A FAST FAILURE DIAGNOSIS METHOD BASED ON STRUCTURE FEATURES OF PLANAR ARRAYS

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Abstract

Array antennas are the basic components which are widely used in radar systems. In this paper, a fast failure diagnosis method is proposed based on structure information of planar arrays when the number of faulty elements is small. The proposed algorithm reduces the time spent locating the failed elements while maintaining the accuracy of the localization. Firstly, the sparse diagnosis model of a failure planar array is introduced. Considering the structure features of planar arrays, the vertical and horizontal elevation plane of the field pattern are used to decide the rows and columns of the failed elements, respectively. Besides, a new smaller subarray is constructed based on the rows and columns above and further diagnosed by the new measured field pattern. With the aid of theoretical analysis and simulation validation, it is shown that the proposed diagnosis method can localize faults faster compared with conventional diagnosis methods.

1 Introduction

With the increasing demand for better abilities of target detection and radar imaging, high-frequency radar systems have been exploited to provide wider bandwidth and higher resolution[1, 2]. In this case, due to shorter wavelengths, more antennas with smaller size are installed in an area, which constitute an array. This technology is applied in many applications such as adaptive beamforming, multiple target detection and synthetic aperture radar images[3].

Planar arrays are the simplest array structure in which the array base elements are laid out along the vertical and horizontal axes, so it can obtain high resolution both in azimuth and elevation, respectively [4]. However, once base elements in planar arrays fail, the whole array structure will change, such as elements' spacing, which results in the modification of the Array Factor. Although the variation of physical structure is slight, it leads to severe consequences, for example the alteration of beam directions and increased sidelobes, which further affect the performance of subsequent applications. Therefore, it is necessary to find failed array elements. Instead of inefficient detection of elements one by one, field pattern measurements are used to infer the locations of faulty elements by solving an inverse electromagnetic problem.

In general, there are several ways to achieve the goal of failure localization by using field patterns. The first is called the matrix method, which uses linear algebra and matrix theory to calculate the values of excitation[5]. The second is the theory of parameter estimation and optimization. In this case, compressed sensing and its variants are utilized for array diagnosis when the number of faults is far smaller than that of the total array elements[6–9]. Moreover, when the transformation from the excitation of array elements to the field pattern is not linear, optimization theory is used to find the optimal solution of excitation[10, 11]. The third type of failure diagnosis is Artificial Intelligence, which uses the strategies of learning, including support vector machines and artificial neural networks[12, 13].

In addition to the classification from the aspects of diagnosis methods, failure detection can be categorized according to the different scenarios of practical applications. The first type of diagnosis scenario is to use all information of either near-field or far-field measurements, including amplitude and phase. There is no difference of diagnosis between using near-field and far-field measurements, except that their radiation matrices are not the same. Considering the limitations of real measurements, the second scenario focuses on the amplitude-only data to avoid phase measuring because of its complex process and high cost [10, 11, 14]. The third category is about some particular situations that diagnosis methods are applied in, such as array mismatch and impluse noise, which are more challenging for researchers and engineers[10, 11].

In spite of the fact that many methods of array diagnosis have been utilized to determine the positions of faulty elements and have achieved good results, there is still a practical and urgent problem unsolved, that is the large amount of time for diagnosis when the size of planar arrays increases. In addition, even though the existing methods are widely applied to various shapes of antenna arrays, they do not explore the physical structure features of planar arrays, which can bring about opportunities to improve diagnosed time. Therefore, a research question arises: how to save the time of failure diagnosis by using structure features of planar arrays? The old way was to consider the sparse characteristics of excitation and use compressed sensing to diagnose faults. Inspired by the cross approximation techniques of matrix decomposition[15–17], another feasible approach is to explore and exploit the structure properties of planar arrays by dividing into several subarrays vertically and horizontally respectively, and to downsize the area that faults lie in to further reduce the time cost of failure diagnosis.

The contributions of this paper are summarized as follows.

1) A diagnosis framework based on the structure features of planar arrays is proposed to accelerate failure diagnosis when the number of faults is far smaller than the total number of elements.
2) The proposed fast diagnosis method is compared with another diagnostic method as a reference and works on the premise of Gaussian noise.

The remaining sections of this paper are organized as follows. Section II presents the diagnosis model of failure arrays and the problem statement. Section III gives the proposed fast diagnosis framework based on the structure features of planar arrays. Section IV shows the comparisons of diagnostic performance between the proposed fast method and Sparse Bayesian Learning, including the time cost and diagnosis error. Finally, conclusions are drawn in section V.

2 Diagnosis model and problem statement

2.1 Diagnosis model

A planar array of size $N_x \times N_y$ is given in Fig. 1(a), where N_x and N_y is the number of elements along the x-axis and y-axis, respectively. The circles marked in green represent the normal elements while the black circles are failed. According to the differential operation[7], the original problem of failure diagnosis is equal to finding out the positions of excitation in Fig. 1(b), where the black circles represent the failed elements while the red elements are working properly.

Assuming that the far-field pattern is measured at spherical angles $\boldsymbol{\theta}, \boldsymbol{\varphi}$, where $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_M]^{\mathsf{T}}$, $\boldsymbol{\varphi} = [\varphi_1, \varphi_2, \cdots, \varphi_N]^{\mathsf{T}}$, the far-field radiation model in Gaussian noise is given by

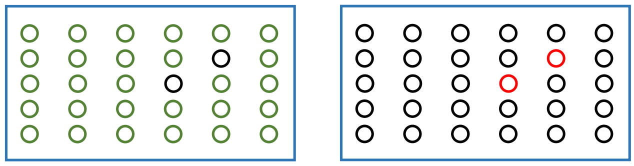
$$\boldsymbol{b} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n_e}, \tag{1}$$

where $\boldsymbol{b}$ is a vector representation of the far-field pattern with respect to variables $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$, and its i^{th} component b_i corresponds to the measurement at angles $(\theta_m, \varphi_n), i = M(n-1) + m$. $\boldsymbol{A}$ is a far-field radiation matrix, whose elements are given by

$$a_{ij} = e^{jkd_j^x \sin\theta_m \cos\varphi_n + jkd_j^y \sin\theta_m \sin\varphi_n}, \tag{2}$$

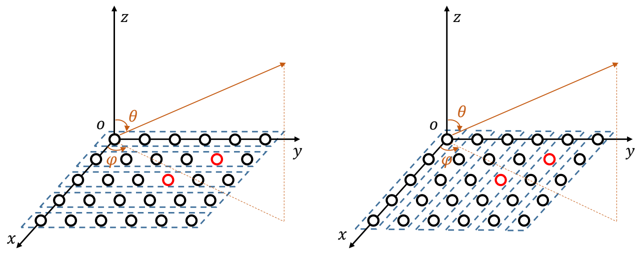
where (d_j^x, d_j^y) represents the coordinates of the j^{th} array element. $\boldsymbol{s}$ is a 0-1 excitation vector and $\boldsymbol{n_e}$ is zero-mean Gaussian noise.

The problem of failure diagnosis is to retrieve the excitation $\boldsymbol{s}$ and further localize faults according to the obtained far-field measurements. Although many existing methods have been utilized to recover $\boldsymbol{s}$, compressed sensing is widely used because



(a) Original array antenna (b) Differential array antenna

Fig. 1. Planar array



(a) A linear array along x-axis (b) A linear array along y-axis

Fig. 2. Divide a planar array into subarrays

of its high efficiency. Since the number of faults in real situations is far smaller than that of the total elements, the diagnosis model is built under the hypothesis of the sparse excitation $\boldsymbol{s}$. Therefore, it is possible to provide the unique recovery of $\boldsymbol{s}$, which is given by

$$\begin{aligned} &\min_{\boldsymbol{s}} \|\boldsymbol{s}\|_1 \\ &s.t.\ \|\boldsymbol{b} - \boldsymbol{A}\boldsymbol{s}\|_2 \le \epsilon, \end{aligned} \tag{3}$$

where $\|\cdot\|_p$ represents the l_p-norm and ϵ is the error brought about by Gaussian noise. It can be seen that (3) is the diagnosis model to localize faulty elements and this problem can be solved via the algorithms of l_1-norm minimization such as Sparse Bayesian Learning (SBL).

2.2 Problem statement

Based on the antenna and array theory[18], a planar array can be seen as a linear array with several subarrays as its base elements. And if all subarrays have the same field pattern, the total far-field beam pattern of a planar array is equal to the far-field pattern of a single subarray multiplied by an array factor. Considering a horizontal subarray composed of a row of elements, as shown in Fig. 2(a), the planar array is converted to a linear array along the x-axis, whose far-field beam pattern with respect to spherical angles (θ_p, φ_q) is written by

$$b(\theta_p, \varphi_q) = \sum_{m=1}^{Nx} f_m(\theta_p, \varphi_q) e^{jkx_m \sin\theta_p \cos\varphi_q} + n_e, \tag{4}$$

where $f_m(\theta_p, \varphi_q)$ is the beam pattern of the m^{th} subarray with respect to θ_p and φ_q, k is the wavenumber, x_m is the position of the m^{th} subarray and n_e is a zero-mean Gaussian random variable. Moreover, the planar array is seen as a linear array along the y-axis with several vertical subarrays, as shown in Fig. 2(b). Then its far-field beam pattern is given by

$$b(\theta_p, \varphi_q) = \sum_{n=1}^{Ny} g_n(\theta_p, \varphi_q) e^{jky_n \sin\theta_p \sin\varphi_q} + n_e, \tag{5}$$

where $g_n(\theta_p, \varphi_q)$ is the beam pattern of the n^{th} subarray with respect to θ_p and φ_q, and y_n is the position of the n^{th} subarray. Obviously, the far-field pattern b is unchanged no matter how the planar array is divided into subarrays. Accordingly, the research problem of failure diagnosis is how to diagnose failed planar arrays by using its structure information in order to save time. A possible solution is to find out the area that failed elements belong to by exploiting the vertical and horizontal division of planar arrays.

3 Proposed diagnosis framework based on structure features of planar arrays

In order to reduce the time of array diagnosis, a fast diagnosis framework based on the structure information of planar arrays is proposed and described in this section. Here, the structure features of planar arrays indicate that a planar array can be cut horizontally and vertically into several subarrays and be seen as two linear arrays, respectively. Although these two ways of cutting lead to the different types of subarrays, they contain the same failed array elements, which reveals the information of rows and columns that faults lie in. By taking advantage of this property, the failure diagnosis of planar arrays can accelerate.

First, consider a planar array of size $N_x \times N_y$ and split it evenly into several rows as shown in Fig. 2(a), so the planar array now is seen as a linear array with the length of N_x. It is obvious that the failed elements are located in these subarrays, so the original problem of recovering $\boldsymbol{s}$ is modified to reconstruct the excitation of subarrays, f_i, and to find out in which subarrays that the failed elements lie.

Now rewrite (4) as

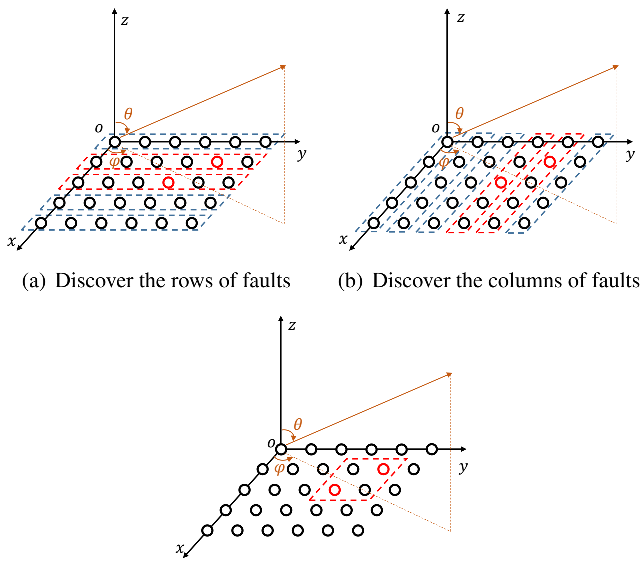
$$\boldsymbol{b}^u = \boldsymbol{A}^u \boldsymbol{f}^u + \boldsymbol{n_e}, \tag{6}$$

where $\boldsymbol{b}^u$ represents the elevation plane of the far-field pattern at the azimuth $0°$, $\boldsymbol{A}^u$ is the radiation matrix composed of the elements $a^u_{ij} = e^{jkx_j \sin\theta_i}$, and $\boldsymbol{f}^u$ is a vector, whose elements f^u_j denote the excitation of each subarray. Since each subarray is a linear array along the y-axis and the field pattern is at the azimuth $\varphi = 0°$, $\boldsymbol{f}^u$ is indepedent of θ_i.

Then the diagnosis problem of failed subarrays is to reconstruct $\boldsymbol{f}^u$, given by

$$\begin{aligned} &\min_{\boldsymbol{f}^u} \|\boldsymbol{f}^u\|_1 \\ &s.t. \ \|\boldsymbol{b}^u - \boldsymbol{A}^u \boldsymbol{f}^u\|_2 \le \epsilon. \end{aligned} \tag{7}$$

The positions of nonzero components in $\boldsymbol{f}^u$ correspond to the rows of the planar array that failed elements lie in. As shown in Fig. 3(a), the rows of faults are marked by the red dashed rectangles.



(a) Discover the rows of faults (b) Discover the columns of faults

(c) Construct and diagnose a new subarray

Fig. 3. Proposed fast diagnosis framework

Similarly, divide the planar array evenly into N_y columns, which form a linear array along y-axis as shown in Fig. 2(b). Without doubt, the diagnosis problem of array elements in a plane is converted to the detection of failed subarrays.

Equation (5) is rewritten as

$$\boldsymbol{b}^v = \boldsymbol{A}^v \boldsymbol{g}^v + \boldsymbol{n_e}, \tag{8}$$

where the elevation plane of the field pattern at the azimuth $90°$ is denoted by $\boldsymbol{b}^v$, the radiation matrix is denoted by $\boldsymbol{A}^v = (a^v_{ij}), a^v_{ij} = e^{jky_j \sin\theta_i}$ and the excitation of vertical subarrays is represented by $\boldsymbol{g}^v$. Here $\boldsymbol{g}^v$ is also indepedent of θ_i because each subarray is perpendicular to the elevation plane of the far-field pattern. Now the recovery of $\boldsymbol{g}^v$ is written by

$$\begin{aligned} &\min_{\boldsymbol{g}^v} \|\boldsymbol{g}^v\|_1 \\ &s.t. \ \|\boldsymbol{b}^v - \boldsymbol{A}^v \boldsymbol{g}^v\|_2 \le \epsilon. \end{aligned} \tag{9}$$

The failed elements are located in the columns which are decided by the corresponding nonzero components in $\boldsymbol{g}^v$. It can be seen in Fig. 3(b) that the columns of faults are marked by the red dashed rectangles.

After the rows and columns of failed elements are obtained, a smaller area with failed elements is determined, as shown in Fig. 3(c), which means it is closer to finding out the faults. Construct a new subarray with the rows and columns provided above and measure its far-field beam pattern, which is given by

$$\boldsymbol{b}^w = \boldsymbol{A}^w \boldsymbol{s}^w + \boldsymbol{n_e}, \tag{10}$$

where $\boldsymbol{A}^w$ denotes the radiation matrix of the new subarray and $\boldsymbol{s}^w$ represents the excitation of its elements. Hence the problem

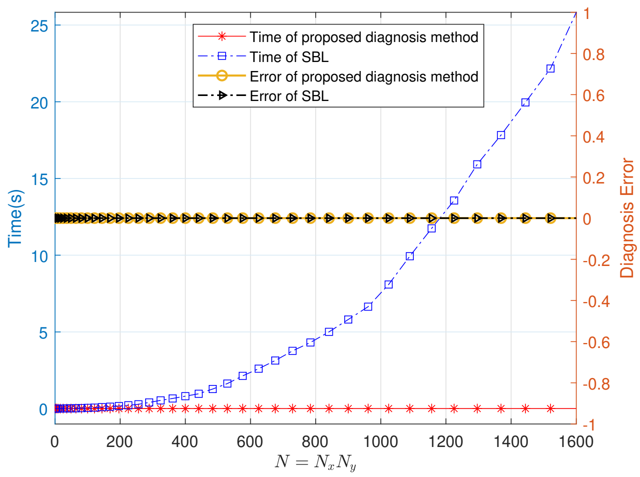


Fig. 4. Diagnosis performance vs. size of planar array

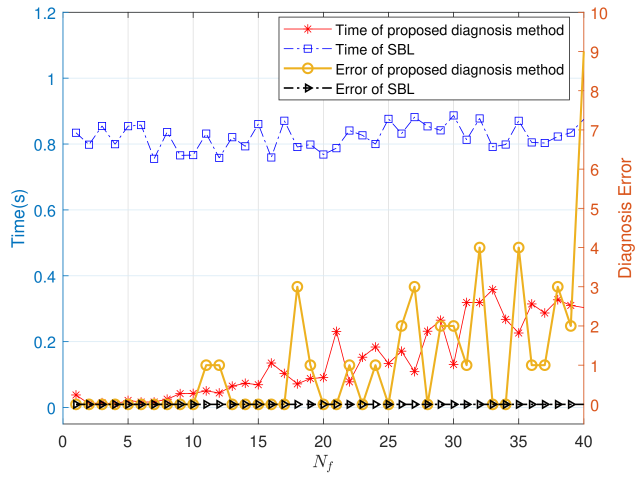


Fig. 5. Diagnosis performance vs. number of faults

of failure diagnosis of this subarray is solved by using sparse recovery in the following form:

$$\begin{aligned} &\min_{\boldsymbol{s}^w} \|\boldsymbol{s}^w\|_1 \\ &s.t.\ \|\boldsymbol{b}^w - \boldsymbol{A}^w \boldsymbol{s}^w\|_2 \le \epsilon. \end{aligned} \tag{11}$$

The approach mentioned above is the fast diagnosis framework of planar arrays. It converts the problem of finding out failed elements into the determinnation of the rectangular area that faults lie in. The procedure of the proposed diagnosis framework is summarized as follows.

1) Measure the elevation plane of the far-field pattern at the azimuth $0°$ $[b(\theta), \varphi = 0°]$, and find out the rows that failed subarrays are in.
2) Measure the elevation plane of the far-field pattern at the azimuth $90°$ $[b(\theta), \varphi = 90°]$, and find out the columns that failed subarrays are in.
3) Constructing a new planar subarray based on the intersection of rows and columns above, measure its 3D far-field pattern and diagnose the faulty elements of the new subarray.

In general, the proposed diagnosis framework decides the rectangular area that faulty elements lie in by just measuring a few points of the 3D far-field pattern of a planar array, and further determines the positions of faulty elements by measuring the field pattern of a new constructed subarray in this region. Through a series of steps, diagnosis time can be reduced largely. Moreover, it should be pointed out that the proposed fast diagnosis framework only provides a quick procedure for the diagnosis of planar arrays. It does not limit the specific diagnosis algorithms, so any algorithms of failure diagnosis can be used within the proposed fast diagnosis framework, including inverse Fourier transform, matrix method, MUSIC and compressed sensing, which only depend on the diagnosis model [5, 7, 19].

4 Simulation and performance analysis

According to the sparse diagnosis model, this section presents the simulation of two diagnosis methods from Sparse Bayesian Learning (SBL) and the proposed fast diagnosis framework based on SBL. First, similar to [7], the error of failure diagnosis is defined by the number of unsuccessful judgment of excitation, that is

$$N_{ERR} = \|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_1, \tag{12}$$

where $\boldsymbol{s}$ is an unknown 0-1 excitation vector and $\hat{\boldsymbol{s}}$ is a 0-1 vector of its estimate.

The first simulation is about diagnosis performance versus the size of a square planar array. Assuming that the operating frequency of an $N_x \times N_y$ square planar array is $f = 3$ GHz and the number of failed elements is $N_f = 1$, measure its far-field beam pattern on a spherical surface at $\theta = 0 : 0.5 : 90°$ and $\varphi = 0 : 6 : 360°$ with no Gaussian noise. Diagnosis results of the time and error with respect to the array size ($N_x = 1 : 40$, $N_x = N_y$), are shown in Fig. 4, where the left y-axis corresponds to the diagnosis time and the right y-axis corresponds to the diagnosis error. It illustrates that with the increase of array size, the proposed fast diagnosis method achieves the correct localization of failed elements with the error $N_{ERR} = 0$ and negligible diagnosis time. Although SBL achieves the same detection error $N_{ERR} = 0$, its diagnosis time increases exponentially with the array size. Given the trend in radar and telecommunications systems to increase the array sizes in millimetre-Wave and Terahertz applications, the fast diagnosis method will be an asset for array failure analysis.

Moreover, the behaviour of diagnosis versus the number of failed elements is also analysed. Keep the operating frequency and spherical sampling surface unchanged with no Gaussian noise. Diagnosis results of a planar array of size $N_x = N_y = 20$ versus the number of faults are shown in Fig. 5. It illustrates that SBL achieves the completely correct diagnosis while

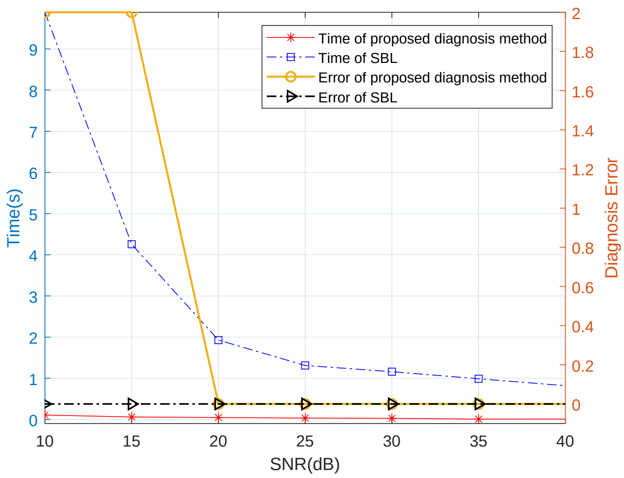


Fig. 6. Diagnosis performance vs. SNR

the proposed fast diagnosis method detects the faults successfully under the condition that the number of failed elements $N_f \leq 10$; for the time cost, SBL takes approximately 0.8 s while the proposed fast algorithm takes no more than 0.05 s when $N_f \leq 10$. At first glance, it seems that the diagnosis time spent by SBL is not very large compared to the proposed algorithm. But the diagnosis results are acquired under the condition of the small size of planar arrays. When a planar array consisting of an enormous amount of antenna elements requires testing in production, the diagnosis time performed by SBL is considerable, which is analysed in Fig. 4. It is important to note that there exists a limitation on the number of faults for the accurate diagnosis of the proposed method. When N_f increases, the assumption of sparse excitation is not guaranteed. To be more specific, N_f is large enough relative to the number of elements in the new constructed subarray.

What's more, diagnosis performance versus SNR (Signal-To-Noise Ratio) in Gaussian noise is also analysed. All parameters remain unchanged but $N_f = 1$ and SNR is tested from 10 to 40 dB in 5 dB steps. Fig. 6 shows the diagnosis error and time cost with respect to SNR in Gaussian noise. It can be seen that both SBL and the proposed fast diagnosis method achieve the accurate localization of faults when SNR is no less than 20 dB. In a diagnosis scenario, a high SNR will be guaranteed by the proximity of the test equipment to the array. Therefore, this method is viable for array diagnosis in real conditions. However, if SNR is less than 20 dB, for example, SNR equals to 15 dB, the proposed method misses 2 failed elements, $N_{ERR} = 2$. Regarding diagnosis time, SBL almostly achieves an exponential decrease to about 1 sec with the increase of SNR while the time of the proposed method is negligible.

5 Conclusion

In this paper, a fast diagnosis framework is proposed for the localization of failed elements based on the structure properties of planar arrays. The main methodological innovation in failure diagnosis is utilizing the physical structure of planar arrays to find out the rows and columns of faults. Hence the area of failed elements in a plane is reduced, which leads to the substantial reduction of diagnosis time if the size of planar arrays is large.

From a simulation point of view, the proposed fast diagnosis method reduces the diagnosis time against SBL in the following cases:

i. Given the number of failed elements $N_f = 1$ and the array size $N_x = N_y = 40$, the diagnosis time of the proposed method is negligible while the time of SBL is about 25 s.

ii. In the case of a 20×20 planar array with 10 failed elements, i.e., a failure rate of 2.5%, the diagnosis time of the proposed method is nearly 0.05 s while the time of SBL is about 0.8 s.

iii. Given SNR $= 20$ dB and a 20×20 planar array with the number of failed elements $N_f = 1$, the diagnosis time of the proposed method is approximately negligible while the time of SBL is about 2 s.

Since high SNR can be guaranteed by laying out absorbing materials to suppress noise in real conditions, the proposed fast diagnosis method is feasible in a test facility. It would be the case for a manufacturing plant to test planar arrays that were produced to see if there are any faults .

It is important to note that the diagnosis model in this paper is based on the least l_1-norm , so it should meet the requirements of the sparsity of exictation. For future works, other diagnosis methods without the hypothesis of sparse structure will be explored and exploited within the proposed failure diagnosis framework.

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