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Analysis of Harvesting Energy from Mistuned Multiple Harvesters with and without Coupling

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Abstract

Energy harvesting has received a lot of attention in the recent past. At present a single device does not harvested energy enough to power up an electronic sensors. In order to increase the power output multiple identical harvesters are used. When multiple harvesters are used, they bring in non-uniformity in their physical parameters due to variability during manufacturing or even during deployment. Therefore, '*n*' numbers of harvesters do not necessary produce '*n*' times the harvested power of a single device. The variability in parameters is less enough to be coined as *mistuning*. In this paper, an analysis of multiple energy harvesters is studied. The harvesters are assumed to show mistuning. The study is further extended to understand the effect of mechanical coupling between the harvesters. For simplification, pendulums are considered as the harvesters, with magnetic tip masses for the electromagnetic energy harvesting. Mistuning is achieved by varying the length of the pendulums. A generalized mathematical model for *n* coupled harvesters with mistuning is developed. Simulations are performed with the number of harvesters varying from 2 to 6 with ±1% non-repetitive mistuning in the lengths of the harvesters, and a comparison of the power harvested between mechanically coupled and uncoupled harvesters is presented.

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1. Introduction

Twenty first century has paved the way for consumer electronics and recent past there has been a boom in the development of miniature electronic systems. Labs in chip, printed electronics, bio-MEMs and micro fluidics have shown tremendous advancement. One thing common to all the applications to miniature electronics is the serviceability, which extensively depends on the battery life. As battery technology has not shown similar potential to cater the need of miniature electronics, leave apart the consideration of environmental threat, engineers have sought for alternate way to power these electronics. There is considerable interest in the development of battery-free mobile electronics systems such as wireless sensors which are used for the condition monitoring of engineering assets, some of which are located in hostile environments. The main focus has been on the development of techniques for the harvesting of energy from ambient sources. One of the main sources of ambient energy is mechanical vibration [1]. Number of vibration energy harvesting devices have been developed using electromagnetic, electrostatic or piezoelectric principles. A conventional harvester generates significant energy at the resonance peak of the vibration system. However, off the resonance peak, the harvested vibration energy is relatively small [2]. Later it was reported that a slight uncertainty or mismatch between the host vibrating frequency and the natural frequency of the harvester can reduce the power harvested to a high extent [3]. Multiple harvesters, in contrast to single harvesters with intentional mistuning, are used to obtain the wider bandwidth [4-9]. Each harvester is mistuned to a slightly different resonance frequency to extend the bandwidth.

If the maximum energy is to be harvested at a given excitation frequency, all of the harvesters should be tuned to same resonance. This means that each harvester must have exactly the same dimensions and properties, and slight deviations will reduce the maximum energy drastically [1-3, 10]. In the ideal case all harvesters are identical and have same properties, but in reality there will be slight differences between substructures because of geometry, material properties, tolerances in manufacturing, or in-service degradation [11-13].

In this present work, the effect of mistuning, mechanical coupling of harvesters with springs on total power generated is analyzed. A comparison between coupled and uncoupled harvesters with and without mistuning for n number of pendulums is carried out.

2. SYSTEM MODEL

Consider *n* pendulums with magnetic end masses as shown in Fig. 1. At one end, the pendulum is hinged and at the other end magnetic mass is attached. Each of the pendulums are connected by linear spring of stiffness k_i at a distance *a* from the hinge. Coils are placed under each magnetic mass. The coils are shown by shaded area, each coil is aligned with pendulum. Parallel circuit connection is used for the voltage measurement. Voltage across each coil is measured by connecting a resistor. A harmonic base excitation of amplitude x_g is applied to the structure, which makes pendulums with magnetic mass to oscillate. Due to this oscillation, relative motion develops between the magnetic mass and the coil. This in turn generates voltage as per Faraday's law of induction. The difference between Fig. 1(a) and (b) is the mechanical coupling; mechanical coupling is absent in Fig. 1(a). Mistuning is introduced by keeping the length (nominal length l_1) of first pendulum constant and varying the length of other pendulums.

Assuming small angular displacement of harvesters and neglecting the magnetic interaction, equations of the motion for the system shown in Fig. 1(b) is given as;

$$\left.\begin{aligned}
&ml_1^2\ddot{\theta}_1 + (c_m + c_e)_1 l_1^2\dot{\theta}_1 + mgl_1\theta_1 + K_1a^2\theta_1 - K_1a^2\theta_2 = -ml_1\ddot{x}_g \\
&ml_i^2\ddot{\theta}_i + (c_m + c_e)_i l_i^2\dot{\theta}_i + mgl_i\theta_i - K_{i-1}a^2\theta_{i-1} + (K_{i-1} + K_i)a^2\theta_i - K_ia^2\theta_{i+1} = -ml_i\ddot{x}_g \\
&ml_n^2\ddot{\theta}_n + (c_m + c_e)_n l_n^2\dot{\theta}_n + mgl_n\theta_n - K_{n-1}a^2\theta_{n-1} + K_{n-1}a^2\theta_n = -ml_n\ddot{x}_g
\end{aligned}\right\} \tag{1}$$

Where m_i is mass, l_i is length of each pendulum. c_m is linear proportional mechanical damping constant and c_e is linear proportional electrical damping constant defined as [7].

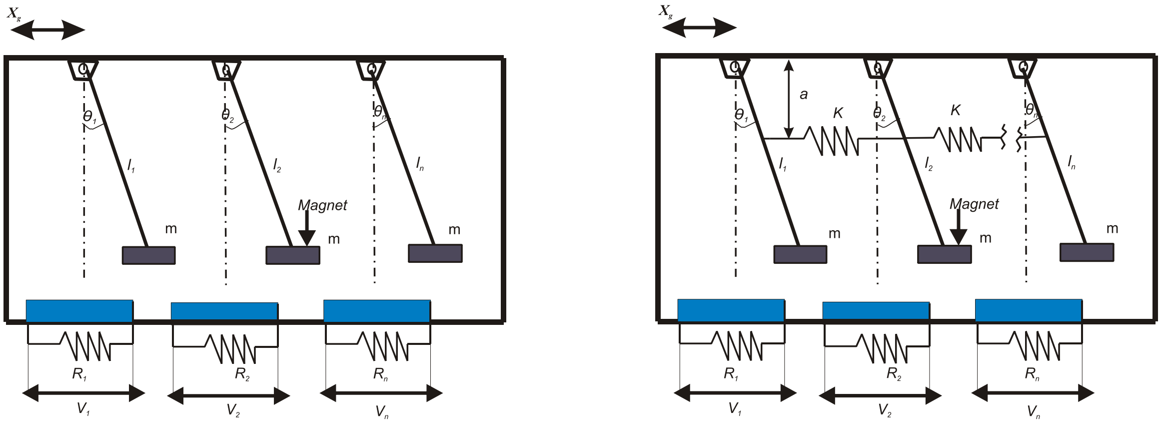

Figure 1: Energy harvesting model a) without coupling b) with coupling

$$c_e = \frac{(BL)^2}{R} \tag{2}$$

Neglecting coil inductance the voltage induced by the coil is defined by.

$$v_i = BLr\dot{\theta}_i \qquad (i=1,2,....,n) \tag{3}$$

Where B is the magnetic field strength, L is practical coil length and R resistance. For harmonic excitation, the steady state solution of Eq. 1 can be assumed to be,

$$\theta_i = \Theta_i e^{i\omega t},\ x_g = X_g e^{i\omega t},\ v_i = V_i e^{i\omega t} \tag{4}$$

Introducing dimensionless parameters and variables as below:

$$\alpha_i = \frac{l_i}{l_1},\ \omega_1 = \sqrt{\frac{g}{l_1}},\ \Gamma_m = \frac{c_m}{m\omega_1},\ \Gamma_e = \frac{c_e}{m\omega_1},\ \beta = \frac{ka^2}{mgl_1},\ f = \frac{x_g}{l_1},\ \Omega = \frac{\omega}{\omega_1} \tag{5}$$

Where ω₁ represents natural frequency of pendulum-1. Introducing, Eqs. 4 and 5 in Eqs. 1 and 3, leads to the following nondimensional form,

$$\left.\begin{array}{l}(-\Omega^2\alpha_1^2 + j\Omega\alpha_1^2(\Gamma_m+\Gamma_e)+\alpha_1+\beta_1)\Theta_1-\beta_1\Theta_2=\Omega^2 f\alpha_1\\(-\Omega^2\alpha_i^2 + j\Omega\alpha_i^2(\Gamma_m+\Gamma_e)+\alpha_i+\beta_{i-1}+\beta_i)\Theta_i-\beta_i\Theta_{i+1}-\beta_{i-1}\Theta_{i+1}=\Omega^2 f\alpha_1\\(-\Omega^2\alpha_n^2 + j\Omega\alpha_n^2(\Gamma_m+\Gamma_e)+\alpha_n+\beta_{n-1})\Theta_n-\beta_{n-1}\Theta_{n-1}=\Omega^2 f\alpha_n\end{array}\right\} \tag{6}$$

$$V_i = j\omega BLr\Theta_i \tag{7}$$

Power harvested by individual harvester with load resistance R,

$$p_i = \frac{V_i^2}{R} = (m\omega_1^3 r^2)\Omega^2 \Gamma_e \Theta_i^2 \tag{8}$$

Normalized power is given as,

$$P_i = \frac{p_i}{(m\omega_1^3 r^2)} = \Omega^2 \Gamma_e \Theta_i^2 \tag{9}$$

The total power is given as;

$$P_t = \Sigma P_i \tag{10}$$

3. RESULTS AND DISCUSSION

In this section effect of mistuning, coupling and damping on the power harvested are investigated. To start with two harvester with and without coupling is investigate. The length of first harvester is taken as nominal length, therefore, (α_1=1).

3.1 Effect of length ratio (mistuning)

Figure 2(a) shows the total power for the coupled and uncoupled cases with identical harvesters. The total power harvested in both the cases is same. Coupling two harvesters by spring will convert the system from two single degrees of systems into a two degrees of freedom, which should exhibit two peaks. As both the harvesters are subjected to base excitation and are identical only first peak which is dominating is observed. Figure 2(b) shows total power curve with 1% mistuning present in the second harvester. Because of this mistuning, the total power is less than the power with identical harvesters (comparing Figure 2(a) and (b)). However, the coupled harvesters gives slightly more power than the uncoupled harvesters. The uncoupled harvesters with mistuning will have their peaks at different frequencies as shown in Figure 2(c), which will reduce the total power of the system with increase in bandwidth. Coupling two mistuned harvesters by spring will reduce the distance between peaks of harvesters and hence increasing the total power as shown in Figure 2(d).

3.2 Effect of damping

To understand the effect of damping on power harvested with and without coupling, power curves with different mechanical damping parameters are plotted with 1% mistuning of second harvester as shown in Figure 4. With increase in damping ratio the magnitude of power harvested decreases, and also the difference between total power of coupled and uncoupled harvesters decreases significantly.

3.3 Effect of coupling ratio and length ratio

The effect of coupling parameter on power harvested with 1% mistuning in harvester-2 is shown in Figure 4(a). When coupling ratio is *0* (uncoupled) power magnitude will have the least value and magnitude keeps increasing as the coupling ratio is increased. Rate of increment of power magnitude up to $\beta=0.03$ is more and after that rate of increment is very less. The value of coupling ratio to obtain maximum power depends on the amount of mistuning exist in the system. As the amount of mistuning increases the coupling ratio has to be increased in order to obtain he maximum power as shown in Figure 4(b). Figure 4(b) exhibits a asymmetry with respect to length ratio (mistuning) i.e., the amount of coupling ratio required to obtain maximum power when mistuning 0f -1% is present is less than that of when mistuning +1%.

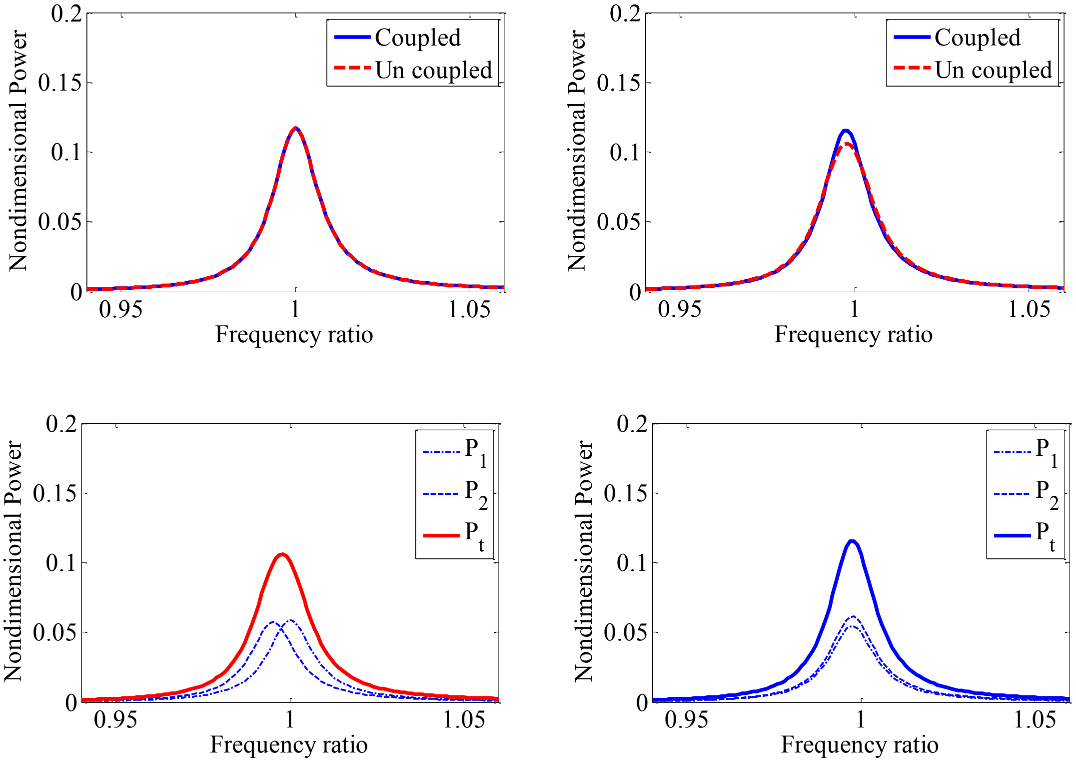

Figure 2: Power curves for two harvesters with Γ=0.01, f=0.05, Γe=0.006 and β=0.1 a) total power for equal length harvesters (α_1=α_2=1), b) total power for mistuned harvesters (α_1=1, α_2=1.01), c) and d) individual and total power for uncoupled and coupled system (α_1=1, α_2=1.01).

3.4 Effect on multiple harvesters

Since the number of harvesters are to be used to harvest maximum power the effect of mistuning on total power for multiple harvesters needs to be investigated. Therefore generalized equations for harvesting energy from *n* pendulums have been given in system model section. The maximum total power and total power at resonance for number of harvesters more than two are obtained for coupled mistuned (CM), uncoupled mistuned (UCM), coupled tuned (CT) and uncoupled tuned cases (UCT). As the number of harvesters increase the computation time to find maximum total power and total power at nominal resonance will increase tremendously.

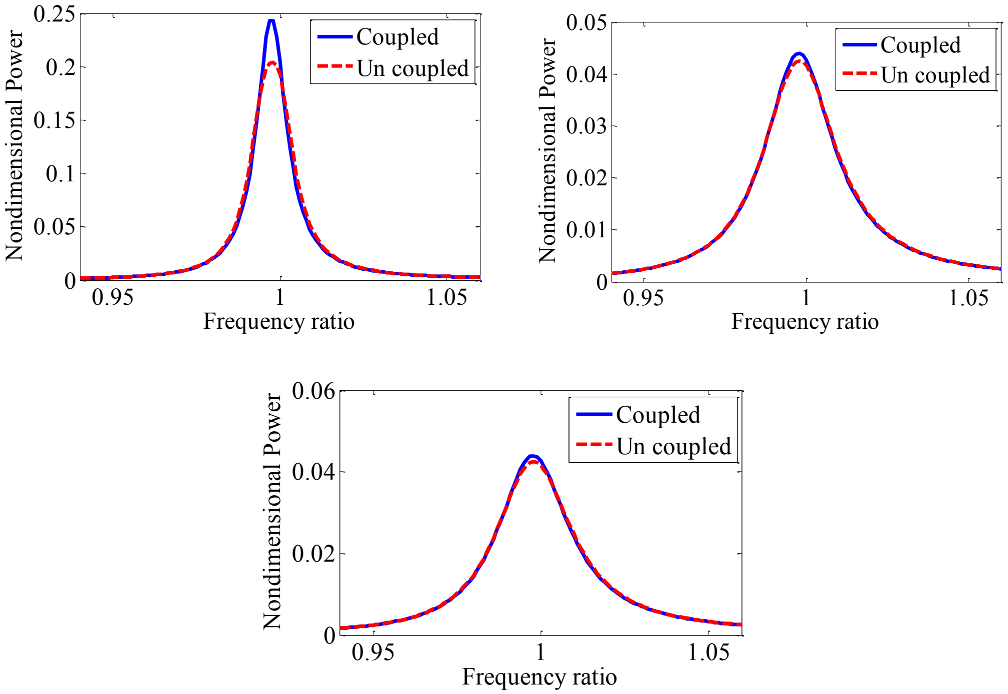

Figure 3: Effect of damping on total power f=0.05, Γe=0.006, β=0.1, α_1=1 and α_2=1.01 a) Γ=0.005, b) Γ=0.01 and c) Γ=0.02

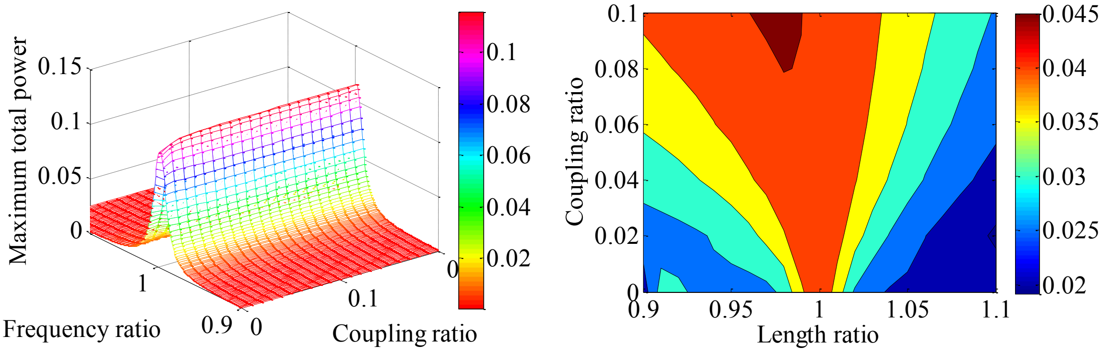

Figure 4. a) Effect of coupling on total power and b) combined effect of length ratio and coupling ratio on total power with f=0.05, Γ=0.01, Γe=0.006, α_1=1 and α_2=1.01.

Figure 5(a) shows how the maximum total power changes with number of harvesters with ±1% and Γ=0.01. The total power obtained with coupled tuned, uncoupled tuned and coupled mistuned are almost same where as the total power from uncoupled mistuned harvesters is less than the other configuration. Due to mistuning there will be a shift in frequency of occurrence of maximum power. Figure 5(b) shows total power at nominal resonance against number of harvesters. The total power obtained with coupled tuned, uncoupled tuned are same where as for coupled mistuned system total power is less than that of tuned initially but as number of harvesters increases the difference is reducing. For uncoupled mistuned case the difference keeps on increasing with increase in number of harvesters. Figure 5(c) and (d) shows the plot for maximum total power and total power at nominal resonance with damping parameter $\Gamma=0.01$, here the difference of total power between uncoupled mistuned and remaining three configuration is less but still the difference is increasing with increase in number of harvesters.

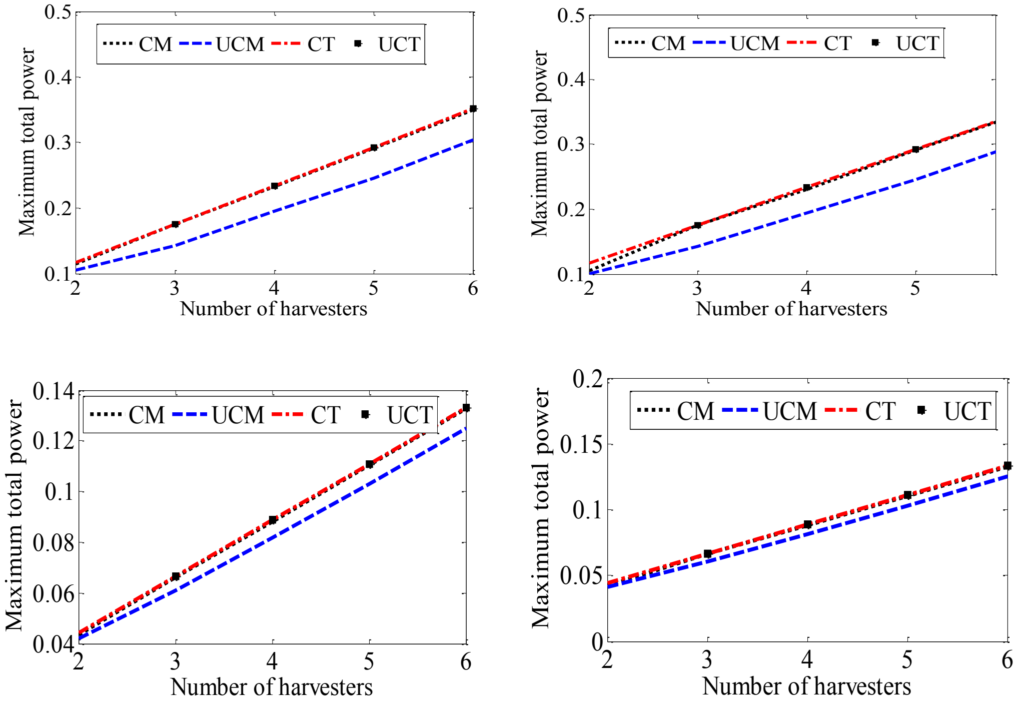

Figure 5: Number of pendulums vs total power for different cases (CM- Coupled Mistuned, UCM- Uncoupled mistuned, CT-Coupled Tuned, UCT- Uncoupled Tuned) $\alpha_1=1$, $\alpha_2=1.01$, $\alpha_3=0.99$, $\alpha_4=1.006$, $\alpha_5=0.994$, $\alpha_6=1.001$, $f=0.05$ and $\beta=0.1$. a) and b) maximum total power and total power at frequency ratio $\Omega=1$ with $\Gamma=0.01$, c) and d) maximum total power and total power at frequency ratio $\Omega=1$ with $\Gamma=0.02$, $\beta=0.1$

4. CONCLUSION

In the present work, the effect of mistuning, coupling and damping on total power is investigated. A generalized model of *n* pendulums with magnetic end mass for electromagnetic harvesting is developed. Numerical simulations are carried out to compare coupled and uncoupled harvesters. Length of first pendulum is taken as nominal and length of others are varied within ±1% with non repetitive lengths ratios. Tuned harvesters with or without coupling generates same amount of power, but when mistuning is present the coupled harvesters produces more power than uncoupled. With increase in damping the difference between power output of coupled and uncoupled harvesters decreases. It is also found that the difference keeps on increasing with increase in number of harvesters with mistuning present. The coupling parameter should be increased with increase in mistuning to increase the total power. These results provide an insight into effect of coupling on power harvested when mistuning due to variability in manufacturing exists and how the power harvested can be improved.

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