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## Modelling and Optimisation on scroll expander for Waste Heat Recovery Organic Rankine Cycle

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### Abstract

Scroll expander has demonstrated high efficiency at low power range. In this paper, a generic model of a scroll expander has been developed. It can calculate the ideal expander parameters to give the optimal efficiency and prevent under- or over-expansion at any given operating conditions or fluids. The dynamic model was validated by predicting the ideal volumetric expansion ratio with ideal expansion ratio of 4.03 at 0.7 MPa pressure, and showed agreement with experimental data. The results suggested that the rate of scroll increase  $K$  in the geometric model has little effect on volumetric expansion ratio or ideal scroll length of the expander, but when expansion ratio is kept constant, lower  $K$  value results in lower leakage losses.

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### 1. Introduction

Scroll expander has been applied in Organic Rankine Cycle (ORC) based Waste Heat Recovery (WHR) due to its high efficiency, compactness and low cost [1]. As early as 1993, Toyota used a scroll expander as an energy conversion device on an Internal Combustion engine WHR system. The results indicated that, when using R123 under ambient temperature of 25°C, 3% overall efficiency improvement was achieved [2]. Prediction of scroll expander operation is important to analyse and design the equipment. With the aim to develop the dynamic model of scroll expander, it is important to describe the geometry of scroll wraps. Orosz et al. [3] built a set of scroll geometries to give the method for selecting optimal scroll geometry. It organised the scrolls by compactness factor. The result showed a positive correlation between

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isentropic efficiency and compactness factor. Bell et al. [4] investigated a comprehensive framework and geometric solutions, which can be used to analyse scroll machines.

In this study, a dynamic scroll expander model was developed based on the geometry and thermodynamic processes of the expander [5, 6]. The aim of the model is to aid manufacturing a scroll of the optimum geometry, which is calculated according to the predetermined inlet conditions. Furthermore, it can also predict the maximum efficiency under the predetermined conditions. But the flank leakage and radial leakage were determined by the manufacturing tolerances and working fluid selection, so they were the function of the operating environment and cannot be optimised. Frictional losses were supposed to be very small, so despite of input conditions they were kept relatively constant and were not optimised.

## 2. Expander Model

The model is comprised of three sub models which were developed in MATLAB/Simulink: a geometric model, a thermodynamic model and a mechanical model. The geometric model has two parts. One is to generate series of expander chambers and calculate geometric characteristics which are useful to the other two models; the other is to optimise scroll geometry for avoiding under-/over-expansion. The thermodynamic model is used to calculate the thermodynamic properties of working fluid in each time step. The mechanical model can determine the gas-driven force, pressure and frictional forces on each chamber to get the power of the expander.

### 2.1 Geometry

Considering the ease of machining, the scroll geometry in this paper was based on circle involute. The expander chambers are divided between moving scroll and fixed scroll curves symmetrically and in pairs. The number of chamber pairs is determined by the scroll rolling angle. The volumes of the central chamber and side chamber,  $V_c$  and  $V_s$ , can be deduced by Green's formula:

$$V_c(\alpha) = \frac{z}{2} \left[ \int_{\alpha}^{\alpha+\pi} ((-y_A dx_A) + (x_A dy_A)) + \int_{\alpha-\pi}^{\alpha} ((-y_B dx_B) + (x_B dy_B)) \right] \quad (1)$$

$$V_s(\alpha) = \frac{z}{2} \left[ \int_{\alpha+2\pi}^{\alpha} ((-y_A dx_A) + (x_A dy_A)) + \int_{\alpha}^{\alpha+2\pi} ((-y_B dx_B) + (x_B dy_B)) \right] \quad (2)$$

Where,  $z$  is the scroll height,  $(x_A, y_A)$ ,  $(x_B, y_B)$  are the family curves of moving scroll and fixed scroll, respectively.  $\alpha$  is the orbit angle of moving scroll, which is shown in Fig.1. The range of  $\alpha$  is decided by the scroll length,  $L_{\text{scroll}}$ . When the initial involute angle set to 0,  $\alpha_{\text{max}} = 2\pi N$ ; where,  $N$  is the number of turns for moving scroll, The solution of (1) and (2) shows that for the suction chamber  $V_c$  is proportional to  $\alpha^2$ , and the side chambers  $V_s$  is proportional to  $\alpha$ . These result in a parabolic increase in volume with respect to orbit angle for the suction chamber, followed by a linear increase in the following side chambers. Therefore, equation (1) and (2) can be simplified as followed:

$$V_c(\alpha) = z \int_0^{\alpha} \pi(K\rho)^2 d\alpha \quad (3)$$

$$V_s(\alpha) = z \int_{\alpha}^{\alpha+2\pi} \pi(K\rho)^2 d\alpha \quad (4)$$

Where,  $\rho$  is radius of orbit.  $K$  is defined as the rate of the effective radius increase. It is irrelevant to location of expansion chamber. The relationship between  $V_c(2\pi)$  and  $K$  is described as:

$$K = \frac{\sqrt{\frac{V_c(2\pi)}{\pi}}}{2\pi} \quad (5)$$

In the preliminary optimisation, the wall thickness of scroll is assumed to be 0. The scroll geometry parameters are defined by  $z$ ,  $N$  and  $K$  in the model.

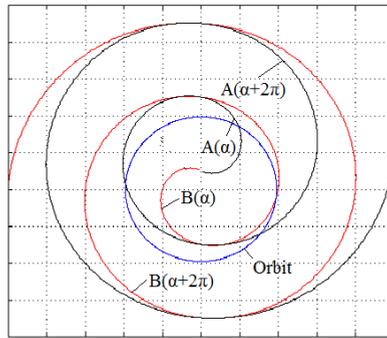


Fig.1. Diagram of a simplified scroll wraps

## 2.2 Model Description

The conceptual expansion process is divided into 6 sections seen in Fig.2 (a) [1]: adiabatic supply pressure change ( $su \rightarrow 1$ ), isobaric heat rejection ( $1 \rightarrow 2$ ), isentropic expansion ( $2 \rightarrow 3$ ), isochoric expansion ( $3 \rightarrow 4$ ), isobaric heat exchange ( $4 \rightarrow ex$ ), and leakage ( $1 \rightarrow ex$ ). After the first two stages, the working fluid passes through the geometric block for calculating leakage, expansion ratio, isentropic expansion efficiency, etc. The optimisation procedure is found in the isentropic expansion stage.

5 pairs of chambers were built in the geometric model. That means  $N$  will not be greater than 5. Each pair of chambers (except the central chamber) uses the same calculation steps, and the number of chambers could be expanded if necessary with the addition of more identical calculation blocks. Every chamber must only be activated after its previous one has completed a cycle. This is achieved with the use of dead zones which turn on each chamber at the correct time. Through the geometric model, each chamber length of radial and axial leak line can be calculated in a non-dimensional model in consideration of low leakage value and accelerating computation[5]. They are given by:

$$L_{leak, ai} = 2z \quad (6)$$

$$L_{leak, ri} = 2\pi K(\alpha - 2i\pi) \cdot \frac{r_c}{2} \quad (7)$$

With the final values of temperature and pressure calculated in leakage calculation block, the model can begin the optimisation process, which is shown in Fig.2 (b). The output pressure was compared to the desired exhaust pressure and the error calculated and used to calculate a new prediction of scroll length. The model was then re-run with the new value of scroll length and a solution for optimum scroll length would be found iteratively.

During the optimisation process of the model, temperature and pressure at each time step can also be calculated. Once the desired scroll length was achieved, the volume in the exit chamber would be set constant. Therefore, it would not affect downstream temperature and pressure calculation in isochoric expansion block. The efficiency of the model was calculated by finding the enthalpy at the inlet and exhaust of the expander and also the inlet and exit of the isentropic expansion stage:

$$\eta_s = \frac{h_{2-3}}{h_{1-ex}} \quad (8)$$

Moreover, the axial force is calculated by:

$$F_a = A_c p_0 + 2\pi r_c \sum_{i=1}^N L_{leak, ai} (p_i - p_{i-1}) \quad (9)$$

The radial force is the force applied by working fluid along the connecting line of base circle centre for moving and fixed scroll. And it is given by:

$$F_r = \sum_{i=1}^N L_{\text{leak}, ri} r_c (p_i - p_{i-1}) \quad (10)$$

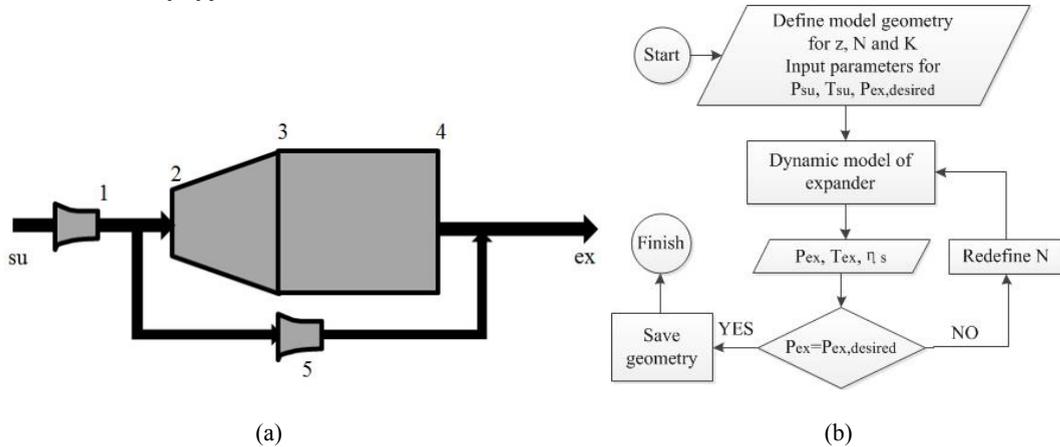


Fig.2. (a) Conceptual schematic expander stages (b) Flow chart of the geometry optimised process

The frictional forces are supposed to be very small, so despite of input conditions they keep relatively constant and are calculated by suitable friction coefficient with 2%. The model is set to use R134a and chooses the Runge-Kutta fixed step solver with  $10^{-4}$  second step-size as default.

### 2.3 Model Validation

This model is designed to optimise the geometry of the expander to a determined set of conditions, but experimental data so far has studied single expander geometry under different conditions. The ideal validation for this model would be to manufacture a range of optimised expanders and test them under their designed operating conditions to examine the accuracy of the model. Therefore, to determine the validity of expander model, the model could only use the data which was involved by V. Lemort [7]. It took three former compressors at conditions of  $P_{in}=0.7\text{MPa}$ ,  $P_{ex}=0.1\text{MPa}$  and  $T_{in}=417\text{K}$  for test. The tests indicated the highest efficiency was at the expansion ratio of 4.1, as opposed to the other 2 expansion ratios of 3.12 and 2.6. Following optimisation at these conditions, the model predicted the optimum expansion ratio to be 4.03 which was close to the 4.1 expansion ratio. The efficiency predicted by the model was notably higher than that was shown in testing (0.778 compared to 0.55). However the paper notes that the low efficiency in the test data may stem from problems with the leakage seals and therefore a higher efficiency should be expected using a similar device. Wang et al. [8] tested the scroll expander using R134a, and the isentropic efficiency reached to 77.5%, when the scroll sealing pressure was increased to 712kPa.

### 3. Numerical simulations and analysis

The validated model can be used to predict the input parameters, expander geometry design and their impact on the expander performance. Furthermore, in preliminary optimisation, the model gives the recommended scroll geometry to obtain better performance.

With an initial scroll length of 2.5 revolutions and desired  $P_{ex}$  of 0.1MPa, Fig.3 shows the pressure history in the expansion chambers for the non-optimised geometry in comparison with the optimised expansion process at  $T_{in} = 373\text{K}$  and  $P_{in} = 1.0\text{MPa}$ . After optimisation, a new scroll length of 3.3 revolutions with the expansion ratio of 5.51 was recommended. The figure illustrated that  $P_{ex}$  with optimised parameters is closer to 0.1MPa than the non-optimised one. In the non-optimised expansion, the exhaust pressure is 0.14MPa which is much higher than desired  $P_{ex}$ . That suggests an under-expansion and consequently higher losses.

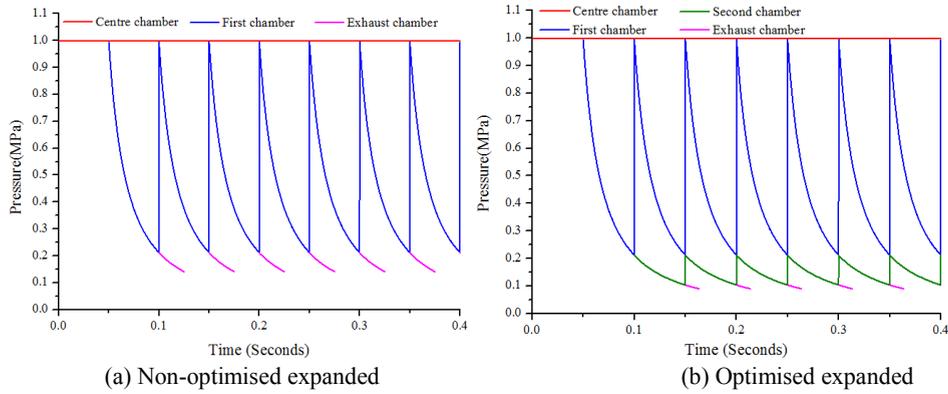


Fig.3. Pressure variation for Non-optimised expanded and Optimised expanded

Every inlet condition corresponds to optimum expander geometry. As shown in Fig. 4, there is a nearly linear relationship between these two parameters. An increase in inlet pressure requires a longer expander to reduce the exhaust pressure and prevent under-expansion. The results also suggest that different inlet conditions may result in different expansion ratios for the same expander. As known, in ORC based WHR for internal combustion engine exhaust gas, the operating conditions of working fluid usually vary frequently. When designing an expander for this application, it is necessary to define an optimum range of operating conditions for the specific expander geometry.

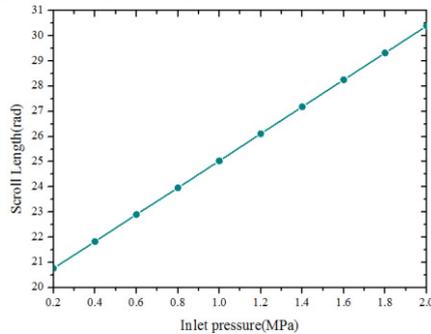


Fig.4. Optimum scroll length against inlet pressure

As mentioned above,  $K$  determines the rate of expansion and it keeps unchanged if  $r_c$  and  $d$  remain unchanged. Hence changing the value of  $K$  has no effect on scroll optimum length or expansion ratio. As shown in Fig. 5, the model operating with lower  $K$  shows lower mass flow rate.  $K$  is similar to compactness factor which is defined by Orosz et al [3]. It suggests that if remains the unchanged expansion ratio, lower leakage losses for lower  $K$  value (higher compactness factor) can be expected.

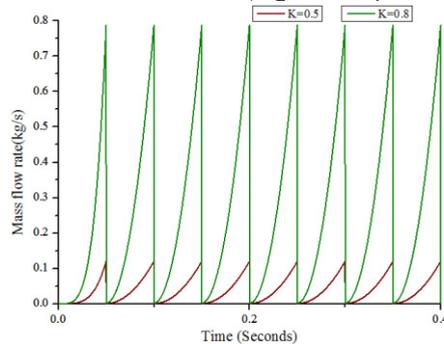


Fig. 5. Mass flow rate for different  $K$  values

#### 4. Conclusions

This paper introduced an idealised dynamic model for a scroll expander. The model can be used in any operating condition and optimise the geometry of the scroll. It is also suggested that the optimum geometry is dependent on the inlet conditions, exhaust pressure, leakage gaps and the working fluids. The model was validated at certain conditions and it gave the simulation result of optimum expansion ratio at 4.03 which agreed with experimental result.

The optimised scroll is demonstrated to avoid the under- or over-expansion, and it suggests that it is important to obtain the optimum range of operating conditions for designing an special expander, which is suitable for ORC based WHR. It is also concluded that the rate of scroll increase determined by the K value in the geometric model has little effect on volumetric expansion ratio or ideal scroll length of the expander. The increase in mass flow rate due to larger volumes for higher K value causes higher leakage losses in the expander; implying scrolls with low K values will provide higher efficiencies.

However, the leakage and frictional losses calculation blocks are static and have not been optimised. More improvements must be done to provide the foundation of geometry optimum design for ORC system.

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#### Biography

Dr G Tian received his PhD degree in Engineering Thermo Physics from Tsinghua University in 2007. He then joined University of Birmingham as a lead researcher conducting a TSB funded bio-diesel research project and an EPSRC funded DMF research project investigating this chemical as a promising gasoline fuel replacement for the first time. In 2010, he joined Swan Centre, School of MSE at Newcastle University as a Lecturer.