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Nonlinear MEMS Piezoelectric Harvesters in the presence of geometric and structural variabilities

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Abstract

This paper investigates the use of an electrostatic device to improve the performance of MEMS piezoelectric harvesters in the presence of geometric and structural variabilities due to the manufacturing process. Different types of uncertain parameters including material and geometric uncertainties have been considered. The variability of these parameters are estimated based on available existing experimental data in the literature. Monte Carlo simulation (MCS) is used for uncertainty propagation and it is shown that the resonance frequencies of the majority of the samples are far away from the excitation frequency and consequently this results in less harvested power. This paper identifies these samples and uses electrostatic devices to improve the performance of the harvester. The proposed device is composed of an unsymmetric arrangement of two electrodes to decrease the resonance frequency of samples through a softening nonlinearity. The unsymmetric arrangement of two electrodes is inevitable and due to geometric variability of the harvester. There are also two arch shape electrodes which can be used to create a hardening effect to increase the resonance frequency of samples which have resonance frequencies smaller than the nominal value.

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1. Introduction

Harvesting energy from ambient vibration sources is one of the promising ways to provide power for Micro Electro Mechanical Systems (MEMS). The most common types of transduction methods are electrostatic [1],

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electromagnetic [2] and piezoelectric [3]. Due to the compatibility between piezoelectric material deposition and the MEMS fabrication process, piezoelectric convertors have been recognized to offer significant benefits. Generally, MEMS piezoelectric harvesters are designed to work at resonance in order to maximize the output power. However, in most cases, there is a mismatch between the natural and excitation frequencies because of manufacturing uncertainties. Therefore, the performance of the harvester will be affected by tolerances that can be higher than ±10% of nominal values [4]. Uncertainty analysis of MEMS devices has been studied by several authors in the literature. Agarwal and Aluru [5] presented a framework to quantify different outputs in MEMS structures, such as deformation and electrostatic pressure. In another study by Agarwal and Aluru [6], the effect of variations in Young's modulus, induced as a result of variations in the manufacturing process parameters or heterogeneous measurements on the performance of a MEMS switch was investigated.

In this paper, a model based on electrostatic forces has been proposed to compensate the effect of manufacturing uncertainties on the performance of MEMS piezoelectric harvesters. In this model, the resonance frequency of samples has been tuned using an arch shaped electrode and two straight electrodes. Based on applying voltage to these electrodes, the resonance frequency of the samples can be adjusted through hardening and softening mechanisms. Applying DC voltage to the arch shaped electrode creates a tensile follower force which can increase the resonance frequency of samples linearly. On the other hand, the resonance frequency of samples can be decreased by applying voltage to the straight electrodes which creates a softening nonlinearity. The Galerkin method has been used to discretise the equations of motion. In order to show the results, a single mode approximation of the harvester has been considered and the shooting method has been used to solve the equations. The distributions of the resonance frequency and the harvested power of the samples have been estimated using MCS in the presence of structural uncertainty.

2. Model description and mathematical modeling

Fig. 1 shows the proposed model in this paper. The model is an isotropic micro-beam of length L, width a, thickness h, density ρ and Young's modulus E, sandwiched with piezoceramic layers having thickness h_0, Young's modulus E_0 and density ρ_0 throughout the micro-beam length and located between two straight-shaped electrodes and one arc-shaped electrode. As illustrated in Fig. 1, the piezoceramic layers are connected to the resistance (R) and the coordinate system is attached to the middle of the left end of the micro-beam where x and z refer to the horizontal and vertical coordinates respectively. The free end of the micro-beam is attached to the two arc-shaped comb fingers which subtend angle α at the base of the beam and remain parallel to the fixed arc-shaped electrode. The governing equation of transverse motion can be written as [7]

$$\begin{aligned}(EI)_{eq}\frac{\partial^4 w(x,t)}{\partial x^4}+(\rho A)_{eq}\frac{\partial^2 w(x,t)}{\partial t^2}+c_a\frac{\partial w(x,t)}{\partial t}+F_f\frac{\partial^2 w(x,t)}{\partial x^2}\\ -\vartheta_p v_p(t)\left(\frac{d\delta(x)}{dx}-\frac{d\delta(x-L)}{dx}\right)=F_e-\left((\rho A)_{eq}+M_t\delta(x-L)\right)\frac{\partial^2 z(t)}{\partial t^2}\end{aligned} \tag{1}$$

$$C_p\frac{dv_p(t)}{dt}+\frac{v_p(t)}{2R}+\frac{\overline{e}_{31}a(h_0+h)}{2}\int_0^L\frac{\partial^3 w(x,t)}{\partial x^2\partial t}dx=0 \tag{2}$$

where

$$(EI)_{eq}=\frac{2a}{3}\left(\frac{Eh^3}{8}+E_0\left(\left(h_0+\frac{h}{2}\right)^3-\frac{h^3}{8}\right)\right),\quad (\rho A)_{eq}=a(\rho h+2\rho_0 h_0) \tag{3}$$

$$C_p = \frac{\bar{\varepsilon}_{33}^s a L}{h_0}, \quad \vartheta_p = \frac{\bar{e}_{31} a}{h_0}\left(\left(h_0 + \frac{h}{2}\right)^2 - \frac{h^2}{4}\right)$$

and subjected to the following boundary conditions

$$w(0,t) = 0, \quad \frac{\partial w(0,t)}{\partial x} = 0, \quad \frac{\partial}{\partial x}\left((EI)_{eq}\frac{\partial^2 w(L,t)}{\partial x^2}\right) = M_t \frac{\partial^2 w(L,t)}{\partial t^2}, \quad \frac{\partial^2 w(L,t)}{\partial x^2} = 0 \tag{4}$$

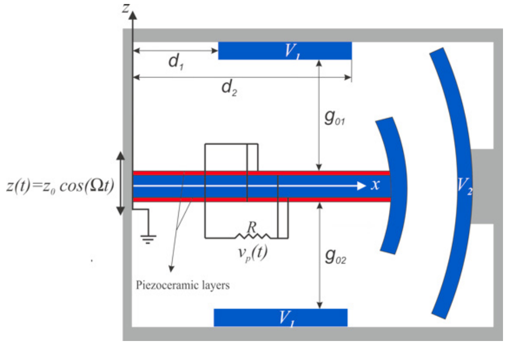


Fig. 1. Schematic of the proposed energy harvester

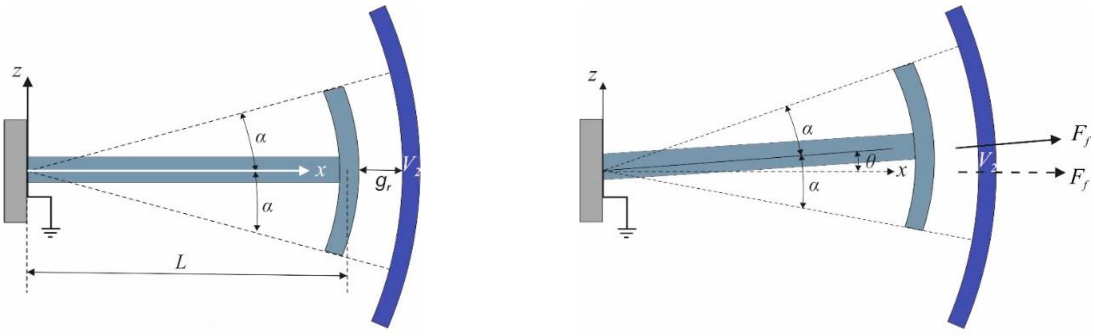
In Equations (1) and (2), $w(x,t)$ is the transverse deflection of the beam relative to its base at the position x and time t, c_a is the viscous air damping coefficient, $\delta(x)$ is the Dirac delta function, $z(x)$ is the base excitation function, F_f is the follower force which is applied to the harvester by the arched-shaped electrode, F_e is the electrostatic force which is applied to the harvester by the straight-shaped electrodes, $v_p(t)$ is the voltage across the electrodes of each piezoceramic layer, ϑ_p is the coupling term and $\bar{\varepsilon}_{33}^s$ is the permittivity component at constant strain with the plane stress assumption for the beam. Using electrostatic principles, the electrostatic force between the micro-beam and the straight electrodes (F_e) can be written as

$$F_e = \frac{\varepsilon_0 a H(x)}{2}\left(\frac{V_1^2}{(g_{01} - w)^2} - \frac{V_1^2}{(g_{02} + w)^2}\right) \tag{5}$$

where

$$H(x) = H(x - d_1) - H(x - d_2) \tag{6}$$

In Equation (5), ε_0 is the permittivity of free space, $H(x)$ is the Heaviside function, V_1 is the applied DC voltage to the straight electrodes, g_{01} and g_{02} are the air gaps between the micro-beam and the straight electrodes. By applying voltage to the arched-shaped electrode shown in Fig. 2, the amplitude of F_f can be tuned. Fig. 2 shows for a small angular deflection of the micro-beam (θ), the angular overlap between the fingers and the arched-shaped electrode is always 2α and the force remains a follower force in all conditions.



(a) (b)

Fig. 2: A micro-beam with an arc-shaped comb fingers in its (a) unperturbed state and (b) perturbed state.

Based on electrostatic principles, the amplitude of the follower force $\left(F_f\right)$ can be written as

$$F_f = \frac{\varepsilon_0 a L \alpha}{g_r^2} V_2^2 \tag{7}$$

In order to eliminate the spatial dependence in Equations (1) and (2), the Galerkin decomposition method is used. The deflection of the micro-beam can be represented as a series expansion in terms of the eigenfunctions of the micro-beam. By considering a single-mode approximation, the model is converted into a system of differential equations. Due to the electrostatic nonlinearity in Equation (5), finding an analytical solution to study the dynamic behaviour of the system is quite complicated. However, there are different methods to find an approximated analytical solution of Equations (1) and (2). Previously the authors (Madinei et al. [7]) used the harmonic balance method to study the dynamic behaviour of the system by considering the approximate electrostatic force. They assumed a symmetric electrostatic force in their approximation, but in the presence of manufacturing uncertainties the electrostatic force is unsymmetric. Therefore, more effort is needed to find an approximate analytical solution. In this work, the shooting method has been used to study the dynamic behaviour of the system [8].

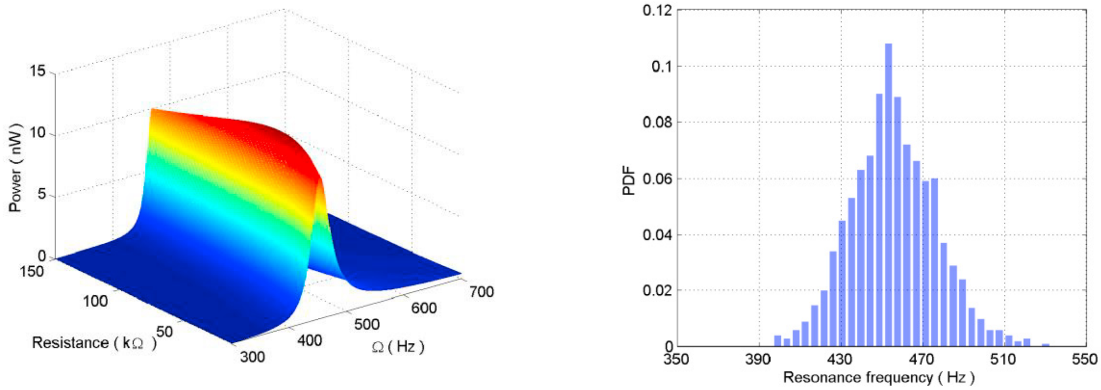
3. Numerical results and discussion

To demonstrate the approach, a bimorph piezoelectric micro cantilever beam and a Gaussian distribution of parameters are assumed. The parameters are given in Table 1

Table 1. Geometrical and material properties of the samples [4].

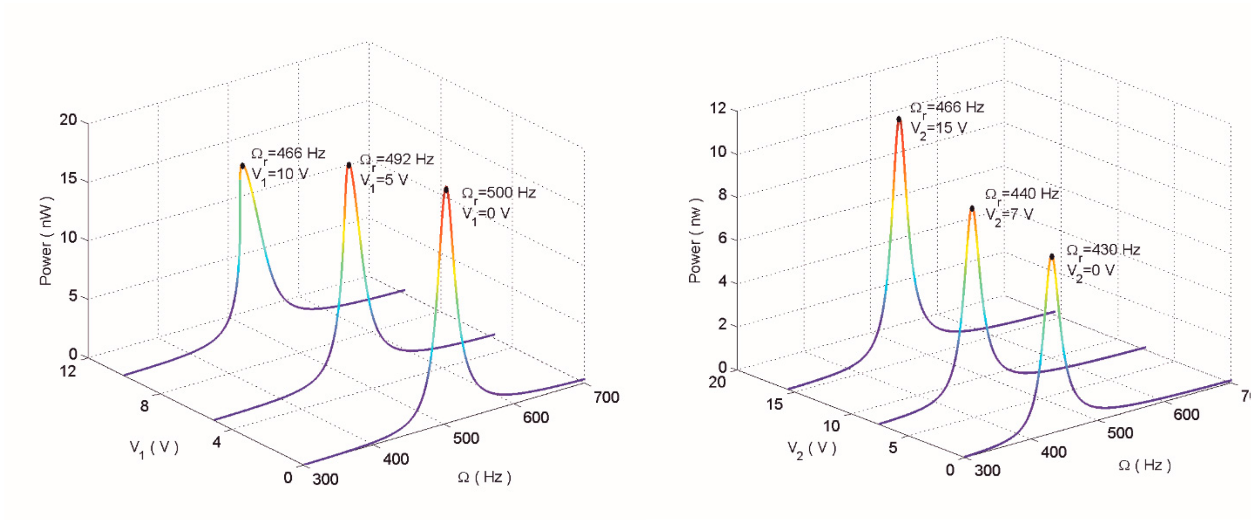
Data	Mean	Standard Deviation	COV (%)
Length, $L\left(\mu m\right)$	3000	4.2	0.14
Width, $a\left(\mu m\right)$	500	2.3	0.46
Thickness, $h\left(\mu m\right)$	4	0.35	8.75
Thickness, $h_0\left(\mu m\right)$	2	0.175	8.75
Young's modulus, $E\left(GPa\right)$	169.6	16.58	9.78
Young's modulus, $E_0\left(GPa\right)$	65	6.35	9.78
Air gap, $g_0\left(\mu m\right)$	40	3.5	6.3
Air gap, $g_r\left(\mu m\right)$	2	0.175	6.3

Based on the mean parameters of the micro-beam, the resonance frequency of the harvester is 466 Hz. As shown in Fig. 3a, by exciting harvester at its resonance frequency, the output power can be maximized at the optimal resistance. However, due to manufacturing uncertainties the resonance frequencies of the samples have a large deviation from one sample to another (see Fig. 3b) and this can decrease the performance of the harvester significantly. Therefore, by tuning the resonance frequency of the samples, the efficiency of the harvester can be improved. Using the proposed electrostatic device in Fig. 1, the resonance frequencies of the samples can be adjusted to the mean resonance frequency (i.e. 466 Hz). By applying voltage to the straight electrodes, the electrostatic field induces the softening nonlinearity. Therefore, this mechanism can be used to decrease the resonance frequency of samples which are greater than 466 Hz. Fig. 4a shows that by applying voltage to straight electrodes the resonance frequency of a sample can be changed from 500 Hz to 466 Hz. On the other hand, by applying tensile follower force to the harvester through the arch electrodes, the resonance frequency of samples can be increased to match the frequency of the vibration source. As shown in Fig. 4b, by applying voltage up to 15 V, the resonance frequency of a sample can be changed from 430 Hz to 466 Hz. In addition to the harvested power through the piezoelectric layers, there is an energy cycle between the beam and straight electrodes which uses an electrostatic conversion mechanism to charge voltage source which is connected to straight electrodes [7]. Also, due to the insignificant deflection of the beam in the horizontal direction the loss of voltage in arch electrodes is negligible.



(a) (b)

Fig. 3: (a) Variation of the piezoelectric peak power with load resistance (b) Probability density function of resonance frequency



(a) (b)

Fig. 4: Tuning resonance frequency of harvester using (a) softening and (b) hardening mechanism

4. Conclusions

In this paper, the effect of geometric and structural variabilities on the performance of MEMS piezoelectric harvesters was investigated. The steady state solution was obtained by using a shooting method and many samples were considered based on the Monte Carlo simulation. The results showed that the resonance frequency of the samples have a large deviation from one sample to another and results in lower harvested power. In order to adjust the resonance frequency of the samples, hardening and softening mechanisms were used. Based on this approach, it was observed that the harvested power can be increased by applying DC voltage to the straight electrodes and arch shaped electrodes, respectively. Furthermore, it was pointed out that the voltages sources can be charge through the electrostatic mechanism.

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