time period. The KP-AR model is of the form

$$y_{t+h} = x_t \beta_{s_t} + \sigma_{s_t} \varepsilon_{t+h}, \tag{1}$$

$$\beta_{s_t} = \beta_{s_{t-1}} + \eta_{s_t}, \tag{2}$$

$$\log \sigma_{s_t} = \log \sigma_{s_{t-1}} + \zeta_{s_t}, \tag{3}$$

where $x_t$ includes only an intercept and lags of $y_t$, $\varepsilon_{t+h}$ is an error following the standard normal distribution and $s_t \in \{1, 2, ..., K\}$ is a Markov switching process with $K$ states. This specification follows much of the Bayesian structural breaks literature and assume that the transition probabilities matrix is block diagonal, such that we can move from one regime to the next and never come back (which is the distinguishing feature of structural breaks compared to standard regime-switching specifications). We follow Bauwens et al. (2015) and specify a maximum number of $K_{max} = 10$ and allow the Gibbs sampler to determine how many structural breaks are relevant (up to the maximum of $K_{max}$). We also use fairy reasonable priors and initial conditions as in Bauwens et al. (2015) and Korobilis (2021), and the reader is referred to this paper and its Appendix for all the tedious computational details.

- **Unobserved Components Stochastic Volatility, Stock and Watson (2007)**: The Stock and Watson (2007) unobserved components stochastic volatility (UC-SV) model only allows for a time-varying intercept, that is, it is a local level specification of the form

$$\begin{aligned} y_{t+h} &= \tau_t + \sigma_t^{\varepsilon} \varepsilon_{t+h}, && (4) \\ \tau_t &= \tau_{t-1} + \sigma_t^{\eta} \eta_t, && (5) \\ \log \sigma_t^{\varepsilon} &= \log \sigma_{t-1}^{\varepsilon} + \zeta_t, && (6) \\ \log \sigma_t^{\eta} &= \log \sigma_{t-1}^{\eta} + \xi_t, && (7) \end{aligned}$$

where we observe that not only the measurement error $\varepsilon_{t+h}$ features stochastic volatility, but also the variance of state error $\eta_t$. This model has been specifically proposed for forecasting inflation, but it is a parsimonious and flexible time-varying parameter specification that can fit other series as well. This model is the most parsimonious among all other time-varying parameter specifications presented in this Section, as it only requires specification of initial conditions and priors for the scalar variances of the volatility parameters. In any case, selection of these hyperparameters needed for estimation follows again the exact implementation of Bauwens et al. (2015).

- **Time Varying Dimensions, Chan et al. (2012)**: The time-varying dimension (TVD) model of Chan et al. (2012) takes the following form

$$\begin{aligned} y_{t+h} &= \sum_{j=1}^{p} x_{j,t} s_{j,t} \beta_{j,t} + \varepsilon_{t+h}, && (8) \\ \beta_t &= \beta_{t-1} + \eta_t, && (9) \end{aligned}$$

where $s_{j,t}$ is an indicator variable such that when $s_{j,t} = 0$ the $j^{th}$ predictor is removed from the regression model in period $t$ only, and when $s_{j,t} = 1$ it is included in the regression. This is a very flexible specification that generalizes the Giordani and Kohn (2008) specification to allow for a predictor to exit the regression only in certain periods. We use the default settings and priors suggested by Chan et al. (2012)[1]. Following the exact implementations by the authors, also means that for computational reasons, the $s_{j,t}$ are not allowed to index all possible $2^p$ models available in each time period. Instead only models with one variable at a time, the full model, and the model with no predictors are estimated and then the optimal model is chosen among this reduced number of specifications. Finally, the authors define three different ways of specifying a TVD model. The specification used here is the first model presented in that paper. Priors, initial conditions and posterior computation for this first model can be found in Section 1.1 of the online Appendix of Chan et al. (2012).

[1] Joshua Chan kindly provided code for their model, which I gratefully acknowledge.

# B Data Appendix

The following high-dimensional dataset combines several popular datasets used in macroeconomics and finance. The core part builds on the FRED-QD dataset compiled in McCracken and Ng (2020), and the financial (portfolio) data used in Jurado et al. (2015) to extract a popular uncertainty index that are originally provided by Kenneth French (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). These are augmented with additional consumer survey indicators from University of Michigan (https://data.sca.isr.umich.edu/); predictors of stock returns used in Welch and Goyal (2007) provided by Amit Goyal (http://www.hec.unil.ch/agoyal/); Commodity prices from the World Bank's Pink Sheet database (https://www.worldbank.org/en/research/commodity-markets); and key macroeconomic indicators for key economies, obtained from Federal Reserve Economic Data (FRED) of St Louis Federal Reserve Bank (https://fred.stlouisfed.org/).

Table B1 presents the 444 variables used in the empirical exercise. These are measured quarterly and cover the period 1960Q1-2018Q4. Where a variable is measured originally in higher frequency (e.g. monthly) quarterly values are obtained by taking the average over the quarter. Column $F$ in Table B1 denotes whether the variable is used or not (1 or 0, respectively) to extract factors (principal components). The idea is that where some variables are aggregates of disaggregated series in the dataset, we only use the disaggregated series to extract factors. Column $F$ denotes the code used in order to transform each variable to be approximately stationary. The transformation codes are the following 1: level (no transformation); 2: first difference; 3: second difference; 4: natural logarithm; 5: first difference of natural logarithm; 6: second difference of natural logarithm; 7: first difference of percent change. The mnemonics used are those provided by the respective resources. For the Welch and Goyal (2007) data in particular, the mnemonics are those provided in the data Appendix of that paper. In the *Source* column of Table B1, this paper is abbreviated as GW2008.

Table B1: Quarterly large dataset

| No | Mnemonic | F | T | Long description | Source |
|---|---|---|---|---|---|
| 1 | GDPC1 | 0 | 5 | Real Gross Domestic Product | FRED-QD |
| 2 | PCECC96 | 0 | 5 | Consumption Real Personal Consumption Expenditures | FRED-QD |
| 3 | PCDGx | 1 | 5 | Real personal consumption expenditures: Durable goods | FRED-QD |
| 4 | PCESVx | 1 | 5 | Real Personal Consumption Expenditures: Services | FRED-QD |
| 5 | PCNDx | 1 | 5 | Real Personal Consumption Expenditures: Nondurable Goods | FRED-QD |
| 6 | GPDIC1 | 0 | 5 | Real Gross Private Domestic Investment | FRED-QD |
| 7 | FPIx | 0 | 5 | Real private fixed investment | FRED-QD |
| 8 | Y033RC1Q027SBEAx | 1 | 5 | Real Gross Private Domestic Fixed Investment: Nonresidential: Equipment | FRED-QD |
| 9 | PNFIx | 1 | 5 | Real private fixed investment: Nonresidential | FRED-QD |
| 10 | PRFIx | 1 | 5 | Real private fixed investment: Residential | FRED-QD |
| 11 | A014RE1Q156NBEA | 1 | 1 | Gross private domestic investment: Change in private inventories | FRED-QD |
| 12 | GCEC1 | 0 | 5 | Real Government Consumption Expenditures & Gross Investment | FRED-QD |
| 13 | A823RL1Q225SBEA | 1 | 1 | Real Government Consumption Expenditures and Gross Investment: Federal | FRED-QD |
| 14 | FGRECPTx | 1 | 5 | Real Federal Government Current Receipts | FRED-QD |
| 15 | SLCEx | 1 | 5 | Real government state and local consumption expenditures | FRED-QD |
| 16 | EXPGSC1 | 1 | 5 | Real Exports of Goods & Services | FRED-QD |
| 17 | IMPGSC1 | 1 | 5 | Real Imports of Goods & Services | FRED-QD |
| 18 | DPIC96 | 0 | 5 | Real Disposable Personal Income | FRED-QD |
| 19 | OUTNFB | 0 | 5 | Real Disposable Personal Income | FRED-QD |
| 20 | OUTBS | 0 | 5 | Business Sector: Real Output | FRED-QD |
| 21 | INDPRO | 0 | 5 | Industrial Production Index | FRED-QD |
| 22 | IPFINAL | 0 | 5 | Industrial Production: Final Products | FRED-QD |
| 23 | IPCONGD | 0 | 5 | Industrial Production: Consumer Goods | FRED-QD |
| 24 | IPMAT | 0 | 5 | Industrial Production: Materials | FRED-QD |
| 25 | IPDMAT | 1 | 5 | Industrial Production: Durable Materials | FRED-QD |
| 26 | IPNMAT | 1 | 5 | Industrial Production: Nondurable Materials | FRED-QD |
| 27 | IPDCONGD | 1 | 5 | Industrial Production: Durable Consumer Goods | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 28 | IPB51110SQ | 1 | 5 | Industrial Production: Durable Goods: Automotive products | FRED-QD |
| 29 | IPNCONGD | 1 | 5 | Industrial Production: Durable Goods: Automotive products | FRED-QD |
| 30 | IPBUSEQ | 1 | 5 | Industrial Production: Business Equipment | FRED-QD |
| 31 | IPB51220SQ | 1 | 5 | Industrial Production: Consumer energy products | FRED-QD |
| 32 | CUMFNS | 1 | 1 | Capacity Utilization: Manufacturing (SIC) | FRED-QD |
| 33 | PAYEMS | 0 | 5 | All Employees: Total nonfarm | FRED-QD |
| 34 | USPRIV | 0 | 5 | All Employees: Total Private Industries | FRED-QD |
| 35 | MANEMP | 0 | 5 | All Employees: Manufacturing | FRED-QD |
| 36 | SRVPRD | 0 | 5 | All Employees: Service-Providing Industries | FRED-QD |
| 37 | USGOOD | 0 | 5 | All Employees: Goods-Producing Industries | FRED-QD |
| 38 | DMANEMP | 1 | 5 | All Employees: Durable goods | FRED-QD |
| 39 | NDMANEMP | 0 | 5 | All Employees: Nondurable goods | FRED-QD |
| 40 | USCONS | 1 | 5 | All Employees: Construction | FRED-QD |
| 41 | USEHS | 1 | 5 | All Employees: Education & Health Services | FRED-QD |
| 42 | USFIRE | 1 | 5 | All Employees: Education & Health Services | FRED-QD |
| 43 | USINFO | 1 | 5 | All Employees: Information Services | FRED-QD |
| 44 | USPBS | 1 | 5 | All Employees: Professional & Business Services | FRED-QD |
| 45 | USLAH | 1 | 5 | All Employees: Leisure & Hospitality | FRED-QD |
| 46 | USSERV | 1 | 5 | All Employees: Other Services | FRED-QD |
| 47 | USMINE | 1 | 5 | All Employees: Mining and logging | FRED-QD |
| 48 | USTPU | 1 | 5 | All Employees: Trade, Transportation & Utilities | FRED-QD |
| 49 | USGOVT | 0 | 5 | All Employees: Government | FRED-QD |
| 50 | USTRADE | 1 | 5 | All Employees: Retail Trade | FRED-QD |
| 51 | USWTRADE | 1 | 5 | All Employees: Wholesale Trade | FRED-QD |
| 52 | CES9091000001 | 1 | 5 | All Employees: Government: Federal | FRED-QD |
| 53 | CES9092000001 | 1 | 5 | All Employees: Government: State Government | FRED-QD |
| 54 | CES9093000001 | 1 | 5 | All Employees: Government: Local Government | FRED-QD |
| 55 | CE16OV | 0 | 5 | Civilian Employment | FRED-QD |
| 56 | CIVPART | 0 | 2 | Civilian Labor Force Participation Rate | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 57 | UNRATE | 0 | 2 | Civilian Unemployment Rate | FRED-QD |
| 58 | UNRATESTx | 0 | 2 | Unemployment Rate less than 27 weeks | FRED-QD |
| 59 | UNRATELTx | 0 | 2 | Unemployment Rate for more than 27 weeks | FRED-QD |
| 60 | LNS14000012 | 1 | 2 | Unemployment Rate - 16 to 19 years | FRED-QD |
| 61 | LNS14000025 | 1 | 2 | Unemployment Rate - 20 years and over, Men | FRED-QD |
| 62 | LNS14000026 | 1 | 2 | Unemployment Rate - 20 years and over, Women | FRED-QD |
| 63 | UEMPLT5 | 1 | 5 | Number of Civilians Unemployed - Less Than 5 Weeks | FRED-QD |
| 64 | UEMP5TO14 | 1 | 5 | Number of Civilians Unemployed for 5 to 14 Weeks | FRED-QD |
| 65 | UEMP15T26 | 1 | 5 | Number of Civilians Unemployed for 15 to 26 Weeks | FRED-QD |
| 66 | UEMP27OV | 1 | 5 | Number of Civilians Unemployed for 27 Weeks and Over | FRED-QD |
| 67 | LNS12032194 | 1 | 5 | Employment Level - Part-Time for Economic Reasons, All Industries | FRED-QD |
| 68 | HOABS | 0 | 5 | Business Sector: Hours of All Persons | FRED-QD |
| 69 | HOANBS | 0 | 5 | Nonfarm Business Sector: Hours of All Persons | FRED-QD |
| 70 | AWHMAN | 1 | 1 | Average Weekly Hours of Production and Nonsupervisory Employees: Manufacturing | FRED-QD |
| 71 | AWOTMAN | 1 | 2 | Average Weekly Hours Of Production And Nonsupervisory Employees: Total private | FRED-QD |
| 72 | HWIx | 0 | 1 | Help-Wanted Index | FRED-QD |
| 73 | HOUST | 0 | 5 | Housing Starts: Total: New Privately Owned Housing Units Started | FRED-QD |
| 74 | HOUST5F | 0 | 5 | Housing Starts: Total: New Privately Owned Housing Units Started | FRED-QD |
| 75 | PERMIT | 1 | 5 | New Private Housing Units Authorized by Building Permits | FRED-QD |
| 76 | HOUSTMW | 1 | 5 | Housing Starts in Midwest Census Region | FRED-QD |
| 77 | HOUSTNE | 1 | 5 | Housing Starts in Northeast Census Region | FRED-QD |
| 78 | HOUSTS | 1 | 5 | Housing Starts in South Census Region | FRED-QD |
| 79 | HOUSTW | 1 | 5 | Housing Starts in West Census Region | FRED-QD |
| 80 | CMRMTSPLx | 0 | 5 | Real Manufacturing and Trade Industries Sales | FRED-QD |
| 81 | RSAFSx | 1 | 5 | Real Retail and Food Services Sales | FRED-QD |
| 82 | AMDMNOx | 1 | 5 | Real Manufacturers' New Orders: Durable Goods | FRED-QD |
| 83 | AMDMUOx | 1 | 5 | Real Manufacturers' Unfilled Orders for Durable Goods | FRED-QD |
| 84 | PCECTPI | 0 | 6 | Personal Consumption Expenditures: Chain-type Price Index | FRED-QD |
| 85 | PCEPILFE | 0 | 6 | Personal Consumption Expenditures Excluding Food and Energy | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 86 | GDPCTPI | 0 | 6 | Gross Domestic Product: Chain-type Price Index | FRED-QD |
| 87 | GPDICTPI | 1 | 6 | Gross Private Domestic Investment: Chain-type Price Index | FRED-QD |
| 88 | IPDBS | 1 | 6 | Business Sector: Implicit Price Deflator | FRED-QD |
| 89 | DGDSRG3Q086SBEA | 0 | 6 | Goods Personal consumption expenditures: Goods | FRED-QD |
| 90 | DDURRG3Q086SBEA | 0 | 6 | Personal consumption expenditures: Durable goods | FRED-QD |
| 91 | DSERRG3Q086SBEA | 0 | 6 | Personal consumption expenditures: Services | FRED-QD |
| 92 | DNDGRG3Q086SBEA | 0 | 6 | Personal consumption expenditures: Nondurable goods | FRED-QD |
| 93 | DHCERG3Q086SBEA | 0 | 6 | Personal consumption expenditures: Nondurable goods | FRED-QD |
| 94 | DMOTRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Durable goods: Motor vehicles and parts | FRED-QD |
| 95 | DFDHRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Durable goods: Furnishings and durable equipment | FRED-QD |
| 96 | DREQRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Durable goods: Recreational goods and vehicles | FRED-QD |
| 97 | DODGRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Durable goods: Other durable goods | FRED-QD |
| 98 | DFXARG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Nondurable goods: Food and beverages | FRED-QD |
| 99 | DCLORG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Nondurable goods: Clothing and footwear | FRED-QD |
| 100 | DGOERG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Nondurable goods: Gasoline and other energy goods | FRED-QD |
| 101 | DONGRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Nondurable goods: Other nondurable goods | FRED-QD |
| 102 | DHUTRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Services: Housing and utilities | FRED-QD |
| 103 | DHLCRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Services: Health care | FRED-QD |
| 104 | DTRSRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Transportation services | FRED-QD |
| 105 | DRCARG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Recreation services | FRED-QD |
| 106 | DFSARG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Services: Food services and accommodations | FRED-QD |
| 107 | DIFSRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Financial services and insurance | FRED-QD |
| 108 | DOTSRG3Q086SBEA | 1 | 6 | Personal consumption expenditures: Other services | FRED-QD |
| 109 | CPIAUCSL | 0 | 6 | Consumer Price Index for All Urban Consumers: All Items | FRED-QD |
| 110 | CPILFESL | 0 | 6 | Consumer Price Index for All Urban Consumers: All Items Less Food & Energy | FRED-QD |
| 111 | WPSFD49207 | 0 | 6 | Producer Price Index by Commodity for Final Demand: Finished Goods | FRED-QD |
| 112 | PPIACO | 0 | 6 | Producer Price Index for All Commodities | FRED-QD |
| 113 | WPSFD49502 | 1 | 6 | Producer Price Index by Commodity for Finished Consumer Goods | FRED-QD |
| 114 | WPSFD4111 | 1 | 6 | Producer Price Index by Commodity for Finished Consumer Foods | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 115 | PPIIDC | 1 | 6 | Producer Price Index by Commodity Industrial Commodities | FRED-QD |
| 116 | WPSID61 | 1 | 6 | Producer Price Index by Commodity Intermediate Materials: Supplies & Components | FRED-QD |
| 117 | WPU0561 | 1 | 5 | Producer Price Index by Commodity for Fuels and Related Products and Power: Crude Petroleum | FRED-QD |
| 118 | OILPRICEx | 0 | 5 | Real Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma | FRED-QD |
| 119 | CES2000000008x | 0 | 5 | Real Average Hourly Earnings of Production and Nonsupervisory Employees: Construction | FRED-QD |
| 120 | CES3000000008x | 0 | 5 | Real Average Hourly Earnings of Production and Nonsupervisory Employees: Manufacturing | FRED-QD |
| 121 | COMPRNFB | 1 | 5 | Manufacturing Sector: Real Compensation Per Hour | FRED-QD |
| 122 | RCPHBS | 1 | 5 | Business Sector: Real Compensation Per Hour | FRED-QD |
| 123 | OPHNFB | 1 | 5 | Nonfarm Business Sector: Real Output Per Hour of All Persons | FRED-QD |
| 124 | OPHPBS | 0 | 5 | Business Sector: Real Output Per Hour of All Persons | FRED-QD |
| 125 | ULCBS | 0 | 5 | Business Sector: Unit Labor Cost | FRED-QD |
| 126 | ULCNFB | 1 | 5 | Nonfarm Business Sector: Unit Labor Cost | FRED-QD |
| 127 | UNLPNBS | 1 | 5 | Nonfarm Business Sector: Unit Nonlabor Payments | FRED-QD |
| 128 | FEDFUNDS | 1 | 2 | Effective Federal Funds Rate | FRED-QD |
| 129 | TB3MS | 1 | 2 | 3-Month Treasury Bill: Secondary Market Rate | FRED-QD |
| 130 | TB6MS | 0 | 2 | 6-Month Treasury Bill: Secondary Market Rate | FRED-QD |
| 131 | GS1 | 0 | 2 | 1-Year Treasury Constant Maturity Rate | FRED-QD |
| 132 | GS10 | 0 | 2 | 10-Year Treasury Constant Maturity Rate | FRED-QD |
| 133 | AAA | 0 | 2 | Moody's Seasoned Aaa Corporate Bond Yield | FRED-QD |
| 134 | BAA | 0 | 2 | Moody's Seasoned Baa Corporate Bond Yield | FRED-QD |
| 135 | BAA10YM | 1 | 1 | Moody's Seasoned Baa Corporate Bond Yield Relative to Yield on 10-Year Treasury Constant Maturity | FRED-QD |
| 136 | TB6M3Mx | 1 | 1 | 6-Month Treasury Bill Minus 3-Month Treasury Bill, secondary market | FRED-QD |
| 137 | GS1TB3Mx | 1 | 1 | 1-Year Treasury Constant Maturity Minus 3-Month Treasury Bill, secondary market | FRED-QD |
| 138 | GS10TB3Mx | 1 | 1 | 10-Year Treasury Constant Maturity Minus 3-Month Treasury Bill, secondary market | FRED-QD |
| 139 | CPF3MTB3Mx | 1 | 1 | 3-Month Commercial Paper Minus 3-Month Treasury Bill, secondary market | FRED-QD |
| 140 | AMBSLREAL | 1 | 5 | St. Louis Adjusted Monetary Base | FRED-QD |
| 141 | M1REAL | 1 | 5 | Real M1 Money Stock | FRED-QD |
| 142 | M2REAL | 1 | 5 | Real M2 Money Stock | FRED-QD |
| 143 | MZMREAL | 1 | 5 | Real MZM Money Stock | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 144 | BUSLOANSx | 1 | 5 | Real Commercial and Industrial Loans, All Commercial Banks | FRED-QD |
| 145 | CONSUMERx | 1 | 5 | Consumer Loans at All Commercial Banks | FRED-QD |
| 146 | NONREVSLx | 1 | 5 | Total Real Nonrevolving Credit Owned and Securitized, Outstanding | FRED-QD |
| 147 | REALLNx | 1 | 5 | Real Real-Estate Loans, All Commercial Banks | FRED-QD |
| 148 | TOTALSLx | 1 | 5 | Total Consumer Credit Outstanding | FRED-QD |
| 149 | TABSHNOx | 1 | 5 | Real Total Assets of Households and Nonprofit Organizations | FRED-QD |
| 150 | TLBSHNOx | 1 | 5 | Real Total Liabilities of Households and Nonprofit Organizations | FRED-QD |
| 151 | LIABPIx | 0 | 5 | Liabilities of Households and Nonprofit Organizations Relative to Personal Disposable Income | FRED-QD |
| 152 | TNWBSHNOx | 1 | 5 | Real Net Worth of Households and Nonprofit Organizations | FRED-QD |
| 153 | NWPIx | 0 | 1 | Net Worth of Households and Nonprofit Organizations Relative to Disposable Personal Income | FRED-QD |
| 154 | TARESAx | 1 | 5 | Real Assets of Households and Nonprofit Organizations excluding Real Estate Assets | FRED-QD |
| 155 | HNOREMQ027Sx | 1 | 5 | Real Real-Estate Assets of Households and Nonprofit Organizations | FRED-QD |
| 156 | TFAABSHNOx | 1 | 5 | Real Total Financial Assets of Households and Nonprofit Organizations | FRED-QD |
| 157 | TWEXMMTH | 1 | 5 | Trade Weighted U.S. Dollar Index: Major Currencies, Goods | FRED-QD |
| 158 | EXSZUSx | 1 | 5 | Switzerland / U.S. Foreign Exchange Rate | FRED-QD |
| 159 | EXJPUSx | 1 | 5 | Japan / U.S. Foreign Exchange Rate | FRED-QD |
| 160 | EXUSUKx | 1 | 5 | U.S. / U.K. Foreign Exchange Rate | FRED-QD |
| 161 | EXCAUSx | 1 | 5 | Canada / U.S. Foreign Exchange Rate | FRED-QD |
| 162 | UMCSENTx | 0 | 1 | University of Michigan: Consumer Sentiment | FRED-QD |
| 163 | PAGO | 1 | 1 | Current Financial Situation Compared with a Year Ago | UofMich |
| 164 | PEXP | 1 | 1 | Expected Change in Financial Situation in a Year | UofMich |
| 165 | NEWS | 1 | 1 | News Heard of Recent Changes in Business Conditions | UofMich |
| 166 | BAGO | 1 | 1 | Current Business Conditions Compared with a Year Ago | UofMich |
| 167 | BEXP | 1 | 1 | Expected Change in Business Conditions in a Year | UofMich |
| 168 | BUS12 | 1 | 1 | Business Conditions Expected During the Next Year | UofMich |
| 169 | BUS5 | 1 | 1 | Business Conditions Expected During the Next 5 Years | UofMich |
| 170 | INFEXP | 1 | 1 | Expected Change in Prices During the Next Year | UofMich |
| 171 | DUR | 1 | 1 | Buying Conditions for Large Household Durables | UofMich |
| 172 | VEH | 1 | 1 | Buying Conditions for Vehicles | UofMich |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 173 | HOM | 1 | 1 | Buying Conditions for Houses | UofMich |
| 174 | USASACRQISMEI | 1 | 1 | Passenger Car Registrations in United States | FRED-QD |
| 175 | USALOLITONOSTSAM | 1 | 1 | Leading indicators: CLI: Normalized for the United States | FRED-QD |
| 176 | BSCICP03USM665S | 1 | 1 | Composite Indicators: OECD Indicator for the United States | FRED-QD |
| 177 | B020RE1Q156NBEA | 0 | 2 | Shares of gross domestic product: Exports of goods and services | FRED-QD |
| 178 | B021RE1Q156NBEA | 0 | 2 | Shares of gross domestic product: Imports of goods and services | FRED-QD |
| 179 | IPMANSICS | 0 | 5 | Industrial Production: Manufacturing (SIC) | FRED-QD |
| 180 | IPB51222S | 0 | 5 | Industrial Production: Residential Utilities | FRED-QD |
| 181 | IPFUELS | 0 | 5 | Industrial Production: Fuels | FRED-QD |
| 182 | UEMPMEAN | 1 | 2 | Duration of Unemployment | FRED-QD |
| 183 | CES0600000007 | 1 | 2 | Average Weekly Hours of Production and Nonsupervisory Employees: Goods-Producing | FRED-QD |
| 184 | TOTRESNS | 0 | 6 | Total Reserves of Depository Institutions | FRED-QD |
| 185 | NONBORRES | 0 | 7 | Reserves of Depository Institutions, Nonborrowed | FRED-QD |
| 186 | GS5 | 0 | 2 | 5-Year Treasury Constant Maturity Rate | FRED-QD |
| 187 | TB3SMFFM | 1 | 1 | 3-Month Treasury Constant Maturity Minus Federal Funds Rate | FRED-QD |
| 188 | T5YFFM | 1 | 1 | 5-Year Treasury Constant Maturity Minus Federal Funds Rate | FRED-QD |
| 189 | AAAFFM | 1 | 1 | Moody's Seasoned Aaa Corporate Bond Minus Federal Funds Rate | FRED-QD |
| 190 | WPSID62 | 1 | 6 | Producer Price Index: Crude Materials for Further Processing | FRED-QD |
| 191 | PPICMM | 0 | 6 | Producer Price Index: Commodities: Metals and metal products: Primary nonferrous metals | FRED-QD |
| 192 | CPIAPPSL | 0 | 6 | Producer Price Index: Commodities: Metals and metal products: Primary nonferrous metals | FRED-QD |
| 193 | CPITRNSL | 1 | 6 | Consumer Price Index for All Urban Consumers: Transportation | FRED-QD |
| 194 | CPIMEDSL | 1 | 6 | Consumer Price Index for All Urban Consumers: Medical Care | FRED-QD |
| 195 | CUSR0000SAC | 1 | 6 | Consumer Price Index for All Urban Consumers: Commodities | FRED-QD |
| 196 | CUSR0000SAD | 1 | 6 | Consumer Price Index for All Urban Consumers: Durables | FRED-QD |
| 197 | CUSR0000SAS | 1 | 6 | Consumer Price Index for All Urban Consumers: Services | FRED-QD |
| 198 | CPIULFSL | 0 | 6 | Consumer Price Index for All Urban Consumers: All Items Less Food | FRED-QD |
| 199 | CUSR0000SA0L2 | 0 | 6 | Consumer Price Index for All Urban Consumers: All items less shelter | FRED-QD |
| 200 | CUSR0000SA0L5 | 0 | 6 | Consumer Price Index for All Urban Consumers: All items less medical care | FRED-QD |
| 201 | CES0600000008 | 0 | 6 | Average Hourly Earnings of Production and Nonsupervisory Employees: Goods-Producing | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 202 | DTCOLNVHFNM | 0 | 6 | Consumer Motor Vehicle Loans Outstanding Owned by Finance Companies | FRED-QD |
| 203 | DTCTHFNM | 0 | 6 | Total Consumer Loans and Leases Outstanding Owned and Securitized by Finance Companies | FRED-QD |
| 204 | INVEST | 1 | 6 | Securities in Bank Credit at All Commercial Banks | FRED-QD |
| 205 | HWIURATIOx | 1 | 2 | Ratio of Help Wanted/No. Unemployed | FRED-QD |
| 206 | CLAIMSx | 1 | 5 | Initial Claims | FRED-QD |
| 207 | BUSINVx | 1 | 5 | Total Business Inventories | FRED-QD |
| 208 | ISRATIOx | 1 | 2 | Total Business: Inventories to Sales Ratio | FRED-QD |
| 209 | CONSPIx | 0 | 2 | Nonrevolving consumer credit to Personal Income | FRED-QD |
| 210 | CP3M | 0 | 2 | 3-Month AA Financial Commercial Paper Rate | FRED-QD |
| 211 | COMPAPFF | 0 | 1 | 3-Month Commercial Paper Minus Federal Funds Rate | FRED-QD |
| 212 | PERMITNE | 0 | 5 | New Private Housing Units Authorized by Building Permits in the Northeast Census Region | FRED-QD |
| 213 | PERMITMW | 0 | 5 | New Private Housing Units Authorized by Building Permits in the Midwest Census Region | FRED-QD |
| 214 | PERMITS | 0 | 5 | New Private Housing Units Authorized by Building Permits in the South Census Region | FRED-QD |
| 215 | PERMITW | 0 | 5 | New Private Housing Units Authorized by Building Permits in the West Census Region | FRED-QD |
| 216 | NIKKEI225 | 0 | 5 | Nikkei Stock Average | FRED-QD |
| 217 | TLBSNNCBx | 0 | 5 | Real Nonfinancial Corporate Business Sector Liabilities | FRED-QD |
| 218 | TLBSNNCBBDIx | 0 | 1 | Nonfinancial Corporate Business Sector Liabilities to Disposable Business Income | FRED-QD |
| 219 | TTAABSNNCBx | 0 | 5 | Real Nonfinancial Corporate Business Sector Assets | FRED-QD |
| 220 | TNWMVBSNNCBx | 0 | 5 | Real Nonfinancial Corporate Business Sector Net Worth | FRED-QD |
| 221 | TNWMVBSNNCBBDIx | 0 | 2 | Nonfinancial Corporate Business Sector Net Worth to Disposable Business Income | FRED-QD |
| 222 | TLBSNNBx | 0 | 5 | Real Nonfinancial Noncorporate Business Sector Liabilities | FRED-QD |
| 223 | TLBSNNBBDIx | 0 | 1 | Nonfinancial Noncorporate Business Sector Liabilities to Disposable Business Income | FRED-QD |
| 224 | TABSNNBx | 0 | 5 | Real Nonfinancial Noncorporate Business Sector Assets | FRED-QD |
| 225 | TNWBSNNBx | 0 | 5 | Real Nonfinancial Noncorporate Business Sector Net Worth | FRED-QD |
| 226 | TNWBSNNBBDIx | 0 | 2 | Nonfinancial Noncorporate Business Sector Net Worth to Disposable Business Income | FRED-QD |
| 227 | CNCFx | 0 | 5 | Real Disposable Business Income, Billions of 2009 Dollars | FRED-QD |
| 228 | S&P 500 | 1 | 5 | S&P's Common Stock Price Index: Composite | FRED-QD |
| 229 | S&P: indust | 0 | 5 | S&P's Common Stock Price Index: Industrials | FRED-QD |
| 230 | S&P div yield | 0 | 2 | S&P's Composite Common Stock: Dividend Yield | FRED-QD |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 231 | S&P PE ratio | 0 | 5 | S&P's Composite Common Stock: Price-Earnings Ratio | FRED-QD |
| 232 | d/p | 1 | 2 | Dividend Price Ratio | GW2008 |
| 233 | d/y | 1 | 2 | Dividend Yield | GW2008 |
| 234 | e/p | 1 | 2 | Earnings Price Ratio | GW2008 |
| 235 | d/e | 1 | 2 | Dividend Payout Ratio | GW2008 |
| 236 | b/m | 1 | 2 | Book-to-Market Ratio | GW2008 |
| 237 | svar | 1 | 1 | Stock Market Variance | GW2008 |
| 238 | ntis | 1 | 1 | Net Equity Expansion | GW2008 |
| 239 | lty | 1 | 1 | Long Term Yield | GW2008 |
| 240 | dfy | 1 | 1 | Default Yield Spread | GW2008 |
| 241 | dfr | 1 | 1 | Default Return Spread | GW2008 |
| 242 | Mkt-RF | 1 | 1 | Market Excess Return (based on NYSE) | K. French |
| 243 | SMB | 1 | 1 | Small Minus Big, Sorted on Size | K. French |
| 244 | HML | 1 | 1 | High Minus Low, Sorted on Book-to-Market | K. French |
| 245 | Agric | 1 | 1 | Agric Industry Portfolio | K. French |
| 246 | Food | 1 | 1 | Food Industry Portfolio | K. French |
| 247 | Beer | 1 | 1 | Beer Industry Portfolio | K. French |
| 248 | Smoke | 1 | 1 | Smoke Industry Portfolio | K. French |
| 249 | Toys | 1 | 1 | Toys Industry Portfolio | K. French |
| 250 | Fun | 1 | 1 | Fun Industry Portfolio | K. French |
| 251 | Books | 1 | 1 | Books Industry Portfolio | K. French |
| 252 | Hshld | 1 | 1 | Hshld Industry Portfolio | K. French |
| 253 | Clths | 1 | 1 | Clths Industry Portfolio | K. French |
| 254 | MedEq | 1 | 1 | MedEq Industry Portfolio | K. French |
| 255 | Drugs | 1 | 1 | Drugs Industry Portfolio | K. French |
| 256 | Chems | 1 | 1 | Chems Industry Portfolio | K. French |
| 257 | Rubbr | 1 | 1 | Rubbr Industry Portfolio | K. French |
| 258 | Txtls | 1 | 1 | Txtls Industry Portfolio | K. French |
| 259 | BldMt | 1 | 1 | BldMt Industry Portfolio | K. French |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 260 | Cnstr | 1 | 1 | Cnstr Industry Portfolio | K. French |
| 261 | Steel | 1 | 1 | Steel Industry Portfolio | K. French |
| 262 | Mach | 1 | 1 | Mach Industry Portfolio | K. French |
| 263 | ElcEq | 1 | 1 | ElcEq Industry Portfolio | K. French |
| 264 | Autos | 1 | 1 | Autos Industry Portfolio | K. French |
| 265 | Aero | 1 | 1 | Aero Industry Portfolio | K. French |
| 266 | Ships | 1 | 1 | Ships Industry Portfolio | K. French |
| 267 | Mines | 1 | 1 | Mines Industry Portfolio | K. French |
| 268 | Coal | 1 | 1 | Coal Industry Portfolio | K. French |
| 269 | Oil | 1 | 1 | Oil Industry Portfolio | K. French |
| 270 | Util | 1 | 1 | Util Industry Portfolio | K. French |
| 271 | Telcm | 1 | 1 | Telcm Industry Portfolio | K. French |
| 272 | PerSv | 1 | 1 | PerSv Industry Portfolio | K. French |
| 273 | BusSv | 1 | 1 | BusSv Industry Portfolio | K. French |
| 274 | Hardw | 1 | 1 | Hardw Industry Portfolio | K. French |
| 275 | Chips | 1 | 1 | Chips Industry Portfolio | K. French |
| 276 | LabEq | 1 | 1 | LabEq Industry Portfolio | K. French |
| 277 | Paper | 1 | 1 | Paper Industry Portfolio | K. French |
| 278 | Boxes | 1 | 1 | Boxes Industry Portfolio | K. French |
| 279 | Trans | 1 | 1 | Trans Industry Portfolio | K. French |
| 280 | Whlsl | 1 | 1 | Whlsl Industry Portfolio | K. French |
| 281 | Rtail | 1 | 1 | Rtail Industry Portfolio | K. French |
| 282 | Meals | 1 | 1 | Meals Industry Portfolio | K. French |
| 283 | Banks | 1 | 1 | Banks Industry Portfolio | K. French |
| 284 | Insur | 1 | 1 | Insur Industry Portfolio | K. French |
| 285 | RlEst | 1 | 1 | RlEst Industry Portfolio | K. French |
| 286 | Fin | 1 | 1 | Fin Industry Portfolio | K. French |
| 287 | Other | 1 | 1 | Other Industry Portfolio | K. French |
| 288 | ME1 BM2 | 1 | 1 | (1, 2) portfolio sorted on (size, book-to-market) | K. French |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 289 | ME1 BM3 | 1 | 1 | (1, 3) portfolio sorted on (size, book-to-market) | K. French |
| 290 | ME1 BM4 | 1 | 1 | (1, 4) portfolio sorted on (size, book-to-market) | K. French |
| 291 | ME1 BM5 | 1 | 1 | (1, 5) portfolio sorted on (size, book-to-market) | K. French |
| 292 | ME1 BM6 | 1 | 1 | (1, 6) portfolio sorted on (size, book-to-market) | K. French |
| 293 | ME1 BM7 | 1 | 1 | (1, 7) portfolio sorted on (size, book-to-market) | K. French |
| 294 | ME1 BM8 | 1 | 1 | (1, 8) portfolio sorted on (size, book-to-market) | K. French |
| 295 | ME1 BM9 | 1 | 1 | (1, 9) portfolio sorted on (size, book-to-market) | K. French |
| 296 | ME1 BM10 | 1 | 1 | (1, 10) portfolio sorted on (size, book-to-market) | K. French |
| 297 | ME2 BM1 | 1 | 1 | (2, 1) portfolio sorted on (size, book-to-market) | K. French |
| 298 | ME2 BM2 | 1 | 1 | (2, 2) portfolio sorted on (size, book-to-market) | K. French |
| 299 | ME2 BM3 | 1 | 1 | (2, 3) portfolio sorted on (size, book-to-market) | K. French |
| 300 | ME2 BM4 | 1 | 1 | (2, 4) portfolio sorted on (size, book-to-market) | K. French |
| 301 | ME2 BM5 | 1 | 1 | (2, 5) portfolio sorted on (size, book-to-market) | K. French |
| 302 | ME2 BM6 | 1 | 1 | (2, 6) portfolio sorted on (size, book-to-market) | K. French |
| 303 | ME2 BM7 | 1 | 1 | (2, 7) portfolio sorted on (size, book-to-market) | K. French |
| 304 | ME2 BM8 | 1 | 1 | (2, 8) portfolio sorted on (size, book-to-market) | K. French |
| 305 | ME2 BM9 | 1 | 1 | (2, 9) portfolio sorted on (size, book-to-market) | K. French |
| 306 | ME2 BM10 | 1 | 1 | (2, 10) portfolio sorted on (size, book-to-market) | K. French |
| 307 | ME3 BM1 | 1 | 1 | (3, 1) portfolio sorted on (size, book-to-market) | K. French |
| 308 | ME3 BM2 | 1 | 1 | (3, 2) portfolio sorted on (size, book-to-market) | K. French |
| 309 | ME3 BM3 | 1 | 1 | (3, 3) portfolio sorted on (size, book-to-market) | K. French |
| 310 | ME3 BM4 | 1 | 1 | (3, 4) portfolio sorted on (size, book-to-market) | K. French |
| 311 | ME3 BM5 | 1 | 1 | (3, 5) portfolio sorted on (size, book-to-market) | K. French |
| 312 | ME3 BM6 | 1 | 1 | (3, 6) portfolio sorted on (size, book-to-market) | K. French |
| 313 | ME3 BM7 | 1 | 1 | (3, 7) portfolio sorted on (size, book-to-market) | K. French |
| 314 | ME3 BM8 | 1 | 1 | (3, 8) portfolio sorted on (size, book-to-market) | K. French |
| 315 | ME3 BM9 | 1 | 1 | (3, 9) portfolio sorted on (size, book-to-market) | K. French |
| 316 | ME3 BM10 | 1 | 1 | (3, 10) portfolio sorted on (size, book-to-market) | K. French |
| 317 | ME4 BM1 | 1 | 1 | (4, 1) portfolio sorted on (size, book-to-market) | K. French |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 318 | ME4 BM2 | 1 | 1 | (4, 2) portfolio sorted on (size, book-to-market) | K. French |
| 319 | ME4 BM3 | 1 | 1 | (4, 3) portfolio sorted on (size, book-to-market) | K. French |
| 320 | ME4 BM4 | 1 | 1 | (4, 4) portfolio sorted on (size, book-to-market) | K. French |
| 321 | ME4 BM5 | 1 | 1 | (4, 5) portfolio sorted on (size, book-to-market) | K. French |
| 322 | ME4 BM6 | 1 | 1 | (4, 6) portfolio sorted on (size, book-to-market) | K. French |
| 323 | ME4 BM7 | 1 | 1 | (4, 7) portfolio sorted on (size, book-to-market) | K. French |
| 324 | ME4 BM8 | 1 | 1 | (4, 8) portfolio sorted on (size, book-to-market) | K. French |
| 325 | ME4 BM9 | 1 | 1 | (4, 9) portfolio sorted on (size, book-to-market) | K. French |
| 326 | ME4 BM10 | 1 | 1 | (4, 10) portfolio sorted on (size, book-to-market) | K. French |
| 327 | ME5 BM1 | 1 | 1 | (5, 1) portfolio sorted on (size, book-to-market) | K. French |
| 328 | ME5 BM2 | 1 | 1 | (5, 2) portfolio sorted on (size, book-to-market) | K. French |
| 329 | ME5 BM3 | 1 | 1 | (5, 3) portfolio sorted on (size, book-to-market) | K. French |
| 330 | ME5 BM4 | 1 | 1 | (5, 4) portfolio sorted on (size, book-to-market) | K. French |
| 331 | ME5 BM5 | 1 | 1 | (5, 5) portfolio sorted on (size, book-to-market) | K. French |
| 332 | ME5 BM6 | 1 | 1 | (5, 6) portfolio sorted on (size, book-to-market) | K. French |
| 333 | ME5 BM7 | 1 | 1 | (5, 7) portfolio sorted on (size, book-to-market) | K. French |
| 334 | ME5 BM8 | 1 | 1 | (5, 8) portfolio sorted on (size, book-to-market) | K. French |
| 335 | ME5 BM9 | 1 | 1 | (5, 9) portfolio sorted on (size, book-to-market) | K. French |
| 336 | ME5 BM10 | 1 | 1 | (5, 10) portfolio sorted on (size, book-to-market) | K. French |
| 337 | ME6 BM1 | 1 | 1 | (6, 1) portfolio sorted on (size, book-to-market) | K. French |
| 338 | ME6 BM2 | 1 | 1 | (6, 2) portfolio sorted on (size, book-to-market) | K. French |
| 339 | ME6 BM3 | 1 | 1 | (6, 3) portfolio sorted on (size, book-to-market) | K. French |
| 340 | ME6 BM4 | 1 | 1 | (6, 4) portfolio sorted on (size, book-to-market) | K. French |
| 341 | ME6 BM5 | 1 | 1 | (6, 5) portfolio sorted on (size, book-to-market) | K. French |
| 342 | ME6 BM6 | 1 | 1 | (6, 6) portfolio sorted on (size, book-to-market) | K. French |
| 343 | ME6 BM7 | 1 | 1 | (6, 7) portfolio sorted on (size, book-to-market) | K. French |
| 344 | ME6 BM8 | 1 | 1 | (6, 8) portfolio sorted on (size, book-to-market) | K. French |
| 345 | ME6 BM9 | 1 | 1 | (6, 9) portfolio sorted on (size, book-to-market) | K. French |
| 346 | ME6 BM10 | 1 | 1 | (6, 10) portfolio sorted on (size, book-to-market) | K. French |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 347 | ME7 BM1 | 1 | 1 | (7, 1) portfolio sorted on (size, book-to-market) | K. French |
| 348 | ME7 BM2 | 1 | 1 | (7, 2) portfolio sorted on (size, book-to-market) | K. French |
| 349 | ME7 BM3 | 1 | 1 | (7, 3) portfolio sorted on (size, book-to-market) | K. French |
| 350 | ME7 BM4 | 1 | 1 | (7, 4) portfolio sorted on (size, book-to-market) | K. French |
| 351 | ME7 BM5 | 1 | 1 | (7, 5) portfolio sorted on (size, book-to-market) | K. French |
| 352 | ME7 BM6 | 1 | 1 | (7, 6) portfolio sorted on (size, book-to-market) | K. French |
| 353 | ME7 BM7 | 1 | 1 | (7, 7) portfolio sorted on (size, book-to-market) | K. French |
| 354 | ME7 BM8 | 1 | 1 | (7, 8) portfolio sorted on (size, book-to-market) | K. French |
| 355 | ME7 BM9 | 1 | 1 | (7, 9) portfolio sorted on (size, book-to-market) | K. French |
| 356 | ME7 BM10 | 1 | 1 | (7, 10) portfolio sorted on (size, book-to-market) | K. French |
| 357 | ME8 BM1 | 1 | 1 | (8, 1) portfolio sorted on (size, book-to-market) | K. French |
| 358 | ME8 BM2 | 1 | 1 | (8, 2) portfolio sorted on (size, book-to-market) | K. French |
| 359 | ME8 BM3 | 1 | 1 | (8, 3) portfolio sorted on (size, book-to-market) | K. French |
| 360 | ME8 BM4 | 1 | 1 | (8, 4) portfolio sorted on (size, book-to-market) | K. French |
| 361 | ME8 BM5 | 1 | 1 | (8, 5) portfolio sorted on (size, book-to-market) | K. French |
| 362 | ME8 BM6 | 1 | 1 | (8, 6) portfolio sorted on (size, book-to-market) | K. French |
| 363 | ME8 BM7 | 1 | 1 | (8, 7) portfolio sorted on (size, book-to-market) | K. French |
| 364 | ME8 BM8 | 1 | 1 | (8, 8) portfolio sorted on (size, book-to-market) | K. French |
| 365 | ME8 BM9 | 1 | 1 | (8, 9) portfolio sorted on (size, book-to-market) | K. French |
| 366 | ME8 BM10 | 1 | 1 | (8, 10) portfolio sorted on (size, book-to-market) | K. French |
| 367 | ME9 BM1 | 1 | 1 | (9, 1) portfolio sorted on (size, book-to-market) | K. French |
| 368 | ME9 BM2 | 1 | 1 | (9, 2) portfolio sorted on (size, book-to-market) | K. French |
| 369 | ME9 BM3 | 1 | 1 | (9, 3) portfolio sorted on (size, book-to-market) | K. French |
| 370 | ME9 BM4 | 1 | 1 | (9, 4) portfolio sorted on (size, book-to-market) | K. French |
| 371 | ME9 BM5 | 1 | 1 | (9, 5) portfolio sorted on (size, book-to-market) | K. French |
| 372 | ME9 BM6 | 1 | 1 | (9, 6) portfolio sorted on (size, book-to-market) | K. French |
| 373 | ME9 BM7 | 1 | 1 | (9, 7) portfolio sorted on (size, book-to-market) | K. French |
| 374 | ME9 BM8 | 1 | 1 | (9, 8) portfolio sorted on (size, book-to-market) | K. French |
| 375 | ME9 BM10 | 1 | 1 | (9, 10) portfolio sorted on (size, book-to-market) | K. French |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 376 | ME10 BM1 | 1 | 1 | (10, 1) portfolio sorted on (size, book-to-market) | K. French |
| 377 | ME10 BM2 | 1 | 1 | (10, 2) portfolio sorted on (size, book-to-market) | K. French |
| 378 | ME10 BM3 | 1 | 1 | (10, 3) portfolio sorted on (size, book-to-market) | K. French |
| 379 | ME10 BM4 | 1 | 1 | (10, 4) portfolio sorted on (size, book-to-market) | K. French |
| 380 | ME10 BM5 | 1 | 1 | (10, 5) portfolio sorted on (size, book-to-market) | K. French |
| 381 | ME10 BM6 | 1 | 1 | (10, 6) portfolio sorted on (size, book-to-market) | K. French |
| 382 | ME10 BM7 | 1 | 1 | (10, 7) portfolio sorted on (size, book-to-market) | K. French |
| 383 | Natural gas index | 1 | 5 | Commodity Prices, Natural Gas Index | World Bank |
| 384 | Cocoa | 1 | 5 | Commodity Prices, Cocoa | World Bank |
| 385 | Coffee, Arabica | 1 | 5 | Commodity Prices, Coffee, Arabica | World Bank |
| 386 | Coffee, Robusta | 1 | 5 | Commodity Prices, Coffee, Robusta | World Bank |
| 387 | Tea | 1 | 5 | Commodity Prices, Tea, avg 3 auctions | World Bank |
| 388 | Tea, Colombo | 1 | 5 | Commodity Prices, Tea, Colombo | World Bank |
| 389 | Tea, Kolkata | 1 | 5 | Commodity Prices, Tea, Kolkata | World Bank |
| 390 | Tea, Mombasa | 1 | 5 | Commodity Prices, Tea, Mombasa | World Bank |
| 391 | Coconut oil | 1 | 5 | Commodity Prices, Coconut Oil | World Bank |
| 392 | Groundnut oil | 1 | 5 | Commodity Prices, Groundnut Oil | World Bank |
| 393 | Palm oil | 1 | 5 | Commodity Prices, Palm Oil | World Bank |
| 394 | Soybeans | 1 | 5 | Commodity Prices, Soybeans | World Bank |
| 395 | Soybean oil | 1 | 5 | Commodity Prices, Soybean Oil | World Bank |
| 396 | Soybean meal | 1 | 5 | Commodity Prices, Soybean Meal | World Bank |
| 397 | Barley | 1 | 5 | Commodity Prices, Barley | World Bank |
| 398 | Maize | 1 | 5 | Commodity Prices, Maize | World Bank |
| 399 | Sorghum | 1 | 5 | Commodity Prices, Sorghum | World Bank |
| 400 | Rice | 1 | 5 | Commodity Prices, Rice, Thai 5% | World Bank |
| 401 | Wheat | 1 | 5 | Commodity Prices, Wheat, US HRW | World Bank |
| 402 | Banana | 1 | 5 | Commodity Prices, Banana, US | World Bank |
| 403 | Orange | 1 | 5 | Commodity Prices, Orange | World Bank |
| 404 | Beef | 1 | 5 | Commodity Prices, Beef | World Bank |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 405 | Chicken | 1 | 5 | Commodity Prices, Meat, Chicken | World Bank |
| 406 | Shrimps | 1 | 5 | Commodity Prices, Shrimps, Mexican | World Bank |
| 407 | Sugar | 1 | 5 | Commodity Prices, Sugar, World | World Bank |
| 408 | Tobacco | 1 | 5 | Commodity Prices, Tobacco, US import u.v. | World Bank |
| 409 | Logs | 1 | 5 | Commodity Prices, Logs, Malaysian | World Bank |
| 410 | Sawnwood | 1 | 5 | Commodity Prices, Sawnwood, Malaysian | World Bank |
| 411 | Cotton | 1 | 5 | Commodity Prices, Cotton, A Index | World Bank |
| 412 | Rubber | 1 | 5 | Commodity Prices, Rubber, SGP/MYS | World Bank |
| 413 | Copper | 1 | 5 | Commodity Prices, Copper | World Bank |
| 414 | Lead | 1 | 5 | Commodity Prices, Lead | World Bank |
| 415 | Tin | 1 | 5 | Commodity Prices, Tin | World Bank |
| 416 | Nickel | 1 | 5 | Commodity Prices, Nickel | World Bank |
| 417 | Zinc | 1 | 5 | Commodity Prices, Zinc | World Bank |
| 418 | Gold | 1 | 5 | Commodity Prices, Gold | World Bank |
| 419 | Platinum | 1 | 5 | Commodity Prices, Platinum | World Bank |
| 420 | Silver | 1 | 5 | Commodity Prices, Silver | World Bank |
| 421 | JPNPROINDQISMEI | 1 | 5 | Production of Total Industry in Japan | FRED |
| 422 | LRHUTTTTJPQ156S | 1 | 5 | Harmonized Unemployment Rate: Total: All Persons for Japan | FRED |
| 423 | JPNCPIALLQINMEI | 1 | 5 | Consumer Price Index of All Items in Japan | FRED |
| 424 | JPNLOLITONOSTSAM | 1 | 1 | Leading indicators: CLI: Normalized for Japan | FRED |
| 425 | DEUPROINDQISMEI | 1 | 5 | Production of Total Industry in Germany | FRED |
| 426 | OPCNRE01DEQ661N | 1 | 5 | Total Cost of Residential Construction for Germany | FRED |
| 427 | IRLTLT01DEQ156N | 1 | 2 | Long-Term (10-year) Government Bond Yields for Germany | FRED |
| 428 | DEUCPIALLQINMEI | 1 | 5 | Consumer Price Index of All Items in Germany | FRED |
| 429 | SPASTT01DEQ661N | 1 | 5 | Total Share Prices for All Shares for Germany | FRED |
| 430 | QDEPAMUSDA | 1 | 5 | Total Credit to Private Non-Financial Sector for Germany | FRED |
| 431 | GBRPROINDQISMEI | 1 | 5 | Production of Total Industry in the United Kingdom | FRED |
| 432 | IRLTLT01GBQ156N | 1 | 2 | Long-Term (10-year) Government Bond Yields for the United Kingdom | FRED |
| 433 | GBRCPIALLQINMEI | 1 | 5 | Consumer Price Index of All Items in the United Kingdom | FRED |

Table B1 (continued)

| | | | | | |
|---|---|---|---|---|---|
| 434 | LMUNRRTTGBQ156S | 1 | 2 | Registered Unemployment Rate for the United Kingdom | FRED |
| 435 | SPASTT01GBQ661N | 1 | 5 | Total Share Prices for All Shares for the United Kingdom | FRED |
| 436 | GBRGFCFQDSMEI | 1 | 5 | Gross Fixed Capital Formation in United Kingdom | FRED |
| 437 | GBRLOLITONOSTSAM | 1 | 1 | Leading indicators: CLI: Normalized for the United Kingdom | FRED |
| 438 | CANPROINDQISMEI | 1 | 5 | Production of Total Industry in Canada | FRED |
| 439 | WSCNDW01CAQ489S | 1 | 4 | Total Dwellings and Residential Buildings by Stage of Construction, Started for Canada | FRED |
| 440 | IRLTLT01CAQ156N | 1 | 2 | Long-Term (10-year) Government Bond Yields for Canada | FRED |
| 441 | LRUNTTTTCAQ156S | 1 | 2 | Unemployment Rate: Aged 15 and Over: All Persons for Canada | FRED |
| 442 | QCAPAM770A | 1 | 5 | Total Credit to Private Non-Financial Sector for Canada | FRED |
| 443 | SPASTT01CAQ661N | 1 | 5 | Total Share Prices for All Shares for Canada | FRED |
| 444 | CANLOLITONOSTSAM | 1 | 1 | Leading indicators: CLI: Normalized for Canada | FRED |

## C Simulation study

In this section we evaluate the performance of the new estimator using artificial data. Although we view the algorithm as primarily a forecasting algorithm, it is also important to investigate its estimation accuracy in an environment where we know the true data generating process (DGP). Thus, we wish to to establish that the VBDVS is able to track time-varying parameters satisfactorily and establish that the dynamic variable selection prior is able to perform shrinkage and selection with high accuracy (at least in cases where we know that the DGP is that of a sparse TVP regression model). We also wish to investigate the computational gains that arise from application of variational Bayes methods on the complex dynamic variable selection prior structure.

In all our experiments we use the following DGP:

$$
\begin{aligned}
y_t &= \beta_{1t}x_{1t} + \beta_{2t}x_{2t} + ... + \beta_{pt}x_{pt} + \sigma_t\varepsilon_t,\ \varepsilon_t \sim N(0,1) && \text{(C.1)}\\
x_{j,t} &\sim N(0,1),\ j = 1, ..., p && \text{(C.2)}\\
\beta_{j,t} &= s_{j,t} \times \theta_{j,t} && \text{(C.3)}\\
\theta_{j,t} &= \underline{\theta}_j + \underline{\rho}\left(\theta_{j,t-1} - \underline{\theta}_j\right) + \underline{\delta}\eta_{j,t},\ \eta_{j,t} \sim N(0,1) && \text{(C.4)}\\
\log\left(\sigma_t^2\right) &= \underline{\sigma}^2 + \underline{\phi}\left(\log\left(\sigma_{t-1}^2\right) - \underline{\sigma}^2\right) + \underline{\xi}\zeta_t,\ \zeta_t \sim N(0,1) && \text{(C.5)}\\
\theta_{j,0} &= \underline{\theta}_j, \qquad \log\left(\sigma_0^2\right) = \underline{\sigma}^2. && \text{(C.6)}
\end{aligned}
$$

Our benchmark specification sets $\underline{\boldsymbol{\theta}} = (-1.7, 2.9, 1.4, -2.3, \mathbf{0})$, $\underline{\sigma}^2 = 0.1$, $\underline{\rho} = \underline{\phi} = 0.99$, $\underline{\delta} = \underline{\xi} = T^{-1/2}$. In the specification above $\boldsymbol{s}_j$ is $T \times 1$ vector of either zeros or ones, such that $\beta_{j,t} = \theta_{j,t}$ when $s_{j,t} = 1$, and zero otherwise. We set $s_{1,t} = 1$ for $t = 1, ..., \lfloor T/3 \rceil - 1$ and zero otherwise, $s_{2,t} = 1\ \forall t = 1, ..., T$, $s_{3,t} = 1$ for $t = 1, ..., \lfloor T/2 \rceil - 1$ and zero otherwise, $s_{4,t} = 0$ for $t = 1, ..., \lfloor T/2 \rceil - 1$ and zero otherwise. These choices mean that $\beta_{1,t}$ is zero during the last third of the sample, $\beta_{2,t}$ is a relevant predictor in all periods, $\beta_{3,t}$ is zero during the last half of the sample, and $\beta_{4,t}$ is zero during the first half of the sample. Any other coefficient for $j = 5, ..., p$ is zero at all periods, i.e. $s_{j,t} = 0\ \forall\ j > 4,\ t = 1, ..., T$. By doing so, we simulate a situation where only one predictor is relevant in all time periods, three predictors are relevant only in certain subsamples of the data, and all remaining $p - 4$ predictors are irrelevant for $y$ at all time periods.

After we generate artificial data, we compare three competing estimation algorithms for TVP models: i) our variational Bayes dynamic variable selection (VBDVS) algorithm, ii) the EM algorithm implementation of the dynamic spike and slab (DSS) of Rockova and McAlinn (2021), and iii) Gibbs sampling (MCMC) estimation of the TVP model using the fast algorithm of Chan and Jeliazkov (2009). While there are numerous other algorithms available for estimating TVP models, our limited choice of algorithms reflects our desire to simulate exclusively high-dimensional models. By doing so, we exclude most of the recently proposed Bayesian methodologies cited in the Introduction. These methodologies introduce various flexible parameterizations (like we do) that result, however, in the need for many tuning parameters and estimation via MCMC, such that they become unreasonably cumbersome for $p > 50$. Our model instead, as we demonstrate in detail later, requires very straightforward tuning. The default prior setting we use for the VBDVS algorithm is based on the case Prior 3 presented in the main text. The settings used in the DSS and MCM algorithms are discussed in Section A of this supplement. In order to compare numerically these algorithms we generate $R = 500$ datasets from the above DGP for various choices of sample size and total number of predictors, namely $T = 100, 200, 500$ and $p = 50, 100, 200$. Subsequently squared deviations between true and estimated parameters are calculated, and then averaged over the $T$ time periods, and $p$ predictors. To be precise, if we let $\left(\beta_t^{true}\right)$ denote the true artificially generated coefficients and $\left(\beta_t^j, \sigma_t^j\right)$, for $j = VBDVS, DSS, MCMC$, the estimates of these coefficients, we calculate the sum of mean squared deviations (MSD) statistic as

$$MSD_{\beta}^{j} = \frac{1}{R}\sum_{r=1}^{R}\left(\frac{1}{T \times p}\sum_{t=1}^{T}\sum_{i=1}^{p}\left(\beta_{it}^{true,(r)} - \beta_{it}^{j,(r)}\right)^2\right), \tag{C.7}$$

where $r = 1, ..., R$, $R = 500$, denotes the number of Monte Carlo iterations.

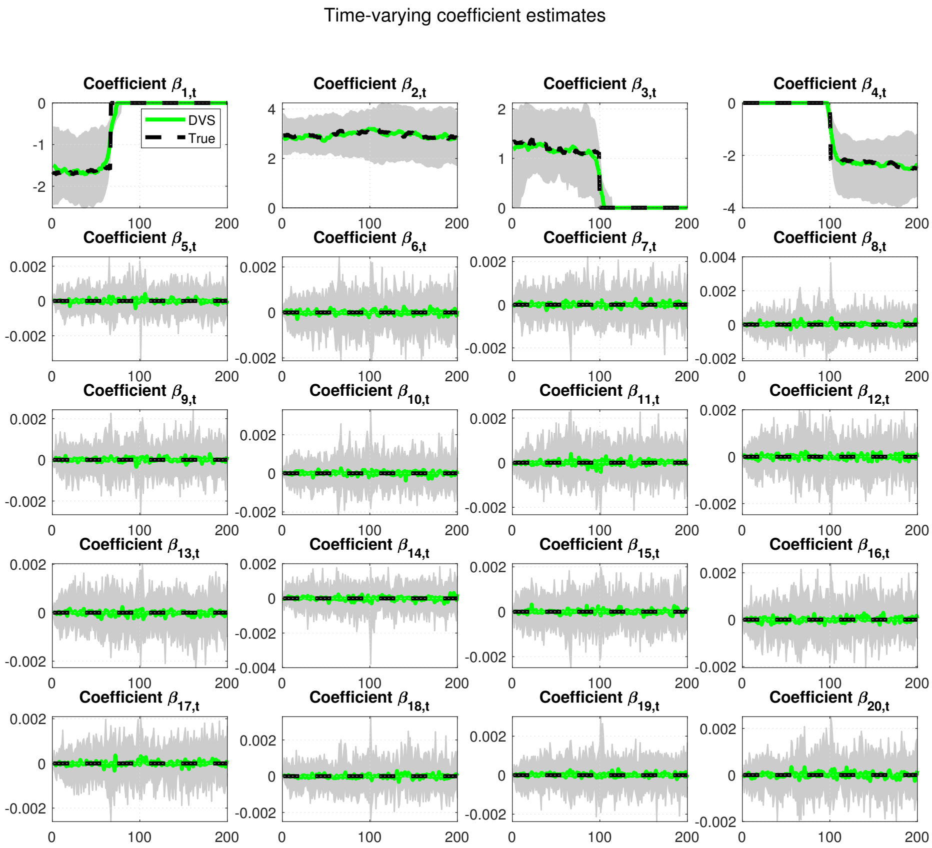


Figure C1: *VBDVS coefficient estimates of the first 20 predictors generated from the DGP with $T = 200$ and $p = 200$. Black dashed lines are the true generated coefficients. Posterior medians (over the 100 Monte Carlo iterations) of VBDVS estimates are shown with green solid lines, and grey areas denote $16^{th}$ and $84^{th}$ percentiles.*

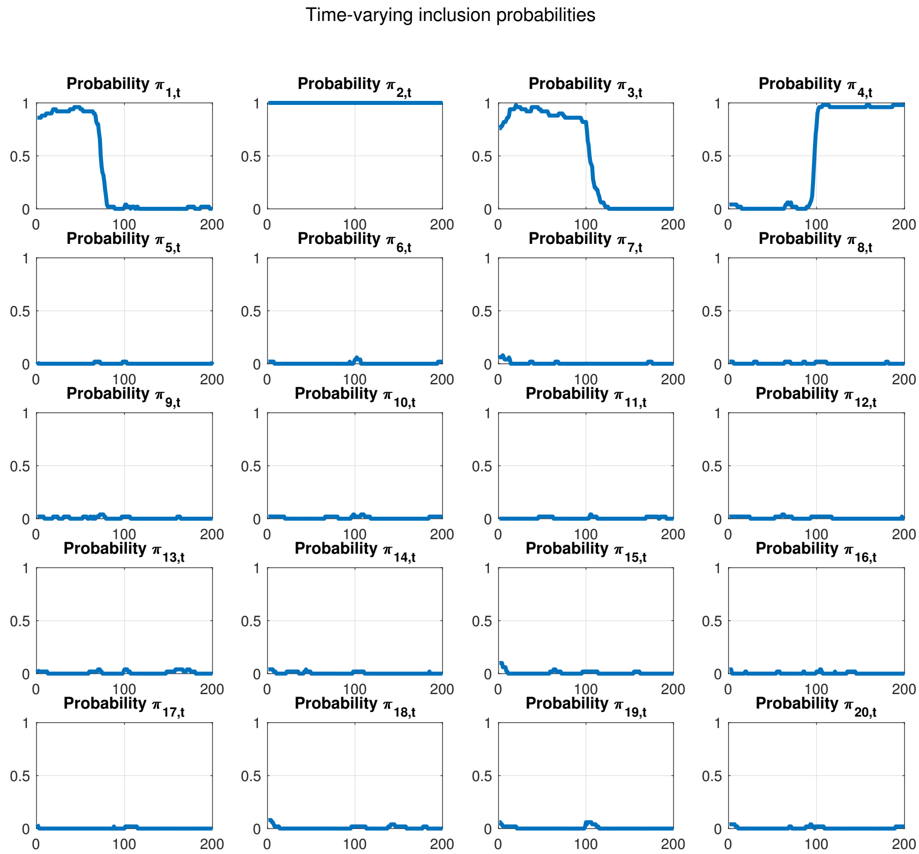


Figure C2: *Time-varying posterior inclusion probabilities (expected value of $\gamma_{j,t}$ estimates) of the first 20 predictors generated from the DGP with $T = 200$ and $p = 200$. These probabilities are means over the 100 Monte Carlo iterations.*

Figure C1 shows the coefficient estimates from VBDVS for the case $T = p = 200$. This plot compares the posterior median (green solid lines) versus the true generated coefficients (black dashed lines). The $16^{th}$ and $84^{th}$ percentiles over the 100 Monte Carlo iterations are also shown as a shaded area around the posterior median. Only the first 20 coefficients, out of the possible 200, are plotted. The first row shows the four coefficients that, at least in some periods, are non-zero, followed by 16 coefficients that are exactly zero. It is impossible to plot the remaining 180 coefficients in the DGP that are exactly zero, but their estimates are represented fairly well by the estimates of coefficients $\beta_{5,t}$ - $\beta_{20,t}$ shown in Figure C1. Under the assumption of sparsity in the DGP, the VBDVS algorithm is able to recover the true coefficients with accuracy. Not only the coefficients that are zero in the DGP in all periods are correctly estimated to be zero, but also the three coefficients that are zero only in certain subsamples are estimated precisely. When a coefficient is initially zero and later in the sample becomes important (see coefficient $\beta_{4,t}$), and vice-versa (see coefficients $\beta_{1,t}$ and $\beta_{3,t}$), the dynamic variable selection algorithm is able to identify and jump quickly to the new state. Figure C2 shows that the true reason why estimation is so precise – even in such a demanding case with 200 time-varying coefficients for only 200 observations – is because the estimates of the time-varying posterior inclusion probabilities (PIPs) of each predictor are recovered with precision in the first instance. By identifying correctly which variables should be excluded from the regression model in each period results in shrinking many coefficients to zero and allowing to preserve enough degrees of freedom for estimation of non-zero coefficients.

Table C1 shows the values of the MSD statistics for the three algorithms under the different combinations of $T$ and $p$. Given that the MSD statistics measure deviation from the true coefficient, lower values imply that a certain estimation algorithm has done better recovery of the coefficients generated by the DGP. In all cases VBDVS has the best performance among all competing algorithms. The estimation error of the MCMC algorithm is quite large mainly because the algorithm is unable to shrink all $p-4$ coefficients in the DGP that are exactly zero. The DSS algorithm provides a better fit since it is also an algorithm that does dynamic variable selection and shrinkage. Its performance is slightly inferior to VBDVS, but the results should not be taken as final evidence. While we have put every effort to follow closely the settings suggested by Rockova and McAlinn (2021), there might be other priors that could improve the performance of this algorithm.

Another important feature of the VBDVS algorithm is its fast computing time. While it is not surprising that our algorithm is faster compared to MCMC, our algorithm can provide substantial savings in high-dimensional settings compared to the DSS that relies on the EM algorithm. Columns 6-8 in Table C1 reveals that VBDVS can be multiple times faster than both DSS and MCMC algorithms.

Table C1: *MSD statistics and computing time for Monte Carlo exercise*

| | | MSD statistic | | | Computing time (secs) | | |
|---|---|---|---|---|---|---|---|
| | | VBDVS | DSS | MCMC | VBDVS | DSS | MCMC |
| | $p = 50$ | 0.203 | 0.419 | 7.979 | 1.2 | 8.3 | 22.6 |
| $T = 100$ | $p = 100$ | 0.469 | 1.014 | 11.787 | 7.2 | 20.1 | 106.6 |
| | $p = 200$ | 0.536 | 1.915 | 14.628 | 29.9 | 45.8 | 402.0 |
| | $p = 50$ | 0.047 | 0.256 | 5.825 | 5.5 | 19.9 | 49.9 |
| $T = 200$ | $p = 100$ | 0.088 | 0.789 | 10.583 | 10.1 | 40.1 | 232.2 |
| | $p = 200$ | 0.165 | 1.780 | 17.983 | 38.6 | 91.9 | 841.4 |
| | $p = 50$ | 0.019 | 0.147 | 4.613 | 8.3 | 51.1 | 125.2 |
| $T = 500$ | $p = 100$ | 0.043 | 0.819 | 9.095 | 50.9 | 125.1 | 555.6 |
| | $p = 200$ | 0.085 | 1.679 | 18.398 | 83.6 | 220.6 | 2127.8 |

*Notes: Computing times are based on a Windows 10 laptop running MATLAB 2020a, featuring an Intel i7-8665U processor and 32GB of RAM.*

# D Additional forecasting exercise: Tracking the Weekly Economic Index (WEI)

In this Section we provide the results of an additional macroeconomic forecasting exercise using weekly US data. During the 2020 Global Pandemic it was made clear to policy-makers that there was a need for real-time tracking of the macroeconomy. Quarterly releases of GDP that are subject to numerous revisions are excessively slow in order to allow for real-time decision-making (e.g. measuring the impacts of lockdowns to the economy). For that reason the New York Federal Reserve Bank released in April 2020 the Weekly Economic Index (WEI, https://www.newyorkfed.org/research/policy/weekly-economic-index/) that tracks movements in GDP on a weekly basis.

While there is no theoretical or empirical reason to forecast the WEI (it is a very incomplete weekly tracker of quarterly GDP), here we engage into forecasting WEI just for the sake of illustrating the numerical stability of our algorithm. Estimating regressions where parameters evolve as non-stationary random walks is challenging enough for quarterly data, but for high-frequency and highly volatile weekly series (e.g. commodity prices) many things can go wrong numerically (in larger samples there is higher probability for the random walk specification to lead to explosive behavior for $\boldsymbol{\beta}_t$). Therefore, our main focus is not to establish that our TVP approach is the best in forecasting also weekly macro data (there is no past evidence on WEI to suggest so, anyway), rather we want to provide additional empirical evidence that our algorithm provides results that are numerically comparable to MCMC. For that reason, we keep exactly the same models we defined for the quarterly data, with exactly the same default prior hyperparameter settings for all models.

Since the main motivation of our algorithm is also the use of many predictors, we collect a novel weekly dataset with 117 potential predictors of WEI. The total 118-variable dataset can be thought of as a weekly variant of the FRED-QD dataset we used for the main forecasting exercise (see Appendix A). This new "FRED-WD" dataset collects many (mainly financial/fast-moving) time series from FRED, adds some key financial series from Bloomberg, and also adds the Aruoba-Diebold-Scotti weekly business conditions index that is maintained by the Philadelphia Federal Reserve Bank (https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/ads). Some of the available series are available from 1971W1, but most series, including the WEI, have much shorter sample availability. For that reason we collect a balanced panel with all 118 variables that spans the period 2008W1 to 2021W52. The names and description of all series, their stationarity transformations, and sources are provided in Table D3.

We use the last two years of the sample, 2020W1 - 2022W52, to evaluate weekly forecasts of WEI at horizons $h = 1, 2, 4, 8, 12, 26$ weeks. This period coincides with WEI plummeting during the pandemic-related lockdowns, and then rebounding and overshooting abruptly. The recession induced by the 2020 pandemic might be the shortest ever recorded by NBER (2 months) but it has definitely changed time series dynamics of macroeconomic variables for good. When looking at weekly data the changes are extremely noisy and volatile, which raises the question about which models are best for forecasting WEI using the 117 predictors. For that reason we use the same class of forecasting models we had before, in the main forecasting exercise for inflation, with the only change that we now include four own lags of weekly WEI (while when forecasting quarterly inflation we always included two own lags).

The results of this forecasting exercise are shown in Table D1. Entries are again MSFEs relative to the MSFE of the benchmark AR(4) specification. It becomes immediately obvious that all TVP regressions (UCSV, TVPAR, TVD, DMA, and the three VBDVS specification) are beaten by specification that feature constant parameters, regardless of how these treat exogenous predictors (i.e. whether they use 5 or 60 factors or all predictors). While for low-frequency macro data TVP models are among the best for capturing abrupt structural change, the same might not be true for high-frequency data.[2] However, within the class of TVP regressions it is again obvious that the dynamic variable selection prior of the VBDVS algorithm does a good job at making good use of additional information, while at the same time preventing overparametrization: Forecast accuracy clearly improves as we move from VBDVS/FAC5 to VBDVS/FAC60 to VBDVS/X. The results from all VBDVS specifications are comparable to TVPAR, which provides additional evidence that the algorithm is stable numerically even in the presence of high-frequency data.

[2] Indeed in Korobilis and Yilmaz (2018), Measuring Dynamic Connectedness with Large Bayesian VAR Models, mimeo, http://repository.essex.ac.uk/20937/, we find that time-varying parameter vector autoregressive (TVP-VAR) models for daily financial return data are not able to beat simple rolling OLS estimators of VARs.

Table D1: *Relative MSFEs for Weekly Economic Index (WEI)*

| | WEI | | | | | |
|---|---|---|---|---|---|---|
| | $h = 1$ | $h = 2$ | $h = 4$ | $h = 8$ | $h = 12$ | $h = 26$ |
| AR | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| SBAR | 0.92 | 1.46 | 0.94 | 0.71 | 1.10 | 0.95 |
| UCSV | 1.40 | 2.45 | 1.05 | 1.08 | 1.19 | 0.84 |
| TVPAR | 1.43 | 2.17 | 1.18 | 1.10 | 1.01 | 0.73 |
| | Models with five factors | | | | | |
| FAC5 | 0.63 | 0.81 | 0.87 | 0.90 | 0.91 | 0.80 |
| BAG/FAC5 | 0.69 | 0.80 | 0.86 | 0.87 | 0.88 | 0.78 |
| DMA/FAC5 | 1.76 | 2.11 | 1.00 | 1.08 | 1.02 | 0.75 |
| TVD/FAC5 | 1.51 | 1.76 | 1.14 | 1.05 | 0.97 | 0.84 |
| VBDVS/FAC5 | 0.89 | 1.66 | 1.23 | 1.58 | 1.11 | 0.79 |
| GPR/FAC5 | 0.53 | 0.91 | 1.01 | 0.90 | 0.94 | 0.90 |
| | Models with 60 factors | | | | | |
| SSVS/FAC60 | 0.67 | 0.79 | 0.88 | 0.85 | 0.83 | 0.70 |
| ELN/FAC60 | 0.61 | 0.80 | 0.98 | 0.88 | 0.89 | 0.81 |
| VBDVS/FAC60 | 0.80 | 1.06 | 1.20 | 1.16 | 1.02 | 0.75 |
| | Models with 443 predictors | | | | | |
| ELN/X | 0.81 | 0.75 | 0.90 | 0.93 | 0.86 | 0.73 |
| PLS/X | 0.64 | 0.60 | 1.10 | 1.10 | 0.80 | 0.62 |
| VBDVS/X | 0.61 | 1.03 | 1.09 | 1.13 | 1.05 | 0.77 |

As a final check of our ability of the algorithm to produce sensible results, we want to test the evidence that constant parameter specifications provide better forecasts, by shrinking our TVP regressions towards constant parameters. This can be done simply by restricting the state error variance $\boldsymbol{w}_t$ via its prior. By choosing $c_{j,0} = 1000$, instead of the previous value $c_{j,0} = 100$, is a way to ensure smaller state variance which will result in $\boldsymbol{\beta}_t$ moving closer to $\boldsymbol{\beta}_{t-1}$. Therefore, we rerun the VBDVS/FAC5, VBDVS/FAC60 and VBDVS/X forecasts changing only this value such that the models have less time-variation in their parameters. Table D2 shows the results of this experiment. Indeed for $h = 2, 4, 8, 12$ weeks ahead, forecasts are markedly improved. However, forecasts for $h = 1$ deteriorate significantly for the two models with factors but not so much for the model with exogenous predictors, suggesting that there one-step ahead forecasts have quite complex dynamics and a researcher interested in forecasting WEI accurately would need to experiment with different sets of predictors, different regression specifications, different patterns of time-variation in the TVPs, and different amounts of dynamic sparsity.

Table D2: *Relative MSFEs for VBDVS forecasting models under a prior that shrinks towards a constant parameter regression*

| | WEI | | | | | |
|---|---|---|---|---|---|---|
| | $h = 1$ | $h = 2$ | $h = 4$ | $h = 8$ | $h = 12$ | $h = 26$ |
| AR | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| VBDVS/FAC5 | 1.26 | 1.08 | 0.98 | 0.93 | 0.96 | 0.80 |
| VBDVS/FAC60 | 1.85 | 0.96 | 0.98 | 0.92 | 0.96 | 0.82 |
| VBDVS/X | 0.70 | 0.81 | 1.00 | 0.90 | 0.95 | 0.79 |

Table D3: Weekly dataset

| No | Mnemonic | T | Long description | Source |
|---|---|---|---|---|
| 1 | WEI | 2 | Weekly Economic Index (Lewis-Mertens-Stock) Index | FRED |
| 2 | ADS | 1 | Aruoba-Diebold-Scotti Business Conditions Index | Philly Fed |
| 3 | M1 | 5 | M1 Money Stock | FRED |
| 4 | M2 | 5 | M2 Money Stock | FRED |
| 5 | WRMFSL | 2 | Retail Money Funds component of M2 | FRED |
| 6 | CURRENCY | 5 | Currency Component of M1 | FRED |
| 7 | NONM1 | 5 | Non-M1 Components of M2 | FRED |
| 8 | MZM | 5 | MZM Money Stock | FRED |
| 9 | CCSA | 5 | Continued Claims (Insured Unemployment) | FRED |
| 10 | ICSA | 5 | Initial Claims | FRED |
| 11 | IURSA | 5 | Insured Unemployment Rate | FRED |
| 12 | BUSAPPWNSAUS | 1 | Business Applications for the United States | FRED |
| 13 | HBUSAPPWNSAUSCH1 | 1 | High-Propensity Business Applications for the United States | FRED |
| 14 | INFECTDISEMVTRACKD | 1 | Equity Market Volatility: Infectious Disease Tracker | FRED |
| 15 | USEPUINDXD | 1 | Economic Policy Uncertainty Index for United States | FRED |
| 16 | WLEMUINDXD | 1 | Equity Market-related Economic Uncertainty Index | FRED |
| 17 | STLFSI2 | 1 | St. Louis Fed Financial Stress Index | FRED |
| 18 | MBAVCHNG Index | 1 | MBA US mortgage market index | Bloomberg |
| 19 | COMFCOMF Index | 2 | Bloomberg US Consumer Comfort Index | Bloomberg |
| 20 | REDSWYOY Index | 1 | Johnson Redbook Index Same Store Sales | Bloomberg |
| 21 | JFRIUS Index | 1 | JPMorgan Forecast Revision Index US | Bloomberg |
| 22 | NFCI | 2 | Chicago Fed National Financial Conditions Index | FRED |
| 23 | NFCICREDIT | 2 | Chicago Fed National Financial Conditions Credit Subindex | FRED |
| 24 | NFCILEVERAGE | 2 | Chicago Fed National Financial Conditions Leverage Subindex | FRED |
| 25 | NFCINONFINLEVERAGE | 2 | Chicago Fed National Financial Conditions Index Nonfinancial Leveral Subindex | FRED |
| 26 | NFCIRISK | 2 | Chicago Fed National Financial Conditions Risk Subindex | FRED |
| 27 | WIMFSL | 5 | Institutional Money Funds | FRED |

Table B1 (continued)

| | | | | |
|---|---|---|---|---|
| 28 | XAU Curncy | 5 | Gold United States Dollar Spot | Bloomberg |
| 29 | PRMIPRGS Index | 2 | Price Ratio Gold/Silver | Bloomberg |
| 30 | IADMGOLD Index | 5 | Bloomberg Composite Gold Inflation Adjusted Spot Price | Bloomberg |
| 31 | WCOILBRENTEU | 5 | Crude Oil Prices: Brent - Europe Dollars per Barrel | FRED |
| 32 | WCOILWTICO | 5 | Crude Oil Prices: West Texas Intermediate (WTI) - Cushing Oklahoma Dollars per Barrel | FRED |
| 33 | GASDESW | 5 | US Diesel Sales Price Dollars per Gallon | FRED |
| 34 | GASREGW | 5 | US Regular All Formulations Gas Price Dollars per Gallon | FRED |
| 35 | WPROPANEMBTX | 5 | Propane Prices: Mont Belvieu Texas Dollars per Gallon | FRED |
| 36 | WJFUELUSGULF | 5 | Kerosene-Type Jet Fuel Prices: U.S. Gulf Coast Dollars per Gallon | FRED |
| 37 | WHHNGSP | 5 | Henry Hub Natural Gas Spot Price Dollars per Million BTU | FRED |
| 38 | COMPOUT | 2 | Commercial Paper Outstanding | FRED |
| 39 | ABCOMP | 2 | Asset-backed Commercial Paper Outstanding | FRED |
| 40 | NFINCP | 2 | Commercial Paper of Nonfinancial Companies | FRED |
| 41 | VXOCLS | 1 | CBOE S&P 100 Volatility Index: VXO | FRED |
| 42 | VXVCLS | 1 | CBOE S&P 500 3-Month Volatility Index | FRED |
| 43 | OPWVCBOC Index | 5 | CBOE US Equity Call Options Volume | Bloomberg |
| 44 | WILL5000PR | 5 | Wilshire 5000 Price Index | FRED |
| 45 | WILLLRGCAPGR | 5 | Wilshire US Large-Cap Growth Total Market Index | FRED |
| 46 | WILLREITIND | 5 | Wilshire US Real Estate Investment Trust Total Market Index (Wilshire US REIT) | FRED |
| 47 | WILLRESIPR | 5 | Wilshire US Real Estate Securities Price Index (Wilshire US RESI) | FRED |
| 48 | DEXUSEU | 5 | U.S. / Euro Foreign Exchange Rate U.S. Dollars to One Euro | FRED |
| 49 | DTWEXBGS | 5 | Trade Weighted U.S. Dollar Index: Broad Goods and Services | FRED |
| 50 | DTWEXAFEGS | 5 | Trade Weighted U.S. Dollar Index: Advanced Foreign Economies Goods and Services | FRED |
| 51 | DTWEXEMEGS | 5 | Trade Weighted U.S. Dollar Index: Emerging Markets Economies Goods and Services | FRED |
| 52 | FF | 2 | Effective Federal Funds Rate | FRED |
| 53 | WGS1MO | 2 | 1-Month Treasury Constant Maturity Rate | FRED |
| 54 | WGS3MO | 2 | 3-Month Treasury Constant Maturity Rate | FRED |
| 55 | WGS6MO | 2 | 6-Month Treasury Constant Maturity Rate | FRED |
| 56 | WGS1YR | 2 | 1-Year Treasury Constant Maturity Rate | FRED |

Table B1 (continued)

| | | | | |
|---|---|---|---|---|
| 57 | WGS2YR | 2 | 2-Year Treasury Constant Maturity Rate | FRED |
| 58 | WGS3YR | 2 | 3-Year Treasury Constant Maturity Rate | FRED |
| 59 | WGS5YR | 2 | 5-Year Treasury Constant Maturity Rate | FRED |
| 60 | WGS7YR | 2 | 7-Year Treasury Constant Maturity Rate | FRED |
| 61 | WGS10YR | 2 | 10-Year Treasury Constant Maturity Rate | FRED |
| 62 | WGS20YR | 2 | 20-Year Treasury Constant Maturity Rate | FRED |
| 63 | WGS30YR | 2 | 30-Year Treasury Constant Maturity Rate | FRED |
| 64 | WTB4WK | 2 | 4-Week Treasury Bill: Secondary Market Rate | FRED |
| 65 | WTB3MS | 2 | 3-Month Treasury Bill: Secondary Market Rate | FRED |
| 66 | WTB6MS | 2 | 6-Month Treasury Bill: Secondary Market Rate | FRED |
| 67 | WFII5 | 2 | 5-Year Treasury Inflation-Indexed Security Constant Maturity | FRED |
| 68 | WFII7 | 2 | 7-Year Treasury Inflation-Indexed Security Constant Maturity | FRED |
| 69 | WFII10 | 2 | 10-Year Treasury Inflation-Indexed Security Constant Maturity | FRED |
| 70 | WFII20 | 2 | 20-Year Treasury Inflation-Indexed Security Constant Maturity | FRED |
| 71 | WLTIIT | 2 | Treasury Inflation-Indexed Long-Term Average Yield | FRED |
| 72 | MORTGAGE5US | 2 | 5/1-Year Adjustable Rate Mortgage Average in the United States | FRED |
| 73 | MORTGAGE15US | 2 | 15-Year Fixed Rate Mortgage Average in the United States | FRED |
| 74 | MORTGAGE30US | 2 | 30-Year Fixed Rate Mortgage Average in the United States | FRED |
| 75 | WAAA | 2 | Moody's Seasoned Aaa Corporate Bond Yield | FRED |
| 76 | WBAA | 2 | Moody's Seasoned Baa Corporate Bond Yield | FRED |
| 77 | WPCREDIT | 2 | Discount Window Primary Credit Rate | FRED |
| 78 | WPRIME | 2 | Bank Prime Loan Rate | FRED |
| 79 | RESPPANWW | 2 | Assets: Total Assets: Total Assets | FRED |
| 80 | TLAACBW027SBOG | 2 | Total Assets All Commercial Banks | FRED |
| 81 | CASACBW027SBOG | 2 | Cash Assets All Commercial Banks | FRED |
| 82 | SWPT | 2 | Central Bank Liquidity Swaps: Central Bank Liquidity Swaps | FRED |
| 83 | WRESCRT | 2 | Other Factors Supplying Reserve Balances: Reserve Bank Credit | FRED |
| 84 | WREPO | 2 | Repurchase Agreements | FRED |
| 85 | WLCFLL | 2 | Liquidity and Credit Facilities: Loans | FRED |

Table B1 (continued)

| | | | | |
|---|---|---|---|---|
| 86 | WSHOMCB | 2 | Securities Held Outright: Mortgage-Backed Securities | FRED |
| 87 | WSHOSHO | 2 | Securities Held Outright: Securities Held Outright | FRED |
| 88 | TREAS10Y | 2 | Securities Held Outright: U.S. Treasury Securities: Maturing in Over 10 Years | FRED |
| 89 | WSHOTSL | 2 | Securities Held Outright: U.S. Treasury Securities | FRED |
| 90 | ECBASSETSW | 2 | Central Bank Assets for Euro Area (11-19 Countries) | FRED |
| 91 | TOTBKCR | 5 | Bank Credit All Commercial Banks | FRED |
| 92 | H8B3094NCBA | 5 | Borrowings All Commercial Banks | FRED |
| 93 | TOTBORR | 5 | Total Borrowings of Depository Institutions from the Federal Reserve | FRED |
| 94 | LCBACBW027SBOG | 5 | Loans to Commercial Banks All Commercial Banks | FRED |
| 95 | TOTLL | 5 | Loans and Leases in Bank Credit All Commercial Banks | FRED |
| 96 | TOTCI | 5 | Commercial and Industrial Loans All Commercial Banks | FRED |
| 97 | CIBOARD | 5 | Commercial and Industrial Loans Large Domestically Chartered Commercial Banks | FRED |
| 98 | CLSACBW027SBOG | 5 | Consumer Loans All Commercial Banks | FRED |
| 99 | RELACBW027SBOG | 5 | Real Estate Loans All Commercial Banks | FRED |
| 100 | CREACBW027SBOG | 5 | Commercial Real Estate Loans All Commercial Banks | FRED |
| 101 | RREACBW027SBOG | 5 | Residential Real Estate Loans All Commercial Banks | FRED |
| 102 | RHEACBW027SBOG | 5 | Residential Real Estate Loans: Revolving Home Equity Loans All Commercial Banks | FRED |
| 103 | DPSACBW027SBOG | 5 | Deposits All Commercial Banks | FRED |
| 104 | SAVINGS | 5 | Total Savings Deposits at all Depository Institutions | FRED |
| 105 | TCD | 5 | Total Checkable Deposits | FRED |
| 106 | WOCDSL | 5 | Other Checkable Deposits | FRED |
| 107 | WDDSL | 5 | Demand Deposits: Total | FRED |
| 108 | LTDACBW027SBOG | 5 | Large Time Deposits All Commercial Banks | FRED |
| 109 | WSMTIME | 5 | Small Time Deposits - Total | FRED |
| 110 | GDTCBW | 5 | US government deposits: Total cash balance | FRED |
| 111 | WMTSECL1 | 5 | Memorandum Items: Custody Holdings: Marketable U.S. Treasury Securities | FRED |
| 112 | TLBACBW027SBOG | 5 | Total Liabilities All Commercial Banks | FRED |
| 113 | WDTGAL | 5 | Deposits with F.R. Banks Other Than Reserve Balances: U.S. Treasury General Account | FRED |
| 114 | WLODLL | 5 | Deposits: Other Deposits Held by Depository Institutions | FRED |

Table B1 (continued)

| | | | | |
|---|---|---|---|---|
| 115 | WLTLECL | 5 | Total Liabilities (Less Eliminations from Consolidation) | FRED |
| 116 | WLFN | 5 | Federal Reserve es Net of F.R. Bank Holdings | FRED |
| 117 | WLRRAL | 5 | Reverse Repurchase Agreements | FRED |
| 118 | WLRRAFOIAL | 5 | Reverse Repurchase Agreements: Foreign Official and International Accounts | FRED |

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