All functions take as input the four variables:
$\delta$ - the delay
$\alpha$ - the visibility
$\sigma$ - the spectral width
$\omega$p - the pump frequency

The fixed shift probabilities take additional variables for the shifts. For mode-correlated noise, these are the two variables:
$\epsilon$ - the frequency-dependent shift
$\theta$ - the frequency-independent shift

For mode-uncorrelated noise, there are four shift variables:
$\epsilon$1 - the frequency-dependent shift for the c mode
$\epsilon$2- the frequency-dependent shift for the d mode
$\theta$1 - the frequency-independent shift for the c mode
$\theta$2 - the frequency-independent shift for the d mode

The noisy probabilities and Fisher information instead take the additional variables:
$\eta\epsilon$ - the degree of frequency-dependent noise
$\eta\theta$ - the degree of frequency-independent noise

Variables are generally included in function even if the specific expression had no dependence on certain variables (such as HOM having no $\omega$p dependence, or the lack of visibility dependence in certain configurations)

P1, P2, Pc are the probabilities for detection at detector 1, detector 2, or a coincidence at both detectors respectively
F is the Fisher information

# HOM

## Frequency-entangled photons

### Fixed shift

$$\text{P1HOM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{4}\left(1 + e^{-2(\delta-\epsilon)^2\sigma^2}\alpha\right)$$

$$\text{P2HOM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{4}\left(1 + e^{-2(\delta-\epsilon)^2\sigma^2}\alpha\right)$$

$$\text{PcHOM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{2}\left(1 - e^{-2(\delta-\epsilon)^2\sigma^2}\alpha\right)$$

### Noisy

In[ ]:= $P1HOM\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{4}\left(1 + \frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\alpha}{\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)$

$P2HOM\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{4}\left(1 + \frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\alpha}{\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)$

$PcHOM\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{2}\left(1 - \frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\alpha}{\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)$

In[ ]:= $FHOM\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{16\alpha^2\delta^2\sigma^4}{(1+4\eta\epsilon^2\sigma^2)^2\left(-\alpha^2 + e^{\frac{4\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1+4\eta\epsilon^2\sigma^2)\right)}$

# Frequency-independent photons

### Fixed shift

$P1HOMind[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{4}\left(1 + e^{-(\delta-\epsilon)^2\sigma^2}\alpha\right)$

$P2HOMind[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{4}\left(1 + e^{-(\delta-\epsilon)^2\sigma^2}\alpha\right)$

$PcHOMind[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{2} - \frac{1}{2}e^{-(\delta-\epsilon)^2\sigma^2}\alpha$

### Noisy

In[ ]:= $P1HOMind\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{4}\left(1 + \frac{e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\alpha}{\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)$

$P2HOMind\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{4}\left(1 + \frac{e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\alpha}{\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)$

$PcHOMind\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{2}\left(1 - \frac{e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\alpha}{\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)$

In[ ]:= $FHOMind\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{4\alpha^2\delta^2\sigma^4}{(1+2\eta\epsilon^2\sigma^2)^2\left(-\alpha^2 + e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}(1+2\eta\epsilon^2\sigma^2)\right)}$

# MZ1

### Fixed shift

$$\text{P1sing}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{2}\left(1 - e^{-\frac{1}{2}(\delta-\epsilon)^2\sigma^2}\,\text{Cos}\left[\theta - \frac{(\delta-\epsilon)\,\omega p}{2}\right]\right)$$

$$\text{P2sing}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{2}\left(1 + e^{-\frac{1}{2}(\delta-\epsilon)^2\sigma^2}\,\text{Cos}\left[\theta - \frac{(\delta-\epsilon)\,\omega p}{2}\right]\right)$$

### Noisy

In[•]:= $$\text{P1sing}\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{2}\left(1 - \frac{e^{-\frac{4\left(\eta\theta^2+\left(\delta^2+\eta\epsilon^2\,\eta\theta^2\right)\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\,\sigma^2}}\right)$$

In[•]:= $$\text{P2sing}\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{1}{2}\left(1 + \frac{e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\,\sigma^2}}\right)$$

In[•]:= $$\text{Fsing}\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] := \frac{\left(2\,\delta\,\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right] + \omega p\,\text{Sin}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]\right)^2}{4\left(1+\eta\epsilon^2\,\sigma^2\right)^2\left(e^{\frac{4\left(\eta\theta^2+\left(\delta^2+\eta\epsilon^2\,\eta\theta^2\right)\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\,\sigma^2}}\left(1+\eta\epsilon^2\,\sigma^2\right) - \text{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]^2\right)}$$

# MZ2s

## Mode-correlated noise

## Frequency-entangled photons

### Fixed shift

$$\text{P1s}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{8}\left(2 + e^{-2(\delta-\epsilon)^2\sigma^2} + \text{Cos}[2\,\theta - \delta\,\omega p + \epsilon\,\omega p] - 4\,e^{-\frac{1}{2}(\delta-\epsilon)^2\sigma^2}\,\text{Cos}\left[\theta + \frac{1}{2}(-\delta+\epsilon)\,\omega p\right]\right)$$

$$\text{P2s}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{8}\left(2 + e^{-2(\delta-\epsilon)^2\sigma^2} + \text{Cos}[2\,\theta - \delta\,\omega p + \epsilon\,\omega p] + 4\,e^{-\frac{1}{2}(\delta-\epsilon)^2\sigma^2}\,\text{Cos}\left[\theta + \frac{1}{2}(-\delta+\epsilon)\,\omega p\right]\right)$$

$$\text{Pcs}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon\_, \theta\_] := \frac{1}{4}\left(2 - e^{-2(\delta-\epsilon)^2\sigma^2} - \text{Cos}[2\,\theta + (-\delta+\epsilon)\,\omega p]\right)$$

## Noisy

In[•]:= P1sη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=

$$\frac{1}{8}\left(2+\frac{e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}}{\sqrt{1+4\,\eta\epsilon^2\,\sigma^2}}+e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\mathrm{Cos}[\delta\,\omega p]-\frac{4\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\,\sigma^2}}\right)$$

In[•]:= P2sη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=

$$\frac{1}{8}\left(2+\frac{e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}}{\sqrt{1+4\,\eta\epsilon^2\,\sigma^2}}+e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\mathrm{Cos}[\delta\,\omega p]+\frac{4\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\,\sigma^2}}\right)$$

In[•]:= Pcsη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=

$$\frac{1}{4}\left(2-\frac{e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}}{\sqrt{1+4\,\eta\epsilon^2\,\sigma^2}}-e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\mathrm{Cos}[\delta\,\omega p]\right)$$

In[•]:= Fsη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=

$$\frac{\left(\frac{4\,e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}\,\delta\,\sigma^2}{(1+4\,\eta\epsilon^2\,\sigma^2)^{3/2}}+e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\omega p\,\mathrm{Sin}[\delta\,\omega p]\right)^2}{4\left(2-\frac{e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}}{\sqrt{1+4\,\eta\epsilon^2\,\sigma^2}}-e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\mathrm{Cos}[\delta\,\omega p]\right)}+$$

$$\left(\frac{4\,e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}\,\delta\,\sigma^2}{(1+4\,\eta\epsilon^2\,\sigma^2)^{3/2}}-\frac{4\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{(1+\eta\epsilon^2\,\sigma^2)^{3/2}}+\right.$$

$$\left.e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\omega p\,\mathrm{Sin}[\delta\,\omega p]-\frac{2\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{(1+\eta\epsilon^2\,\sigma^2)^{3/2}}\right)^2\Big/$$

$$\left(8\left(2+\frac{e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}}{\sqrt{1+4\,\eta\epsilon^2\,\sigma^2}}+e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\mathrm{Cos}[\delta\,\omega p]-\frac{4\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\,\sigma^2}}\right)\right)+$$

$$\left(\frac{4\,e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}\,\delta\,\sigma^2}{(1+4\,\eta\epsilon^2\,\sigma^2)^{3/2}}+\frac{4\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{(1+\eta\epsilon^2\,\sigma^2)^{3/2}}+\right.$$

$$\left.e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\omega p\,\mathrm{Sin}[\delta\,\omega p]+\frac{2\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{(1+\eta\epsilon^2\,\sigma^2)^{3/2}}\right)^2\Big/$$

$$\left(8\left(2+\frac{e^{-\frac{2\,\delta^2\,\sigma^2}{1+4\,\eta\epsilon^2\,\sigma^2}}}{\sqrt{1+4\,\eta\epsilon^2\,\sigma^2}}+e^{-2\,\eta\theta^2-\frac{\eta\epsilon^2\,\omega p^2}{2}}\,\mathrm{Cos}[\delta\,\omega p]+\frac{4\,e^{-\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\,\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\,\sigma^2}}\right)\right)$$

# Frequency-independent photons

## Fixed shift

In[•]:=
$$\text{P1sind}[\delta\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_] := \frac{1}{4} e^{-(\delta^2+\epsilon^2)\sigma^2}\left(e^{\frac{1}{2}(\delta^2+\epsilon^2)\sigma^2} - e^{\delta\epsilon\sigma^2}\,\text{Cos}\left[\theta+\frac{1}{2}(-\delta+\epsilon)\,\omega p\right]\right)^2$$

In[•]:=
$$\text{P2sind}[\delta\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_] := \frac{1}{4} e^{-(\delta^2+\epsilon^2)\sigma^2}\left(e^{\frac{1}{2}(\delta^2+\epsilon^2)\sigma^2} + e^{\delta\epsilon\sigma^2}\,\text{Cos}\left[\theta+\frac{1}{2}(-\delta+\epsilon)\,\omega p\right]\right)^2$$

In[•]:=
$$\text{Pcsind}[\delta\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_] := -\frac{1}{4} e^{-(\delta^2+\epsilon^2)\sigma^2}\left(e^{2\delta\epsilon\sigma^2} - 2\,e^{(\delta^2+\epsilon^2)\sigma^2} + e^{2\delta\epsilon\sigma^2}\,\text{Cos}\left[2\theta+(-\delta+\epsilon)\,\omega p\right]\right)$$

## Noisy

In[•]:=
$$\text{P1sind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \left(e^{-\frac{\delta^2\sigma^2(3+2\eta\epsilon^2\sigma^2)+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{\frac{\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\delta^2(\sigma^2+2\eta\epsilon^2\sigma^4)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)+ e^{\frac{\delta^2\sigma^2}{2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right] - 4\,e^{\frac{12\eta\theta^2(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+4\delta^2\sigma^2(2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+\eta\epsilon^2(3+2\eta\epsilon^2\sigma^2)\omega p^2}{8(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)}}\sqrt{1+2\eta\epsilon^2\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\right)\right)\Big/\left(8\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)$$

In[•]:=
$$\text{P2sind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \left(e^{-\frac{\delta^2\sigma^2(3+2\eta\epsilon^2\sigma^2)+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{\frac{\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\delta^2(\sigma^2+2\eta\epsilon^2\sigma^4)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)+ e^{\frac{\delta^2\sigma^2}{2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right] + 4\,e^{\frac{12\eta\theta^2(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+4\delta^2\sigma^2(2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+\eta\epsilon^2(3+2\eta\epsilon^2\sigma^2)\omega p^2}{8(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)}}\sqrt{1+2\eta\epsilon^2\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\right)\right)\Big/\left(8\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)$$

In[•]:=
$$\text{Pcsind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \frac{1}{4}\left(2-\frac{e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}}{\sqrt{1+2\eta\epsilon^2\sigma^2}} - \frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)$$

In[•]:=
$$\text{Fsind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] :=$$

$$\frac{\left(\frac{2\,e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\delta\sigma^2}{(1+2\eta\epsilon^2\sigma^2)^{3/2}} + \frac{4\,e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\delta\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}\,(2+4\eta\epsilon^2\sigma^2)} + \frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\omega p\,\text{Sin}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{(1+2\eta\epsilon^2\sigma^2)^{3/2}}\right)^2}{4\left(2-\frac{e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}}{\sqrt{1+2\eta\epsilon^2\sigma^2}} - \frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)} +$$

$$\Bigg(8\,e^{\frac{\delta^2\sigma^2(3+2\eta\epsilon^2\sigma^2)+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\eta\epsilon^2\sigma^2}$$

$$\Bigg(-\frac{1}{4\sqrt{1+\eta\epsilon^2\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\left(2+4\,\eta\epsilon^2\sigma^2\right)}\,e^{-\frac{\delta^2\sigma^2\left(3+2\eta\epsilon^2\sigma^2\right)+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\left(3+2\,\eta\epsilon^2\sigma^2\right)$$

$$\Bigg(e^{\frac{\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\delta^2\left(\sigma^2+2\eta\epsilon^2\sigma^4\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+e^{\frac{\delta^2\sigma^2}{2}}$$

$$\sqrt{1+\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\Big[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\Big]+4\,e^{\frac{12\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(3+2\eta\epsilon^2\sigma^2\right)\omega p^2}{8\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)}}$$

$$\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\Big[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\Big]\Bigg)+\frac{1}{8\sqrt{1+\eta\epsilon^2\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}}$$

$$e^{-\frac{\delta^2\sigma^2\left(3+2\eta\epsilon^2\sigma^2\right)+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\Bigg(\frac{4\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}+\frac{\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\delta^2\left(\sigma^2+2\eta\epsilon^2\sigma^4\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\sqrt{1+\eta\epsilon^2\sigma^2}}{\sqrt{1+2\,\eta\epsilon^2\sigma^2}}+$$

$$\frac{1}{2+4\,\eta\epsilon^2\sigma^2}2\,e^{\frac{\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\delta^2\left(\sigma^2+2\eta\epsilon^2\sigma^4\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\delta\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(\sigma^2+2\,\eta\epsilon^2\sigma^4\right)$$

$$\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+e^{\frac{\delta^2\sigma^2}{2}}\,\delta\,\sigma^2\sqrt{1+\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\Big[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\Big]+$$

$$\Bigg(4\,e^{\frac{12\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(3+2\eta\epsilon^2\sigma^2\right)\omega p^2}{8\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)}}\,\delta\,\sigma^2\sqrt{1+2\,\eta\epsilon^2\sigma^2}$$

$$\left(2+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)\mathrm{Cos}\Big[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\Big]\Bigg)\Big/$$

$$\left(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)-\frac{e^{\frac{\delta^2\sigma^2}{2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\omega p\,\mathrm{Sin}\Big[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\Big]}{1+2\,\eta\epsilon^2\sigma^2}-$$

$$\frac{4\,e^{\frac{12\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(3+2\eta\epsilon^2\sigma^2\right)\omega p^2}{8\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\omega p\,\mathrm{Sin}\Big[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\Big]}{2+2\,\eta\epsilon^2\sigma^2}\Bigg)\Bigg)^2\Bigg)$$

$$\Big/\Bigg(e^{\frac{\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\delta^2\left(\sigma^2+2\eta\epsilon^2\sigma^4\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+$$

$$e^{\frac{\delta^2\sigma^2}{2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\Big[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\Big]+$$

$$4\,e^{\frac{12\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(3+2\eta\epsilon^2\sigma^2\right)\omega p^2}{8\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\Big[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\Big]\Bigg)+$$

$$\Bigg(8\,e^{\frac{\delta^2\sigma^2\left(3+2\eta\epsilon^2\sigma^2\right)+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}$$

$$\Bigg(-\frac{1}{4\sqrt{1+\eta\epsilon^2\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\left(2+4\,\eta\epsilon^2\sigma^2\right)}\,e^{-\frac{\delta^2\sigma^2\left(3+2\eta\epsilon^2\sigma^2\right)+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\left(3+2\,\eta\epsilon^2\sigma^2\right)$$

$$\Bigg(e^{\frac{\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\delta^2\left(\sigma^2+2\eta\epsilon^2\sigma^4\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+e^{\frac{\delta^2\sigma^2}{2}}$$

$$\sqrt{1+\eta\epsilon^2\sigma^2}\ \mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]-4\,e^{\frac{12\,\eta\theta^2\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+4\,\delta^2\sigma^2\,(2+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+\eta\epsilon^2\,(3+2\,\eta\epsilon^2\sigma^2)\,\omega p^2}{8\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)}}$$

$$\left.\left.\sqrt{1+2\,\eta\epsilon^2\sigma^2}\ \mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\right]\right)\right)+\frac{1}{8\,\sqrt{1+\eta\epsilon^2\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}}$$

$$e^{-\frac{\delta^2\sigma^2\,(3+2\,\eta\epsilon^2\sigma^2)+\eta\theta^2\,(4+8\,\eta\epsilon^2\sigma^2)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\left(\frac{4\,e^{\frac{\delta^2\sigma^2}{1+2\,\eta\epsilon^2\sigma^2}+\frac{\eta\theta^2\,(4+8\,\eta\epsilon^2\sigma^2)+\delta^2\,(\sigma^2+2\,\eta\epsilon^2\sigma^4)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\,\sqrt{1+\eta\epsilon^2\sigma^2}}{\sqrt{1+2\,\eta\epsilon^2\sigma^2}}+\right.$$

$$\frac{1}{2+4\,\eta\epsilon^2\sigma^2}2\,e^{\frac{\eta\theta^2\,(4+8\,\eta\epsilon^2\sigma^2)+\delta^2\,(\sigma^2+2\,\eta\epsilon^2\sigma^4)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\,\delta\,\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(\sigma^2+2\,\eta\epsilon^2\sigma^4\right)$$

$$\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\,\eta\epsilon^2\sigma^2}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+e^{\frac{\delta^2\sigma^2}{2}}\,\delta\,\sigma^2\,\sqrt{1+\eta\epsilon^2\sigma^2}\ \mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]-$$

$$\left(4\,e^{\frac{12\,\eta\theta^2\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+4\,\delta^2\sigma^2\,(2+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+\eta\epsilon^2\,(3+2\,\eta\epsilon^2\sigma^2)\,\omega p^2}{8\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)}}\,\delta\,\sigma^2\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right.$$

$$\left.\left(2+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\right]\right)\Big/$$

$$\left(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)-\frac{e^{\frac{\delta^2\sigma^2}{2}}\,\sqrt{1+\eta\epsilon^2\sigma^2}\ \omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]}{1+2\,\eta\epsilon^2\sigma^2}+$$

$$\left.\left.\frac{4\,e^{\frac{12\,\eta\theta^2\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+4\,\delta^2\sigma^2\,(2+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+\eta\epsilon^2\,(3+2\,\eta\epsilon^2\sigma^2)\,\omega p^2}{8\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)}}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\ \omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\right]}{2+2\,\eta\epsilon^2\sigma^2}\right)\right)^2$$

$$\Big/\left(e^{\frac{\eta\theta^2\,(4+8\,\eta\epsilon^2\sigma^2)+\delta^2\,(\sigma^2+2\,\eta\epsilon^2\sigma^4)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\,\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(1+2\,e^{\frac{\delta^2\sigma^2}{1+2\,\eta\epsilon^2\sigma^2}}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+\right.$$

$$e^{\frac{\delta^2\sigma^2}{2}}\,\sqrt{1+\eta\epsilon^2\sigma^2}\ \mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]-$$

$$\left.4\,e^{\frac{12\,\eta\theta^2\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+4\,\delta^2\sigma^2\,(2+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+\eta\epsilon^2\,(3+2\,\eta\epsilon^2\sigma^2)\,\omega p^2}{8\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)}}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\ \mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\right]\right)$$

# Mode-uncorrelated noise

## Frequency-entangled photons

### Fixed shift

In[•]:=

$$
\begin{aligned}
&\text{P1s}\mu[\delta\_,\ \alpha\_,\ \sigma\_,\ \omega\text{p}\_,\ \epsilon1\_,\ \epsilon2\_,\ \theta1\_,\ \theta2\_] := \frac{1}{8\,(1+\alpha)}\, e^{-\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2} \\
&\Big(2\,e^{\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2} + 2\,e^{\frac{1}{2}\left(8\,\delta\,\epsilon1+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha + 2\,e^{\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha - \\
&\quad 2\,e^{\frac{1}{2}\left(3\,\delta^2+2\,\delta\,\epsilon1+3\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,(1+3\,\alpha)\,\text{Cos}\left[\theta1+\frac{1}{2}\,(-\delta+\epsilon1)\,\omega\text{p}\right] + \\
&\quad 2\,e^{\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha\,\text{Cos}\left[2\,\theta1+(-\delta+\epsilon1)\,\omega\text{p}\right] + e^{\frac{1}{2}\left(3\,\epsilon1^2+4\,\delta\,(\epsilon1+\epsilon2)\right)\sigma^2} \\
&\quad \text{Cos}\left[\theta1-\theta2+\frac{1}{2}\,(\epsilon1-\epsilon2)\,\omega\text{p}\right] - e^{\frac{1}{2}\left(3\,\epsilon1^2+4\,\delta\,(\epsilon1+\epsilon2)\right)\sigma^2}\,\alpha\,\text{Cos}\left[\theta1-\theta2+\frac{1}{2}\,(\epsilon1-\epsilon2)\,\omega\text{p}\right] - \\
&\quad 2\,e^{\frac{1}{2}\left(3\,\delta^2+2\,\delta\,\epsilon2+2\,\epsilon1\,(2\,\epsilon1+\epsilon2)\right)\sigma^2}\,\text{Cos}\left[\theta2+\frac{1}{2}\,(-\delta+\epsilon2)\,\omega\text{p}\right] + 2\,e^{\frac{1}{2}\left(3\,\delta^2+2\,\delta\,\epsilon2+2\,\epsilon1\,(2\,\epsilon1+\epsilon2)\right)\sigma^2}\,\alpha \\
&\quad \text{Cos}\left[\theta2+\frac{1}{2}\,(-\delta+\epsilon2)\,\omega\text{p}\right] + e^{\frac{1}{2}\left(4\,\delta^2+3\,\epsilon1^2+4\,\epsilon1\,\epsilon2\right)\sigma^2}\,\text{Cos}\left[\theta1+\theta2+\frac{1}{2}\,(-2\,\delta+\epsilon1+\epsilon2)\,\omega\text{p}\right] - \\
&\quad e^{\frac{1}{2}\left(4\,\delta^2+3\,\epsilon1^2+4\,\epsilon1\,\epsilon2\right)\sigma^2}\,\alpha\,\text{Cos}\left[\theta1+\theta2+\frac{1}{2}\,(-2\,\delta+\epsilon1+\epsilon2)\,\omega\text{p}\right]\Big)
\end{aligned}
$$

In[•]:=

$$
\begin{aligned}
&\text{P2s}\mu[\delta\_,\ \alpha\_,\ \sigma\_,\ \omega\text{p}\_,\ \epsilon1\_,\ \epsilon2\_,\ \theta1\_,\ \theta2\_] := \frac{1}{8\,(1+\alpha)}\, e^{-\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2} \\
&\Big(2\,e^{\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2} + 2\,e^{\frac{1}{2}\left(8\,\delta\,\epsilon1+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha + 2\,e^{\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha + \\
&\quad 2\,e^{\frac{1}{2}\left(3\,\delta^2+2\,\delta\,\epsilon1+3\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,(1+3\,\alpha)\,\text{Cos}\left[\theta1+\frac{1}{2}\,(-\delta+\epsilon1)\,\omega\text{p}\right] + \\
&\quad 2\,e^{\frac{1}{2}\left(4\,\delta^2+4\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha\,\text{Cos}\left[2\,\theta1+(-\delta+\epsilon1)\,\omega\text{p}\right] + e^{\frac{1}{2}\left(3\,\epsilon1^2+4\,\delta\,(\epsilon1+\epsilon2)\right)\sigma^2} \\
&\quad \text{Cos}\left[\theta1-\theta2+\frac{1}{2}\,(\epsilon1-\epsilon2)\,\omega\text{p}\right] - e^{\frac{1}{2}\left(3\,\epsilon1^2+4\,\delta\,(\epsilon1+\epsilon2)\right)\sigma^2}\,\alpha\,\text{Cos}\left[\theta1-\theta2+\frac{1}{2}\,(\epsilon1-\epsilon2)\,\omega\text{p}\right] + \\
&\quad 2\,e^{\frac{1}{2}\left(3\,\delta^2+2\,\delta\,\epsilon2+2\,\epsilon1\,(2\,\epsilon1+\epsilon2)\right)\sigma^2}\,\text{Cos}\left[\theta2+\frac{1}{2}\,(-\delta+\epsilon2)\,\omega\text{p}\right] - 2\,e^{\frac{1}{2}\left(3\,\delta^2+2\,\delta\,\epsilon2+2\,\epsilon1\,(2\,\epsilon1+\epsilon2)\right)\sigma^2}\,\alpha \\
&\quad \text{Cos}\left[\theta2+\frac{1}{2}\,(-\delta+\epsilon2)\,\omega\text{p}\right] + e^{\frac{1}{2}\left(4\,\delta^2+3\,\epsilon1^2+4\,\epsilon1\,\epsilon2\right)\sigma^2}\,\text{Cos}\left[\theta1+\theta2+\frac{1}{2}\,(-2\,\delta+\epsilon1+\epsilon2)\,\omega\text{p}\right] - \\
&\quad e^{\frac{1}{2}\left(4\,\delta^2+3\,\epsilon1^2+4\,\epsilon1\,\epsilon2\right)\sigma^2}\,\alpha\,\text{Cos}\left[\theta1+\theta2+\frac{1}{2}\,(-2\,\delta+\epsilon1+\epsilon2)\,\omega\text{p}\right]\Big)
\end{aligned}
$$

In[•]:=

$$
\begin{aligned}
&\text{Pcs}\mu[\delta\_,\ \alpha\_,\ \sigma\_,\ \omega\text{p}\_,\ \epsilon1\_,\ \epsilon2\_,\ \theta1\_,\ \theta2\_] := \frac{1}{4\,(1+\alpha)}\, e^{-\frac{1}{2}\left(8\,\delta^2-8\,\delta\,\epsilon1+5\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2} \\
&\Big(2\,e^{\frac{1}{2}\left(8\,\delta^2-8\,\delta\,\epsilon1+5\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2} + 2\,e^{\frac{1}{2}\left(8\,\delta^2-8\,\delta\,\epsilon1+5\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha - 2\,e^{\frac{1}{2}\left(4\,\delta^2+(\epsilon1+\epsilon2)^2\right)\sigma^2}\,\alpha - \\
&\quad 2\,e^{\frac{1}{2}\left(8\,\delta^2-8\,\delta\,\epsilon1+5\,\epsilon1^2+2\,\epsilon1\,\epsilon2+\epsilon2^2\right)\sigma^2}\,\alpha\,\text{Cos}\left[2\,\theta1+(-\delta+\epsilon1)\,\omega\text{p}\right] + e^{2\left(\delta^2-\delta\,\epsilon1+\epsilon1^2+\delta\,\epsilon2\right)\sigma^2}\,(-1+\alpha) \\
&\quad \text{Cos}\left[\theta1-\theta2+\frac{1}{2}\,(\epsilon1-\epsilon2)\,\omega\text{p}\right] - e^{2\left(\delta^2+(\delta-\epsilon1)^2+\epsilon1\,\epsilon2\right)\sigma^2}\,\text{Cos}\left[\theta1+\theta2+\frac{1}{2}\,(-2\,\delta+\epsilon1+\epsilon2)\,\omega\text{p}\right] + \\
&\quad e^{2\left(\delta^2+(\delta-\epsilon1)^2+\epsilon1\,\epsilon2\right)\sigma^2}\,\alpha\,\text{Cos}\left[\theta1+\theta2+\frac{1}{2}\,(-2\,\delta+\epsilon1+\epsilon2)\,\omega\text{p}\right]\Big)
\end{aligned}
$$

## Noisy

In[∘]:= $$\text{P1s}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \frac{1}{4+4\alpha}+\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\alpha\left(\frac{1}{4+4\alpha}-\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}}{(4+4\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)+\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(-e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)+2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{2\sqrt{1+\eta\epsilon^2\sigma^2}}$$

In[∘]:= $$\text{P2s}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \frac{1}{4+4\alpha}+\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\alpha\left(\frac{1}{4+4\alpha}-\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}}{(4+4\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)+\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(-e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)+2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{2\sqrt{1+\eta\epsilon^2\sigma^2}}$$

In[∘]:= $$\text{Pcs}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \frac{1}{2+2\alpha}-\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\alpha\left(\frac{1}{2+2\alpha}+\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}-\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}}{(2+2\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)+\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}$$

In[∘]:= $$\text{Fs}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] := \left(\frac{4\,e^{-\frac{8\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}\delta\,\sigma^2}{(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}\,(4+8\eta\epsilon^2\sigma^2)}+\right.$$

$$\alpha\left(\frac{4\,e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(2+2\alpha)\left(1+4\eta\epsilon^2\sigma^2\right)^{3/2}}-\frac{4\,e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}\left(4+8\eta\epsilon^2\sigma^2\right)}\right)-$$

$$\left.\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\omega p\,\mathrm{Sin}[\delta\,\omega p]}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)^2\Big/$$

$$\left(\frac{1}{2+2\alpha}-\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\right.$$

$$\alpha\left(\frac{1}{2+2\alpha}+\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}-\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}}{(2+2\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)+$$

$$\left.\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\mathrm{Cos}[\delta\,\omega p]}{4(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)+$$

$$\left(-\frac{2\,e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}\left(4+8\eta\epsilon^2\sigma^2\right)}+\alpha\left(-\frac{4\,e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(4+4\alpha)\left(1+4\eta\epsilon^2\sigma^2\right)^{3/2}}+\right.\right.$$

$$\left.\frac{2\,e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}\left(4+8\eta\epsilon^2\sigma^2\right)}\right)-\frac{4\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\left(8+8\eta\epsilon^2\sigma^2\right)}-$$

$$\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(-e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)+2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\omega p\,\mathrm{Sin}[\delta\,\omega p]}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}-$$

$$\left.\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{2\sqrt{1+\eta\epsilon^2\sigma^2}\left(2+2\eta\epsilon^2\sigma^2\right)}\right)^2\Big/\left(\frac{1}{4+4\alpha}+\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\right.$$

$$\alpha\left(\frac{1}{4+4\alpha}-\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}}{(4+4\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}}\right)+$$

$$\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(-e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)+2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\mathrm{Cos}[\delta\,\omega p]}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+$$

$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{2\sqrt{1+\eta\epsilon^2\sigma^2}}\Bigg) +$$

$$\Bigg(-\frac{2\,e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}\,\left(4+8\eta\epsilon^2\sigma^2\right)}+\alpha\Bigg(-\frac{4\,e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(4+4\alpha)\left(1+4\eta\epsilon^2\sigma^2\right)^{3/2}} +$$

$$\frac{2\,e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}\,\left(4+8\eta\epsilon^2\sigma^2\right)}\Bigg)+\frac{4\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(8+8\eta\epsilon^2\sigma^2\right)} -$$

$$\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(-e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)+2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\omega p\,\text{Sin}[\delta\,\omega p]}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}} +$$

$$\left(\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\omega p\,\text{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{2\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(2+2\eta\epsilon^2\sigma^2\right)}\right)^2\Bigg/\Bigg(\frac{1}{4+4\alpha}+\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}} +$$

$$\alpha\left(\frac{1}{4+4\alpha}-\frac{e^{-\frac{8\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+8\eta\epsilon^2\sigma^2}}}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}}+\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}}{(4+4\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}}\right) +$$

$$\frac{e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}\left(-e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)+2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]}{8(1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}} -$$

$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{2\sqrt{1+\eta\epsilon^2\sigma^2}}\Bigg)$$

# Frequency-independent photons

## Fixed shift

In[∘]:=

$$\text{P1sind}\mu[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon 1\_,\epsilon 2\_,\theta 1\_,\theta 2\_] := \frac{1}{4(1+\alpha)}$$
$$e^{-\frac{1}{2}\left(2\delta^2+2\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\left(e^{\frac{1}{2}\left(\delta^2+\epsilon 1^2\right)\sigma^2}-e^{\delta\,\epsilon 1\,\sigma^2}\,\text{Cos}\left[\theta 1+\frac{1}{2}(-\delta+\epsilon 1)\,\omega p\right]\right)$$
$$\left(e^{\frac{1}{2}\left(\delta^2+\epsilon 1^2+\epsilon 2^2\right)\sigma^2}(1+\alpha)-2\,e^{\frac{1}{2}\left(2\delta\,\epsilon 1+\epsilon 2^2\right)\sigma^2}\,\alpha\,\text{Cos}\left[\theta 1+\frac{1}{2}(-\delta+\epsilon 1)\,\omega p\right]+\right.$$
$$\left.e^{\frac{1}{2}\left(\epsilon 1^2+2\delta\,\epsilon 2\right)\sigma^2}(-1+\alpha)\,\text{Cos}\left[\theta 2+\frac{1}{2}(-\delta+\epsilon 2)\,\omega p\right]\right)$$

In[•]:= 

$$\mathrm{P2sind}\mu[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon 1\_,\epsilon 2\_,\theta 1\_,\theta 2\_] := \frac{1}{4\,(1+\alpha)}$$

$$e^{-\frac{1}{2}\left(2\,\delta^2+2\,\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\left(e^{\frac{1}{2}\left(\delta^2+\epsilon 1^2\right)\sigma^2}+e^{\delta\,\epsilon 1\,\sigma^2}\,\mathrm{Cos}\left[\theta 1+\frac{1}{2}\left(-\delta+\epsilon 1\right)\omega p\right]\right)$$

$$\left(e^{\frac{1}{2}\left(\delta^2+\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\left(1+\alpha\right)+2\,e^{\frac{1}{2}\left(2\,\delta\,\epsilon 1+\epsilon 2^2\right)\sigma^2}\,\alpha\,\mathrm{Cos}\left[\theta 1+\frac{1}{2}\left(-\delta+\epsilon 1\right)\omega p\right]-\right.$$

$$\left.e^{\frac{1}{2}\left(\epsilon 1^2+2\,\delta\,\epsilon 2\right)\sigma^2}\left(-1+\alpha\right)\mathrm{Cos}\left[\theta 2+\frac{1}{2}\left(-\delta+\epsilon 2\right)\omega p\right]\right)$$

In[•]:= 

$$\mathrm{Pcsind}\mu[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon 1\_,\epsilon 2\_,\theta 1\_,\theta 2\_] :=$$

$$\frac{1}{4\,(1+\alpha)}\,e^{-\frac{1}{2}\left(4\,\delta^2-4\,\delta\,\epsilon 1+3\,\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\left(2\,e^{\frac{1}{2}\left(4\,\delta^2-4\,\delta\,\epsilon 1+3\,\epsilon 1^2+\epsilon 2^2\right)\sigma^2}-2\,e^{\frac{1}{2}\left(2\,\delta^2+\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\,\alpha+\right.$$

$$2\,e^{\frac{1}{2}\left(4\,\delta^2-4\,\delta\,\epsilon 1+3\,\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\,\alpha-2\,e^{\frac{1}{2}\left(2\,\delta^2+\epsilon 1^2+\epsilon 2^2\right)\sigma^2}\,\alpha\,\mathrm{Cos}\left[2\,\theta 1+\left(-\delta+\epsilon 1\right)\omega p\right]+$$

$$e^{\left(\delta^2-\delta\,\epsilon 1+\epsilon 1^2+\delta\,\epsilon 2\right)\sigma^2}\left(-1+\alpha\right)\mathrm{Cos}\left[\theta 1-\theta 2+\frac{1}{2}\left(\epsilon 1-\epsilon 2\right)\omega p\right]-$$

$$e^{\left(\delta^2-\delta\,\epsilon 1+\epsilon 1^2+\delta\,\epsilon 2\right)\sigma^2}\,\mathrm{Cos}\left[\theta 1+\theta 2+\frac{1}{2}\left(-2\,\delta+\epsilon 1+\epsilon 2\right)\omega p\right]+$$

$$\left.e^{\left(\delta^2-\delta\,\epsilon 1+\epsilon 1^2+\delta\,\epsilon 2\right)\sigma^2}\,\alpha\,\mathrm{Cos}\left[\theta 1+\theta 2+\frac{1}{2}\left(-2\,\delta+\epsilon 1+\epsilon 2\right)\omega p\right]\right)$$

## Noisy

In[•]:= 

$$\mathrm{P1sind}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_] :=$$

$$\frac{1}{16\,(1+\alpha)\left(1+\eta\epsilon^2\,\sigma^2\right)^{3/2}\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}}\,e^{-\frac{8\,\eta\theta^2\left(1+3\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)+4\,\delta^2\,\sigma^2\left(3+6\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)+\eta\epsilon^2\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}$$

$$\left(-2\,e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}\left(-1+\alpha\right)\sqrt{1+\eta\epsilon^2\,\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\,\sigma^2}\right]+\right.$$

$$4\,e^{\frac{4\,\delta^2\,\sigma^2\left(2+5\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\left(1+\eta\epsilon^2\,\sigma^2\right)^{3/2}\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\,\sigma^2}\right]+$$

$$2\,e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}\left(\sqrt{1+\eta\epsilon^2\,\sigma^2}\left(2\,e^{\eta\theta^2+\frac{\eta\epsilon^2\left(4\,\delta^2\,\sigma^4+\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\omega p^2\right)}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\left(1+\eta\epsilon^2\,\sigma^2\right)-\right.\right.$$

$$\left.\left(-1+\alpha\right)\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}+2\,e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\,\sigma^2}}\left(1+\alpha\right)\left(1+\eta\epsilon^2\,\sigma^2\right)\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\right)-$$

$$\left.\left.4\,e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\left(1+\alpha\right)\left(1+\eta\epsilon^2\,\sigma^2\right)\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]\right)\right)$$

In[•]:= **P2sindμη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=**

$$\frac{1}{16\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\sqrt{1+2\,\eta\epsilon^2\sigma^2}}\,\mathrm{e}^{-\frac{8\,\eta\theta^2\,\left(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)+4\,\delta^2\sigma^2\,\left(3+6\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\,\left(1+2\,\eta\epsilon^2\sigma^2\right)\,\omega p^2}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}$$

$$\left(-2\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\,\sqrt{1+\eta\epsilon^2\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]+\right.$$

$$4\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2\,\left(2+5\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4\right)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]+$$

$$2\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\,\eta\epsilon^2\sigma^2}}\,\left(\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(2\,\mathrm{e}^{\eta\theta^2+\frac{\eta\epsilon^2\,\left(4\,\delta^2\sigma^4+\left(1+2\,\eta\epsilon^2\sigma^2\right)\,\omega p^2\right)}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\sigma^2\right)-\right.\right.$$

$$\left.(-1+\alpha)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}+2\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\right)+$$

$$\left.\left.4\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\right]\right)\right)$$

In[•]:= **Pcsindμη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=**

$$\frac{1}{2+2\,\alpha}+\frac{\alpha}{2+2\,\alpha}-\frac{\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}}{4\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)}+\frac{\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,\alpha}{4\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)}-\frac{\mathrm{e}^{-\frac{\delta^2\sigma^2}{1+2\,\eta\epsilon^2\sigma^2}}\,\alpha}{(2+2\,\alpha)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}}+$$

$$\frac{\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{4\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)}-\frac{\mathrm{e}^{-\frac{2\,\delta^2\sigma^2+\eta\theta^2\,\left(4+8\,\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\,\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]}{2\,(1+\alpha)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}}$$

In[•]:= **Fsindμη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=**

$$\left(\frac{2\,\mathrm{e}^{-\frac{\delta^2\sigma^2}{1+2\,\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2}{(2+2\,\alpha)\,\left(1+2\,\eta\epsilon^2\sigma^2\right)^{3/2}}+\frac{2\,\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)\,\left(4+4\,\eta\epsilon^2\sigma^2\right)}-\right.$$

$$\frac{2\,\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2}{(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)\,\left(4+4\,\eta\epsilon^2\sigma^2\right)}-\frac{2\,\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)\,\left(4+4\,\eta\epsilon^2\sigma^2\right)}+$$

$$\frac{2\,\mathrm{e}^{-\frac{2\,\delta^2\sigma^2+\eta\theta^2\,\left(4+8\,\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]}{(1+\alpha)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\left(2+4\,\eta\epsilon^2\sigma^2\right)}-$$

$$\frac{\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{4\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)^2}+$$

$$\left.\frac{\mathrm{e}^{-\frac{2\,\delta^2\sigma^2+\eta\theta^2\,\left(4+8\,\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{2+4\,\eta\epsilon^2\sigma^2}}\,\alpha\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\right]}{2\,(1+\alpha)\,\left(1+2\,\eta\epsilon^2\sigma^2\right)^{3/2}}\right)^2\Big/$$

$$\left(\frac{1}{2+2\,\alpha}+\frac{\alpha}{2+2\,\alpha}-\frac{\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}}{4\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)}+\frac{\mathrm{e}^{-\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,\alpha}{4\,(1+\alpha)\,\left(1+\eta\epsilon^2\sigma^2\right)}-\frac{\mathrm{e}^{-\frac{\delta^2\sigma^2}{1+2\,\eta\epsilon^2\sigma^2}}\,\alpha}{(2+2\,\alpha)\,\sqrt{1+2\,\eta\epsilon^2\sigma^2}}+\right.$$

$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\operatorname{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{4\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)}-\frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\alpha\operatorname{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{2\left(1+\alpha\right)\sqrt{1+2\eta\epsilon^2\sigma^2}}\Bigg)+$$

$$\Bigg(16\,e^{\frac{8\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(3+6\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\sqrt{1+2\eta\epsilon^2\sigma^2}$$

$$\Bigg(-\Bigg(\Bigg(e^{-\frac{8\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(3+6\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\delta\,\sigma^2\left(3+6\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)$$

$$\Bigg(-2\,e^{\eta\theta^2+\frac{2\delta^2\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\eta\epsilon^2\sigma^2}\operatorname{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]+$$

$$4\,e^{\frac{4\delta^2\sigma^2\left(2+5\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)-\eta\epsilon^2\omega p^2}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\alpha\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\operatorname{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]+$$

$$2\,e^{\eta\theta^2+\frac{2\delta^2\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\eta\epsilon^2\sigma^2}}\Bigg(\sqrt{1+\eta\epsilon^2\sigma^2}\Bigg(2\,e^{\eta\theta^2+\frac{\eta\epsilon^2\left(4\delta^2\sigma^4+\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2\right)}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\alpha\left(1+\eta\epsilon^2\sigma^2\right)-$$

$$\left(-1+\alpha\right)\sqrt{1+2\eta\epsilon^2\sigma^2}+2\,e^{\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(1+\alpha\right)$$

$$\left(1+\eta\epsilon^2\sigma^2\right)\sqrt{1+2\eta\epsilon^2\sigma^2}\Bigg)+4\,e^{\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}$$

$$\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)\sqrt{1+2\eta\epsilon^2\sigma^2}\operatorname{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\Bigg)\Bigg)\Bigg)\Big/$$

$$\Bigg(2\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\sqrt{1+2\eta\epsilon^2\sigma^2}\left(4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4\right)\Bigg)\Bigg)+$$

$$\frac{1}{16\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\sqrt{1+2\eta\epsilon^2\sigma^2}}\,e^{-\frac{8\eta\theta^2\left(1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+4\delta^2\sigma^2\left(3+6\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)+\eta\epsilon^2\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}$$

$$\Bigg(-\frac{8\,e^{\eta\theta^2+\frac{2\delta^2\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\delta\,\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\operatorname{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}+$$

$$\Bigg(32\,e^{\frac{4\delta^2\sigma^2\left(2+5\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)-\eta\epsilon^2\omega p^2}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\alpha\,\delta\,\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)^{3/2}\left(2+5\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4\right)$$

$$\operatorname{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]\Bigg)\Big/\left(4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4\right)+$$

$$\frac{1}{1+2\eta\epsilon^2\sigma^2}\,8\,e^{\eta\theta^2+\frac{2\delta^2\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\eta\epsilon^2\sigma^2}}\delta\,\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)\Bigg(\sqrt{1+\eta\epsilon^2\sigma^2}$$

$$\Bigg(2\,e^{\eta\theta^2+\frac{\eta\epsilon^2\left(4\delta^2\sigma^4+\left(1+2\eta\epsilon^2\sigma^2\right)\omega p^2\right)}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\alpha\left(1+\eta\epsilon^2\sigma^2\right)-\left(-1+\alpha\right)\sqrt{1+2\eta\epsilon^2\sigma^2}+$$

$$2\,e^{\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)\sqrt{1+2\eta\epsilon^2\sigma^2}\Bigg)+$$

$$4\,e^{\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\left(1+\alpha\right)\left(1+\eta\epsilon^2\sigma^2\right)\sqrt{1+2\eta\epsilon^2\sigma^2}\operatorname{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\Bigg)+$$

$$\frac{2\,e^{\eta\theta^2+\frac{2\delta^2\sigma^2\left(1+\eta\epsilon^2\sigma^2\right)}{1+2\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\sqrt{1+2\eta\epsilon^2\sigma^2}\,\omega p\operatorname{Sin}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}-$$

$$\frac{4\, e^{\frac{4\,\delta^2\,\sigma^2\,\left(2+5\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\,\sigma^2\right)^{3/2}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\,\sigma^2}\right]}{1+2\,\eta\epsilon^2\,\sigma^2}+2\, e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}$$

$$\left(\sqrt{1+\eta\epsilon^2\,\sigma^2}\,\left(\frac{16\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\,\sigma^2}}\,(1+\alpha)\,\delta\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}}{4+4\,\eta\epsilon^2\,\sigma^2}+\right.\right.$$

$$\left.\frac{16\, e^{\eta\theta^2+\frac{\eta\epsilon^2\,\left(4\,\delta^2\,\sigma^4+\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\,\omega p^2\right)}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\,\delta\,\eta\epsilon^2\,\sigma^4\,\left(1+\eta\epsilon^2\,\sigma^2\right)}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}\right)+$$

$$\frac{32\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,(1+\alpha)\,\delta\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{8+8\,\eta\epsilon^2\,\sigma^2}-$$

$$\left.\left.\left.\left.\frac{4\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,(1+\alpha)\,\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]}{2+2\,\eta\epsilon^2\,\sigma^2}\right)\right)\right)^2\right)$$

$$\Bigg/\left(-2\, e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}\,(-1+\alpha)\,\sqrt{1+\eta\epsilon^2\,\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\,\sigma^2}\right]+\right.$$

$$4\, e^{\frac{4\,\delta^2\,\sigma^2\,\left(2+5\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\,\sigma^2\right)^{3/2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\,\sigma^2}\right]+$$

$$2\, e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}\,\left(\sqrt{1+\eta\epsilon^2\,\sigma^2}\,\left(2\, e^{\eta\theta^2+\frac{\eta\epsilon^2\,\left(4\,\delta^2\,\sigma^4+\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\,\omega p^2\right)}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\,\sigma^2\right)-\right.\right.$$

$$\left.(-1+\alpha)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}+2\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\,\sigma^2}}\,(1+\alpha)\,\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\right)+$$

$$\left.\left.4\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}\,(1+\alpha)\,\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]\right)\right)+$$

$$\left(16\, e^{\frac{8\,\eta\theta^2\,\left(1+3\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)+4\,\delta^2\,\sigma^2\,\left(3+6\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)+\eta\epsilon^2\,\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\,\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,(1+\alpha)\,\left(1+\eta\epsilon^2\,\sigma^2\right)^{3/2}\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\right.$$

$$\left(-\left(\left(e^{-\frac{8\,\eta\theta^2\,\left(1+3\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)+4\,\delta^2\,\sigma^2\,\left(3+6\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)+\eta\epsilon^2\,\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\,\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\delta\,\sigma^2\,\left(3+6\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)\right.\right.\right.$$

$$\left(-2\, e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}\,(-1+\alpha)\,\sqrt{1+\eta\epsilon^2\,\sigma^2}\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\,\sigma^2}\right]+\right.$$

$$4\, e^{\frac{4\,\delta^2\,\sigma^2\,\left(2+5\,\eta\epsilon^2\,\sigma^2+2\,\eta\epsilon^4\,\sigma^4\right)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\,\sigma^2\right)^{3/2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\,\sigma^2}\right]+$$

$$2\, e^{\eta\theta^2+\frac{2\,\delta^2\,\sigma^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)}{1+2\,\eta\epsilon^2\,\sigma^2}}\,\left(\sqrt{1+\eta\epsilon^2\,\sigma^2}\,\left(2\, e^{\eta\theta^2+\frac{\eta\epsilon^2\,\left(4\,\delta^2\,\sigma^4+\left(1+2\,\eta\epsilon^2\,\sigma^2\right)\,\omega p^2\right)}{4+12\,\eta\epsilon^2\,\sigma^2+8\,\eta\epsilon^4\,\sigma^4}}\,\alpha\,\left(1+\eta\epsilon^2\,\sigma^2\right)-\right.\right.$$

$$(-1+\alpha)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}+2\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\,\sigma^2}}\,(1+\alpha)$$

$$\left.\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\right)-4\, e^{\frac{4\,\delta^2\,\sigma^2+4\,\eta\theta^2\,\left(1+\eta\epsilon^2\,\sigma^2\right)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\,\sigma^2}}$$

$$\left.\left.\left.\left.(1+\alpha)\,\left(1+\eta\epsilon^2\,\sigma^2\right)\,\sqrt{1+2\,\eta\epsilon^2\,\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\,\sigma^2}\right]\right)\right)\right)\right)\Bigg/$$

$$\Big(2\,(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)^{3/2}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,(4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4)\Big)\Big)+$$

$$\frac{1}{16\,(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)^{3/2}\sqrt{1+2\,\eta\epsilon^2\sigma^2}}\,\mathrm{e}^{-\frac{8\,\eta\theta^2\,(1+3\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+4\,\delta^2\sigma^2\,(3+6\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)+\eta\epsilon^2\,(1+2\,\eta\epsilon^2\sigma^2)\,\omega p^2}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}$$

$$\Bigg(-\frac{8\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,(1+\eta\epsilon^2\sigma^2)}{1+2\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\,\delta\,\sigma^2\,(1+\eta\epsilon^2\sigma^2)^{3/2}\,\text{Cos}\Big[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\Big]}{\sqrt{1+2\,\eta\epsilon^2\sigma^2}}+$$

$$\Big(32\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2\,(2+5\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,\delta\,\sigma^2\,(1+\eta\epsilon^2\sigma^2)^{3/2}\,(2+5\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)$$

$$\text{Cos}\Big[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\Big]\Big)\Big/\,(4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4)+$$

$$\frac{1}{1+2\,\eta\epsilon^2\sigma^2}\,8\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,(1+\eta\epsilon^2\sigma^2)}{1+2\,\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\,(1+\eta\epsilon^2\sigma^2)\,\Big(\sqrt{1+\eta\epsilon^2\sigma^2}$$

$$\Big(2\,\mathrm{e}^{\eta\theta^2+\frac{\eta\epsilon^2\,(4\,\delta^2\sigma^4+(1+2\,\eta\epsilon^2\sigma^2)\,\omega p^2)}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,(1+\eta\epsilon^2\sigma^2)-(-1+\alpha)\sqrt{1+2\,\eta\epsilon^2\sigma^2}+$$

$$2\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\,\eta\epsilon^2\sigma^2}\Big)-$$

$$4\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\text{Cos}\Big[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\Big]\Big)+$$

$$\frac{2\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,(1+\eta\epsilon^2\sigma^2)}{1+2\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\omega p\,\text{Sin}\Big[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\Big]}{\sqrt{1+\eta\epsilon^2\sigma^2}}-$$

$$\frac{4\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2\,(2+5\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,(1+\eta\epsilon^2\sigma^2)^{3/2}\,\omega p\,\text{Sin}\Big[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\Big]}{1+2\,\eta\epsilon^2\sigma^2}+2\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,(1+\eta\epsilon^2\sigma^2)}{1+2\,\eta\epsilon^2\sigma^2}}$$

$$\Bigg(\sqrt{1+\eta\epsilon^2\sigma^2}\,\Bigg(\frac{16\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\,\omega p^2}{4+4\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,\delta\,\sigma^2\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\,\eta\epsilon^2\sigma^2}}{4+4\,\eta\epsilon^2\sigma^2}+$$

$$\frac{16\,\mathrm{e}^{\eta\theta^2+\frac{\eta\epsilon^2\,(4\,\delta^2\sigma^4+(1+2\,\eta\epsilon^2\sigma^2)\,\omega p^2)}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,\delta\,\eta\epsilon^2\,\sigma^4\,(1+\eta\epsilon^2\sigma^2)}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}\Bigg)-$$

$$\frac{32\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,\delta\,\sigma^2\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\text{Cos}\Big[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\Big]}{8+8\,\eta\epsilon^2\sigma^2}+$$

$$\frac{4\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2+4\,\eta\theta^2\,(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\,\omega p^2}{8+8\,\eta\epsilon^2\sigma^2}}\,(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\omega p\,\text{Sin}\Big[\frac{\delta\,\omega p}{2+2\,\eta\epsilon^2\sigma^2}\Big]}{2+2\,\eta\epsilon^2\sigma^2}\Bigg)\Bigg)\Bigg)^2\Bigg)$$

$$\Big/\,\Big(-2\,\mathrm{e}^{\eta\theta^2+\frac{2\,\delta^2\sigma^2\,(1+\eta\epsilon^2\sigma^2)}{1+2\,\eta\epsilon^2\sigma^2}}\,(-1+\alpha)\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\,\eta\epsilon^2\sigma^2}\,\text{Cos}\Big[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\Big]+$$

$$4\,\mathrm{e}^{\frac{4\,\delta^2\sigma^2\,(2+5\,\eta\epsilon^2\sigma^2+2\,\eta\epsilon^4\sigma^4)-\eta\epsilon^2\,\omega p^2}{4+12\,\eta\epsilon^2\sigma^2+8\,\eta\epsilon^4\sigma^4}}\,\alpha\,(1+\eta\epsilon^2\sigma^2)^{3/2}\,\text{Cos}\Big[\frac{\delta\,\omega p}{1+2\,\eta\epsilon^2\sigma^2}\Big]+$$

$$2\,e^{\eta\theta^2+\frac{2\delta^2\sigma^2(1+\eta\epsilon^2\sigma^2)}{1+2\eta\epsilon^2\sigma^2}}\left(\sqrt{1+\eta\epsilon^2\sigma^2}\left(2\,e^{\eta\theta^2+\frac{\eta\epsilon^2(4\delta^2\sigma^4+(1+2\eta\epsilon^2\sigma^2)\omega p^2)}{4+12\eta\epsilon^2\sigma^2+8\eta\epsilon^4\sigma^4}}\alpha\,(1+\eta\epsilon^2\sigma^2)-\right.\right.$$
$$\left.(-1+\alpha)\sqrt{1+2\eta\epsilon^2\sigma^2}+2\,e^{\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\eta\epsilon^2\sigma^2}\right)-$$
$$\left.\left.4\,e^{\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}(1+\alpha)\,(1+\eta\epsilon^2\sigma^2)\sqrt{1+2\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\right)\right)$$

# MZ2d

## Mode-correlated noise

### Frequency-entangled photons

#### Fixed shift

In[•]:= 
$$\mathrm{P1d}[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_]:=\frac{1}{8}\left(e^{-2(\delta-\epsilon)^2\sigma^2}\left(-1+2\,e^{2(\delta-\epsilon)^2\sigma^2}+\alpha\right)-(1+\alpha)\,\mathrm{Cos}[2\theta+(-\delta+\epsilon)\,\omega p]\right)$$

In[•]:= 
$$\mathrm{P2d}[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_]:=\frac{1}{8}\left(e^{-2(\delta-\epsilon)^2\sigma^2}\left(-1+2\,e^{2(\delta-\epsilon)^2\sigma^2}+\alpha\right)-(1+\alpha)\,\mathrm{Cos}[2\theta+(-\delta+\epsilon)\,\omega p]\right)$$

In[•]:= 
$$\mathrm{Pcd}[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_]:=\frac{1}{4}\left(2-e^{-2(\delta-\epsilon)^2\sigma^2}(-1+\alpha)+(1+\alpha)\,\mathrm{Cos}[2\theta+(-\delta+\epsilon)\,\omega p]\right)$$

#### Noisy

In[•]:= 
$$\mathrm{P1d}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{1}{8}\left(2-\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1-\alpha)}{\sqrt{1+4\eta\epsilon^2\sigma^2}}-e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}(1+\alpha)\,\mathrm{Cos}[\delta\,\omega p]\right)$$

In[•]:= 
$$\mathrm{P2d}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{1}{8}\left(2-\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1-\alpha)}{\sqrt{1+4\eta\epsilon^2\sigma^2}}-e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}(1+\alpha)\,\mathrm{Cos}[\delta\,\omega p]\right)$$

In[•]:= 
$$\mathrm{Pcd}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{1}{4}\left(2+\frac{e^{-\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1-\alpha)}{\sqrt{1+4\eta\epsilon^2\sigma^2}}+e^{-2\eta\theta^2-\frac{\eta\epsilon^2\omega p^2}{2}}(1+\alpha)\,\mathrm{Cos}[\delta\,\omega p]\right)$$

In[•]:=
$$\text{Fd}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=-\left(\left(-4\,e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2}}(-1+\alpha)\,\delta\,\sigma^2+e^{\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1+\alpha)\left(1+4\eta\epsilon^2\sigma^2\right)^{3/2}\omega p\,\text{Sin}[\delta\,\omega p]\right)^2\Big/\left(\left(1+4\eta\epsilon^2\sigma^2\right)\left(e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2}}\left(-(-1+\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}+e^{\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\left(2+8\eta\epsilon^2\sigma^2\right)\right)+e^{\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1+\alpha)\left(1+4\eta\epsilon^2\sigma^2\right)\text{Cos}[\delta\,\omega p]\right)\left(-e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2}}\left((-1+\alpha)\sqrt{1+4\eta\epsilon^2\sigma^2}+e^{\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}\left(2+8\eta\epsilon^2\sigma^2\right)\right)+e^{\frac{2\delta^2\sigma^2}{1+4\eta\epsilon^2\sigma^2}}(1+\alpha)\left(1+4\eta\epsilon^2\sigma^2\right)\text{Cos}[\delta\,\omega p]\right)\right)\right)$$

# Frequency-independent photons

## Fixed shift

In[•]:=
$$\text{P1dind}[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_]:=\frac{1}{8}\left(2+e^{-(\delta-\epsilon)^2\sigma^2}\left(-1+\alpha-(1+\alpha)\,\text{Cos}[2\,\theta+(-\delta+\epsilon)\,\omega p]\right)\right)$$

In[•]:=
$$\text{P2dind}[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_]:=\frac{1}{8}\left(2+e^{-(\delta-\epsilon)^2\sigma^2}\left(-1+\alpha-(1+\alpha)\,\text{Cos}[2\,\theta+(-\delta+\epsilon)\,\omega p]\right)\right)$$

In[•]:=
$$\text{Pcdind}[\delta\_,\alpha\_,\sigma\_,\omega p\_,\epsilon\_,\theta\_]:=\frac{1}{4}\left(2+e^{-(\delta-\epsilon)^2\sigma^2}\left(1-\alpha+(1+\alpha)\,\text{Cos}[2\,\theta+(-\delta+\epsilon)\,\omega p]\right)\right)$$

## Noisy

In[•]:=
$$\text{P1dind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)-(1+\alpha)\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]\right)}{8\sqrt{1+2\eta\epsilon^2\sigma^2}}$$

In[•]:=
$$\text{P2dind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)-(1+\alpha)\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]\right)}{8\sqrt{1+2\eta\epsilon^2\sigma^2}}$$

In[•]:=
$$\text{Pcdind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(1-\alpha+2\,e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)+(1+\alpha)\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]\right)}{4\sqrt{1+2\eta\epsilon^2\sigma^2}}$$

In[•]:=
$$\text{Fdind}\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=-\left(\left(-2\,e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}(-1+\alpha)\,\delta\,\sigma^2+(1+\alpha)\left(2\,\delta\,\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]+\omega p\,\text{Sin}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]\right)\right)^2\Big/\left(\left(1+2\eta\epsilon^2\sigma^2\right)^2\left(-4\,e^{\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{1+2\eta\epsilon^2\sigma^2}}\left(1+2\eta\epsilon^2\sigma^2\right)+\left(e^{2\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}(-1+\alpha)-(1+\alpha)\,\text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]\right)^2\right)\right)\right)$$

# Mode-uncorrelated noise

## Frequency-entangled photons

### Fixed shift

In[∘]:=

$$
\begin{aligned}
&\text{P1d}\mu[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon 1\_, \epsilon 2\_, \theta 1\_, \theta 2\_] := \\
&\frac{1}{8} e^{-\frac{1}{2}\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} \Bigg(2 e^{\frac{1}{2}\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} + 2 e^{\frac{1}{2}\left(3\delta^2+2\delta\,\epsilon 1+2\,\epsilon 1\,\epsilon 2+\epsilon 2^2\right)\sigma^2} (-1+\alpha) \\
&\quad \text{Cos}\left[\theta 1 + \frac{1}{2}(-\delta+\epsilon 1)\,\omega p\right] - 2 e^{\frac{1}{2}\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} \alpha\, \text{Cos}\left[2\,\theta 1 + (-\delta+\epsilon 1)\,\omega p\right] - \\
&\quad e^{2\delta(\epsilon 1+\epsilon 2)\sigma^2} \text{Cos}\left[\theta 1 - \theta 2 + \frac{1}{2}(\epsilon 1-\epsilon 2)\,\omega p\right] + e^{2\delta(\epsilon 1+\epsilon 2)\sigma^2} \alpha\, \text{Cos}\left[\theta 1 - \theta 2 + \frac{1}{2}(\epsilon 1-\epsilon 2)\,\omega p\right] + \\
&\quad 2 e^{\frac{1}{2}\left(3\delta^2+\epsilon 1^2+2\delta\,\epsilon 2+2\,\epsilon 1\,\epsilon 2\right)\sigma^2} \text{Cos}\left[\theta 2 + \frac{1}{2}(-\delta+\epsilon 2)\,\omega p\right] - \\
&\quad 2 e^{\frac{1}{2}\left(3\delta^2+\epsilon 1^2+2\delta\,\epsilon 2+2\,\epsilon 1\,\epsilon 2\right)\sigma^2} \alpha\, \text{Cos}\left[\theta 2 + \frac{1}{2}(-\delta+\epsilon 2)\,\omega p\right] - e^{2\left(\delta^2+\epsilon 1\,\epsilon 2\right)\sigma^2} \\
&\quad \text{Cos}\left[\theta 1 + \theta 2 + \frac{1}{2}(-2\delta+\epsilon 1+\epsilon 2)\,\omega p\right] + e^{2\left(\delta^2+\epsilon 1\,\epsilon 2\right)\sigma^2} \alpha\, \text{Cos}\left[\theta 1 + \theta 2 + \frac{1}{2}(-2\delta+\epsilon 1+\epsilon 2)\,\omega p\right]\Bigg)
\end{aligned}
$$

In[∘]:=

$$
\begin{aligned}
&\text{P2d}\mu[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon 1\_, \epsilon 2\_, \theta 1\_, \theta 2\_] := \\
&\frac{1}{8} e^{-\frac{1}{2}\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} \Bigg(2 e^{\frac{1}{2}\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} - 2 e^{\frac{1}{2}\left(3\delta^2+2\delta\,\epsilon 1+2\,\epsilon 1\,\epsilon 2+\epsilon 2^2\right)\sigma^2} (-1+\alpha) \\
&\quad \text{Cos}\left[\theta 1 + \frac{1}{2}(-\delta+\epsilon 1)\,\omega p\right] - 2 e^{\frac{1}{2}\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} \alpha\, \text{Cos}\left[2\,\theta 1 + (-\delta+\epsilon 1)\,\omega p\right] - \\
&\quad e^{2\delta(\epsilon 1+\epsilon 2)\sigma^2} \text{Cos}\left[\theta 1 - \theta 2 + \frac{1}{2}(\epsilon 1-\epsilon 2)\,\omega p\right] + e^{2\delta(\epsilon 1+\epsilon 2)\sigma^2} \alpha\, \text{Cos}\left[\theta 1 - \theta 2 + \frac{1}{2}(\epsilon 1-\epsilon 2)\,\omega p\right] - \\
&\quad 2 e^{\frac{1}{2}\left(3\delta^2+\epsilon 1^2+2\delta\,\epsilon 2+2\,\epsilon 1\,\epsilon 2\right)\sigma^2} \text{Cos}\left[\theta 2 + \frac{1}{2}(-\delta+\epsilon 2)\,\omega p\right] + \\
&\quad 2 e^{\frac{1}{2}\left(3\delta^2+\epsilon 1^2+2\delta\,\epsilon 2+2\,\epsilon 1\,\epsilon 2\right)\sigma^2} \alpha\, \text{Cos}\left[\theta 2 + \frac{1}{2}(-\delta+\epsilon 2)\,\omega p\right] - e^{2\left(\delta^2+\epsilon 1\,\epsilon 2\right)\sigma^2} \\
&\quad \text{Cos}\left[\theta 1 + \theta 2 + \frac{1}{2}(-2\delta+\epsilon 1+\epsilon 2)\,\omega p\right] + e^{2\left(\delta^2+\epsilon 1\,\epsilon 2\right)\sigma^2} \alpha\, \text{Cos}\left[\theta 1 + \theta 2 + \frac{1}{2}(-2\delta+\epsilon 1+\epsilon 2)\,\omega p\right]\Bigg)
\end{aligned}
$$

In[∘]:=

$$
\begin{aligned}
&\text{Pcd}\mu[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon 1\_, \epsilon 2\_, \theta 1\_, \theta 2\_] := \\
&\frac{1}{4} e^{-\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} \Bigg(2 e^{\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} + 2 e^{\left(4\delta^2+(\epsilon 1+\epsilon 2)^2\right)\sigma^2} \alpha\, \text{Cos}[2\,\theta 1 - \delta\,\omega p + \epsilon 1\,\omega p] - \\
&\quad e^{\frac{1}{2}(2\delta+\epsilon 1+\epsilon 2)^2\sigma^2} (-1+\alpha)\, \text{Cos}\left[\theta 1 - \theta 2 + \frac{1}{2}(\epsilon 1-\epsilon 2)\,\omega p\right] + \\
&\quad e^{\frac{1}{2}\left(8\delta^2+\epsilon 1^2+6\,\epsilon 1\,\epsilon 2+\epsilon 2^2\right)\sigma^2} \text{Cos}\left[\theta 1 + \theta 2 + \frac{1}{2}(-2\delta+\epsilon 1+\epsilon 2)\,\omega p\right] - \\
&\quad e^{\frac{1}{2}\left(8\delta^2+\epsilon 1^2+6\,\epsilon 1\,\epsilon 2+\epsilon 2^2\right)\sigma^2} \alpha\, \text{Cos}\left[\theta 1 + \theta 2 + \frac{1}{2}(-2\delta+\epsilon 1+\epsilon 2)\,\omega p\right]\Bigg)
\end{aligned}
$$

# Noisy

In[•]:=

$$\text{P1d}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{1}{8\sqrt{1+2\eta\epsilon^2\sigma^2}}e^{-\frac{4\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}\left(-1+\alpha+2e^{\eta\theta^2+\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}+\frac{\eta\epsilon^2\omega p^2}{4}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)+e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]\right)$$

In[•]:=

$$\text{P2d}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{1}{8\sqrt{1+2\eta\epsilon^2\sigma^2}}e^{-\frac{4\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}\left(-1+\alpha+2e^{\eta\theta^2+\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}+\frac{\eta\epsilon^2\omega p^2}{4}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)+e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]\right)$$

In[•]:=

$$\text{Pcd}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\frac{1}{4\sqrt{1+2\eta\epsilon^2\sigma^2}}e^{-\frac{4\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2(1+2\eta\epsilon^2\sigma^2)\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}\left(1-\alpha+2e^{\eta\theta^2+\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}+\frac{\eta\epsilon^2\omega p^2}{4}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)-e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]\right)$$

In[•]:=

$$\text{Fd}\mu\eta[\delta\_,\alpha\_,\sigma\_,\omega p\_,\eta\epsilon\_,\eta\theta\_]:=\left(4e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)\delta\sigma^2+e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}(1+2\eta\epsilon^2\sigma^2)\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\omega p\,\text{Sin}[\delta\,\omega p]\right)^2\Big/\left((1+2\eta\epsilon^2\sigma^2)^2\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}\left(1-\alpha+2e^{\eta\theta^2+\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}+\frac{\eta\epsilon^2\omega p^2}{4}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)-e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]\right)\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}\left(-1+\alpha+2e^{\eta\theta^2+\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}+\frac{\eta\epsilon^2\omega p^2}{4}}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)+e^{\frac{2\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}\left(e^{\eta\theta^2+\frac{\eta\epsilon^2\omega p^2}{4}}(-1+\alpha)-2\alpha\sqrt{1+2\eta\epsilon^2\sigma^2}\right)\text{Cos}[\delta\,\omega p]\right)\right)$$

# Frequency-independent photons

## Fixed shift

In[ ]:=

$$\mathrm{P1dind}\mu[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon 1\_, \epsilon 2\_, \theta 1\_, \theta 2\_] := \frac{1}{8} e^{-\frac{1}{2}\left(2\left(\delta^2+\epsilon 1^2\right)+\epsilon 2^2\right)\sigma^2} \left(2 e^{\frac{1}{2}\left(2\left(\delta^2+\epsilon 1^2\right)+\epsilon 2^2\right)\sigma^2} - 2 e^{\frac{1}{2}\left(4\delta\epsilon 1+\epsilon 2^2\right)\sigma^2}\alpha\,\mathrm{Cos}\left[2\theta 1+\left(-\delta+\epsilon 1\right)\omega p\right] + \left(-1+\alpha\right)\left(2 e^{\frac{1}{2}\left(\left(\delta+\epsilon 1\right)^2+\epsilon 2^2\right)\sigma^2}\mathrm{Cos}\left[\theta 1+\frac{1}{2}\left(-\delta+\epsilon 1\right)\omega p\right] - 2 e^{\frac{1}{2}\left(\delta^2+2\epsilon 1^2+2\delta\epsilon 2\right)\sigma^2}\mathrm{Cos}\left[\theta 2+\frac{1}{2}\left(-\delta+\epsilon 2\right)\omega p\right] + e^{\frac{1}{2}\left(\epsilon 1^2+2\delta\left(\epsilon 1+\epsilon 2\right)\right)\sigma^2}\left(\mathrm{Cos}\left[\theta 1-\theta 2+\frac{1}{2}\left(\epsilon 1-\epsilon 2\right)\omega p\right] + \mathrm{Cos}\left[\theta 1+\theta 2+\frac{1}{2}\left(-2\delta+\epsilon 1+\epsilon 2\right)\omega p\right]\right)\right)\right)$$

In[ ]:=

$$\mathrm{P2dind}\mu[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon 1\_, \epsilon 2\_, \theta 1\_, \theta 2\_] := \frac{1}{8} e^{-\frac{1}{2}\left(2\left(\delta^2+\epsilon 1^2\right)+\epsilon 2^2\right)\sigma^2} \left(2 e^{\frac{1}{2}\left(2\left(\delta^2+\epsilon 1^2\right)+\epsilon 2^2\right)\sigma^2} - 2 e^{\frac{1}{2}\left(4\delta\epsilon 1+\epsilon 2^2\right)\sigma^2}\alpha\,\mathrm{Cos}\left[2\theta 1+\left(-\delta+\epsilon 1\right)\omega p\right] + \left(-1+\alpha\right)\left(-2 e^{\frac{1}{2}\left(\left(\delta+\epsilon 1\right)^2+\epsilon 2^2\right)\sigma^2}\mathrm{Cos}\left[\theta 1+\frac{1}{2}\left(-\delta+\epsilon 1\right)\omega p\right] + 2 e^{\frac{1}{2}\left(\delta^2+2\epsilon 1^2+2\delta\epsilon 2\right)\sigma^2}\mathrm{Cos}\left[\theta 2+\frac{1}{2}\left(-\delta+\epsilon 2\right)\omega p\right] + e^{\frac{1}{2}\left(\epsilon 1^2+2\delta\left(\epsilon 1+\epsilon 2\right)\right)\sigma^2}\left(\mathrm{Cos}\left[\theta 1-\theta 2+\frac{1}{2}\left(\epsilon 1-\epsilon 2\right)\omega p\right] + \mathrm{Cos}\left[\theta 1+\theta 2+\frac{1}{2}\left(-2\delta+\epsilon 1+\epsilon 2\right)\omega p\right]\right)\right)\right)$$

In[ ]:=

$$\mathrm{Pcdind}\mu[\delta\_, \alpha\_, \sigma\_, \omega p\_, \epsilon 1\_, \epsilon 2\_, \theta 1\_, \theta 2\_] := \frac{1}{4} e^{-\frac{1}{2}\left(2\left(\delta^2+\epsilon 1^2\right)+\epsilon 2^2\right)\sigma^2} \left(2 e^{\frac{\epsilon 2^2\sigma^2}{2}}\left(e^{\left(\delta^2+\epsilon 1^2\right)\sigma^2} + e^{2\delta\epsilon 1\sigma^2}\alpha\,\mathrm{Cos}\left[2\theta 1+\left(-\delta+\epsilon 1\right)\omega p\right]\right) - e^{\frac{1}{2}\left(\epsilon 1^2+2\delta\left(\epsilon 1+\epsilon 2\right)\right)\sigma^2}\left(-1+\alpha\right)\left(\mathrm{Cos}\left[\theta 1-\theta 2+\frac{1}{2}\left(\epsilon 1-\epsilon 2\right)\omega p\right] + \mathrm{Cos}\left[\theta 1+\theta 2+\frac{1}{2}\left(-2\delta+\epsilon 1+\epsilon 2\right)\omega p\right]\right)\right)$$

## Noisy

In[ ]:= P1dind$\mu\eta$[$\delta$_, $\alpha$_, $\sigma$_, $\omega$p_, $\eta\epsilon$_, $\eta\theta$_] :=

$$\frac{1}{8}\left(2-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}}{1+\eta\epsilon^2\sigma^2}+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}+\frac{3e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha\,\eta\epsilon^2\sigma^2}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}+\right.$$
$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}(-1+\alpha)\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{1+\eta\epsilon^2\sigma^2}-\frac{2e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}-$$
$$\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}+\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}+$$
$$\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}-$$
$$\left.\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)$$

In[ ]:= P2dind$\mu\eta$[$\delta$_, $\alpha$_, $\sigma$_, $\omega$p_, $\eta\epsilon$_, $\eta\theta$_] :=

$$\frac{1}{8}\left(2-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}}{1+\eta\epsilon^2\sigma^2}+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}+\frac{3e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha\,\eta\epsilon^2\sigma^2}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}+\right.$$
$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}(-1+\alpha)\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{1+\eta\epsilon^2\sigma^2}-\frac{2e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}-$$
$$\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}+\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}+$$
$$\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}-$$
$$\left.\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)$$

In[∘]:= Pcdindμη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=

$$\frac{1}{4}\left(2+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}}{1+\eta\epsilon^2\sigma^2}-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}-\frac{3e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha\eta\epsilon^2\sigma^2}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}(-1+\alpha)\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{1+\eta\epsilon^2\sigma^2}+\frac{2e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}+\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}-\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}-\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}+\frac{2e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)$$

In[∘]:= Fdindμη[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] :=

$$\left(-\frac{8e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\delta\sigma^2}{(1+\eta\epsilon^2\sigma^2)(4+4\eta\epsilon^2\sigma^2)}+\frac{8e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha\delta\sigma^2}{(4+4\eta\epsilon^2\sigma^2)(1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4)}+\frac{24e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\alpha\delta\eta\epsilon^2\sigma^4}{(4+4\eta\epsilon^2\sigma^2)(1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4)}+\frac{8e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}(-1+\alpha)\delta\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{(1+\eta\epsilon^2\sigma^2)(4+4\eta\epsilon^2\sigma^2)}-\frac{8e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\alpha\delta\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}\,(2+4\eta\epsilon^2\sigma^2)}-\frac{16e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\delta\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,(8+8\eta\epsilon^2\sigma^2)}+\frac{16e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\delta\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,(8+8\eta\epsilon^2\sigma^2)}+\frac{16e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\delta\sigma^2\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{(8+8\eta\epsilon^2\sigma^2)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}-\frac{16e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\alpha\delta\sigma^2\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{(8+8\eta\epsilon^2\sigma^2)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2(1+\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}(-1+\alpha)\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{(1+\eta\epsilon^2\sigma^2)^2}-\frac{2e^{-\frac{2\delta^2\sigma^2+\eta\theta^2(4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\alpha\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{(1+2\eta\epsilon^2\sigma^2)^{3/2}}-$$

$$\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(2+2\eta\epsilon^2\sigma^2\right)}+\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(2+2\eta\epsilon^2\sigma^2\right)}+$$

$$\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\sqrt{1+4\eta\epsilon^2\sigma^2}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\left(2+2\eta\epsilon^2\sigma^2\right)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}-$$

$$\left.\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\sqrt{1+4\eta\epsilon^2\sigma^2}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\left(2+2\eta\epsilon^2\sigma^2\right)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)^2\Bigg/$$

$$\left(4\left(2+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}}{1+\eta\epsilon^2\sigma^2}-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\alpha}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}-\frac{3\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\alpha\,\eta\epsilon^2\sigma^2}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}-\right.\right.$$

$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{1+\eta\epsilon^2\sigma^2}+\frac{2\,e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}+$$

$$\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}-\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}-$$

$$\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}+$$

$$\left.\left.\frac{2\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)\right)+$$

$$\left(\frac{8\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2}{\left(1+\eta\epsilon^2\sigma^2\right)\left(4+4\eta\epsilon^2\sigma^2\right)}-\frac{8\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2}{\left(4+4\eta\epsilon^2\sigma^2\right)\left(1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4\right)}-\right.$$

$$\frac{24\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\eta\epsilon^2\sigma^4}{\left(4+4\eta\epsilon^2\sigma^2\right)\left(1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4\right)}-\frac{8\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{\left(1+\eta\epsilon^2\sigma^2\right)\left(4+4\eta\epsilon^2\sigma^2\right)}+$$

$$\frac{8\,e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}\,\left(2+4\eta\epsilon^2\sigma^2\right)}+\frac{16\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(8+8\eta\epsilon^2\sigma^2\right)}-$$

$$\frac{16\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\,\left(8+8\eta\epsilon^2\sigma^2\right)}-$$

$$\frac{16\,e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\delta\,\sigma^2\,\sqrt{1+4\eta\epsilon^2\sigma^2}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\left(8+8\eta\epsilon^2\sigma^2\right)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}+$$

$$\frac{16\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\delta\,\sigma^2\sqrt{1+4\eta\epsilon^2\sigma^2}\;\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\left(8+8\eta\epsilon^2\sigma^2\right)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}-$$

$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{\left(1+\eta\epsilon^2\sigma^2\right)^2}+\frac{2\, e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\alpha\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\left(1+2\eta\epsilon^2\sigma^2\right)^{3/2}}+$$

$$\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\;\left(2+2\eta\epsilon^2\sigma^2\right)}-\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}\;\left(2+2\eta\epsilon^2\sigma^2\right)}-$$

$$\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\sqrt{1+4\eta\epsilon^2\sigma^2}\;\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\left(2+2\eta\epsilon^2\sigma^2\right)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}+$$

$$\left.\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\sqrt{1+4\eta\epsilon^2\sigma^2}\;\omega p\,\mathrm{Sin}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\left(2+2\eta\epsilon^2\sigma^2\right)\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)^2\Bigg/$$

$$\left(4\left(2-\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}}{1+\eta\epsilon^2\sigma^2}+\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\alpha}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}+\frac{3\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\,\alpha\,\eta\epsilon^2\sigma^2}{1+4\eta\epsilon^2\sigma^2+3\eta\epsilon^4\sigma^4}+\right.\right.$$

$$\frac{e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{4+4\eta\epsilon^2\sigma^2}}\left(-1+\alpha\right)\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+\eta\epsilon^2\sigma^2}\right]}{1+\eta\epsilon^2\sigma^2}-\frac{2\, e^{-\frac{2\delta^2\sigma^2+\eta\theta^2\left(4+8\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\,\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}-$$

$$\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}+\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\,\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+\eta\epsilon^2\sigma^2}}+$$

$$\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\sqrt{1+4\eta\epsilon^2\sigma^2}\;\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}-$$

$$\left.\left.\frac{2\, e^{-\frac{4\delta^2\sigma^2+4\eta\theta^2\left(1+\eta\epsilon^2\sigma^2\right)+\eta\epsilon^2\omega p^2}{8+8\eta\epsilon^2\sigma^2}}\,\alpha\sqrt{1+4\eta\epsilon^2\sigma^2}\;\mathrm{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+5\eta\epsilon^2\sigma^2+4\eta\epsilon^4\sigma^4}}\right)\right)$$

# Classical correlations

In[ ]:= $$\text{P1CC}\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] :=$$

$$\left(e^{-\frac{\delta^2 \sigma^2 (3+2\eta\epsilon^2\sigma^2)+\eta\theta^2 (4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2 \omega p^2}{2+4\eta\epsilon^2\sigma^2}} \left(e^{\frac{\eta\theta^2 (4+8\eta\epsilon^2\sigma^2)+\delta^2 (\sigma^2+2\eta\epsilon^2\sigma^4)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}} \sqrt{1+\eta\epsilon^2\sigma^2} \left(1+2\, e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}} \sqrt{1+2\eta\epsilon^2\sigma^2}\right) + e^{\frac{\delta^2\sigma^2}{2}} \sqrt{1+\eta\epsilon^2\sigma^2}\, \text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right] - 4\, e^{\frac{12\eta\theta^2 (1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+4\delta^2\sigma^2 (2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+\eta\epsilon^2 (3+2\eta\epsilon^2\sigma^2)\,\omega p^2}{8 (1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)}} \sqrt{1+2\eta\epsilon^2\sigma^2}\, \text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\right)\right) \Big/ \left(8\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)$$

In[ ]:= $$\text{P2CC}\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] :=$$

$$\left(e^{-\frac{\delta^2 \sigma^2 (3+2\eta\epsilon^2\sigma^2)+\eta\theta^2 (4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2 \omega p^2}{2+4\eta\epsilon^2\sigma^2}} \left(e^{\frac{\eta\theta^2 (4+8\eta\epsilon^2\sigma^2)+\delta^2 (\sigma^2+2\eta\epsilon^2\sigma^4)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}} \sqrt{1+\eta\epsilon^2\sigma^2} \left(1+2\, e^{\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}} \sqrt{1+2\eta\epsilon^2\sigma^2}\right) + e^{\frac{\delta^2\sigma^2}{2}} \sqrt{1+\eta\epsilon^2\sigma^2}\, \text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right] + 4\, e^{\frac{12\eta\theta^2 (1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+4\delta^2\sigma^2 (2+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)+\eta\epsilon^2 (3+2\eta\epsilon^2\sigma^2)\,\omega p^2}{8 (1+3\eta\epsilon^2\sigma^2+2\eta\epsilon^4\sigma^4)}} \sqrt{1+2\eta\epsilon^2\sigma^2}\, \text{Cos}\left[\frac{\delta\,\omega p}{2+2\eta\epsilon^2\sigma^2}\right]\right)\right) \Big/ \left(8\sqrt{1+\eta\epsilon^2\sigma^2}\sqrt{1+2\eta\epsilon^2\sigma^2}\right)$$

In[ ]:= $$\text{PcCC}\eta[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\epsilon\_, \eta\theta\_] :=$$

$$\frac{1}{4}\left(2 - \frac{e^{-\frac{\delta^2\sigma^2}{1+2\eta\epsilon^2\sigma^2}}}{\sqrt{1+2\eta\epsilon^2\sigma^2}} - \frac{e^{-\frac{2\delta^2\sigma^2+\eta\theta^2 (4+8\eta\epsilon^2\sigma^2)+\eta\epsilon^2\omega p^2}{2+4\eta\epsilon^2\sigma^2}}\, \text{Cos}\left[\frac{\delta\,\omega p}{1+2\eta\epsilon^2\sigma^2}\right]}{\sqrt{1+2\eta\epsilon^2\sigma^2}}\right)$$

# von Mises distribution

The noisy probabilities calculated in the paper and throughout this notebook assume Gaussian distributions for noise. For $\theta$ noise, this produces probabilities equivalent to the wrapped Gaussian distribution (see Appendix E).

An alternative circular distribution for $\theta$ noise is the von Mises distribution, this produces qualitatively the same behaviour but with slight differences in noise dependence.
A reader who wishes to obtain the noisy probabilities with a von Mises distribution may do so by taking the fixed shift probabilities in this notebook, and performing the integral in Eq. (25) replacing the $\theta$ distribution with

In[ ]:= $$\text{J}\theta\text{VM}[\theta\_, \eta\theta\_] := \frac{\text{Exp}\left[\frac{\text{Cos}[\theta]}{\eta\theta\text{^}2}\right]}{2\,\pi\,\text{BesselI}\left[0, \frac{1}{\eta\theta\text{^}2}\right]}$$

and integrating $\theta$ only between $-\pi$ and $\pi$.
(If $\epsilon$ noise is also being considered, its Gaussian distribution should remain and $\epsilon$ integrated between $-\infty$ and $\infty$)

Appendix E also includes comparison between Gaussian and von Mises distributions for $\theta$ noise for a few limited cases: MZ1, MZ2s, and MZ2d with frequency-entangled photons and mode-correlated noise.
As the noisy probabilities with von Mises have been calculated explicitly for these three cases, they are included below.

(Note all cases here assume no $\epsilon$ noise, so the $\eta\epsilon$ variable is absent from these expressions)

# MZ1

In[∘]:= $\text{P1sing}\eta\text{VM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\theta\_] := \frac{1}{2}\left(1 - \frac{e^{-\frac{1}{2}\delta^2\sigma^2}\,\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right]}{\text{BesselI}\left[0, \frac{1}{\eta\theta^2}\right]}\right)$

In[∘]:= $\text{P2sing}\eta\text{VM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\theta\_] := \frac{1}{2}\left(1 + \frac{e^{-\frac{1}{2}\delta^2\sigma^2}\,\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right]}{\text{BesselI}\left[0, \frac{1}{\eta\theta^2}\right]}\right)$

In[∘]:= $\text{Fsing}\eta\text{VM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\theta\_] :=$

$$-\frac{\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]^2\left(2\,\delta\,\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right] + \omega p\,\text{Sin}\left[\frac{\delta\,\omega p}{2}\right]\right)^2}{-4\,e^{\delta^2\sigma^2}\,\text{BesselI}\left[0, \frac{1}{\eta\theta^2}\right]^2 + 2\,\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]^2\left(1 + \text{Cos}[\delta\,\omega p]\right)}$$

# MZ2s

In[∘]:= $\text{P1s}\eta\text{VM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\theta\_] :=$

$$\frac{1}{8}\left(2 + e^{-2\delta^2\sigma^2} + \text{Cos}[\delta\,\omega p] + \frac{2\,\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]\left(-2\,e^{-\frac{1}{2}\delta^2\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right] - \eta\theta^2\,\text{Cos}[\delta\,\omega p]\right)}{\text{BesselI}\left[0, \frac{1}{\eta\theta^2}\right]}\right)$$

In[∘]:= $\text{P2s}\eta\text{VM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\theta\_] :=$

$$\frac{1}{8}\left(2 + e^{-2\delta^2\sigma^2} + \text{Cos}[\delta\,\omega p] + \frac{2\,\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]\left(2\,e^{-\frac{1}{2}\delta^2\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right] - \eta\theta^2\,\text{Cos}[\delta\,\omega p]\right)}{\text{BesselI}\left[0, \frac{1}{\eta\theta^2}\right]}\right)$$

In[∘]:= $\text{Pcs}\eta\text{VM}[\delta\_, \alpha\_, \sigma\_, \omega p\_, \eta\theta\_] := \frac{1}{4}\left(2 - e^{-2\delta^2\sigma^2} - \text{Cos}[\delta\,\omega p] + \frac{2\,\eta\theta^2\,\text{BesselI}\left[1, \frac{1}{\eta\theta^2}\right]\,\text{Cos}[\delta\,\omega p]}{\text{BesselI}\left[0, \frac{1}{\eta\theta^2}\right]}\right)$

In[•]:= FsηVM[δ_, α_, σ_, ωp_, ηθ_] :=

$$\frac{1}{8}\left(-\frac{2\left(4\,e^{-2\,\delta^2\,\sigma^2}\,\delta\,\sigma^2+\omega p\,\text{Sin}[\delta\,\omega p]-\frac{2\,\eta\theta^2\,\omega p\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\text{Sin}[\delta\,\omega p]}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}\right)^2}{-2+e^{-2\,\delta^2\,\sigma^2}+\text{Cos}[\delta\,\omega p]-\frac{2\,\eta\theta^2\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\text{Cos}[\delta\,\omega p]}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}}+\right.$$

$$\frac{\left(4\,e^{-2\,\delta^2\,\sigma^2}\,\delta\,\sigma^2+\omega p\,\text{Sin}[\delta\,\omega p]+\frac{2\,e^{-\frac{1}{2}\delta^2\,\sigma^2}\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\left(2\,\delta\,\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right]+\omega p\left(\text{Sin}\left[\frac{\delta\,\omega p}{2}\right]-e^{\frac{\delta^2\,\sigma^2}{2}}\,\eta\theta^2\,\text{Sin}[\delta\,\omega p]\right)\right)}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}\right)^2}{2+e^{-2\,\delta^2\,\sigma^2}+\text{Cos}[\delta\,\omega p]+\frac{2\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\left(2\,e^{-\frac{1}{2}\delta^2\,\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right]-\eta\theta^2\,\text{Cos}[\delta\,\omega p]\right)}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}}+$$

$$\left.\frac{\left(4\,e^{-2\,\delta^2\,\sigma^2}\,\delta\,\sigma^2+\omega p\,\text{Sin}[\delta\,\omega p]-\frac{2\,e^{-\frac{1}{2}\delta^2\,\sigma^2}\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\left(2\,\delta\,\sigma^2\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right]+\omega p\left(\text{Sin}\left[\frac{\delta\,\omega p}{2}\right]+e^{\frac{\delta^2\,\sigma^2}{2}}\,\eta\theta^2\,\text{Sin}[\delta\,\omega p]\right)\right)}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}\right)^2}{2+e^{-2\,\delta^2\,\sigma^2}+\text{Cos}[\delta\,\omega p]+\frac{2\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\left(-2\,e^{-\frac{1}{2}\delta^2\,\sigma^2}\,\text{Cos}\left[\frac{\delta\,\omega p}{2}\right]-\eta\theta^2\,\text{Cos}[\delta\,\omega p]\right)}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}}\right)$$

# MZ2d

In[•]:= P1dηVM[δ_, α_, σ_, ωp_, ηθ_] :=

$$\frac{1}{8}\left(2+e^{-2\,\delta^2\,\sigma^2}\,(-1+\alpha)-(1+\alpha)\,\text{Cos}[\delta\,\omega p]+\frac{2\,(1+\alpha)\,\eta\theta^2\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\text{Cos}[\delta\,\omega p]}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}\right)$$

In[•]:= P2dηVM[δ_, α_, σ_, ωp_, ηθ_] :=

$$\frac{1}{8}\left(2+e^{-2\,\delta^2\,\sigma^2}\,(-1+\alpha)-(1+\alpha)\,\text{Cos}[\delta\,\omega p]+\frac{2\,(1+\alpha)\,\eta\theta^2\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\text{Cos}[\delta\,\omega p]}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}\right)$$

In[•]:= PcdηVM[δ_, α_, σ_, ωp_, ηθ_] :=

$$\frac{1}{4}\left(2-e^{-2\,\delta^2\,\sigma^2}\,(-1+\alpha)+(1+\alpha)\,\text{Cos}[\delta\,\omega p]-\frac{2\,(1+\alpha)\,\eta\theta^2\,\text{BesselI}\left[1,\frac{1}{\eta\theta^2}\right]\text{Cos}[\delta\,\omega p]}{\text{BesselI}\left[0,\frac{1}{\eta\theta^2}\right]}\right)$$

```
In[•]:= FdηVM[δ_, α_, σ_, ωp_, ηθ_] :=
  -((-2 e^(2 δ^2 σ^2) (1 + α) ηθ^2 ωp BesselI[1, 1/ηθ^2] Sin[δ ωp] + BesselI[0, 1/ηθ^2]
        (-4 (-1 + α) δ σ^2 + e^(2 δ^2 σ^2) (1 + α) ωp Sin[δ ωp]))^2 /
     ((-2 e^(2 δ^2 σ^2) (1 + α) ηθ^2 BesselI[1, 1/ηθ^2] Cos[δ ωp] + BesselI[0, 1/ηθ^2]
         (1 - 2 e^(2 δ^2 σ^2) - α + e^(2 δ^2 σ^2) (1 + α) Cos[δ ωp])) (-2 e^(2 δ^2 σ^2) (1 + α) ηθ^2 BesselI[1, 1/ηθ^2]
         Cos[δ ωp] + BesselI[0, 1/ηθ^2] (1 + 2 e^(2 δ^2 σ^2) - α + e^(2 δ^2 σ^2) (1 + α) Cos[δ ωp]))))
```

# Adding Loss

This section allows for the generation of lossy probabilities for a given protocol configuration. Loss is represented by $\gamma$, the per photon loss rate.

First, run the earlier sections of this notebook to define all the non-lossy expressions.

Choose a protocol and configuration and run the cells for that choice. This will set the generic probability functions equal to the relevant probabilities for your chosen configuration.

Then, lossy expressions can be generated for either bucket detectors (i.e. those detectors that cannot distinguish between photon numbers) or number-resolving detectors.

Choose a detector type, and run the code for that section to generate probabilities and Fisher information.

Probabilities must be generated first before Fisher information, as the latter calls the probability functions.

The code will attempt to simplify the Fisher information expression. Sometimes this can take some time, the “TimeConstraint” parameter ensures simplification will cancel for cases where it takes too long (usually MZ2s cases). Increasing this parameter may allow more complicated simplifications to complete, though this may take some time to evaluate and there is no guarantee the automatic simplification will ever be possible for every configuration.

# Protocol configuration choice

## HOM

### Frequency-entangled photons

```
In[•]:= P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1HOMη[δ, α, σ, ωp, ηϵ, ηθ]
```

```
In[•]:= P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2HOMη[δ, α, σ, ωp, ηϵ, ηθ]
```

```
In[]:= Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := PcHOMη[δ, α, σ, ωp, ηϵ, ηθ]
```

### Frequency-independent photons

```
P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1HOMindη[δ, α, σ, ωp, ηϵ, ηθ]
P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2HOMindη[δ, α, σ, ωp, ηϵ, ηθ]
Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := PcHOMindη[δ, α, σ, ωp, ηϵ, ηθ]
```

# MZ2s

## Mode-correlated noise

### Frequency-entangled photons

```
In[]:= P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1sη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2sη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcsη[δ, α, σ, ωp, ηϵ, ηθ]
```

### Frequency-independent photons

```
P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1sindη[δ, α, σ, ωp, ηϵ, ηθ]
P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2sindη[δ, α, σ, ωp, ηϵ, ηθ]
Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcsindη[δ, α, σ, ωp, ηϵ, ηθ]
```

## Mode-uncorrelated noise

### Frequency-entangled photons

```
P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1sμη[δ, α, σ, ωp, ηϵ, ηθ]
P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2sμη[δ, α, σ, ωp, ηϵ, ηθ]
Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcsμη[δ, α, σ, ωp, ηϵ, ηθ]
```

### Frequency-independent photons

```
P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1sindμη[δ, α, σ, ωp, ηϵ, ηθ]
P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2sindμη[δ, α, σ, ωp, ηϵ, ηθ]
Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcsindμη[δ, α, σ, ωp, ηϵ, ηθ]
```

# MZ2d

## Mode-correlated noise

### Frequency-entangled photons

```
In[]:= P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1dη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2dη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcdη[δ, α, σ, ωp, ηϵ, ηθ]
```

### Frequency-independent photons

```
In[]:= P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1dindη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2dindη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcdindη[δ, α, σ, ωp, ηϵ, ηθ]
```

## Mode-uncorrelated noise

### Frequency-entangled photons

```
P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1dμη[δ, α, σ, ωp, ηϵ, ηθ]
P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2dμη[δ, α, σ, ωp, ηϵ, ηθ]
Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcdμη[δ, α, σ, ωp, ηϵ, ηθ]
```

### Frequency-independent photons

```
In[]:= P1[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P1dindμη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= P2[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := P2dindμη[δ, α, σ, ωp, ηϵ, ηθ]
In[]:= Pc[δ_, α_, σ_, ωp_, ηϵ_, ηθ_] := Pcdindμη[δ, α, σ, ωp, ηϵ, ηθ]
```

# Generate lossy probabilities and Fisher information

## Bucket detectors

### Probabilities

#### generate

```
In[∘]:= Pcɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := (1 - ɣ)^2 Pc[δ, α, σ, ωp, ηϵ, ηθ] // Simplify;
  (*coincidence*)

In[∘]:= P1ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := (1 - ɣ)^2 P1[δ, α, σ, ωp, ηϵ, ηθ] + ɣ (1 - ɣ)
        (2 P1[δ, α, σ, ωp, ηϵ, ηθ] + Pc[δ, α, σ, ωp, ηϵ, ηθ]) // Simplify; (*click at D1*)

In[∘]:= P2ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := (1 - ɣ)^2 P2[δ, α, σ, ωp, ηϵ, ηθ] + ɣ (1 - ɣ)
        (2 P2[δ, α, σ, ωp, ηϵ, ηθ] + Pc[δ, α, σ, ωp, ηϵ, ηθ]) // Simplify; (*click at D2*)

In[∘]:= P0ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := ɣ^2; (*no clicks*)
```

#### output

```
In[∘]:= Pcɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]

In[∘]:= P1ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]

In[∘]:= P2ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]

In[∘]:= P0ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

### Fisher information

#### generate

```
In[∘]:= D[Pcɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / Pcɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ] +
  D[P1ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / P1ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ] + D[P2ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / P2ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ];

In[∘]:= Simplify[%, TimeConstraint → 1];

In[∘]:= Fɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] = %;
```

#### output

```
In[∘]:= Fɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

# Number-resolving detectors detectors

## Probabilities

### generate

In[ ]:=
```
Pcɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := (1 - ɣ)^2 Pc[δ, α, σ, ωp, ηϵ, ηθ] // Simplify;
(*coincidence*)
```

In[ ]:=
```
P12ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := (1 - ɣ)^2 P1[δ, α, σ, ωp, ηϵ, ηθ] // Simplify;
(*two clicks at D1*)
```

In[ ]:=
```
P11ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] :=
  ɣ (1 - ɣ) (2 P1[δ, α, σ, ωp, ηϵ, ηθ] + Pc[δ, α, σ, ωp, ηϵ, ηθ]) //
   Simplify; (*one click at D1*)
```

In[ ]:=
```
P22ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := (1 - ɣ)^2 P2[δ, α, σ, ωp, ηϵ, ηθ] // Simplify;
(*two clicks at D2*)
```

In[ ]:=
```
P21ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] :=
  ɣ (1 - ɣ) (2 P2[δ, α, σ, ωp, ηϵ, ηθ] + Pc[δ, α, σ, ωp, ηϵ, ηθ]) // Simplify;
(*two clicks at D1*)
```

In[ ]:=
```
P0ɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] := ɣ^2; (*no clicks*)
```

### output

In[ ]:=
```
Pcɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

In[ ]:=
```
P12ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

In[ ]:=
```
P11ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

In[ ]:=
```
P22ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

In[ ]:=
```
P21ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

In[ ]:=
```
P0ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ]
```

## Fisher information

### generate

In[ ]:=
```
D[Pcɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / Pcɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ] +
  D[P12ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / P12ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ] + D[P11ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / P11ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ] +
  D[P22ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / P22ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ] + D[P21ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ], δ]^2 / P21ɣ[δ, α, σ, ωp, ηϵ, ηθ, ɣ];
```

In[ ]:=
```
Simplify[%, TimeConstraint → 1];
```

In[ ]:=
```
Fɣ[δ_, α_, σ_, ωp_, ηϵ_, ηθ_, ɣ_] = %;
```

## output

In[ ]:=
```
Fγ[δ, α, σ, ωp, ηϵ, ηθ, γ]
```