Dynamic Asymmetric Dependence and Portfolio Management in Cryptocurrency Markets

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Abstract

As a new form of digital asset based on blockchain technology, the cryptocurrency has received increasing attention from researchers and practitioners. However, less attention has been paid to their joint dynamics from the perspective of portfolio management. This paper investigates the dependence dynamics across four major cryptocurrencies and their economic importance in portfolio management using the data from January 2014 to June 2020. Our empirical analysis shows that significant economic gains can be obtained from modelling dynamic asymmetric dependence among cryptocurrencies. We show that our results are robust to the period of the recent market fluctuations caused by COVID-19.

Keywords: Cryptocurrency, dependence structure, portfolio management, COVID-19

JEL Codes: G15, F21, F37

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1. Introduction

Cryptocurrencies are digital currencies not backed by real assets or tangible securities but instead are based on the security of an algorithm. The cryptocurrency prices are highly volatile compared with the traditional assets. With the rapid growth and increasing importance of the cryptocurrency market, a growing number of studies have been conducted in recent years. This paper is motivated by three streams of literature on the cryptocurrency market. The first stream of literature focuses on the market integration in the cryptocurrency market, such as (Bouri et al., 2021; Ji et al., 2019; Corbet et al., 2020c). Limited attention has been paid to the non-linear dependence, especially tail dependence among the various cryptocurrencies. Thus, one primary object of this paper is to model the joint dynamics among cryptocurrencies.

This paper is also motivated by another stream of literature focusing on the portfolio management in the cryptocurrency market, including investment strategies in cryptocurrency (Corbet et al., 2019a), and its diversification benefits (Corbet et al., 2018a,b; Guesmi et al., 2019; Shahzad et al., 2019; Bouri et al., 2020). Therefore, another object of this paper is to apply a copula approach to model the joint distribution of cryptocurrencies and then use constant relative risk aversion (CRRA) utility functions to construct portfolios of cryptocurrencies.

The global spread of COVID-19 has had severe impacts on financial markets worldwide. The cryptocurrency market has attracted increasing attention due to its hedging property. However, the contagion between the financial market and cryptocurrency market is confirmed when suffering the shocks of COVID-19 (Conlon et al., 2020; Corbet et al., 2020a,b; Vidal-Tomás, 2021). Although the cryptocurrency market is not connected with the real economy, the price of cryptocurrencies is still related to the behaviour of the traders, such as the panic and fear of investors (Vidal-Tomás, 2021). Motivated by this stream of literature, this paper aims to conduct a robustness test by evaluating the performance of the cryptocurrency portfolios constructed by our copula models during the COVID-19 crisis.

Compared to the above literature, we contribute to the literature of cryptocurrency by modelling the tail dependence and joint distribution of the cryptocurrencies, which could help the investors to avoid the underestimation of the risk. It is notable that our paper confirms the existence of non-linear and asymmetric dependence between cryptocurrency pairs. In addition, we propose new strategies in the cryptocurrency market, which can provide outstanding and robust performances, especially during the COVID-19 crisis.

This paper is organized as follows. In Section 2, we discuss the data and methodology to model the tail dependence and build the portfolios. Section 3 reports the portfolio investment results in different scenarios, and Section 4 concludes.

2. Data and methodology

2.1. Data and preliminary analysis

We choose the four most long-standing, liquid and large market cap cryptocurrencies, namely Bitcoin, Dash, Litecoin, and XRP, which contribute almost 75% to the total market capitalization.[1] Following Guesmi et al. (2019), we collect the daily data from CoinMarketCap over the period from February 14, 2014 to June 26, 2020, totalling 2,325 daily observations. The logarithmic return of cryptocurrency i at day t in the long position is given by $r_{i,t} = \ln(p_{i,t+1}/p_{i,t}) \times 100$, where $p_{i,t}$ denotes the close price of cryptocurrency i at day t.

[INSERT TABLE 1 ABOUT HERE]

We find that four cryptocurrencies have the non-normality and auto-correlation in Table 1. Thus, we apply the AR-GARCH model in the next subsection to model the return dynamics. The correlation matrix shows that all four cryptocurrencies are significantly correlated at 1% level, which coincides with findings of Bouri et al. (2021). It is interesting to investigate the tail risk connectedness in the cryptocurrency markets, because their performances during extreme events are of great importance for investors who seek to diversify their cryptocurrency portfolios.

2.2. Return dynamics

To capture the return dynamics of cryptocurrencies, we allow each return series to have time-varying conditional mean $\mu_{i,t}$ and variance $\sigma^2_{i,t}$, and we also assume that the standardized returns $z_{i,t} = (r_{i,t} - \mu_{i,t})/\sigma_{i,t}$ are identically distributed. We apply the AR model and the GJR-GARCH(1,1,1) model to the capture the return dynamics. Given the large values of skewness and kurtosis of cryptocurrency returns, we assume that the residual follows the skewed t distribution of Hansen (1994). Our marginal model specifications are shown in Appendix A.

[INSERT TABLE 2 ABOUT HERE]

Table 2 reports parameter estimates for AR-GARCH model. The leverage effect parameter β_i for Bitcoin is significantly positive, suggesting the existence of leverage effects. This indicates that negative shocks have a stronger impact than positive shocks. The estimates of skewness parameters show that filtered residuals of Bitcoin are negatively skewed, while others are positively skewed, which is consistent with the results in Table 1. Our results suggest that the skewed t distribution is suitable for modelling filtered residuals. Thus, the diagnosis provides evidences that our marginal distribution models are well-specified.

[1] Notably, Bitcoin trading takes the dominant role since Dash, Litecoin, and XRP only contribute 6% to the whole market capitalization.

2.3. Asymmetric dependence

According to Table 1, the four cryptocurrencies are significantly correlated. However, we are more interested in their co-movements in the tail distributions, since the tail dependence which is not captured by the linear correlation measure may lead to the underestimation of the portfolio risk.

[INSERT TABLE 3 AND FIGURE 1 ABOUT HERE]

First, we follow Christoffersen and Langlois (2013) to characterise the correlations between two variables in the joint lower and upper tails. The threshold correlations are shown in Figure 1. We compare the bivariate normal distribution correlation (dashed line) and threshold correlation (solid line) in different quantiles for six cryptocurrency pairs. All the solid lines show considerable deviations from the dashed lines when the threshold is less than 0.5, indicating that cryptocurrencies are more correlated in the lower tail. Then, we test the asymmetric tail dependence of the cryptocurrencies by the method described in Patton (2013). The asymmetric dependence between four cryptocurrencies is shown in Table 3. This table reports the lower and upper tail dependence coefficients from the Student's t copula. The striking higher lower tail dependence confirms the finding from the threshold correlation. Both threshold correlation and tail dependence highlight the significance of the nonlinear dependence and multivariate asymmetry of cryptocurrencies.

2.4. Dependence structure of cryptocurrencies

After modelling marginal distributions, we focus on the cryptocurrencies dependence and estimate the parameters of copula models. We consider three kinds of widely used copulas with different types of tail dependence, namely the Normal copula, the Student's t copula, and the skewed t copula of Demarta and McNeil (2005). More details about the skewed t copula can be found in Appendix B. All the copulas are estimated using maximum likelihood estimation.

[INSERT FIGURE 2 ABOUT HERE]

Following Christoffersen and Langlois (2013), we rely on the dynamic conditional correlation (DCC) model of Engle (2002) to estimate the dynamic copula correlations. We estimate the dynamic correlation for copulas using the data from February 14, 2014 to June 26, 2020. We find that the correlations between cryptocurrency pairs vary significantly through time (see Figure 2). The cryptocurrencies become more correlated with each other during the period from 2018 to 2020. The notable variations in Figure 2 highlight the importance of using the dynamic copula model to capture the evaluations of dependence between cryptocurrencies.

2.5. Portfolio construction

We start constructing investment portfolios by estimating the AR-GARCH model on each cryptocurrency using the first 500-day returns[2], and then estimate their dynamic dependence using various copula models. We re-estimate the parameters of AR-GARCH and copula models quarterly with the expanding window following Christoffersen and Langlois (2013). Then, we can simulate the next term returns and obtain the optimal weighting vector w_t at time t, and re-balance cryptocurrency portfolios on a daily basis. In addition, we consider the simple diversified portfolios (the naïve 1/N portfolio) as the benchmark.[3] Together, our horse race contains four optimal CRRA portfolios and the naïve 1/N portfolio. The optimal w_t could be obtained by maximizing the CRRA utility function of the investor:

$$U(\gamma) \quad = \quad (1-\gamma)^{-1}\left(P_0\left(1+w_t^{\top}r_{t+1}\right)^{1-\gamma}\right), \tag{1}$$

where P_0 is the initial wealth which we set \$1 here, r_t is the vector of cryptocurrency returns at time t, w_t is the weighting vector, and γ denotes the degree of relative risk aversion (RRA). We consider three different levels of RRA: $\gamma = 3, 7, 10$. The weighting vector for each time t is obtained by maximizing the expected utility function given different assumptions for the cryptocurrencies joint distribution.

$$\begin{aligned} w_t^* &\equiv \underset{w\in W}{\arg\max}\, E_{\hat{f}_{t+1}}\left(U\left(1+w_t^{\top}r_{t+1}\right)\right) \\ &= \underset{w\in W}{\arg\max} \int \frac{\left(1+w_t^{\top}r_{t+1}\right)^{1-\gamma}}{1-\gamma} f_{t+1}\left(r_{t+1}\right) dr_{t+1}, \end{aligned} \tag{2}$$

where $f_{t+1}(r_{t+1})$ denotes the joint distribution of four cryptocurrencies. We assume that each cryptocurrency i at time t has weight $w_{i,t} \in [0, 1]$, and the total weight of four cryptocurrencies invested is one. Note that the short-selling is assumed to be prohibited which means all weights are positive.

[2] The 500-day is a reasonably long sample period for the robust estimation of copula-based models. We also tested 1-year (365 days) and 2-year (730 days) returns as alternative estimation periods and obtained similar results. For parsimonious reasons, we only report our main results based on the 500-day data in this paper. The results for the 1- and 2-year periods are available upon request.

[3] Note that we also set initial values of the w_t as 25% in the optimization of CRRA utility functions.

3. Results

3.1. Investment results

Table 4 reports the out-of-sample performances of different portfolios over the period July 2, 2015 to June 26, 2020. We use the naïve 1/N portfolio as the benchmark, compared with the portfolios based on the multivariate normal distribution, the Normal copula, the Student's t copula, and the skewed t copula. The three panels in Table 4 show the investment results with different levels of risk aversion ($\gamma = 3, 7, 10$). As Equation 2 shows, a higher level of risk aversion would lead to less variations in the estimation of the weight for each term t. CDB could measure the diversification benefits, which takes into account higher-order moments and non-linear dependence (More details on CBD can be found in Appendix C.). In our case, the values of daily CDB are calculated with a one-year rolling window.

[INSERT TABLE 4 AND FIGURE 3 ABOUT HERE]

The left panel of Table 4 reports the results of the performance from the different portfolios without transaction costs. The Sharpe ratios of all the CRRA portfolios are significantly higher than the naïve 1/N portfolio. The copula model portfolios yield higher Sharpe ratios than the ones of the multivariate normal distribution model, which indicates the economic importance of acknowledging nonlinear dependence. The CRRA portfolios are rebalanced on a daily basis, which increase the average turnover and portfolio volatility. Although the values of CDB for copula-based CRRA portfolios are lower than that of the naïve 1/N portfolios, all the CRRA portfolios could still manage a high CDB (close to 0.9).[4] The skewed t copula model has the highest CDB among the optimal portfolios via in Panel A.

More trading (high turnover) could increase the transaction costs and reduce the profits of portfolios. Thus, to check the robustness of our results, we further compare the performance of all the portfolios after taking into account transaction costs. Following the Lintilhac and Tourin (2017), we use bid-ask spread of BTC/USD exchange rate from Bitstamp to represent the transaction costs of cryptocurrencies.[5] The transaction costs, following Barroso and Santa-Clara (2015), can be calculated from the following equation:

$$tc_{i,t} = \frac{F^{ask}_{i,t,t+1} - F^{bid}_{i,t,t+1}}{F^{ask}_{i,t,t+1} + F^{bid}_{i,t,t+1}}, \tag{3}$$

where $tc_{i,t}$ represents the transaction costs of cryptocurrency i at time t. $F^{ask}_{i,t,t+1}$ and $F^{bid}_{i,t,t+1}$ denote the forward ask and bid price for cryptocurrency i at time t, respectively.

The right panel of Table 4 presents the performance of portfolios that take into account transaction costs. The CRRA portfolios experience a decrease in the annualized return and the Sharpe ratio, but they still significantly outperform the naïve 1/N portfolio. Next, we plot the portfolio performance as well as the performance of four cryptocurrencies using the mean-variance framework in the upper panel of Figure 3. The CRRA portfolios are significantly benefited by modelling their dependence. The skewed t copula model (with $\gamma = 3$) yields the highest return among all the candidates.

Overall, we find that the optimal CRRA portfolios clearly outperform the benchmark. In addition, the CRRA portfolios based on copula models can provide better performance than the ones formed by the multivariate normal distribution, which indicates the evidence of the non-linear correlation. The skewed t copula model yields the best performance in terms of the Sharpe ratio, which highlights the importance of modelling the asymmetric dependence among cryptocurrencies.

3.2. Portfolio performance during the COVID-19 pandemic

We further check whether the results are robust to sample selection by applying the above framework to the period of the COVID-19 pandemic. Our sample of the COVID-19 crisis is from February 12, 2020 to June 26, 2020, where the starting point is the day when the US reported the first COVID-19 case (World-Health-Organization, 2020). Since our dataset only includes a short period of the COVID-19 pandemic, we change the frequency of the models' estimation from quarterly to monthly.

[INSERT TABLE 5 ABOUT HERE]

Table 5 reports the portfolio losses during the period of the COVID-19 pandemic. The CRRA portfolios have smaller losses than the naïve 1/N portfolio, even considering transaction costs. The skewed t copula model shows the best performance in the COVID-19 with the minimum loss at the each level of risk aversion. The lower panel of

[4] The optimal portfolios in this paper are obtained by maximizing an investor's CRRA utility. Christoffersen and Langlois (2013) suggest that CRRA functions are locally mean-variance preferences. Thus, the CRRA portfolios could yield significantly higher Sharpe ratios than the naïve 1/N portfolio. The CDB is a dynamic measure of diversification benefits that takes into account higher-order moments and nonlinear dependence. Therefore, we mainly use the CDB to compare the performance of copula-based portfolios with the portfolios based on the multivariate normal distribution.

[5] We use the same transaction costs for different cryptocurrencies for simplicity following literature (Brauneis and Mestel, 2019; Platanakis and Urquhart, 2019). The bid and ask prices are from Quandl over the period April 15, 2014 to June 26, 2020.

Figure 3 shows that the CRRA portfolios consistently outperform the naïve 1/N portfolio in this period. Note that the CRRA portfolios provide higher CDB than the naïve 1/N portfolio during the pandemic period. Interestingly, the CRRA portfolios yield higher returns during the period of market upturn. In comparison, these portfolios turn to avoid loss and provides better risk management during the period of market downturn.

4. Conclusion

This paper investigates the joint dependence across cryptocurrencies and explores the economic importance of acknowledging their dependence in portfolio management. First, we confirm the existence of non-linear and asymmetric dependence between cryptocurrencies. Second, we model the return dynamics and the joint distribution of cryptocurrencies using AR-GRACH and various copulas. Finally, we use the dependence structure to identify the optimal cryptocurrency portfolios in real-time investment.

Our analysis reveals several interesting findings. First, we verify the existence of non-linear and asymmetric dependence between cryptocurrency pairs, which may influence not only the estimation of portfolio risk, but also the diversification benefits. Second, we show that significant economic value can be obtained by modelling the asymmetric and dynamic dependence between cryptocurrencies in real-time investing. Our empirical results suggest that the optimal portfolios which consider the dynamic asymmetric dependence perform reasonably better than benchmark portfolios. We show that our results are robust during the period of the COVID-19 pandemic.

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Table 1: Descriptive Statistics of Daily Cryptocurrencies Return

Notes: This table presents summary statistics and other results for daily returns of four cryptocurrencies over the period of February 14, 2014 to June 26, 2020, which corresponds to a sample of 2,325 observations for each series. The top panel presents summary statistics; the second panel presents diagnostic statistics; the third panel presents the correlation matrix for four cryptocurrencies. Significance at the 5% and 1% levels is denoted by * and **.

	Bitcoin	Dash	Litecoin	XRP
Mean	0.0011	0.0023	0.0004	0.0010
Medium	0.1387	-0.1743	-0.0087	-0.2836
Std.	0.0396	0.0758	0.0571	0.0655
Skewness	-0.9045	2.8575	0.3706	2.2667
Kurtosis	15.6859	46.8488	16.5970	43.3267
Diagnostics				
JB test	0.0000	0.0000	0.0000	0.0000
LB Q	0.0614	0.0006	0.0005	0.0000
LM	0.0000	0.0000	0.0000	0.0000
First-order	-0.0214	-0.0013	-0.0111	0.0158
Second-order	-0.0078	-0.0563*	-0.0283	0.0679**
Cross Correlations				
Bitcoin	1.0000			
Dash	0.4365**	1.0000		
Litecoin	0.6548**	0.3794**	1.0000	
XRP	0.3692**	0.2097**	0.3860**	1.0000

Table 2: Estimation for Univariate Distribution

Notes: This table reports parameter estimations and model diagnostics for the AR-GARCH model with normal shocks. Panel A reports parameter estimates from AR(0) and AR(2) models for the conditional mean; Panel B reports parameter estimates from GJR-GARCH(1,1) models for the conditional variance; Panel C reports parameter estimates from skew t models for the distribution of the standardized residuals; Panel D presents simulation-based p-values from two Kolmogorov-Smirnov and Cramervon Mises goodness-of-fit tests for the models of the conditional marginal distributions. We estimate all parameters using the sample from February 14, 2014 to June 26, 2020, which corresponds to a sample of 2,325 observations for each series. The values in parenthesis represent the standard errors of the parameters.

	Bitcoin	Dash	Litecoin	XRP
Panel A: Conditional mean				
ϕ_0	0.0011	0.0023	0.0004	0.0010
	(0.0008)	(0.0015)	(0.0011)	(0.0013)
ϕ_1		0.0019		
		(0.0206)		
ϕ_2		-0.0563		
		(0.0206)		
Panel B: Conditional variance				
ω	0.0001	0.0002	0.0001	0.0003
	(0.0000)	(0.0001)	(0.0000)	(0.0001)
α	0.1126	0.2282	0.0896	0.2387
	(0.0106)	(0.0114)	(0.0073)	(0.0209)
γ	0.0722	-0.0234	-0.0122	-0.0183
	(0.0092)	(0.0124)	(0.0065)	(0.0016)
β	0.8172	0.7589	0.8661	0.7110
	(0.0098)	(0.0089)	(0.0086)	(0.0313)
Log-likelihood	4429.78	3299.65	3585.26	3632.31
Variance persistence	0.876	0.782	0.933	0.745
Panel C: Skew t density				
υ	2.9485	3.2793	2.6852	2.8018
κ	-0.0220	0.1203	0.0417	0.0768
Panel D: GoF tests				
KS p-value	0.126	0.123	0.129	0.122
CvM p-value	0.334	0.297	0.346	0.338

Table 3: Tests of Asymmetric Tail Dependence

Notes: This table presents the coefficient of lower tail dependence (“Lower”), the coefficient of upper tail dependence (“Upper”) and their difference for each pair of cryptocurrencies. We estimate the tail dependence from Student’s t copula using the sample from February 14, 2014 to June 26, 2020, which correspond to a sample of 2,325 observations for each series. We use both parametric estimation methods developed in Patton (2012). The p-values of testing a zero difference are computed by a bootstrapping with 1000 replications.

	Lower	Upper	Difference	p-value
Bitcoin-Dash	0.2851	0.0686	0.2165	0.0231
Bitcoin-Litecoin	0.5021	0.1726	0.3295	0.0174
Bitcoin-XRP	0.2723	0.0469	0.2254	0.0000
Dash-Litecoin	0.2899	0.0635	0.2264	0.0390
Dash-XRP	0.2295	0.0200	0.2095	0.0000
Litecoin-XRP	0.2878	0.0397	0.2481	0.0690

Table 4: Out-of-Sample Portfolio Performance

Notes: This table reports the investment results for five selected portfolios, including 1/N (naïve 1/N portfolio), Linear (Multivariate normal distribution), Norm-Cop (Normal copula), T-Cop (Student's t copula), and Skew-Cop (Skewed t copula) from July 2, 2015 to June 26, 2020. We present the portfolio's annualized return (mean), volatility, annual turnover, Sharpe ratio and CDB's mean, volatility and median for each portfolio. Panels A, B and C present the investment results for relative risk aversion (RRA) coefficients $\gamma = 3$, 7 and 10, respectively. The left panel shows the investment results without transaction costs, while the right panel reports the portfolio performance with transaction costs.

	Portfolio without Transaction Costs					Portfolio with Transaction Costs				
	1/N	Linear	Norm-Cop	T-Cop	Skew-Cop	1/N	Linear	Norm-Cop	T-Cop	Skew-Cop
Panel A. γ=3										
Annualized mean	0.5969	1.1447	1.1878	1.1564	1.2168	0.5969	1.0391	1.0841	1.0588	1.1162
Annualized volatility(%)	81.2020	104.3204	103.7222	101.3425	101.2862	81.2020	104.3590	103.7555	101.3922	101.3596
Average turnover(%)	0.0000	13.5723	13.0477	12.3480	12.7515	0.0000	13.5723	13.0477	12.3480	12.7515
Sharpe ratio	0.7351	1.0973	1.1451	1.1411	1.2013	0.7351	0.9957	1.0448	1.0443	1.1012
CDB mean	0.9122	0.8789	0.8775	0.8804	0.8840	0.9122	0.8786	0.8774	0.8800	0.8837
CDB volatility	0.0135	0.0376	0.0366	0.0360	0.0377	0.0135	0.0376	0.0367	0.0362	0.0377
CDB median	0.9120	0.8888	0.8874	0.8899	0.8869	0.9120	0.8887	0.8887	0.8901	0.8864
Panel B. γ=7										
Annualized mean	0.5969	1.0290	1.0553	1.0504	1.1040	0.5969	0.9520	0.9817	0.9789	1.0188
Annualized volatility(%)	81.2020	96.7627	96.5812	89.3713	104.7343	81.2020	96.8320	96.6391	89.3787	104.7985
Average turnover(%)	0.0000	9.5613	8.9618	9.2267	10.9223	0.0000	9.5613	8.9618	9.2267	10.9223
Sharpe ratio	0.7351	1.0634	1.0927	1.1754	1.0541	0.7351	0.9832	1.0158	1.0952	0.9722
CDB mean	0.9122	0.8867	0.8880	0.9076	0.8752	0.9122	0.8864	0.8878	0.9074	0.8752
CDB volatility	0.0135	0.0337	0.0340	0.0225	0.0359	0.0135	0.0337	0.0340	0.0225	0.0361
CDB median	0.9120	0.8851	0.8835	0.9113	0.8802	0.9120	0.8856	0.8834	0.9115	0.8797
Panel C. γ=10										
Annualized mean	0.5969	0.9473	0.9684	0.9056	1.0081	0.5969	0.8826	0.9063	0.8414	0.9423
Annualized volatility(%)	81.2020	93.7227	93.2138	90.0530	93.2494	81.2020	93.7977	93.2816	90.1208	93.3225
Average turnover(%)	0.0000	7.8804	7.4504	8.4727	8.3360	0.0000	7.8804	7.4504	8.4727	8.3360
Sharpe ratio	0.7351	1.0107	1.0389	1.0056	1.0810	0.7351	0.9410	0.9716	0.9336	1.0098
CDB mean	0.9122	0.8906	0.8916	0.8974	0.8950	0.9122	0.8902	0.8912	0.8972	0.8947
CDB volatility	0.0135	0.0333	0.0336	0.0307	0.0315	0.0135	0.0336	0.0338	0.0309	0.0316
CDB median	0.9120	0.8891	0.8851	0.8901	0.8950	0.9120	0.8878	0.8839	0.8897	0.8946

Table 5: Out-of-Sample Portfolio Performance during the COVID-19 Pandemic

Notes: This table reports the investment results for five selected portfolios during the COVID-19 pandemic from February 12, 2020 to June 26, 2020. Specifically, these portfolios include 1/N (naïve 1/N portfolio), Linear (multivariate normal distribution), Norm-Cop (Normal copula), T-Cop (Student's t copula), and Skew-Cop (Skewed t copula). We present the portfolio's annualized return (mean), volatility, annual turnover, Sharpe ratio and CDB's mean, volatility and median for each portfolio. Panels A, B and C present the investment results for relative risk aversion (RRA) coefficients $\gamma = 3$, 7 and 10, respectively. The left panel shows the investment results without transaction costs, while the right panel reports the portfolio performance with transaction costs.

	Portfolio without Transaction Costs					Portfolio with Transaction Costs				
	1/N	Linear	Norm-Cop	T-Cop	Skew-Cop	1/N	Linear	Norm-Cop	T-Cop	Skew-Cop
Panel A. γ=3										
Annualized mean	-1.1439	-0.2725	-0.2693	-0.2676	-0.1942	-1.1439	-0.3327	-0.3296	-0.3426	-0.2788
Annualized volatility(%)	105.1813	106.5975	106.7765	107.0015	107.3354	105.1813	106.6086	106.7820	106.9472	107.3317
Average turnover(%)	0.0000	8.0184	8.0818	9.8752	11.1877	0.0000	8.0184	8.0818	9.8752	11.1877
Sharpe ratio	-1.0875	-0.2556	-0.2522	-0.2501	-0.1809	-1.0875	-0.3120	-0.3086	-0.3203	-0.2598
CDB mean	0.8666	0.8710	0.8713	0.8715	0.8653	0.8666	0.8720	0.8724	0.8725	0.8658
CDB volatility	0.0290	0.0348	0.0344	0.0330	0.0247	0.0290	0.0338	0.0333	0.0321	0.0239
CDB median	0.8556	0.8587	0.8592	0.8600	0.8587	0.8556	0.8604	0.8610	0.8616	0.8595
Panel B. γ=7										
Annualized mean	-1.1439	-0.3621	-0.3578	-0.3508	-0.3159	-1.1439	-0.4030	-0.3988	-0.4271	-0.3884
Annualized volatility(%)	105.1813	106.1838	106.1648	106.5008	105.5035	105.1813	106.1897	106.1696	106.5371	105.4865
Average turnover(%)	0.0000	5.5988	5.6469	10.1310	9.2874	0.0000	5.5988	5.6469	10.1310	9.2874
Sharpe ratio	-1.0875	-0.3410	-0.3370	-0.3294	-0.2994	-1.0875	-0.3795	-0.3756	-0.4009	-0.3682
CDB mean	0.8666	0.8712	0.8713	0.8684	0.8675	0.8666	0.8725	0.8726	0.8696	0.8686
CDB volatility	0.0290	0.0339	0.0339	0.0335	0.0320	0.0290	0.0329	0.0330	0.0332	0.0311
CDB median	0.8556	0.8594	0.8595	0.8579	0.8573	0.8556	0.8612	0.8613	0.8594	0.8589
Panel C. γ=10										
Annualized mean	-1.1439	-0.4343	-0.4296	-0.4274	-0.3428	-1.1439	-0.4711	-0.4661	-0.5093	-0.4072
Annualized volatility(%)	105.1813	105.3945	105.3606	103.0134	104.4005	105.1813	105.4089	105.3762	103.0101	104.3908
Average turnover(%)	0.0000	4.9072	4.8529	11.9955	9.2869	0.0000	4.9072	4.8529	11.9955	9.2869
Sharpe ratio	-1.0875	-0.4121	-0.4077	-0.4149	-0.3283	-1.0875	-0.4470	-0.4423	-0.4944	-0.3900
CDB mean	0.8666	0.8709	0.8710	0.8716	0.8723	0.8666	0.8722	0.8723	0.8724	0.8729
CDB volatility	0.0290	0.0333	0.0332	0.0332	0.0331	0.0290	0.0323	0.0323	0.0325	0.0326
CDB median	0.8556	0.8593	0.8595	0.8600	0.8606	0.8556	0.8612	0.8614	0.8613	0.8615

Figure 1: Threshold Correlations Between Cryptocurrencies

Notes: This figure presents threshold correlations between four cryptocurrencies. Our sample consists of daily returns from February 14, 2014 to June 26, 2020. The continuous line represents the correlations when both cryptocurrency returns are below (above) a threshold. The dashed line represents the threshold correlation implied by a bivariate normal distribution using the linear correlation coefficient between two cryptocurrencies.

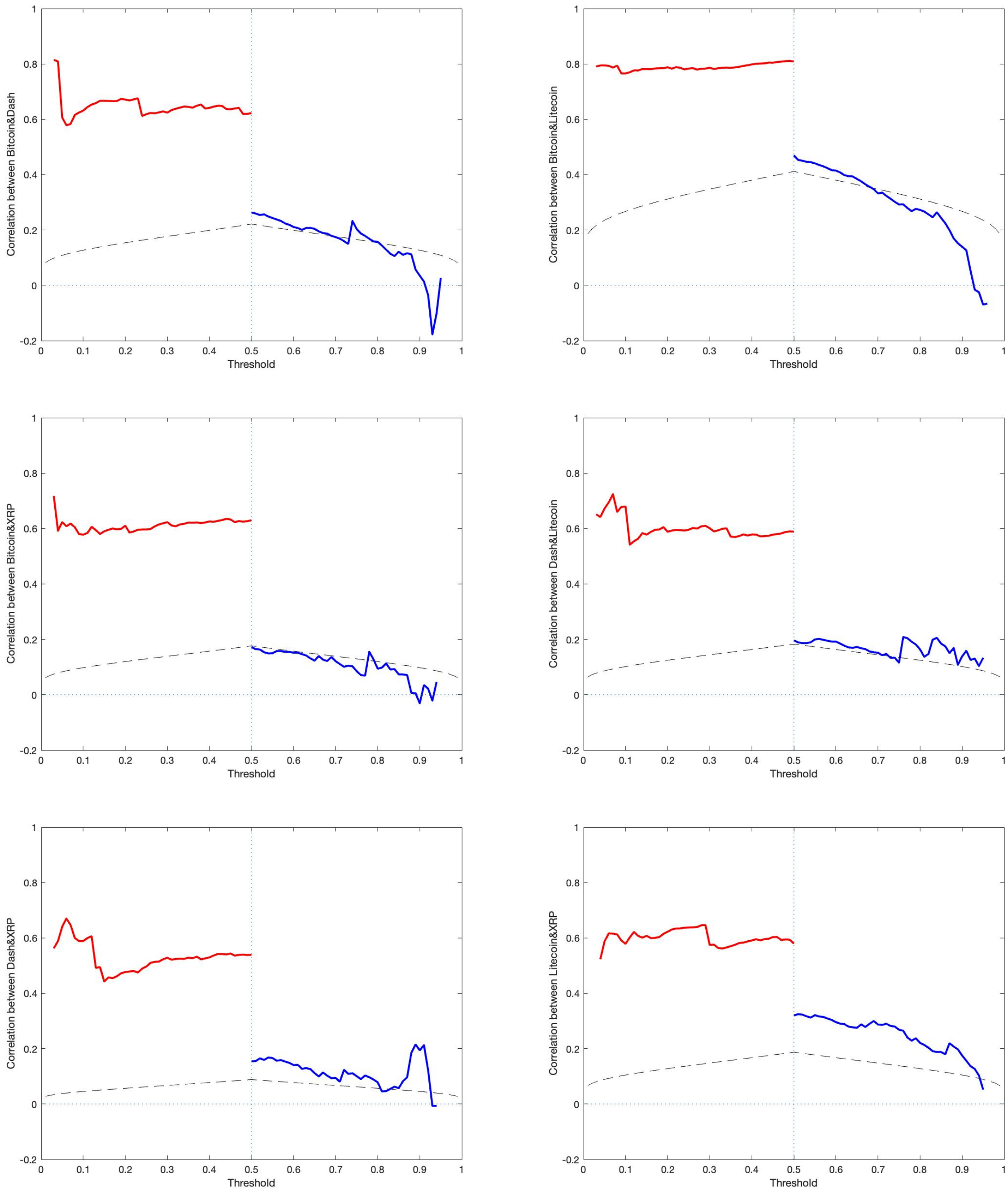


Figure 2: Dynamic Correlations Implied by the skewed t Copula

Notes: This figure presents dynamic correlations between four cryptocurrencies. The correlations are implied in the skewed t copula model on residuals of cryptocurrencies from the AR-GARCH model. Our sample consists of daily returns from February 14, 2014 to June 26, 2020.

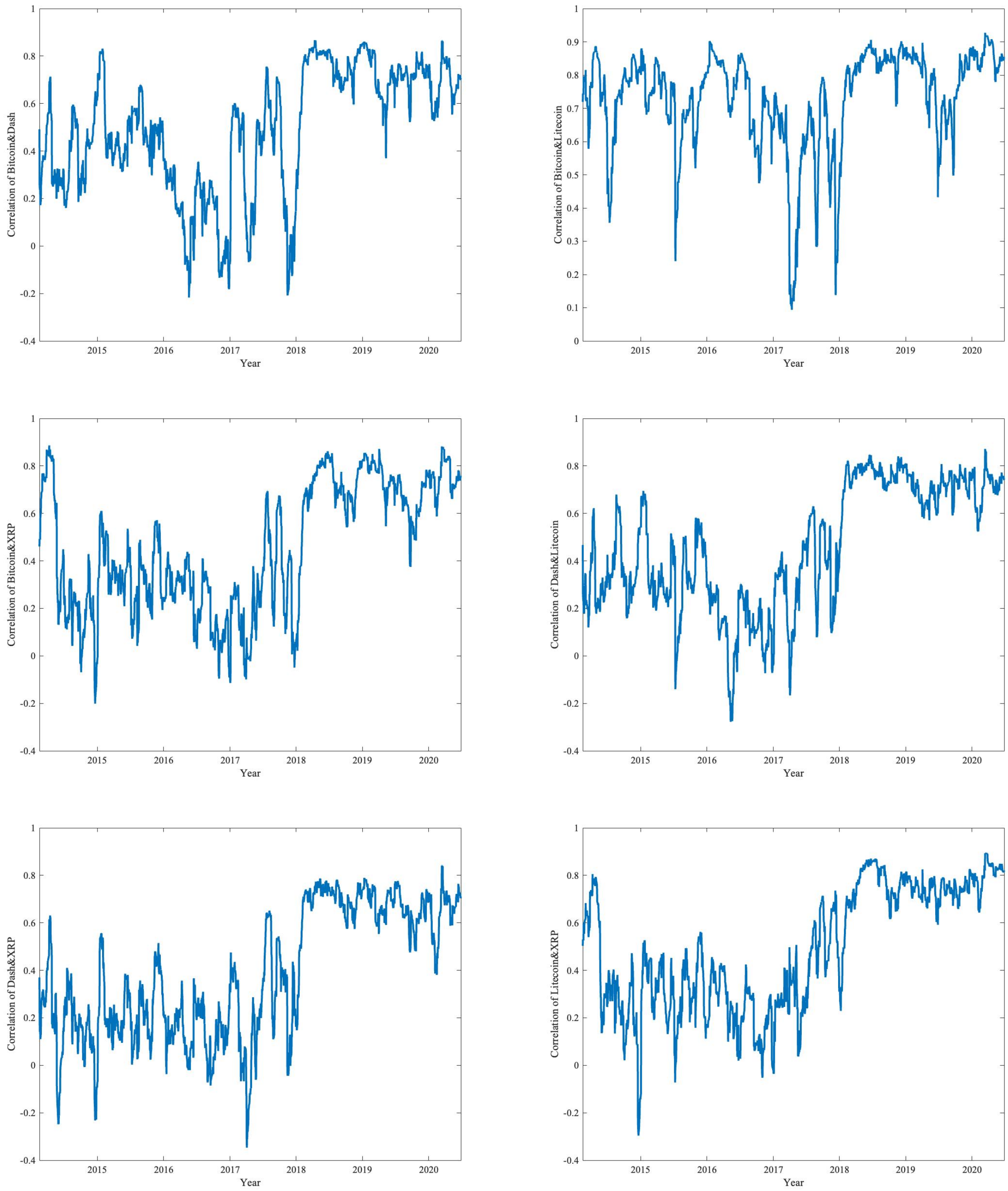
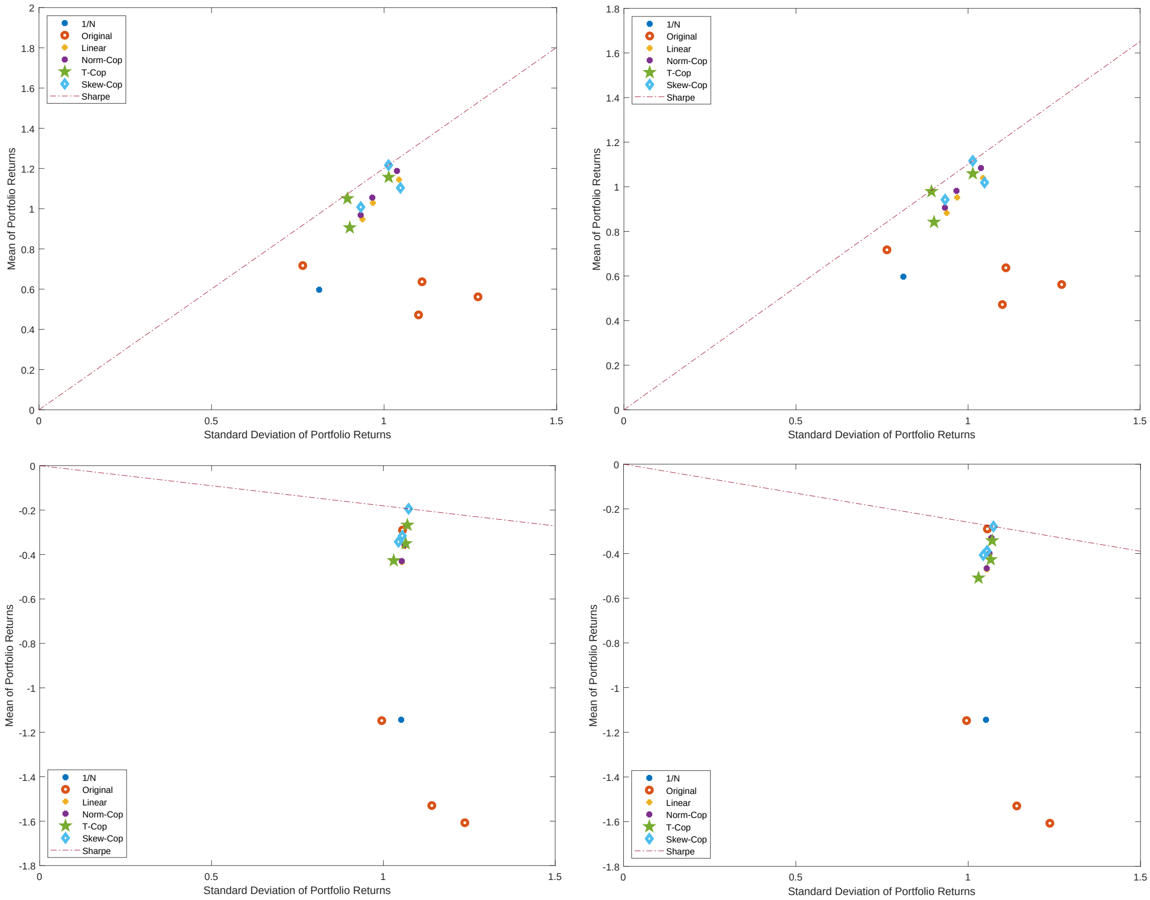


Figure 3: Portfolio Performance

Notes: This figure presents the mean (annualized return) and standard deviation (annualized volatility) of different portfolios. Our sample consists of daily returns from July 2, 2015, to June 26, 2020. The red dashed line is a straight line with the slope of the highest Sharpe ratio among all portfolios and zero intercept. Overall, there are six styles of points in this figure, representing the risk-return performance of 1/N (naïve 1/N portfolio), Original (four cryptocurrencies), Linear (Normal distribution with linear correlation), Norm-Cop (Normal copula), T-Cop (Student's t copula) and Skew-Cop (Skewed t copula) portfolios. The upper panel shows the portfolio performance with the full sample, while the lower panel reports the performance with the data during the COVID-19 pandemic. The left panel presents the performance without the transaction cost. The right panel plots the performance with the transaction cost.



Appendix

A. AR-GARCH model

We fit an AR model to the conditional mean

$$r_{i,t} = \phi_{0,i} + \sum_{k=1}^{p} \phi_{i,k} r_{i,t-1} + \varepsilon_{i,t}, \text{ where } \varepsilon_{i,t} = \sigma_{i,t} z_{i,t}, \tag{.1}$$

where $\phi_{0,i}$ denotes the constant term, $\phi_{1,i}$ denotes the coefficient of AR(1), and $\epsilon_{i,t}$ denotes the residuals of AR process. We employ the GJR-GARCH(1,1,1) model to capture volatility persistence, heteroskedasticity and the leverage effect:

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \gamma_i \varepsilon_{i,t-1}^2 I_{i,t-1}, \tag{.2}$$

where $I_{i,t-1}$ captures the leverage effect. $I_{i,t-1} = 1$ if $\varepsilon_{i,t-1} < 0$, and $I_{i,t-1} = 0$ if $\varepsilon_{i,t-1} \geq 0$. β_i denotes the GARCH parameter while α_i denotes the ARCH parameter.

B. Copula model

The skewed t copula model is shown below. The multivariate probability density function (PDF) of the multivariate skewed t distribution is given by Demarta and McNeil (2005):

$$f_t(r; v, \lambda, \Psi) = \frac{2^{\frac{2-(v+N)}{2}} K_{\frac{v+N}{2}} \left(\sqrt{(v + z^{*\top} \Psi^{-1} z^*) \lambda^\top \Psi^{-1} \lambda} \right) e^{z^{*\top} \Psi^{-1} \lambda}}{\Gamma\left(\frac{v}{2}\right) (\pi v)^{\frac{N}{2}} \mid \Psi \mid^{\frac{1}{2}} \left(\sqrt{(v + z^{*\top} \Psi^{-1} z^*) \lambda^\top \Psi^{-1} \lambda} \right)^{-\frac{v+N}{2}} \left(1 + \frac{z^{*\top} \Psi^{-1} z^*}{v} \right)^{\frac{v+N}{2}}} \tag{.3}$$

where v is the degree of freedom, Ψ denotes the correlation matrix, λ is the skewness parameters. N is the dimension of data. z^* denotes the standardized residuals from the AR-GARCH model.

Given the PDF of the univariate and multivariate skewed t distributions, the PDF of the skewed t copula can be derived as follows:

$$c_t(\eta;\lambda,v,\Psi) = \frac{2^{\frac{(v-2)(N-1)}{2}} K_{\frac{v+N}{2}}\left(\sqrt{(v+z^{*\top}\Psi^{-1}z^{*})\lambda^{\top}\Psi^{-1}\lambda}\right) e^{z^{*\top}\Psi^{-1}\lambda}}{\Gamma\left(\frac{v}{2}\right)^{1-N} \mid \Psi \mid^{\frac{1}{2}} \left(\sqrt{(v+z^{*\top}\Psi^{-1}z^{*})\lambda^{\top}\Psi^{-1}\lambda}\right)^{-\frac{v+N}{2}} \left(1+\frac{z^{*\top}\Psi^{-1}z^{*}}{v}\right)^{\frac{v+N}{2}}} \times \prod_{j=1}^{N} \frac{\left(\sqrt{\left(v+\left(z_j^{*}\right)^2\right)\lambda_j^2}\right)^{-\frac{v+1}{2}} \left(1+\frac{\left(z_j^{*}\right)^2}{v}\right)^{\frac{v+1}{2}}}{K_{\frac{v+1}{2}}\left(\sqrt{\left(v+\left(z_j^{*}\right)^2\right)\lambda_j^2}\right) e^{z_j^{*}\lambda_j}} \tag{.4}$$

where K$(\cdot)$ denotes the modified Bessel function of the second kind, and $z^{*} = t^{-1}_{\lambda,v}(\eta_i)$ denotes the copula shocks, where $t_{\lambda,v}(\eta_i)$ is univariate skewed t distribution:

$$t_{\lambda,v}(\eta_i) = \int_{-\infty}^{\eta_i} \frac{2^{1-\frac{v+1}{2}} K_{\frac{v+1}{2}}\left(\sqrt{(v+x^2)\lambda_i^2}\right) e^{x\lambda_i}}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi v}\left(\sqrt{(v+x^2)\lambda_i^2}\right)^{-\frac{v+1}{2}} \left(1+\frac{x^2}{v}\right)^{\frac{v+1}{2}}} dx \tag{.5}$$

However, a closed-form solution for skewed t quantile function is not available. We use simulation to obtain the quantile estimation and employ 1,000,000 replications of the equation below. More details about the skewed t copula can be found in Christoffersen et al. (2012) and Christoffersen and Langlois (2013).

$$X = \sqrt{W}Z + \lambda W \tag{.6}$$

where W follows an inverse Gamma $IG(v/2, v/2)$ distribution which could be calculate from the parameter v; Z denotes a N-dimensional normal distribution with mean 0 and correlation matrix Ψ; λ denotes a $N \times 1$ asymmetry parameter vector.

C. *Conditional diversification benefits*

We followed Christoffersen et al. (2012) to calculate the CDB of each portfolio as follows.

Firstly, we calculate the expected shortfall ES:

$$ES_t^q(R_{i,t}) = -E\left[R_{i,t} \mid R_{i,t} \leq F_{i,t}^{-1}(q)\right] \tag{.7}$$

where $R_{i,t}$ is the return of cryptocurrency i at term t, $F_{i,t}^{-1}(q)$ denotes the inverse cumulative distribution function and q is the probability which we set 5% here. $ES_t^q(R_{i,t})$ denotes the expected shortfall of cryptocurrency i at term t with the percentile q.

$$ES_t^q(w_t) \leq \sum_{i=1}^{N} w_{i,t} ES_t^q(R_{i,t}) \; for\, all\, w_t \tag{.8}$$

where $ES_t^q(w_t)$ is the expected shortfall for the portfolios with the weight w_t. Hence, we could set the upper bound as

$$\overline{ES_t^q}(w_t) \equiv \sum_{i=1}^{N} w_{i,t} ES_t^q(R_{i,t}) \tag{.9}$$

as for the lower bound, we set the value at risk of the portfolio with the weight w_t:

$$\underline{ES_t^q}(w_t) \equiv -F_{i,t}^{-1}(w_t, q) \tag{.10}$$

where the $F_{i,t}^{-1}(w_t, q)$ denotes the inverse cumulative distribution function of the portfolio with the weight w_t. Therefore, we can calculate the CDB for a portfolio using following function

$$CDB_t(w_t, q) \equiv \frac{\overline{ES_t^q}(w_t) - ES_t^q(w_t)}{\overline{ES_t^q}(w_t) - \underline{ES_t^q}(w_t)} \tag{.11}$$

The higher CDB provides a better risk management.