Optimal Multi-user Transmission based on a Single Intelligent Reflecting Surface

Yihong Liu*, Lei Zhang*, Paulo Valente Klaine* and Muhammad Ali Imran*

* James Watt School of Engineering, University of Glasgow, Glasgow, G12 8QQ, United Kingdom
Email: y.liu.6@research.gla.ac.uk, lei.zhang@glasgow.ac.uk,
paulo.valenteklaine@glasgow.ac.uk, muhammad.imran@glasgow.ac.uk

***Abstract*—Intelligent Reflecting Surface (IRS) is envisioned as a revolutionary solution for providing high data rate communication, better signal coverage and physical layer security through controlling the transmission environment of electromagnetic (EM) wave. In this paper, based on a single IRS and millimeter channel model, the optimal multi-user transmission sum rate is derived with the maximal ratio combining (MRC) beamformer. The conditions to achieve such a maximum sum rate are expressed as a function of placement of transceivers, number of IRS elements and its spacing. We first transformed the single IRS assisted non-scattering mmWave channel into a high-rank multiple-in multiple-out (MIMO) channel. The result shows that even without rich scattering, each single antenna user can enjoy the maximal passive beamforming gain while keeping spatial orthogonality to leverage the maximal spatial diversity when the corresponding conditions are satisfied. Simulation results verify the existence and accuracy of optimal placement of multi-user transmission and present that the transmission sum rate of IRS channel scales by the number of IRS elements with different spacing settings.**

***Index Terms*—Intelligent reflecting surface, beamforming, mmWave, MIMO, multi-users.**

I. Introduction

Magnificent efforts have been devoted to improving the spectrum efficiency and power efficiency in the generations of wireless communication from all different aspects, including algorithms, hardware, architecture, protocols, etc [1]. However, the performance metrics are bounded due to the uncontrollable and unpredictable wireless channel. Thus, significantly more power and spectrum resources are required to combat the path-loss and selective fading within the wireless channel [2].

Until recently, intelligent reflecting surface (IRS) is proposed to enhance the power and spectrum efficiency of wireless transmission by controlling the wireless channel [3]. IRS is normally comprising of a set of conductive patches, diodes, and conductive power/signal lines. By integrating artificially designed electronic elements, i.e., PIN diode, in the surface of a meta-material, IRS can be produced [4]. Meanwhile, the excitation and phase of each electronic element can be orchestrated by processors and controllers, e.g., Field Programmable Gate Array (FPGA); thus, the phase and amplitude of the electromagnetic (EM) wave impinging on the surface can be manipulated correspondingly [3]. IRS is evolutionary for wireless communication systems because it

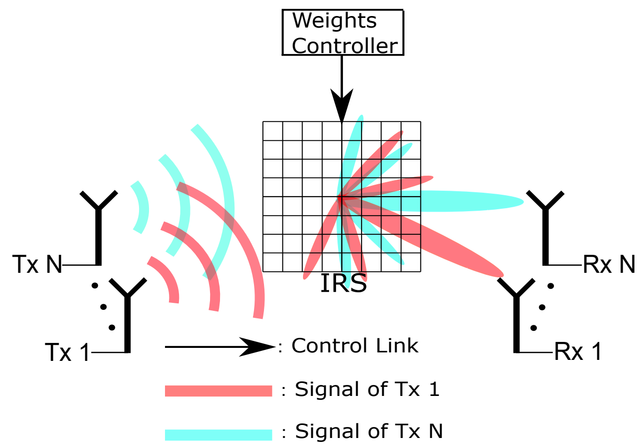

Figure 1. Multi-user transmission base on a single IRS.

essentially provides an extra degree of freedom to control and design the wireless channel instead of compromising to the unpredictable and uncontrollable characteristic. As shown in Fig. 1, controlling the wireless channel is achieved by deploying the IRS between transmitters (Tx) and receivers (Rx), which can steer the impinging EM wave while focus the impinging energy to realize passive beamforming for its receivers. It would be impressively beneficial for wireless communication system working in mmWave band or higher frequency bands, since no direct link between Tx and Rx could be a common and a pressing issue which result in the overhead from the densification of mmWave network [5].

Extensive research has been done for IRS regarding beamforming [6], channel estimation [7] and physical layer security [8], etc. However, the general IRS channel assumption may conflict with realistic situation in mmWave. Specifically, the channel between transmitters (Tx) to the IRS and the IRS to the receivers (Rx) are often assumed to be scattering enough, i.e., Rayleigh fading channel, to assure a high-rank channel condition [6], [9]. However, mmWave communication, which is more demanding on the assistance of IRS, its channel is significantly restricted in terms of directions and the wireless channel rank shall not as rich as we assume when the IRS is in far-field to each transceiver [10]. In fact, the IRS performance based on single-input-single-output (SISO) has been evaluated in mmWave scenario [11]. In addition, multi-user (MU) transmission has been proposed in [12] through space-time-coding of a single IRS. In [13], the author propose to use multiple IRSs to realize MU transmission while maximizing the MU's sum rate. However, it is still unclear how to achieve the MU transmission through a single IRS in the less scattering channels (such as mmWave channel). In addition, if the spatial diversity of a single IRS can be optimally leveraged, the overhead of deploying IRS due to the densification of network can be significantly mitigated. Thus, a thorough analysis on spatial multiplexing of a single IRS is required and it is also necessary to derive the bound of the MU sum rate to guide the deployment of the IRS network in real world applications.

The main contributions of this paper are as follows.

- We propose to optimally leverage the spatial multiplexing of a single IRS, where the maximal beamforming gain and interference nullification can both be achieved in MU transmission.
- The maximum sum rate of MU transmission based on a single IRS is analyzed in the uniform linear array (ULA) scenario. Furthermore, the condition for achieving such a bound is derived.
- The relationship between the number of elements, elements' spacing and rank of the IRS channel is derived by introducing the concept of unified length in an MIMO.

The rest of the paper is organized as follows. In Section II, the general system model of a single IRS is presented. In Section III, the placement condition to achieve the maximum MU sum rate are derived, the bound of sum rate is given, and the relationship between channel ranks and element spacings are discussed in mmWave channel model. Simulations and conclusions are given in Section IV and Section V, respectively.

II. System Model

In this Section, we model a distributed MU system based on a single IRS, which take the role of steering the multiple beams to the desired directions. The considered system contains *N* pairs of transceivers where each Tx and Rx is equipped with a single antenna. Here, a single IRS is assumed to be composed of *M* elements shaping an ULA. Generally, the received signal vector at the receivers can be written as a vector form

$$\mathbf{y} = (\mathbf{A}_{out}^T \mathbf{W} \mathbf{A}_{in} + \mathbf{D})\mathbf{s} + \mathbf{n} \ , \tag{1}$$

where $\mathbf{s} = [s_1, s_2, \ldots, s_N]^T \in \mathbb{C}^{N\times 1}$ is the source signal vector and s_i $(i = 1, 2, \cdots, N)$ is the signal from i-th transmitter $\mathtt{Tx}_i$. The weights matrix $\mathbf{W} \in \mathbb{C}^{M\times M}$ is a diagonal matrix with each entity on the diagonal being the weight value of each element. $\mathbf{D} \in \mathbb{C}^{N\times N}$ is the traditional MIMO channel between *N* pairs of transceivers without considering the IRS, and $\mathbf{n}$ is the noise vector at the receivers. $\mathbf{A}_{in} \in \mathbb{C}^{M\times N}$ and $\mathbf{A}_{out} \in \mathbb{C}^{M\times N}$ are the LOS channel matrix of angle of arrival (AOA) and angle of departure (AOD) for IRS respectively. Specifically,

$$\mathbf{A_{in}} = [\mathbf{a}(\Omega_{in,1}), \mathbf{a}(\Omega_{in,2}), \ldots, \mathbf{a}(\Omega_{in,N})] \ , \tag{2}$$

where $\mathbf{a}(\Omega_{in,i})$ is i-th user's steering vector of incident directions and $\Omega_{in,i}$ is the term containing the spatial information of incident directions of $\mathtt{Tx}_i$. Ω is a function of azimuth and elevation angles for a two-dimensional deployment, e.g., uniform rectangular array (URA), or contains only one-dimensional information of azimuth angle such as ULA.

Since $\mathbf{A_{out}}$ has the same form as $\mathbf{A_{in}}$, the received signal of i-th receiver $\mathtt{Rx}_i$ in (1) can be rewritten as

$$y_{r,i} = (\mathbf{w}^H \mathbf{A}_{C,i} + \mathbf{d_i})\mathbf{s} + n_i \, , i = 1, 2, ..., N \ \ , \tag{3}$$

where $\mathbf{w}$ is a column vector with its elements being the main diagonal elements of $\mathbf{W}$ and $\mathbf{w}^H$ is its conjugate transpose. It is worth to mention that $\mathbf{w}$ is the *single* weight vector employed on the surface to achieve *all* transceiver pairs' desired signal response and mutual interference suppression simultaneously. $\mathbf{d_i}$ is the i-th row vector of matrix $\mathbf{D}$, n_i is the noise at $\mathtt{Rx}_i$, and $\mathbf{A}_{C,i}$ is the combined steering matrix for i-th receiver which can be expressed as

$$\mathbf{A}_{C,i} = [\mathbf{a}_C(\Omega_{out,i} \, , \Omega_{in,1}), \ldots, \mathbf{a}_C(\Omega_{out,i} \, , \Omega_{in,N})] \in \mathbb{C}^{M\times N}, \tag{4}$$

where

$$\mathbf{a}_C(\Omega_{out,v} \, , \Omega_{in,u}) = \sqrt{l_{IRS,uv}}\mathbf{a}(\Omega_{out,v}) \odot \mathbf{a}(\Omega_{in,u}) \, , \tag{5}$$

for $u, v = 1, ..., N$, which is the equivalent steering vector combining AOA and DOA directions. $l_{IRS,uv}$ is the path-loss given by [14]

$$l_{IRS,uv} = \frac{G_t G_r}{(4\pi)^2} (\frac{S}{d_{in,u} d_{out,v}})^2 cos^2(\Omega_{in,u}), \tag{6}$$

where G_t and G_r are the transmitting and receiving antenna gains, respectively. S is the effective area of a single element of the IRS and $d_{in,u}$ and $d_{out,v}$ are the distances from $\mathtt{Tx}_u$ to IRS and from IRS to $\mathtt{Rx}_v$, respectively. It is worth to note the distance between the antennas of transceivers and the IRS must be sufficiently larger than the wavelength of the carrier, such that they are in the far-field of each other, i.e., $d_{in,u} >> \lambda$ and $d_{out,v} >> \lambda$.

III. Optimal MU transmission sum-rate

In [15], we briefly presented that a single piece of the IRS is able to support MU transmission. However, the spatial multiplexing of a single IRS was not discussed in the paper. Here, in this Section, we will analytically derive the sum-rate bound of a single IRS for MU transmission. Considering the mmWave channel, as shown in Fig. 1, the direct link component is omitted here as non-LOS (NLOS) path between Tx and Rx can be a common issue, but it can be reconsidered for scenarios having strong direct path. We omit the path-loss in the analysis since it does not affect the derivations. Noise component at receivers can also be ignored as it does not affect the derivation of the optimal solution but we will consider it when derive the sum rate. Thus, (1) simply becomes:

$$\hat{\mathbf{y}}_r = \mathbf{A}_{out}^T \mathbf{W} \mathbf{A}_{in} \mathbf{s} \ . \tag{7}$$

If we compare the IRS channel to the traditional MIMO channel, it can be seen as a controllable channel where each term is adjusted by the weights vector $\mathbf{w}$ as

$$\mathbf{H} = \mathbf{A}_{out}^T \mathbf{W} \mathbf{A}_{in} = \begin{bmatrix} \mathbf{w}^H \mathbf{a}_C(\Omega_{out,1}, \Omega_{in,1}) & \dots & \mathbf{w}^H \mathbf{a}_C(\Omega_{out,1}, \Omega_{in,N}) \\ \mathbf{w}^H \mathbf{a}_C(\Omega_{out,2}, \Omega_{in,1}) & \dots & \mathbf{w}^H \mathbf{a}_C(\Omega_{out,2}, \Omega_{in,N}) \\ \vdots & \ddots & \vdots \\ \mathbf{w}^H \mathbf{a}_C(\Omega_{out,N}, \Omega_{in,1}) & \dots & \mathbf{w}^H \mathbf{a}_C(\Omega_{out,N}, \Omega_{in,N}) \end{bmatrix}. \tag{8}$$

To be noted, the correlation between each subchannel is essentially the dominating factor for the sum rate, since it directly influences the upper bound of the IRS. I.e., the higher the correlation channel for transmitters or receivers, the lower the channel rank and singular value are, which further results in the lower upper bound of the overall sum rate. Another factor that affects this performance is the effective area of the IRS. Intuitively, to define an optimal upper bound of capacity, we consider a condition about a specific relationship of channels between each transmitter and receiver such that every pair can leverage the maximal power gain brought by the IRS. Besides, the interference between each pair can be nullified if MU pairs are keeping spatial orthogonality. Therefore, we have the following theorems,

Theorem 1. *In order to obtain the maximum power gain and spatial multiplexing gain simultaneously, the required positions for Tx and Rx are*

$$\alpha_j = \cos^{-1}\left(\frac{j}{L} - \zeta_m - \cos\beta_i \pm \frac{1}{\Delta r}\right), \tag{9}$$

and

$$\beta_j = \cos^{-1}\left(\frac{j}{L} - \zeta_m - \cos\alpha_i \pm \frac{1}{\Delta r}\right), \tag{10}$$

where the i-th pair user is located at $\phi_{in,i} = \alpha_i$, $\phi_{out,i} = \beta_i$, and j-th pairs are at $\phi_{in,j} = \alpha_j$, $\phi_{out,j} = \beta_j$ and $i \neq j, j = 1, 2, ..., N$. $\Delta r = \frac{d}{\lambda}$ is the normalized spacing between each IRS element since d is the distance between each element and λ is the carrier wavelength. $L = M\Delta r$ is the unified length of the IRS, and

$$\zeta_m = -cos\alpha_i - cos\beta_i + \frac{K}{\Delta r}, \tag{11}$$

is the optimal factor given by MRC for the i-th pair transceivers.

Theorem 1 reveals that if the position of each transceiver can be coordinated correspondingly, the channel matrix in (8) can be optimized such that the diagonal terms can be maximized and and the off diagonal terms can be nullified at the same time. In other word, each reflected beam towards each Rx is orthogonal to others, even though we are using a single set of weights in the single IRS. Therefore, we define such a link between each transceiver as the optimal link. The proof is omitted due to the length limit of this paper, but it will be presented in the extended version of this work.

The normalized length, L, indicates the maximal number of channel rank for a single IRS, which is M, thus we have

Theorem 2. *Given all transceivers are optimally positioned, the upper bound of the sum rate for a single IRS is*

$$C_{Max} = Nlog(1 + \frac{P_T M^2}{N_0}), if\ N \leq M, \tag{12}$$

where P_T is the power of transmitters, N is the spatial multiplexing gain and N_0 is the noise power at the receivers, given M pairs.

When $N > M$, the interference between users is unavoidable, even if zero-forcing is applied. In this case, the bound of sum rate can be determined by specific placement of transceivers and ratio between N and M and numerical analysis can also apply to calculate the bound. Other multiplexing scheme, e.g, time division multiplexing, is proposed to avoid the interference. However, this case is rare in the real situation since $N < M$ can be guaranteed easily as there can be hundreds of thousands of IRS elements.

Based on Theorem 2, the maximal sum-rate is derived when $M = N$ and each pair receives the power gain of M^2. Considering the effective area of a single element and distance between transceivers and IRS, the lossless received power, which is P_T, should multiply with the path-loss factor in (6). If the direct link is reconsidered, the spatial multiplexing gain becomes a summation of N and the power gain should be modified as $(M + D_i)^2$, where D_i is the gain from i-th direct link between i-th pair, meaning an ideal situation that only those diagonal terms in $\mathbf{D}$ exist. However, the theorems still hold as the ideal assumption to $\mathbf{D}$ will not affect the upper bound we derived.

Remark 1. *Without changing L, the unified length of the IRS, no matter if the number of elements and distance changes, the IRS channel rank will not change. In fact, the channel rank is proportional to L.*

Therefore, we propose to use $d = \frac{\lambda}{2}$, since this is the maximal spacing for a fixed L, which causes no grating lobe of the reflected beam while using the minimal number of elements M to utilize the maximal number of channel rank fully. To be noted, there is a trade-off between energy efficiency and spacing as well. The origin of trade-off is due to smaller spacing resulting in a smaller number of channel rank and larger beamwidth, but the redundant beam that causes energy waste in trivial directions is less likely to occur [15]. Thus, based on specific considerations, the actual spacing can be less than this value. Some discussions about the spacing and the beamwidth of IRS are presented in [14], [15], [16].

IV. Simulations and Discussions

We consider the MRC solution of beamforming to illustrate the optimal position depicted in the section above, where MRC is optimal for the 1st fixed pair, which is located at $\mathbf{a}_C(\phi_{in,1}, \phi_{out,1}) = (30°, 135°)$ for ULA shape's IRS. SNR is assumed to be 10 dB.

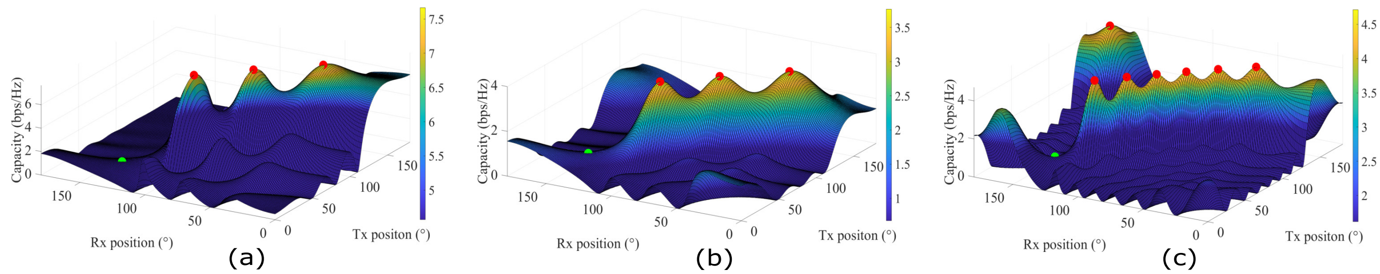

Figure 2. The capacity vs pairs' position. The optimal positions for the 2nd pair are marked by red dots. Green dot marked the position of the 1st pair position.(a) $M = 64$, $L = 2$, (b) $M = 4$, $L = 2$, (c) $M = 8$, $L = 4$.

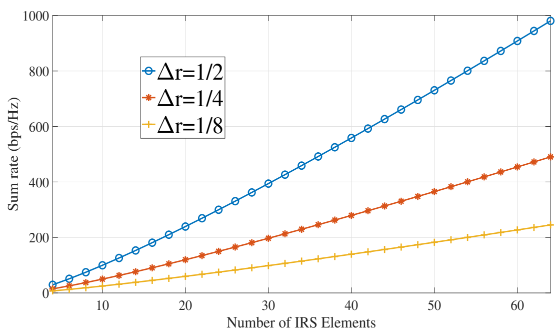

Figure 3. Optimal capacity vs number of IRS elements with different Δr

According to the theorems proposed in this paper, we can analytically calculate the optimal available positions for the 2$^{\text{nd}}$ pair, where it can harvest both maximal spatial multiplexing gain and diversity gain from a single IRS while considering the interference from the 1$^{\text{st}}$ fixed pair. Analytically, these positions are $(68.5°, 102°)$, $(97.7°, 73.0°)$, and $(129.3°, 37.5°)$, respectively when $L = 2$. Both Fig. 2(a) and (b) show that the theories in Section III accurately give the optimal positions for other pairs and we can also observe that increasing the elements under the fixed unified length L will not change the spatial diversity gain, as all the optimal positions remain in the same place but only with narrower beamwidth and higher power gain. Fig. 2(c) shows doubling the normalized length L also doubles the available channel rank, and 8 pairs can be optimally supported in this case. As shown in Fig. 3, larger spacing between IRS elements can have better performance on the optimal sum rate. The sum rate enjoy benefits from scaling of number of IRS elements.

V. Conclusions and Future Works

In this paper, the maximum spatial multiplexing gain and maximum power gain of a single IRS is achieved simultaneously. The condition to reach the maximum MU sum rate is firstly derived. The theoretical upper bound of sum rate given a single IRS is derived secondly. In addition, the relationship between the rank of the IRS channel, IRS element spacing and number of elements is also discussed. Simulation results verify the theorem of optimal placement condition for each transceiver and the concept of unified length related to the spatial multiplexing.

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