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In-plane energy absorption characteristics of a modified re-entrant auxetic structure fabricated via 3D printing

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Abstract

Here, we present the in-plane energy absorption characteristics of modified re-entrant auxetic honeycombs realized via fused filament fabrication in conjunction with parametric analysis and geometry optimization. The influence and interaction effects of the geometrical parameters such as strut-length ratio and joint-angles on the stiffness, strength and energy absorption characteristics of modified re-entrant auxetic honeycombs were evaluated. Subsequently, Finite Element results obtained using ABAQUS/Explicit were corroborated with measured data. Deformation mode, stress-strain response and energy absorption behavior of an optimal re-entrant auxetic honeycomb were studied and compared with conventional re-entrant auxetic structure. Our modified auxetic structure reveals an 36% improvement in the specific energy absorption capacity. Our analysis indicates that due to the introduction of more nodes with low rotational stiffness, the failure strain of the modified re-entrant structure has increased resulting in improved energy absorption capacity.

Keywords: Auxetic re-entrant honeycombs; lightweight cellular structures; energy absorption capacity; fused deposition modeling (FDM); mechanical characteristics

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1. Introduction

Mechanical metamaterials are engineered cellular materials that exhibit unique mechanical properties (such as higher energy absorption, higher indentation resistance, higher fracture toughness, good compressive strength, and lighter weight, etc.) due to their designed nano/micro-architecture [1–5]. Such micro- or nano-architected lattices find applications in a wide range of fields including robotics, medical, soft electronics, sensors, acoustic cloaking, automobile, defense, aerospace and energy harvesting.[6–9]. Metamaterials that exhibit negative Poisson’s ratio (NPR) are known as auxetics. Unlike conventional materials, when auxetic materials are stretched, they expand laterally instead of contracting [10,11]. Recently auxetic structures have attracted tremendous attention due to their unique mechanical properties, which can be tailored by changing the NPR[12,13]. More importantly, 3D printing technologies enable the fabrication of cellular structures with fine geometric features [14–16]. Gibson first introduced 2D re-entrant honeycombs structure[17] and thereafter many re-entrant structures such as star re-entrant [18–20], hierarchical star re-entrant [21], double arrowhead, [22–24] augmented re-entrant honeycomb (ARH) [25–27], graded re-entrant [28,29] and re-entrant chiral auxetic (RCA) [30,31] have been proposed. Numerous studies have attempted to improve the in-plane mechanical response by designing new structures possessing enhanced energy absorption properties. Node connectivity (number of struts connected to a node/point) within the structure plays a key role in determining the deformation behavior; the structures with higher node connectivity are more likely to deform in a stretch-dominated manner and therefore they are stiffer and weight efficient [32]. The energy absorption capacity of a lattice structure can be enhanced by having a combination of bending and stretching modes of deformation with stretching being the dominant deformation mode. To improve the deformation characteristics of a lattice structure there should be a change in its architectural design so that the structure can achieve desired combination of properties.
Weitao Lv et al. proposed a hierarchical design to enhance the energy absorption ability, where the cell wall of the structure was replaced by a triangular lattice. The deformation mechanism and energy absorption characteristics were experimentally studied and compared with FEM results. It was found that for first-order hierarchical octet-truss structure the lattice stiffness, energy absorption capacity and collapse strength were higher than those of second-order hierarchical octet-truss structure. Similarly, a hybrid structure was developed by Ingrole et al. to enhance the energy absorption and compressive strength of the re-entrant honeycomb structure by combining the re-entrant honeycomb with the conventional honeycomb. In another study, a hybrid design was proposed by combining re-entrant with chiral geometry and demonstrated its high specific energy absorption characteristics compared to original structures. Kumar et al. attempted to improve the energy absorption property of re-entrant honeycomb by varying the cell wall thickness in the out-of-plane direction both under quasi-static and low-velocity impact loadings. Realizing architected honeycombs via material jetting additive manufacturing, their study demonstrated that such geometrically tailored designs exhibit energy absorption efficiency as high as 90%. It can be concluded from the above studies that mechanical/energy absorption properties of auxetic honeycombs are primarily governed by their unit-cell architectures.

In this study, modified re-entrant honeycomb structures were designed and realized via fused filament fabrication (FFF) additive manufacturing. The in-plane behavior of these structures was studied through quasi-static compression tests and results are further numerically analyzed and compared. The influence of geometrical parameters such as ‘strut-length ratio’ and ‘joint-angles’ on the mechanical properties of the structure is studied through the response surface methodology (RSM) technique. Numerical models were developed using ABAQUS/Explicit and benchmarked with the experimentally obtained results. Deformation mode, stress-strain
curves and energy absorption characteristics of the optimized structure are studied and compared.

2. Materials and methods

2.1 Auxetic structure design

A new design of the re-entrant auxetic structure is proposed in this study by modifying the regular re-entrant honeycomb geometry without changing the mass of the specimen. In order to enhance the energy absorption capacity through improved bend-dominated deformation, low rotational stiffness nodes are generated in the new design by splitting the inclined struts into two links in such a way that the sum of the link length is equal to the original inclined strut length. Fig. 1 shows the configuration of regular and modified re-entrant honeycomb structures.

The architectural parameters of the regular and modified re-entrant honeycomb structures are described as follows. In re-entrant honeycomb structure, \( h \) represents vertical strut-length of 16mm while \( l \) represents inclined strut-length of 8 mm, \( \theta \) represents the angle of an inclined strut to horizontal of 30\(^\circ\) and \( t \) (2mm) represents the thickness of all struts. Similarly, in the modified structure, \( h \) represents vertical strut length, \( \theta_1 \) indicates the inclination of strut \( l_1 \) to horizontal, \( \theta_2 \) represents the angle between two inclined struts, and \( t \) indicates the thickness of all struts. \( l_1 \) and \( l_2 \) represent the lengths of two inclined struts.
The theoretical relative density of an auxetic lattice structure is defined as the ratio of the area occupied by the cell wall of the unit cell to the area occupied by the unit cell \([36,38]\). For low relative densities, the relative density of both structures can be obtained from the geometry of the unit cell as follows:

The relative density of regular re-entrant honeycomb structure

\[
\bar{\rho} = \frac{(2l+h)t}{2l \cos \theta (h - l \sin \theta)} \tag{1}
\]

The relative density of modified re-entrant honeycomb structure

\[
\bar{\rho} = \frac{2(l_1 + l_2) + ht}{2 [l_1^2 \sin \theta_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \{l_1 \cos \theta_1 (h - 2l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2))\]} \tag{2}
\]

The relative density of the fabricated cellular structures are obtained using the equation given by

\[
\bar{\rho} = \left( \frac{\rho}{\rho_s} \right) \tag{3}
\]

where \(\rho\) and \(\rho_s\) represent the density of the cellular structure and density of the base material respectively.
2.2 Geometrical tailoring and design of experiments

The geometrical tailoring of the modified re-entrant honeycomb structure is achieved by varying three variables namely, inclined strut-length ratio ($l_1:l_2$), $\theta_1$ (deg.) and $\theta_2$ (deg.) while maintaining constancy of mass of the structure. The maximum and minimum limits, as well as center points of these variables (see Table 1), were chosen considering the design feasibility and printing limitations of the FFF 3D printer used in this study.

The details of experiments based on the Box–Behnken approach with three inputs and responses are summarized in Table 2. The mechanical responses measured from the analysis are Young’s modulus, compressive strength and energy absorption capacity. The specific energy absorption (SEA), $\varphi$ [39] and the energy absorption efficiency, $\eta$ of the structures at the onset densification strain $\varepsilon_D$ were calculated by using the equation (4) [40] and equation (5) [41,42] respectively.

$$\varphi = \frac{1}{\rho} \int_0^{\varepsilon_D} \sigma(\varepsilon) d\varepsilon$$ (4)

$$\eta = \frac{1}{\sigma_D} \int_0^{\varepsilon_D} \sigma(\varepsilon) d\varepsilon$$ (5)

Where $\sigma$ represents the axial compressive stress of the structure experienced during quasi-static compression and $\varepsilon$ is its work conjugate. $\varepsilon_D$ and $\sigma_D$ represents the densification strain and compressive strength (maximum stress a structure can resist before densification begins) of the structure respectively.

The energy absorption efficiency of structures for different tailored designs is summarized in Table 2. Due to changes in geometrical parameters, volume and hence the density of the resulting unit-cell geometry changes, while the mass of the overall lattice structure was kept constant. The relative densities of the modified structures, calculated using the equation (3), were also given in Table 2.
Table 1. Design variables with their levels

<table>
<thead>
<tr>
<th>Variables</th>
<th>-1 (low)</th>
<th>0 (mid)</th>
<th>1 (high)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1: l_2 )</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>( \theta_1 ) (deg.)</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>( \theta_2 ) (deg.)</td>
<td>120</td>
<td>150</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 2. Design matrix with input factors and responses

<table>
<thead>
<tr>
<th>Factors</th>
<th>Responses</th>
<th>Energy absorption efficiency (%)</th>
<th>Relative density ( \rho )</th>
<th>SEA (J/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1: l_2 ) ratio (mm)</td>
<td>( \theta_1 ) (deg.)</td>
<td>( \theta_2 ) (deg.)</td>
<td>Young’s modulus (MPa)</td>
<td>Compressive strength (MPa)</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>120</td>
<td>90</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>150</td>
<td>106.3</td>
<td>3.4</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>120</td>
<td>111.2</td>
<td>2.7</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>150</td>
<td>175.14</td>
<td>3.45</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>150</td>
<td>100.65</td>
<td>3.25</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>120</td>
<td>87.32</td>
<td>3.58</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>180</td>
<td>138.05</td>
<td>4.85</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>150</td>
<td>108.65</td>
<td>3.05</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>180</td>
<td>86.86</td>
<td>3.25</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
<td>150</td>
<td>73.73</td>
<td>2.37</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>120</td>
<td>145.17</td>
<td>4.01</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>180</td>
<td>193.93</td>
<td>3.1</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>150</td>
<td>106.45</td>
<td>3.05</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>180</td>
<td>122.75</td>
<td>3.5</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>150</td>
<td>74.50</td>
<td>2.4</td>
</tr>
</tbody>
</table>

2.3 Numerical simulation and validation

2.3.1 Finite element modelling

Finite element (FE) models were developed using ABAQUS 2017 to simulate the behavior of samples under quasi-static compression. In order to replicate actual experimental conditions, the structures were placed in between two rigid surfaces as shown in Fig. S2 (supplementary information: S2). Acrylonitrile butadiene styrene (ABS) polymer exhibited significantly different yield behavior in tension and compression (supplementary information: S1 (see Fig.
S1)), and hence a pressure-dependent plasticity model was used to model the material behavior. Isotropic elasticity was considered for modeling the elastic behavior while plastic behavior was modeled by using Linear Drucker-Prager plasticity model. The ductile damage model was considered for modeling the material failure. “Explicit dynamic” analysis was performed considering geometric, material and contact non-linearities. The elastic properties of the ABS material used in the FE analysis are summarized in Table 3 while the plastic and failure behavior is provided in the supplementary information: S2.

Table 3. Material properties of 3D printed ABS specimen.

<table>
<thead>
<tr>
<th>Material</th>
<th>Elastic modulus (MPa)</th>
<th>Yield stress (MPa)</th>
<th>Poisson’s ratio</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS</td>
<td>2200</td>
<td>31</td>
<td>0.35</td>
<td>1.05</td>
</tr>
</tbody>
</table>

All the calculations were performed in ABAQUS/Explicit with a sufficiently low displacement rate (500 mm/s) to eliminate inertial effects (see, Supplementary Information: S3) and ensure quasi-static deformation. Contact between rigid plates and the surfaces of models is defined as a ‘general contact interaction’ while ‘self contact is considered in between the surfaces of the structure. Contact with a friction coefficient of 0.3 was set in the tangential direction and hard contact was considered in the normal direction[43]. An 8-node hexahedral (C3D8R)[44] element with 0.4 mm mesh size (see the details of mesh sensitivity analysis in Supplementary Information: S4) was used for meshing the structures and a 4-node linear quadrilateral (R3D4) element with 2 mm mesh size was used for the top & bottom rigid plates. Boundary conditions as shown in Fig. S2 (supplementary information: S2) was imposed. The analysis was performed up to a nominal compressive strain of 70% (as in the experiments), and the obtained force vs displacement profiles are used to determine the stress vs strain responses as well as the energy absorption capacities of the structures.
2.3.2. Numerical model validation

The finite element model developed in this work was validated by comparing the FE predictions with the experimentally measured stress-strain response for the optimal structure shown in Fig. 2. More details about the optimal structure are given in section 3.4. It is clear from the figure that the numerical model is capable of predicting both the elastic and plastic behavior (with three distinct stages) as discussed by many researchers [45,46]. In the beginning, a linear elastic response is observed followed by a plateau regime and finally the densification regime. In summary, the FE prediction is in good agreement with experimental results.

Fig. 2. Experiment vs FEA prediction: stress-strain response of the optimal structure under quasi-static compression.

Table 4 below shows specimen morphology at different strains during compression. It was observed that cracking and delamination started much before the onset of densification, nearly
at a strain of 0.4 (Table 4) and it continued till it reaches the densification strain. At the
densification strain, almost every link had cracked at their joints.

<table>
<thead>
<tr>
<th>Strain</th>
<th>Specimen morphology</th>
<th>Experimental morphologies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>Crack</td>
<td>Crack Delamination</td>
</tr>
<tr>
<td>0.55</td>
<td>Crack</td>
<td>Crack Delamination</td>
</tr>
<tr>
<td>0.60</td>
<td>Crack</td>
<td>Crack Delamination</td>
</tr>
</tbody>
</table>

**Table 4.** Comparative analysis of specimen morphology at different strains.

2.4 Modelling and optimization

The methodology model development and identification of an optimal design is shown in Fig.
3. The model was developed by using the Box–Behnken RSM technique which requires a
smaller number of runs to generate accurate response surfaces than a normal factorial
technique [47]. In Box–Behnken RSM technique the midpoints of the edges and center of the
cubical were considered as design points and hence twelve middle edge nodes and three center
nodes were required to fit a polynomial.

The variables, ‘length ratio (l₁: l₂)’, ‘θ₁’ and ‘θ₂’ were chosen as input and three responses i.e.,
‘Young’s modulus’, ‘compressive strength’ and ‘energy absorption capacity’ were selected as
output functions. Simulation data were fitted to generate mathematical models and then the
models were used to study the influence of design variables on mechanical responses of the structure.

Fig. 3. Flow chart of the design optimization process.

The best-fit indicators characterizing the accuracy of the models revealed that the energy absorbed (EA) by the structure can be characterized using quadratic models while Young’s modulus (E), and compressive strength can be characterized through reduced quadratic models (backward) as listed in Eqs. 6, 7 and 8 respectively.

Energy absorbed (EA) = -0.376457 - 1.08269 l₁l₂ + 0.2721 θ₁ - 0.003458 θ₂ + 0.002083 l₁l₂ θ₁ + 0.005497 l₁l₂ θ₂ + 0.001078 θ₁θ₂ + 0.003627(l₁l₂)² - 0.001435 l₁² + 0.000089 θ₂² (6)

Young’s modulus (E) = 757.66246 - 18.12379 l₁l₂ - 12.01085 θ₁ - 6.61854 θ₂ + 0.566050 l₁l₂ θ₁ + 0.04325 θ₁θ₂ + 0.086926 θ₁² + 0.017796 θ₂² (7)

Compressive strength = -9.47763 - 1.36794 l₁l₂ + 0.889121 θ₁ - 0.054632 θ₂ + 0.007917 l₁l₂ θ₁ - 0.002213 θ₁θ₂ + 0.024023(l₁l₂)² - 0.006325 θ₁² + 0.000403 θ₂² (8)

As shown in Table 5, analysis of variance (ANOVA) was used to evaluate the accuracy of the model: probability (p-value), coefficient of determination R², Adjusted R², Predicted R², and adequate precision. All model terms show high R² (>0.75) value with excellent agreement.
between the predicted and adjusted $R^2$, indicating the trustworthiness of the model. It is evident from ANOVA analysis that all the models are significant and can be used to make valid predictions within the range listed in Table 1.

### Table 5. Analysis of variance (ANOVA) of the developed models

<table>
<thead>
<tr>
<th>Model</th>
<th>F-value</th>
<th>P-value</th>
<th>$R^2$</th>
<th>Adj- $R^2$</th>
<th>Pre- $R^2$</th>
<th>Adeq-precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy absorbed (EA)</td>
<td>117.8</td>
<td>&lt;0.0001</td>
<td>0.9837</td>
<td>0.9763</td>
<td>0.9427</td>
<td>41.6573</td>
</tr>
<tr>
<td>Young’s modulus (E)</td>
<td>38.36</td>
<td>&lt;0.0001</td>
<td>0.9746</td>
<td>0.9492</td>
<td>0.8224</td>
<td>20.7162</td>
</tr>
<tr>
<td>Compressive strength</td>
<td>28.27</td>
<td>&lt;0.0003</td>
<td>0.9742</td>
<td>0.9397</td>
<td>0.7852</td>
<td>21.8009</td>
</tr>
</tbody>
</table>

Fig. 4. shows comparative analysis between model prediction and finite element results, suggesting that the developed model (Eqs. (6)–(8)) can adequately capture the non-linear relationship between the input and output variables of the structure.

Fig. 4. Comparison of model predictions and finite element results for (a) Young’s modulus, (b) compressive strength and (c) energy absorption of the structure.

### 2.5 3D printing and experimental testing

#### 2.5.1 3D printing

Fused filament fabrication 3D printer from Divide by Zero, India [48] was used to fabricate all the structures for experimental analysis. A KISSlicer PRO v 1.5 was used to digitally slice stereolithography (STL) file of the structures and generate the tool path for 3D printing. Commercially available acrylonitrile butadiene styrene (ABS) polymer filament was used to
fabricate the structures. “Z” building direction is selected for printing the structures with a 0.2 mm layer thickness so that the printing can be completed without any support material. The extruder and bed temperatures were set at 245°C and 90°C respectively and 100% infill is selected. Mechanical anisotropy of the 3D printed part was minimized by adopting ±45° raster angle for deposition of filament[49]. All printing parameters were kept the same for all the samples to avoid any variation of material properties among the samples. The 3D printed regular re-entrant honeycomb and modified re-entrant honeycombs are shown in Fig. 5.

Fig. 5. FDM 3D printed optimal structure (left) and re-entrant honeycomb structure (right).

2.5.2 Experimental testing procedure

In order to evaluate the mechanical response, quasi-static compression tests were performed at room temperature on the 3D-printed auxetic structures. The testing was carried out using 250 kN UTM (MEDIAN 250) at the crosshead speed of 2 mm/min (strain rate = 0.00133/s) [33,49]. The 250 kN load cell has an accuracy of ≤±1% of its output reading. The samples were loaded in Y-direction and the deformation maps were recorded using a high-definition digital camera. Crushing forces were recorded via a sensor, attached to the loading plate. Load-displacement
data, generated during the compression test were recorded and used for further analysis. Three samples for each model were tested to get reliable and repeatable results.

3. Result and Discussion

3.1. Influence of design variables on energy absorption characteristics

Fig. 6a, b and c show the effect of design variables on the energy absorption behavior of the structure. At a low value of $\theta_2 (120^0)$, the $l_1:l_2$ ratio has a higher influence on EA and the effect weakens as $\theta_2$ increases from $120^0$ to $180^0$ (fig. 6a and b). It can be seen from fig. 6a that as the $l_1:l_2$ ratio increases from 1 to 7, EA ability of structure decreases significantly irrespective of $\theta_1$ value. The RSM model predicted the highest (2.986 MJ/m$^3$) and lowest (0.983 MJ/m$^3$) EA value of the structure for $l_1:l_2$ ratio = 1 and 7 respectively. In order to support these observations, deformation maps are presented in fig. 7. The collapse mechanism of structure at a constant value of $\theta_2 (120^0)$ with a higher $l_1:l_2$ ratio and $\theta_1 (40^0)$ was found to be primarily due to bending with lower percolation of crush bands (fig. 7 (a)). But as the $l_1:l_2$ ratio decreases even with an increase of $\theta_1 (50^0)$ the structure deformation pattern changes; the structure first globally buckles from the center and then progressively deformed (fig. 7 (b)). Similar behavior was observed at $\theta_1$ equals $30^0$ (fig. 7 (c)). Structure with the least value of $l_1:l_2$ ratio (1) and $\theta_1 (30^0)$ showed higher global buckling with the highest percolation of crush bands (fig. 7 (d)). The percolation of crush bands during compression affects the densification strain and hence the energy absorption of the structure. The percolation of crush bands depends on the rotational stiffness of the joints which is explained in section 3.4.

(a) (b)
Fig. 6. Influence of geometrical parameters on energy absorption capacity (a) effect of $l_1:l_2$ and $\theta_1$ when $\theta_2 = 120^0$, (b) effect of $l_1:l_2$ and $\theta_2$ when $\theta_1 = 40^0$ and (c) effect of $\theta_1$ and $\theta_2$ when $l_1:l_2 = 1$

Fig. 6c shows that $\theta_1$ has less effect on the EA property of the structure at the lower value of $\theta_2$ and it becomes more significant with an increase in $\theta_2$. At the highest value of $\theta_2$ (i.e. $180^0$) the structure response is analogous to that of the traditional regular re-entrant model and hence it is no more affected by the $l_1:l_2$ ratio. The lowest EA (0.983 MJ/m$^3$) value of the structure was found for $\theta_1$ and $\theta_2$ values at their highest levels ($50^0$, $180^0$ respectively) as the deformation in such condition was not primarily due to bending, but it was due to stretching of inclined struts.
Fig. 7. Deformation sequences of different structures represented in terms of $(l_1:l_2, \theta_1, \theta_2)$ (a) $(7,40,120)$, (b) $(4,50,120)$, (c) $(4,30,120)$ and (d) $(1,30,120)$.

It can be concluded that the three design variables, $l_1:l_2$ ratio, $\theta_1$ and $\theta_2$ significantly influence EA behavior of the structure and the dependency of EA on the $l_1:l_2$ ratio was found to be linear while for $\theta_1$ & $\theta_2$, the relationship was quadratic. Accordingly, for improving the EA of the
proposed structure, the most significant terms are in order of $l_1: l_2$ ratio followed by the joint angles ($\theta_2, \theta_1$).

3.2. Influence of design variables on compressive strength

Compressive strength determines the load-bearing capacity of a structure. The influence of design variables on the compressive strength of the proposed structure is shown in fig. 8 (a) and (b). It can be seen that the structure exhibits the highest compressive strength at a higher value of $\theta_1$ and $\theta_2$ ($50^0, 180^0$ respectively) and it continuously decreases with a decrease in joint angles while resulting in minimum strength at the lower value of $\theta_1=30^0$ and $\theta_2=120^0$, respectively.

The model shows that $\theta_1$ has the highest influence on compressive strength. The compressive strength increases quadratically with an increase in $\theta_1$ from $30^0$ to $50^0$. $\theta_2$ is the second most influential parameter with a linear relationship with compressive strength, while the $l_1:l_2$ ratio has the least effect on the strength.

![Fig. 8](image-url) Impact of geometrical variables on compressive strength (a) the effect of $l_1:l_2$ ratio and $\theta_2$ when $\theta_1$ is $40^0$ and (b) the effect of $\theta_1$ and $\theta_2$ when $l_1:l_2$ ratio is 1.
3.3. Influence of design variables on Young’s modulus of the structure

Fig. 9 (a) and (b) show the effect of design variables on Young’s modulus of the structure. It is evident from fig. 9 (a) that the modulus is varying linearly with $\theta_1$ and it has the highest influence on the response while $\theta_2$ has the least contribution. The effect of the $l_1:l_2$ ratio is significant only at a higher value of $\theta_1$ and it reduces with a decrease in the $\theta_1$ value (see fig. 9 (b)).

![Graphs showing influence of design variables on Young's modulus](image)

Fig. 9. Influence of geometrical variables on Young’s modulus of the modified auxetic structure (a) the effect of $\theta_1$ and $\theta_2$ when $l_1:l_2$ ratio is 1 and (b). the effect of $l_1:l_2$ ratio and $\theta_1$ when $\theta_2$ is 120°. At higher values of $l_1:l_2$ ratio and $\theta_1$, the model suggests a higher Young’s modulus which is logical. This can be explained by analyzing the elastic collapse of the structure. Fig. 10 represents the Elastic collapse of the structure at the highest value of $\theta_2$ (180°). Due to quasi-static compression along the y-direction, the structure experiences a macroscopic compressive stress $\sigma$ along the y-direction (fig. 10 (a)). Due to compressive load inclined walls bend. The free body diagram of one inclined walls is shown in fig. 10 (b). For equilibrium in inclined wall AB, the force component along the y-direction is zero. The moment acting on the wall AB can be expressed as
where $P$ is the force acting on the wall AB in y-direction due to compressive stress ($\sigma$).

$$P = \sigma bl \cos \Theta_1$$  \hspace{1cm} (10)

where $b$ is the width of the unit cell along the z-direction. The deflection ($\delta$) of wall AB can be expressed as

$$\delta = \frac{Ml^2}{6EI}$$  \hspace{1cm} (11)

where $E_s$ is Young's modulus of basis material (ABS polymer) and $I$ is the second moment of inertia ($\frac{bt^3}{12}$) of the inclined wall. Hence, strain along y-direction can be written as

$$\epsilon = \frac{2\delta \cos \Theta_1}{2(h-l \sin \Theta_1)}$$  \hspace{1cm} (12)

Young's modulus $E$ of structure along y-direction can be expressed as

$$E = \frac{\sigma}{\epsilon}$$  \hspace{1cm} (13)

By plugging all equations (equations-(9-12)) in Young's modulus equation (equation-(13)) and rearranging, Young's modulus can be expressed as

$$E = \frac{E_s t^3 (h-l \sin \Theta_1)}{l^4 \cos^3 \Theta_1}$$  \hspace{1cm} (14)

From Young's modulus expression, it is clear that $E$ varies as $\Theta_1$ changes (for all structures, all other parameters are constant when $\Theta_2$ is $180^\circ$), and as $\Theta_1$ increases Young's modulus $E$ also increases.
Overall, it is clear from the above discussion that the relationship between the three variables (length ratio (l1: l2), θ1 and θ2) affects the mechanical and energy absorption properties of the proposed structure. In some cases, during the analysis, individual parameters effect was found to be dominating over their combined effect and vice-versa. It was observed that at higher levels of l1:l2 ratio, θ1 and θ2 (i.e. 7, 50°, 180° respectively) the structure exhibits higher compressive strength and modulus but EA ability was low (as low plateau area due to lower failure strain). On the other hand, at the lower level of l1:l2 ratio, θ1 and θ2 (i.e. 1, 30°, 120° respectively) the structure shows a high value of EA (higher failure strain) with lower compressive strength and modulus respectively.

Fig. 10. Elastic collapse of structure at the highest value of θ2 (180°): (a) Unit cell with loading condition and (b) bending of the inclined link due to loading.
3.4. Optimal design parameters for higher energy absorption of the structure

After realizing the tunable mechanical properties of modified structure, an optimal combination of design parameters for achieving the highest energy absorption is investigated in this section. Therefore, single-objective optimization approach is used and the resulting optimization problem can be formulated as:

Maximize energy absorption (EA) = \( f (l_r, \theta_1, \theta_2) \) \hspace{1cm} (15)

s.t. \( 1 \leq l_r \leq 7 \)

s.t. \( 30^\circ \leq \theta_1 \leq 50^\circ \)

s.t. \( 120^\circ \leq \theta_2 \leq 180^\circ \)

For optimization, each response must have a low and high value as represented by Eq. (15) and is solved using the desirability approach. Fig. 11 (a-c) shows the desirability plot of the optimum solution considering the three design variables as parameters. It appears that the optimal solution lies close to the lowest value of the \( l_1/l_2 \) ratio and \( \theta_2 \) and at a middle value of \( \theta_1 \). Table 6 summarizes the optimal solution predicted by the model with the highest desirability (0.956).
Fig. 11. Desirability plot of the optimum solution (a) effect of $l_1:l_2$ and $\theta_1$ at $\theta_2 = 120^0$, (b) effect of $l_1:l_2$ and $\theta_1$ at $\theta_2 = 150^0$ and (c) effect of $l_1:l_2$ and $\theta_1$ at $\theta_2 = 180^0$.

Table 6. Optimal solution predicted by the model

<table>
<thead>
<tr>
<th>$l_1:l_2$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>Energy absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40$^0$</td>
<td>120$^0$</td>
<td>2.986 (MJ/m$^3$)</td>
</tr>
</tbody>
</table>

In order to compare the performance of the optimal modified structure, the final design is 3D printed and tested as described in Section 2.5.1. A regular re-entrant honeycomb with the same mass is considered for benchmarking.

Fig. 12 (a) shows a stress-strain curve of the optimal structure. It is marked with numbers to signify its deformation behavior at different stages during compression. Fig. 12 (b) represents the sequential deformation maps during quasi-static compression at the corresponding points shown in the stress-strain curve. The optimal structure also shows three distinct stages as reported by several studies[33,50]. In the beginning, a small region known as the elastic regime was found where structure showed its linear elastic behavior and after an initial peak, a large plateau was observed (plateau regime) while densification regime starts when cell walls begins to come in contact with each other.
Results generated from the quasi-static compression test were used to calculate energy absorption characteristics of the structures (equation- 4) and it is found that the modified optimized structure outperforms regular re-entrant honeycomb (+36.42%).

**Fig. 12.** Regular re-entrant honeycomb vs optimal structure: (a) stress-strain curve and (b) sequential deformation maps at different stages during quasi-static compression (see the corresponding points shown in the stress-strain curve).
To further understand the energy absorption ability of modified optimal structure, sequences of deformed configurations (from optical images captured experimentally) at different strains are analyzed (Fig. 12b). From the experimental evidence (Fig. 12b), we observed that at the start of compression (elastic-regime) both structures show linear elastic behavior up to the yield point. There was a linear increase in stress, as the cells of the structures deform uniformly and reach a maximum value (Fig. 12(a)). As compression continues, further deformation behavior of the structure is influenced by the rotational stiffness of the nodes (joints). Rotational stiffness of the node (joint) is defined as the torque required per degree to rotate about the joints. The node with high rotational stiffness offers more resistance to rotation. The rotational stiffness of nodes is highly influenced by the number of inclined struts as they determine the degree of constraint to rotation[50].

In the case of regular re-entrant honeycomb, it can be seen from fig. 13(a) that all the nodes (marked in blue) connected with 3 inclined links offer high resistance to rotation and hence during loading the cells collapsed due to buckling and bending. Such deformation propagates from one layer to other layers causing a progressive collapse of the structure. Cell collapse started from the bottom end and propagated towards the top of the structure (Fig. 12 (b)). On the other hand, there are some nodes in the modified optimized structure, connected with only two inclined links (marked in Red) (fig. 13 (b)), and they offer less rotational stiffness. Further, due to the low rotational stiffness offered by these nodes, the deformation was not able to propagate to the next layer until the complete layer is collapsed, and therefore the low-stress plateau area increases which can be seen from the density of collapsed layer (higher in optimal structure as compared to regular re-entrant) (fig. 12 (b)). The stress values decrease as the first layer of the structure collapsed and then it again started to increase due to the densification of
the collapsed cells. This behavior continues until all the rows are collapsed. Finally, densification starts at the point when all the rows of the structures are collapsed (fig.12 (a)).

![Diagram](image.png)

**Fig. 13.** Different types of node (joint) and their distribution in the unit cell of (a) regular re-entrant honeycomb (b) modified optimal structure

Our analysis indicates that due to the introduction of more nodes with low rotational stiffness, the failure strain of structure increases, resulting in improved EA of the optimal structure. After densification, compaction of layers due to compression loading continues and hence the load value increases rapidly. It may be noted that within the compaction phase the performance is determined by the base material and not by the structure[45] and the stiffness of the basic material is higher than the structure.

Table 7 shows the comparison of properties obtained experimentally for regular re-entrant honeycomb and our modified optimal structure. It is clear that the modified optimal structure outperforms regular re-entrant honeycomb (+36.42%) without much change in other properties.
<table>
<thead>
<tr>
<th>Properties</th>
<th>Re-entrant honeycomb structure</th>
<th>Optimal structure</th>
<th>Improve (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus (MPa)</td>
<td>86.86</td>
<td>87.32</td>
<td>0.53</td>
</tr>
<tr>
<td>Compressive strength (MPa)</td>
<td>2.95</td>
<td>3.31</td>
<td>12.2</td>
</tr>
<tr>
<td>Energy absorbed (MJ/m³)</td>
<td>2.161</td>
<td>2.948</td>
<td>36.42</td>
</tr>
<tr>
<td>SEA (J/Kg)</td>
<td>2064.56</td>
<td>2816.47</td>
<td>36.42</td>
</tr>
</tbody>
</table>

4. Conclusions

In this study, energy absorption characteristics of a modified re-entrant auxetic structure processed via fused filament fabrication technique were evaluated both experimentally and numerically and its performance was compared with a conventional re-entrant auxetic structure of the same mass. Many topology-tailored designs were investigated using the Design of Experiment (DOE). The statistical models (derived by response surface methodology) were used to estimate their mechanical properties. It is clear from the results of the parametric study that the performance of the proposed auxetic structure is dependent on the strut-length ratio and joint angles. Having captured these relationships, optimization of geometrical parameters for the highest energy absorption was carried out for the proposed auxetic structure. The following conclusions can be drawn from this study:

- The compression response of auxetic structure predicted using the finite element method shows that the link length ratio has the highest influence on the energy absorption properties followed by joint angle interaction effects.
- The analysis shows that for the highest joint angles, the proposed structure evinces the lowest energy absorption capacity. However, it is interesting to note that the joint angles significantly affect the compressive strength but the strength is least affected by the link length ratio.
- It is also clear from the parametric analysis that the structure exhibits the highest Young’s modulus at higher values of $\theta_1$ with the highest value of $\theta_2$ (i.e. 180°), as at the highest value of $\theta_2$ (i.e. 180°) the structure response is analogous to that of the traditional regular re-entrant model and hence it is no more affected by the $l_1:l_2$ ratio.

- The most desirable solution with the highest energy absorption was found to be at $l_1:l_2$ ratio, $\theta_1$ and $\theta_2$ of 1, 40°, and 120° respectively. The crushing analysis indicates that the optimized structure outperforms (+36%) regular re-entrant honeycomb and the improvement can be attributed to the introduction of more nodes with low rotational stiffness. This results in increased compliance of structure and thus the energy absorption capacity.

While energy absorption of the proposed structure is increased, Young’s modulus has slightly decreased and compressive strength has slightly increased. In this regard, spatially material-tailored designs [37,51] of the structure can be explored in the future [52–54] to simultaneously improve all the mechanical properties which are often mutually exclusive.
References


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