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# Quantifying the small-area spatio-temporal dynamics of the Covid-19 pandemic in Scotland during a period with limited testing capacity

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## Abstract

Modelling the small-area spatio-temporal dynamics of the Covid-19 pandemic is of major public health importance, because it allows health agencies to better understand how and why the virus spreads. However, in Scotland during the first wave of the pandemic testing capacity was severely limited, meaning that large numbers of infected people were not formally diagnosed as having the virus. As a result, data on confirmed cases are unlikely to represent the true infection rates, and due to the small numbers of positive tests these data are not available at the small-area level for confidentiality reasons. Therefore to estimate the small-area dynamics in Covid-19 incidence this paper analyses the spatio-temporal trends in telehealth data relating to Covid-19, because during the first wave of the pandemic the public were advised to call the national telehealth provider NHS 24 if they experienced symptoms of the virus. Specifically, we propose a multivariate spatio-temporal correlation model for modelling the proportions of calls classified as either relating to Covid-19 directly or having related symptoms, and provide software for fitting the model in a Bayesian setting using Markov chain Monte Carlo sim-

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ulation. The model was developed in partnership with the national health agency Public Health Scotland, and here we use it to analyse the spatio-temporal dynamics of the first wave of the Covid-19 pandemic in Scotland between March and July 2020, specifically focusing on the spatial variation in the peak and the end of the first wave.

*Keywords:* Covid-19 pandemic, Gaussian Markov random field models, Scotland, Telehealth data.

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## 1. Introduction

Covid-19 represents the biggest public health challenge in decades, and was declared a global pandemic by the World Health Organisation on 11th March 2020. The disease originated in the city of Wuhan in the People's Republic of China in December 2019, and reached the USA and Europe towards the end of January 2020. The first European epicentre for Covid-19 was in northern Italy in February 2020, and in Scotland, the focus of this paper, the first confirmed case occurred on the 2nd March 2020 (Public Health Scotland, <https://www.opendata.nhs.scot/dataset/covid-19-in-scotland>). Since then Covid-19 has spread across the world causing global health and economic devastation, and as of 30th March 2021 there have been over 127 million cases worldwide with over 2.7 million people sadly dying from the disease (Johns Hopkins Coronavirus Resource Centre, <https://coronavirus.jhu.edu/map.html>).

Unsurprisingly, modelling the spread and dynamics of the Covid-19 pandemic has become a research priority, and there is a quickly growing research literature in this area. This literature has focused on a range of important epidemiological topics, including: (i) predicting the spread of the pandemic and its impacts on healthcare systems (Remuzzi and Remuzzi, 2020); (ii) identifying the factors that make people more at risk of displaying severe symptoms (Conticini et al., 2020, Wu et al., 2020 and Konstantinou et al., 2021); (iii) identifying the wider health impacts of the pandemic (Douglas et al., 2020); and (iv) developing surveillance systems for identifying the spatio-temporal dynamics in disease incidence (Dong et al., 2020). Developing a small-area surveillance system for monitoring the spatio-temporal trend in Covid-19 incidence is a vital tool in the fight against the virus, because it allows public health agencies to monitor its spread and identify hot-spots with high incidence, as well as providing vital clues as to how and why the

29 virus spreads more easily in certain areas.

30 The focus of this study is Covid-19 surveillance in Scotland, which is  
31 currently in its second wave of infection since September 2020. During this  
32 second wave the spatio-temporal spread of the pandemic can be measured  
33 using data on positive tests at the small-area scale, which is due to Scot-  
34 land having a wide-spread testing programme during this period. This  
35 programme allows any member of the public to book a test at <https://www.gov.uk/get-coronavirus-test>, and well over 15,000 tests are con-  
36 ducted each day. However, during the first wave of the pandemic between  
37 March and July 2020 Covid-19 testing capacity was strictly limited to priority  
38 groups, because there was a lack of infrastructure to allow large-scale test-  
39 ing. For example, in March 2020 only 350 tests could be conducted each day  
40 (<https://www.gov.scot/publications/foi-202000084813/>), which rose  
41 to 1,900 in April 2020. Therefore in this first wave the public were not able  
42 to access a diagnostic test to determine if they had the virus unless a test  
43 was recommended by a doctor. Instead, anyone experiencing symptoms was  
44 advised to phone the national telehealth service NHS 24 for medical advice,  
45 and was then asked to self-isolate at home. As a result data on confirmed  
46 Covid-19 cases will not provide a detailed picture of the spatio-temporal  
47 spread of the virus during this first wave, because only a very small fraction  
48 of the actual cases were confirmed by a positive test.

49  
50 Due to this massive under-reporting the aim of this paper is to use proxy  
51 indicators of disease incidence to quantify the small-area spatio-temporal  
52 dynamics of the Covid-19 pandemic in Scotland during its first wave of in-  
53 fections. Specifically, we aim to estimate both Scotland-wide and small-area  
54 temporal trends in disease incidence, focusing on both the peak and the end  
55 of this first wave. As people with symptoms during this first wave were ad-  
56 vised to phone NHS 24 for medical advice, we model data on the numbers  
57 of NHS 24 calls categorised as Covid-19 or having related symptoms at the  
58 small-area scale on a weekly basis. The model we developed was run by  
59 analysts in Public Health Scotland (PHS) on this proxy measure of disease  
60 incidence on a weekly basis during the first wave of the pandemic, allowing  
61 them to better understand the spread of the virus and target public health  
62 interventions appropriately at areas likely to exhibit the greatest risks.

63 Our model is a multivariate binomial spatio-temporal random effects  
64 model, with inference in a Bayesian setting using Markov chain Monte Carlo  
65 (MCMC) simulation. It jointly models the spatio-temporal variation in the  
66 numbers of calls to NHS 24 directly categorised as Covid-19, as well as those

67 calls categorised with related symptoms such as fever and difficulty breath-  
68 ing, the latter ensuring that potential local outbreaks are not missed due to  
69 calls being misclassified. In developing this model the key methodological  
70 challenge we address is the complex multivariate spatio-temporal structure  
71 of the data, which means we need to capture spatial, temporal and between  
72 call type correlations.

73 The development of multivariate space-time (MVST) models for disease  
74 risk modelling is a relatively new advance, with Carroll et al. (2017) and Law-  
75 son et al. (2017) proposing innovative mixture models, Quick et al. (2017)  
76 proposing a fully MVST Gaussian Markov Random Field (GMRF, Rue and  
77 Held, 2005) model, while Jack et al. (2019) combine separate simpler multi-  
78 variate spatial and multivariate temporal processes. The model we propose  
79 here is most similar to that proposed by Quick et al. (2017), because it uses  
80 a Gaussian Markov Random Field prior distribution applied to a set of ran-  
81 dom effects to model the multivariate spatio-temporal correlations inherent  
82 in the data. Our model extends that of Quick et al. (2017) by considering  
83 first and second order temporal autoregressive dependence structures, as well  
84 as allowing for varying strengths of spatial correlation via the Leroux spa-  
85 tial correlation model (Leroux et al., 2000). The NHS 24 telehealth data for  
86 the first wave of the pandemic that we analyse are described in Section 2,  
87 while our multivariate spatio-temporal model is presented in Section 3. Our  
88 surveillance model is applied to the Scottish telehealth data in Section 4,  
89 while Section 5 concludes the paper.

## 90 2. Covid-19 telehealth data in Scotland

### 91 2.1. NHS 24 and the study region

92 NHS 24 (<https://www.nhs24.scot/>) is Scotland's national telehealth ser-  
93 vice, and gives the public phone access to non-emergency medical advice  
94 24 hours a day and 7 days a week when their regular primary health care  
95 providers are closed. NHS 24 deals with around 1.5 million calls per year and  
96 serves a population of around 5.4 million people, and at peak demand answers  
97 around 14,500 calls over the course of a weekend. Data were obtained from  
98 Public Health Scotland (PHS, <https://publichealthscotland.scot/>) on  
99 the weekly numbers of calls to NHS 24 for Covid-19 and other similar con-  
100 ditions during the first wave of the pandemic, which spanned  $N = 22$  weeks  
101 from the week beginning 2nd March 2020 to the week beginning 27th July  
102 2020 inclusive. A weekly temporal scale was used because it smooths out the

103 large amount of noise in the daily data caused by small numbers of calls and  
104 known day of the week effects, the latter including the fact that there are  
105 more calls during the weekends when doctors surgeries are closed.

106 The data have been aggregated to the 444 postcode districts (PD) within  
107 Scotland, and a shapefile containing the spatial boundary information for  
108 these PDs was obtained from the National Records for Scotland (<https://www.nrscotland.gov.uk>). This spatial boundary information did not in-  
109 clude 8 of the PDs in the data set, but as these PDs only accounted for  
110 44 NHS 24 calls out of a total of 524,036 calls they were removed from the  
111 study region. After removing these PDs there were 1005 instances (PD and  
112 week combinations) with no NHS 24 calls at all, which were spread relatively  
113 evenly across the 22 weeks with between 34 and 56 instances each week.  
114 Therefore, to ensure a rectangular data set for analysis, only the  $K = 328$   
115 PDs having at least 1 NHS 24 call (about any illness) per week were retained  
116 in the study region. The PDs removed from the data only accounted for  
117 0.7% of the total calls to NHS 24, and were mostly sparsely populated rural  
118 or industrial / commercial areas.

## 120 2.2. Data available

121 For the  $k$ th PD and  $t$ th week the data comprise the following counts of the  
122 numbers of calls to NHS 24: (i)  $N_{kt}$  - the total number of calls to NHS 24; (ii)  
123  $Y_{kt1}$  - the number of calls classified as Covid-19; and (iii)  $Y_{kt2}$  - the number  
124 of calls classified as **Simple Estimate 1** (hereafter **SE1**), which is a set of  
125 symptoms potentially related to Covid-19 including cold, flu, coughs, fever  
126 and difficulty breathing. The latter is modelled here to ensure that potential  
127 local outbreaks are not missed due to a misclassification of calls. The clas-  
128 sification for Covid-19 was only initially available from 14th April onwards,  
129 but was back-predicted to 2nd March using a prediction model developed by  
130 PHS to allow trends to be modelled over the peak of the first wave of the  
131 pandemic. The prediction model was developed using NHS 24 call data from  
132 mid April to the end of May relating to respiratory and gastrointestinal syn-  
133 dromes plus the patients age. The prediction performance of this model had  
134 a specificity of 96% and a sensitivity of 75%, with an area under the curve  
135 (AUC) of 0.88. Therefore to ensure the Covid-19 series covers the peak of  
136 the first wave of the pandemic, we treat these predictions as observed data.

137 *2.3. Limitations with the data*

138 As discussed in the introduction wide-scale testing of Covid-19 was not avail-  
 139 able during the first wave of the pandemic, and the public were instead  
 140 advised to phone NHS 24 if they developed Covid-like symptoms. These  
 141 considerations motivate our use of the NHS 24 data as a proxy measure of  
 142 disease incidence, but one must be cognisant of the issues that arise with  
 143 these data not relating to laboratory confirmed cases. The main issue is  
 144 misclassification of calls, because a person phoning NHS 24 with Covid-like  
 145 symptoms does not mean they actually have the virus. Furthermore, the  
 146 NHS 24 call handler may misdiagnose the patients symptoms, and hence  
 147 wrongly classify them as having or not having Covid-19. This potential for  
 148 misclassification is why we jointly model calls classified as Covid-19 and SE1,  
 149 and examine the similarities and differences in the spatio-temporal dynamics  
 150 of both classifications. Furthermore, each NHS 24 call can actually have mul-  
 151 tiple classifications, and as expected there is substantial overlap in the calls  
 152 classified as Covid-19 and SE1. In fact, the total number of calls classified  
 153 as Covid-19 or SE1 is sometimes greater than the total number of calls, i.e  
 154  $Y_{kt1} + Y_{kt2} > N_{kt}$ , particularly where  $N_{kt}$  is small. Thus in the next section we  
 155 model these two classifications as a correlated multivariate binomial process  
 156 rather than with a multinomial distribution.

157 A further potential issue with using the NHS 24 data as a proxy measure  
 158 of disease incidence is that an individual may call NHS 24 more than once  
 159 during a week, either for different or for the same reason. Hence the data  
 160 we model relate to the numbers of calls to NHS 24 rather than the number  
 161 of individuals who call NHS 24. However, the number of individuals who  
 162 call NHS 24 multiple times for Covid-like symptoms within a week should be  
 163 low, because the NHS 24 call handlers are trained to provide expert medical  
 164 advice, precluding the need for multiple calls by the same individual. Thus  
 165 despite these limitations the NHS 24 data provide the most comprehensive,  
 166 if imperfect, data source for quantifying the spatio-temporal dynamics of the  
 167 first wave of the Covid-19 pandemic across Scotland, which is why we model  
 168 them here.

169 *2.4. Exploratory analysis*

170 The correlations between the proportions of calls,  $\hat{\theta}_{ktj} = Y_{ktj}/N_{kt}$ , classified as  
 171 Covid-19 ( $j = 1$ ) and SE1 ( $j = 2$ ) across all PDs for each week range between  
 172 0.60 and 0.94, suggesting there is a strong relationship between them. This  
 173 is further evidenced by the top panel (A) of Figure 1, which displays the

174 temporal trends in these raw proportions. In the figure jittering has been  
175 added to the week beginning (horizontal) dimension to improve the visibility  
176 of the points, and the proportions for Covid-19 are in red while those for SE1  
177 are in blue. The trend line in each case has been estimated using generalised  
178 additive model (GAM) smoothing. The figure shows a number of key points,  
179 the first of which is large amounts of noise in the data arising from small  
180 numbers of calls in some PDs, with sample proportions equal to 0 or 1 in  
181 6.4% (Covid-19) and 7.4% (SE1) of week and PD combinations respectively.  
182 Secondly, the temporal trends are broadly similar for Covid-19 and SE1,  
183 showing a rise in the proportions from the 2nd March, a peak around 23rd  
184 March, a decrease until 1st June, and a generally steady state since then.  
185 Thirdly, the figure shows that the dominant classification seems to change  
186 around the week beginning 6th April, with more calls classified as SE1 before  
187 that date and more Covid-19 calls after that date. This may be an artifact of  
188 the prediction model used to back-predict the Covid-19 classification before  
189 14th April, or alternatively it may be that as the pandemic became more  
190 prevalent from late March onwards people might be more likely to mention  
191 Covid-19 directly when they called NHS 24.

192 The median lag-1 temporal autocorrelation coefficients across the  $K =$   
193 328 PDs are respectively 0.54 (Covid-19) and 0.70 (SE1), which suggests  
194 these data are likely to exhibit temporal autocorrelation as expected. The  
195 raw proportions also exhibit spatial autocorrelation, which was quantified  
196 for each week and call classification using Moran's I (Moran, 1950) statistics  
197 and a corresponding Monte-Carlo p-value to test the null hypothesis of no  
198 spatial autocorrelation. The computation of Moran's I statistic requires an  
199 adjacency or neighbourhood structure between the  $K$  PDs to be specified,  
200 and details of its construction that accounts for the fact that PDs with no  
201 NHS 24 calls have been removed is given in the model specification in Section  
202 3.2. From these Moran's I tests 41% (Covid-19) and 23% (SE1) of these  
203 weekly p-values were significant at the 5% level, suggesting that despite the  
204 noise in these raw proportions, spatial autocorrelation is likely to be present  
205 in the data.

## 206 2.5. Aims of the analysis

207 Thus as the data exhibit spatio-temporal and between call type correlations  
208 contaminated by noise due to small numbers, a multivariate spatio-temporal  
209 smoothing model is proposed in the next section to estimate the underlying



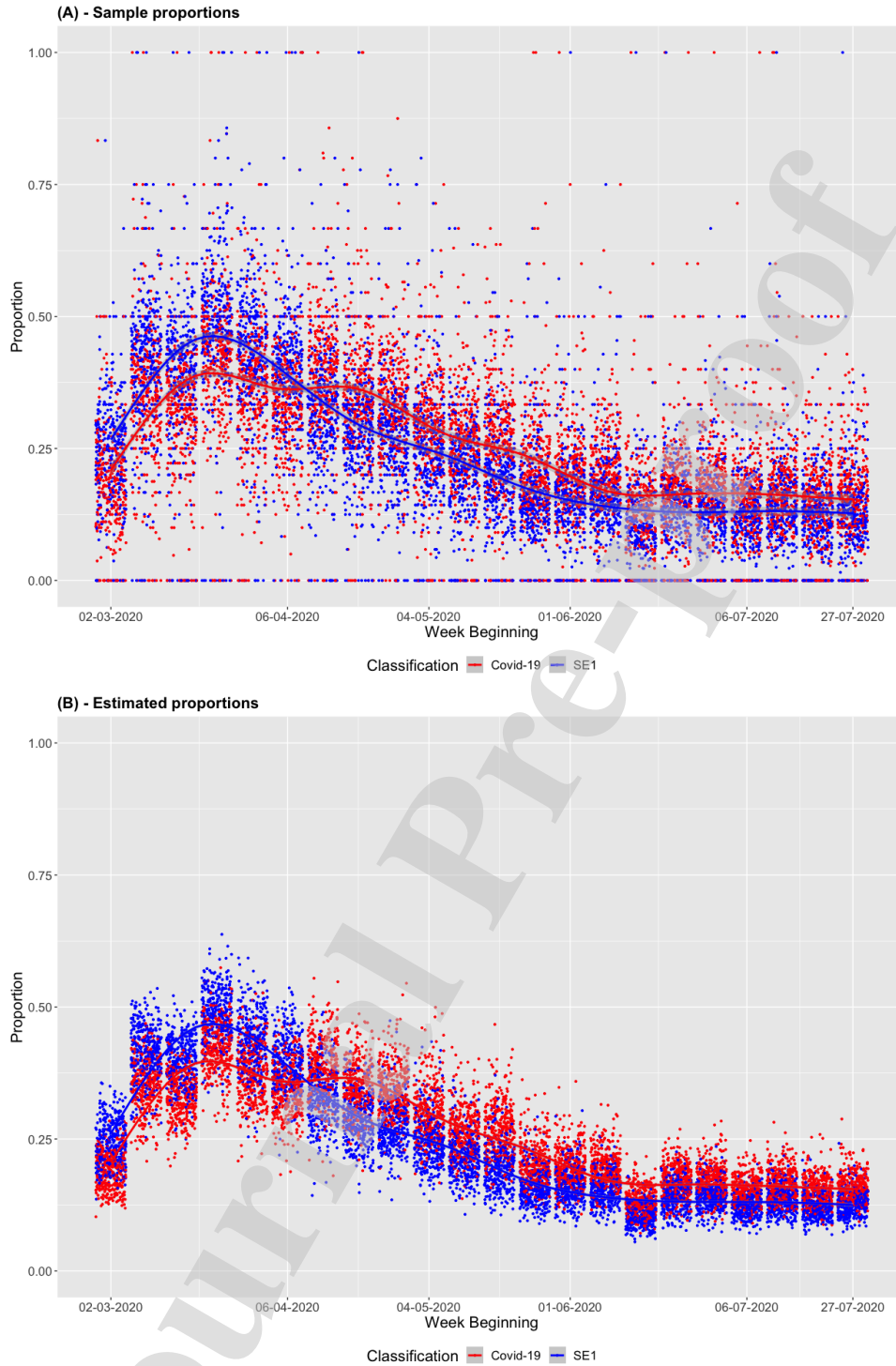


Figure 1: Scatterplots showing the temporal trends in the proportions of calls to NHS 24 that were related to Covid-19 (red) and SE1 (blue) for all PDs as points, with generalised additive model smoothed trend lines superimposed. The points have been jittered in the Week Beginning (horizontal) direction to improve their visibility. Panel (A) relates to the sample proportions and panel (B) to the estimated proportions from the final model (AR(2) Intrinsic CAR model with  $D = 7$ ).

210 trends in these data. Specifically, our 2 underlying goals when modelling  
 211 these data are to:

- 212 (a) Estimate the Scotland-wide spatio-temporal trend in disease incidence  
 213 across the first wave of the pandemic.
- 214 (b) Estimate the spatial variation in this overall trend, particularly the  
 215 extent of the spatial variation in when each PD in Scotland reached its  
 216 peak and the end of its first pandemic wave.

### 217 3. Methodology

218 This section proposes a new multivariate spatio-temporal (MVST) model for  
 219 estimating the spatio-temporal trends in the proportions of NHS 24 calls  
 220 classified as either Covid-19 or having related symptoms (SE1). The model  
 221 is fitted in a Bayesian setting using MCMC simulation, using a combination  
 222 of Gibbs sampling and Metropolis-Hastings steps. Software to implement the  
 223 model in R is available in the `CARBayesST` package (Lee et al., 2018), which  
 224 allows others to apply the MVST models considered here to their own data.

#### 225 3.1. Level 1 - Data likelihood model

226 Let  $Y_{ktj}$  denote the number of calls to NHS 24 in the  $k$ th PD ( $k = 1, \dots, K$ )  
 227 during the  $t$ th week ( $t = 1, \dots, N$ ) for the  $j$ th outcome ( $j = 1, \dots, J$ ), where  
 228 for our data  $j = 1$  is Covid-19 and  $j = 2$  is SE1. Additionally, let  $N_{kt}$  denote  
 229 the total number of NHS 24 calls in the  $k$ th PD and  $t$ th week. Then as the  
 230 two outcomes (call classifications) are not disjoint as described in Section 2,  
 231 a multinomial model is not appropriate for these data. Instead, we model  
 232 these data as conditionally independent binomial distributions, where the  
 233 spatio-temporal and between outcome (auto) correlations are modelled by  
 234 random effects at the second level of the model hierarchy. The first level of  
 235 the hierarchical model is given by:

$$\begin{aligned}
 Y_{ktj} &\sim \text{Binomial}(N_{kt}, \theta_{ktj}) & (1) \\
 \ln \left( \frac{\theta_{ktj}}{1 - \theta_{ktj}} \right) &= \beta_j + \phi_{ktj}.
 \end{aligned}$$

236 Here,  $\theta_{ktj}$  is the true unknown proportion of calls (or probability that a  
 237 single call) to NHS 24 in PD  $k$  during week  $t$  that is due to outcome  $j$ , and the

238 spatio-temporal variation in the estimated  $\{\hat{\theta}_{ktj}\}$  provides a proxy measure of  
 239 the incidence of the virus in the absence of comprehensive testing data. We  
 240 do not include any covariates in our model for two reasons, the first of which  
 241 is that our aim is to estimate the spatio-temporal trends in  $\{\theta_{ktj}\}$  via the  
 242 random effects  $\{\phi_{ktj}\}$ , rather than explaining what factors are associated with  
 243 these trends. Secondly, up-to-date temporally varying covariate information  
 244 is not available on a weekly basis, meaning that it would not be available  
 245 to include in the model. The intercept terms  $\beta_j$  are outcome specific, which  
 246 allows the two call types to have different average proportions over all PD  
 247 and time period combinations. We assign weakly informative independent  
 248 Gaussian prior distributions given by  $\beta_j \sim N(0, 100,000)$  to these outcome  
 249 specific intercept terms, which allow the data to play the dominant role in  
 250 estimating their values.

### 251 3.2. Level 2 - Multivariate spatio-temporal random effects model

252 The remaining term in (1)  $\{\phi_{ktj}\}$  are random effects, which are the mecha-  
 253 nism for estimating the smooth multivariate spatio-temporal trends in  $\{\theta_{ktj}\}$   
 254 for all outcomes. As such, the prior distribution for these random effects  
 255 must induce (auto)correlations in time, space and between outcomes. The  
 256 entire set of random effects are denoted by  $\phi = (\phi_1, \dots, \phi_N)$ , where  $\phi_t =$   
 257  $(\phi_{1t}, \dots, \phi_{Kt})$  denotes the set of  $K \times J$  random effects at time  $t$ , while  
 258  $\phi_{kt} = (\phi_{kt1}, \dots, \phi_{ktJ})$  denotes the subset of these effects at the  $k$ th PD for  
 259 all  $J$  outcomes. As mentioned earlier MVST models are in their infancy for  
 260 areal unit data, and we follow the general approach of Quick et al. (2017)  
 261 and propose a zero-mean multivariate Gaussian Markov random field (Rue  
 262 and Held, 2005) model for  $\phi$ . The general form of the model is given by

$$\phi \sim N\left(\mathbf{0}, [\mathbf{D}(\boldsymbol{\alpha}) \otimes \mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}]^{-1}\right), \quad (2)$$

263 where  $\otimes$  denotes a Kronecker product. The precision matrix is given by  
 264  $\mathbf{P}(\boldsymbol{\alpha}, \rho, \boldsymbol{\Sigma}) = \mathbf{D}(\boldsymbol{\alpha}) \otimes \mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}$ , where  $\mathbf{D}(\boldsymbol{\alpha})_{N \times N}$  controls the tem-  
 265 poral autocorrelations,  $\mathbf{Q}(\mathbf{W}, \rho)_{K \times K}$  controls the spatial autocorrelations  
 266 and  $\boldsymbol{\Sigma}_{J \times J}$  captures the between outcome correlations. The precision ma-  
 267 trix  $\mathbf{P}(\boldsymbol{\alpha}, \rho, \boldsymbol{\Sigma})$  is sparse because both  $[\mathbf{D}(\boldsymbol{\alpha}), \mathbf{Q}(\mathbf{W}, \rho)]$  are sparse as they  
 268 are built from specific cases of GMRFs (described below), which enables  
 269 computationally efficient Bayesian inference by making use of their triplet  
 270 form representation. As the model is defined in terms of its precision matrix

271  $\mathbf{P}(\boldsymbol{\alpha}, \rho, \boldsymbol{\Sigma})$  rather than its covariance matrix, multivariate Gaussian theory  
 272 gives the following partial (auto)correlations for  $(\phi_{ktj}, \phi_{rsi})$  conditional on  
 273 the remaining random effects  $\boldsymbol{\phi}_{-ktj,rsi}$ :

$$\text{Corr}(\phi_{ktj}, \phi_{rsi} | \boldsymbol{\phi}_{-ktj,rsi}) = \frac{-\mathbf{D}(\boldsymbol{\alpha})_{ts} \mathbf{Q}(\mathbf{W}, \rho)_{kr} (\boldsymbol{\Sigma}^{-1})_{ji}}{\sqrt{(\mathbf{D}(\boldsymbol{\alpha})_{tt} \mathbf{Q}(\mathbf{W}, \rho)_{kk}) (\boldsymbol{\Sigma}^{-1})_{jj} (\mathbf{D}(\boldsymbol{\alpha})_{ss} \mathbf{Q}(\mathbf{W}, \rho)_{rr}) (\boldsymbol{\Sigma}^{-1})_{ii}}}. \quad (3)$$

274 In what follows we now discuss the three components of the precision  
 275 matrix in turn.

### 276 3.2.1. Between outcome correlation

277 The between outcome covariance matrix  $\boldsymbol{\Sigma}$  is not assigned a specific struc-  
 278 ture, and is instead assigned the following conjugate Inverse-Wishart prior  
 279 distribution

$$\boldsymbol{\Sigma} \sim \text{Inverse-Wishart}(d, \boldsymbol{\Omega}). \quad (4)$$

280 The hyperparameters are set at  $(d = J + 1, \boldsymbol{\Omega} = 0.01\mathbf{I})$  where  $\mathbf{I}$  is the  
 281 identity matrix, and are chosen to ensure it is only weakly informative.

### 282 3.2.2. Spatial autocorrelation

283 Spatial autocorrelation is modelled by a conditional autoregressive (CAR)  
 284 prior, which is a special case of a GMRF. The prior requires the specification  
 285 of a  $K \times K$  neighbourhood or adjacency matrix  $\mathbf{W} = (w_{kr})$  that quantifies  
 286 the spatial closeness between each pair of PDs. Here we adopt a binary  
 287 specification where  $w_{kr} = 1$  if PDs  $(k, r)$  are spatially close together, and  
 288  $w_{kr} = 0$  otherwise, with  $w_{kk} = 0 \forall k$ . The most common approach in the  
 289 literature is to specify  $\mathbf{W}$  via the border sharing rule, that is  $w_{kr} = 1$  if areas  
 290  $(k, r)$  share a common border and  $w_{kr} = 0$  otherwise. However our study  
 291 region has numerous islands, as well as additionally a number of mainland  
 292 PDs with no NHS 24 calls that have therefore been removed. As a result this  
 293 border sharing specification leads to a corresponding graph with 15 separate  
 294 unconnected components, one main one containing most of the areas, 7 small  
 295 components containing between 2 and 8 areas, and 7 additional isolates with  
 296 no neighbours at all.

297 Therefore to obtain a neighbourhood structure with all the PDs in a single  
 298 connected component we use the  $D$ -nearest neighbours rule (after removing

299 the PDs with no NHS 24 calls), which first represents the location of each  
 300 PD by its centroid (central point). Then based on these centroids it specifies  
 301  $w_{kr} = 1$  if the  $r$ th PD is one of the  $D$  nearest PDs to the  $k$ th PD, and  $w_{kr} = 0$   
 302 otherwise. This leads to an asymmetric  $\mathbf{W}$  matrix, which is made symmetric  
 303 for the purposes of fitting the model by if  $w_{kr} = 1$  and  $w_{rk} = 0$  then setting  
 304  $w_{rk} = 1$ . In the analysis in the next section we consider  $D = 3, 5, 7$  to assess  
 305 the sensitivity of our results to this choice. Further details on specifying  
 306 neighbourhood matrices can be found in Bivand et al. (2013). Based on  $\mathbf{W}$   
 307 we model the spatial autocorrelation via the CAR prior proposed by Leroux  
 308 et al. (2000), which corresponds to the following spatial precision matrix

$$\mathbf{Q}(\mathbf{W}, \rho) = \rho(\text{diag}[\mathbf{W}\mathbf{1}] - \mathbf{W}) + (1 - \rho)\mathbf{I}. \quad (5)$$

309 Here  $(\mathbf{1}, \mathbf{I})$  are a  $K \times 1$  vector of ones and the  $K \times K$  identity matrix re-  
 310 spectively, while  $\text{diag}[\mathbf{W}\mathbf{1}]$  denotes a diagonal matrix with diagonal elements  
 311  $\mathbf{W} \times \mathbf{1}$ , so that the  $k$ th diagonal element is given by  $\sum_{i=1}^K w_{ki}$ . This spec-  
 312 ification models  $(\phi_{ktj}, \phi_{rtj})$  as partially spatially autocorrelated if  $w_{kr} = 1$   
 313 and conditionally independent if  $w_{kr} = 0$ , which can be seen from (3) and  
 314 the fact that for  $k \neq r$   $\mathbf{Q}(\mathbf{W}, \rho)_{kr} = -\rho w_{kr}$ . This also illustrates that  $\rho$  is  
 315 a global spatial dependence parameter, with a value of 0 corresponding to  
 316 spatial independence. In contrast, if  $\rho = 1$  the model captures strong spatial  
 317 autocorrelation and simplifies to the intrinsic CAR model proposed by Besag  
 318 et al. (1991), and this simplification was used to capture spatial correlation  
 319 by Quick et al. (2017) within an MVST setting. We specify a non-informative  
 320 uniform prior on the unit interval for  $\rho$ , i.e.  $\rho \sim \text{Uniform}(0, 1)$ , which pro-  
 321 vides equal prior weight for all allowable values of  $\rho$  and allows the data to  
 322 play the dominant role in estimating its value.

### 323 3.2.3. Temporal autocorrelation

324 Temporal autocorrelation is modelled using either first order or second or-  
 325 der autoregressive processes, which are both special cases of a GMRF. This  
 326 extends the work of Quick et al. (2017) who only consider the first order  
 327 case. The joint distribution for  $\phi$  from (2) in each case can be decomposed  
 328 as described below.

#### 329 330 **A - First-order autoregressive process**

331 For a first-order autoregressive process the joint prior distribution  $f(\phi)$  can  
 332 be decomposed as

$$\begin{aligned}
 f(\boldsymbol{\phi}) &= f(\boldsymbol{\phi}_1) \prod_{t=2}^N f(\boldsymbol{\phi}_t | \boldsymbol{\phi}_{t-1}) \\
 &= N\left(\boldsymbol{\phi}_1 \mid \mathbf{0}, [\mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}]^{-1}\right) \prod_{t=2}^N N\left(\boldsymbol{\phi}_t \mid \alpha \boldsymbol{\phi}_{t-1}, [\mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}]^{-1}\right),
 \end{aligned} \tag{6}$$

333 which is combined with the improper non-informative prior  $f(\alpha) \propto 1$ .  
 334 This specification corresponds to a tridiagonal matrix for  $\mathbf{D}(\boldsymbol{\alpha})$  with entries

$$\begin{aligned}
 \mathbf{D}(\boldsymbol{\alpha})_{t,t} &= \begin{cases} 1 + \alpha^2 & \text{for } t = 1, \dots, N-1 \\ 1 & \text{for } t = N \end{cases}, \\
 \mathbf{D}(\boldsymbol{\alpha})_{t,t-1} &= -\alpha \quad \text{for } t = 2, \dots, N.
 \end{aligned}$$

335 Thus from (3) it is clear that  $(\phi_{ktj}, \phi_{ksj})$  are conditionally independent if  
 336  $s \notin \{t-1, t, t+1\}$ .

337

### 338 **B - Second-order autoregressive process**

339 For a second-order autoregressive process the joint prior distribution  $f(\boldsymbol{\phi})$   
 340 can be decomposed as

$$\begin{aligned}
 f(\boldsymbol{\phi}) &= f(\boldsymbol{\phi}_1) f(\boldsymbol{\phi}_2) \prod_{t=3}^N f(\boldsymbol{\phi}_t | \boldsymbol{\phi}_{t-1}, \boldsymbol{\phi}_{t-2}) \\
 &= N\left(\boldsymbol{\phi}_1 \mid \mathbf{0}, [\mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}]^{-1}\right) N\left(\boldsymbol{\phi}_2 \mid \mathbf{0}, [\mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}]^{-1}\right) \\
 &\quad \times \prod_{t=3}^N N\left(\boldsymbol{\phi}_t \mid \alpha_1 \boldsymbol{\phi}_{t-1} + \alpha_2 \boldsymbol{\phi}_{t-2}, [\mathbf{Q}(\mathbf{W}, \rho) \otimes \boldsymbol{\Sigma}^{-1}]^{-1}\right),
 \end{aligned} \tag{7}$$

341 which is combined with the improper non-informative prior  $f(\alpha_1, \alpha_2) \propto 1$ .  
 342 This specification corresponds to the following sparse matrix for  $\mathbf{D}(\boldsymbol{\alpha})$  with

343 non-zero entries

$$\begin{aligned}
 \mathbf{D}(\boldsymbol{\alpha})_{t,t} &= \begin{cases} 1 + \alpha_2^2 & \text{for } t = 1 \\ 1 + \alpha_1^2 + \alpha_2^2 & \text{for } t = 2, \dots, N - 2 \\ 1 + \alpha_1^2 & \text{for } t = N - 1 \\ 1 & \text{for } t = N \end{cases}, \\
 \mathbf{D}(\boldsymbol{\alpha})_{t,t-1} &= \begin{cases} \alpha_1\alpha_2 & \text{for } t = 2 \\ \alpha_1\alpha_2 - \alpha_1 & \text{for } t = 3, \dots, N - 1 \\ -\alpha_1 & \text{for } t = N \end{cases}, \\
 \mathbf{D}(\boldsymbol{\alpha})_{t,t-2} &= -\alpha_2 \quad \text{for } t = 3, \dots, N.
 \end{aligned}$$

344 Thus from (3) it is clear that  $(\phi_{ktj}, \phi_{ksj})$  are conditionally independent if  
 345  $s \notin \{t - 2, t - 1, t, t + 1, t + 2\}$ .

#### 346 4. Spatio-temporal dynamics of Covid-19 in Scotland

347 This section presents the results of fitting the MVST models to the Covid-  
 348 19 telehealth data in Scotland during the first wave of the pandemic. In  
 349 modelling these data our aims are to: (a) estimate the Scotland-wide spatio-  
 350 temporal trend in disease incidence; and (b) estimate when each PD in Scot-  
 351 land reached the peak and end of its first pandemic wave.

##### 352 4.1. Model fitting

353 We fit 12 different models to the data that have varying spatio-temporal  
 354 correlation structures, because it allows us to examine the sensitivity of the  
 355 results to model choice. Specifically, we fit models with all possible com-  
 356 binations of: (i) first and second order temporal autoregressive structures;  
 357 (ii) spatial autocorrelation structures defined by the Leroux (given by (5))  
 358 and intrinsic (where  $\rho = 1$  in (5)) CAR models; and (iii) the neighbourhood  
 359 matrix  $\mathbf{W}$  defined by the  $D = 3, 5$  and  $7$  nearest neighbours rule. The model  
 360 with a temporal first order autoregressive process and the Intrinsic CAR  
 361 structure is the closest to that proposed by Quick et al. (2017), while the  
 362 models based on a second order autoregressive process and a Leroux CAR  
 363 structure are the extensions considered here. In what follows AR(1) / AR(2)  
 364 respectively denote models with first and second order temporal autoregres-  
 365 sive structures, while (I, L) respectively denote models with intrinsic and  
 366 Leroux CAR spatial structures.

367 Inference for each of these 12 models is based on 3,000 MCMC samples  
 368 generated from 3 independent Markov chains. Each chain was burnt in for

369 50,000 samples by which time convergence was assessed to have been reached,  
 370 and then run for a further 300,000 samples which were thinned by 300 to  
 371 greatly reduce their autocorrelation. Convergence was visually assessed using  
 372 traceplots and numerically assessed using the Gelman-Rubin diagnostic, and  
 373 for the latter none of the values of  $\hat{R}$  were above 1.1, which is suggested as a  
 374 convergence criteria by Gelman et al. (2013).

#### 375 4.2. Model assessment

376 A summary of the fit of each model to the data is presented in Table 1,  
 377 which displays the deviance information criterion (DIC, Spiegelhalter et al.,  
 378 2002), the effective number of independent parameters (p.d), and the log  
 379 marginal predictive likelihood (LMPL, Geisser and Eddy, 1979). The DIC  
 380 measures the overall fit of each model to the data, and the model with an  
 381 intrinsic CAR spatial structure and a second order autoregressive temporal  
 382 structure fits the data best as it minimises the DIC. However, the overall fits  
 383 of all the models are relatively similar, as there is only a 0.8% difference be-  
 384 tween the largest and smallest DIC values. The LMPL measures the predic-  
 385 tive ability of each model and is calculated as  $LMPL = \sum_{ktj} \ln[f(Y_{ktj} | \mathbf{Y}_{-ktj})]$ ,  
 386 where  $\mathbf{Y}_{-ktj}$  denotes all observations except for  $Y_{ktj}$ . The best fitting model is  
 387 the one that maximises the LMPL, which is also achieved by the model with  
 388 an intrinsic CAR spatial structure and a second order autoregressive tempo-  
 389 ral structure. However, in common with the DIC the differences between the  
 390 models by this measure are also small, being at most 1.3%

391 The residuals from all models were assessed for the presence of any re-  
 392 maining spatial autocorrelation using a Moran's I permutation test sepa-  
 393 rately for each year, and in all cases no significant autocorrelation remained.  
 394 The presence of residual temporal autocorrelation was also checked for each  
 395 model and PD, by determining whether the lag 1 autocorrelation coefficient  
 396 was significantly different from zero at the 5% level. We based on our assess-  
 397 ment on the lag one coefficient only because the data only contain  $N = 22$   
 398 time periods making estimation of higher lags less reliable, and also because  
 399 the Moran's I test is also only based on first order neighbours. The models  
 400 with a second order autoregressive process adequately capture the temporal  
 401 autocorrelation in the data, as in all cases only 5% of the sets of tempo-  
 402 ral residuals contain significant (at the 5% level) autocorrelation at lag 1.  
 403 In contrast, the corresponding percentages for the models with a first order  
 404 autoregressive process are between 12% - 14%, suggesting that an AR(1)



Table 1: Summary of all models fitted to the data, including overall fit to the observed data via the DIC, model complexity via the effective number of independent parameters (p.d), and predictive ability via the log marginal predictive likelihood (LMPL).

Quantity	W matrix	Spatio-temporal correlation model			
		AR(1) - I	AR(1) - L	AR(2) - I	AR(2) - L
DIC	D=3	68,424	68,461	62,276	68,313
	D=5	68,139	68,171	68,014	68,057
	D=7	67,982	68,028	67,888	67,915
p.d	D=3	2,330	2,372	2,487	2,524
	D=5	2,579	2,612	2,689	2,720
	D=7	2,735	2,757	2,802	2,834
LMPL	D=3	-34,050	-34,065	-33,928	-33,941
	D=5	-33,828	-33,842	-33,726	-33,739
	D=7	-33,694	-33,722	-33,619	-33,631

405 temporal autocorrelation structure is not entirely sufficient for capturing the  
 406 temporal autocorrelation in the data.

407 Finally, the fitted values from each model were plotted against the ob-  
 408 served values, and in all cases good agreement was seen with no large outliers  
 409 suggesting a lack of fit for individual data points. The estimated proportions  
 410  $\{\hat{\theta}_{ktj}\}$  were also relatively similar for all models, with for example the dif-  
 411 ferences between the AR(1) Leroux CAR model with  $D = 3$  and the AR(2)  
 412 Intrinsic CAR model with  $D = 7$  (the two most dissimilar models) ranging  
 413 between -0.06 and 0.06 on the proportion scale for both Covid-19 and SE1  
 414 call classifications.

#### 415 4.3. Multivariate spatio-temporal correlation structures

416 The spatio-temporal and between outcome correlations estimated by each  
 417 model are summarised in Table 2, which presents point estimates (posterior  
 418 medians) and 95% credible intervals for key model parameters. The table  
 419 shows that the estimated proportions of calls classified as Covid-19 and SE1  
 420 have similar levels of spatio-temporal variation, as the posterior medians of  
 421  $(\Sigma_{11}, \Sigma_{22})$  are similar for both models, albeit slightly larger for SE1 calls in  
 422 all cases. The values of both  $(\Sigma_{11}, \Sigma_{22})$  increase with increasing numbers  
 423 of spatial neighbours  $D$ , which occurs because the conditional distribution  
 424 of  $\phi_{kt}|\phi_{-kt}$  has a covariance matrix including the elements of  $\Sigma$  divided by  
 425 a function of  $\sum_{r=1}^K w_{kr}$ . Thus as the average number of neighbours (con-

426 trolled by  $D$ ) increases the conditional variance is divided by a bigger number,  
 427 leading to the inflation of  $(\Sigma_{11}, \Sigma_{22})$ . The table also shows substantial  
 428 between outcome (call classification) correlations, which are computed by  
 429  $(\Sigma_{12}/\sqrt{\Sigma_{11}\Sigma_{22}})$  and are very close to one for all models.

430 The levels of spatial dependence estimated by the Leroux CAR models  
 431 are high because the posterior medians for  $\rho$  are close to or equal to 1 for all  
 432 models, which corresponds to the intrinsic CAR model (where  $\rho$  is fixed at  
 433 1) for strong spatial dependence. Thus for these data there is little difference  
 434 between the Intrinsic and Leroux CAR models, with the former having a  
 435 better DIC due to it having a lower p.d as it does not need to estimate  $\rho$ .  
 436 Substantial temporal dependence is also present in these data, because in  
 437 the AR(1) and AR(2) models the respective 95% credible intervals for  $\alpha$  and  
 438  $(\alpha_1, \alpha_2)$  are not close to zero which would represent temporal independence.

#### 439 4.4. (a) Scotland-wide spatio-temporal trend in the pandemic

440 The remainder of this section presents the estimated spatio-temporal trend  
 441 in the Covid-19 pandemic during its first wave in Scotland. All results relate  
 442 to the AR(2) Intrinsic CAR model with  $D = 7$ , because this was shown to be  
 443 the best model via both the DIC and LMPL metrics, as well as adequately  
 444 capturing both the temporal and spatial correlations in the data.

445 The estimated (posterior median) proportions of calls  $\{\hat{\theta}_{kt1}, \hat{\theta}_{kt2}\}$  to NHS  
 446 24 classified as Covid-19 and SE1 are displayed in the bottom panel of Figure  
 447 1, which has the same format as the top panel of the same figure, with Covid-  
 448 19 in red and SE1 in blue. The estimated proportions exhibit much less noise  
 449 than the raw proportions due to the spatio-temporal smoothing applied by  
 450 the model, and the peak in the average proportions is 0.42 for Covid-19 and  
 451 0.49 for SE1 in the week beginning 23<sup>rd</sup> March. The trends in the estimated  
 452 proportions are shown by generalised additive model curves, and the curve  
 453 for SE1 is unimodal and has a steeper ascent and descent compared to the  
 454 Covid-19 curve.

455 In contrast, the Covid-19 curve exhibits a second local maximum on  
 456 the week beginning 13<sup>th</sup> April, and the very limited available data on con-  
 457 firmed cases at a national level also suggests the existence of a double peak  
 458 (for details see [https://public.tableau.com/profile/phs.covid.19#!/vizhome/COVID-19DailyDashboard\\_15960160643010/Overview](https://public.tableau.com/profile/phs.covid.19#!/vizhome/COVID-19DailyDashboard_15960160643010/Overview)). This dou-  
 459 ble peak in the confirmed cases occurs slightly later with around a 2 week  
 460 lag compared to the NHS 24 calls, which is likely to be partially caused by  
 461 testing and reporting delays as the testing infrastructure was less advanced  
 462

Table 2: Summary of the posterior medians and 95% credible intervals for the covariance parameters from each of the models.

Quantity	W matrix	Spatio-temporal correlation model		
		AR(1) - I	AR(1) - L	AR(2) - I
$\Sigma_{11}$	D=3	0.059 (0.051, 0.068)	0.060 (0.052, 0.070)	0.074 (0.065, 0.084)
	D=5	0.151 (0.132, 0.172)	0.152 (0.134, 0.173)	0.175 (0.155, 0.195)
	D=7	0.262 (0.231, 0.295)	0.260 (0.230, 0.292)	0.287 (0.257, 0.319)
$\Sigma_{22}$	D=3	0.062 (0.054, 0.072)	0.063 (0.055, 0.074)	0.077 (0.068, 0.087)
	D=5	0.157 (0.136, 0.178)	0.159 (0.140, 0.180)	0.178 (0.158, 0.198)
	D=7	0.271 (0.238, 0.304)	0.272 (0.239, 0.307)	0.293 (0.262, 0.326)
$\Sigma_{12}/\sqrt{\Sigma_{11}\Sigma_{22}}$	D=3	0.997 (0.996, 0.998)	0.994 (0.991, 0.996)	0.997 (0.996, 0.998)
	D=5	0.998 (0.997, 0.999)	0.995 (0.993, 0.997)	0.998 (0.998, 0.999)
	D=7	0.999 (0.998, 0.999)	0.996 (0.993, 0.997)	0.999 (0.998, 0.999)
$\rho$	D=3	-	1.000 (1.000, 1.000)	-
	D=5	-	1.000 (0.999, 1.000)	-
	D=7	-	0.999 (0.999, 1.000)	0.999 (0.999, 1.000)
$\alpha$	D=3	$\alpha$ 0.770 (0.724, 0.810)	$\alpha$ 0.762 (0.713, 0.802)	$\alpha_1$ 0.459 (0.394, 0.529) $\alpha_2$ 0.346 (0.272, 0.419)
	D=5	$\alpha$ 0.689 (0.637, 0.739)	$\alpha$ 0.687 (0.639, 0.730)	$\alpha_1$ 0.419 (0.360, 0.480) $\alpha_2$ 0.333 (0.267, 0.396)
	D=7	$\alpha$ 0.638 (0.583, x0.687)	$\alpha$ 0.640 (0.591, 0.687)	$\alpha_1$ 0.401 (0.342, 0.459) $\alpha_2$ 0.319 (0.255, 0.379)
	D=3	-	-	$\alpha_1$ 0.461 (0.394, 0.527) $\alpha_2$ 0.337 (0.269, 0.407)
	D=5	-	-	$\alpha_1$ 0.419 (0.362, 0.479) $\alpha_2$ 0.324 (0.259, 0.387)
	D=7	-	-	$\alpha_1$ 0.403 (0.346, 0.457) $\alpha_2$ 0.311 (0.251, 0.369)
	D=7	-	-	-

463 than it is now. The average (over Scotland) estimated proportions of calls  
 464 classified as Covid-19 for the weeks beginning 15<sup>th</sup> June onwards are lower  
 465 than the average for 2<sup>nd</sup> March (the first week of the data), suggesting that  
 466 the majority of the first wave of the pandemic had come to an end by this  
 467 point.

468 The spatio-temporal trend in the Covid-19 classifications is summarised  
 469 in Figure 2, which displays maps for the first and last week of the study as  
 470 well as for the two peaks in the estimated proportions (23<sup>rd</sup> March and 13<sup>th</sup>  
 471 April) highlighted above. The figure shows that most PDs have relatively  
 472 low proportions of calls in the first and last weeks below 0.2, while most PDs  
 473 have increased proportions between 0.3 and 0.6 during the two weeks of peak  
 474 Covid-19 activity. The figure also shows that the proportions of NHS 24 calls  
 475 classified as Covid-19 do not show a pronounced spatial trend for any of the  
 476 weeks, and instead show pockets of higher proportions in different parts of  
 477 the country.

#### 478 4.5. (b) PD specific temporal trends

479 The previous section suggested that on average the first wave of the pandemic  
 480 peaked in Scotland in the week beginning 23<sup>rd</sup> March, and had reduced back  
 481 to baseline levels seen at the beginning of March by 15<sup>th</sup> June. However, our  
 482 second motivating question is to assess whether the pandemic hit some parts  
 483 of Scotland earlier than other parts. Our hypothesis is that the pandemic  
 484 would be likely to affect more connected urban areas before it affected more  
 485 remote rural ones, due to the former's greater levels of population density  
 486 (and hence mixing) and easier access to travel via proximity to airports.

487 To assess this Figure 3 displays maps for each PD displaying: (A) the  
 488 week that  $\hat{\theta}_{kt1}$  was at its highest, which represents the peak of its first wave;  
 489 and (B) the first week after this peak that  $\hat{\theta}_{kt1}$  was smaller than its value in  
 490 the first week (i.e. smaller than  $\hat{\theta}_{k11}$ ), which approximately represents the  
 491 end of its first wave of infection. The maps relate to Covid-19 rather than the  
 492 SE1 classification, because the previous section highlighted that the double  
 493 peak observed in the Covid-19 trend (see Figure 1) resembles the limited  
 494 testing data at a national level more closely than the single peak from the  
 495 SE1 trend.

496 The figure shows that 62% of the PDs exhibited their peak in Covid-  
 497 related calls during the week beginning 23<sup>rd</sup> March, with the 7% of the  
 498 PDs that exhibited their peak two weeks earlier mainly being located around  
 499 the largest city of Glasgow. In contrast, those PDs exhibiting later peaks

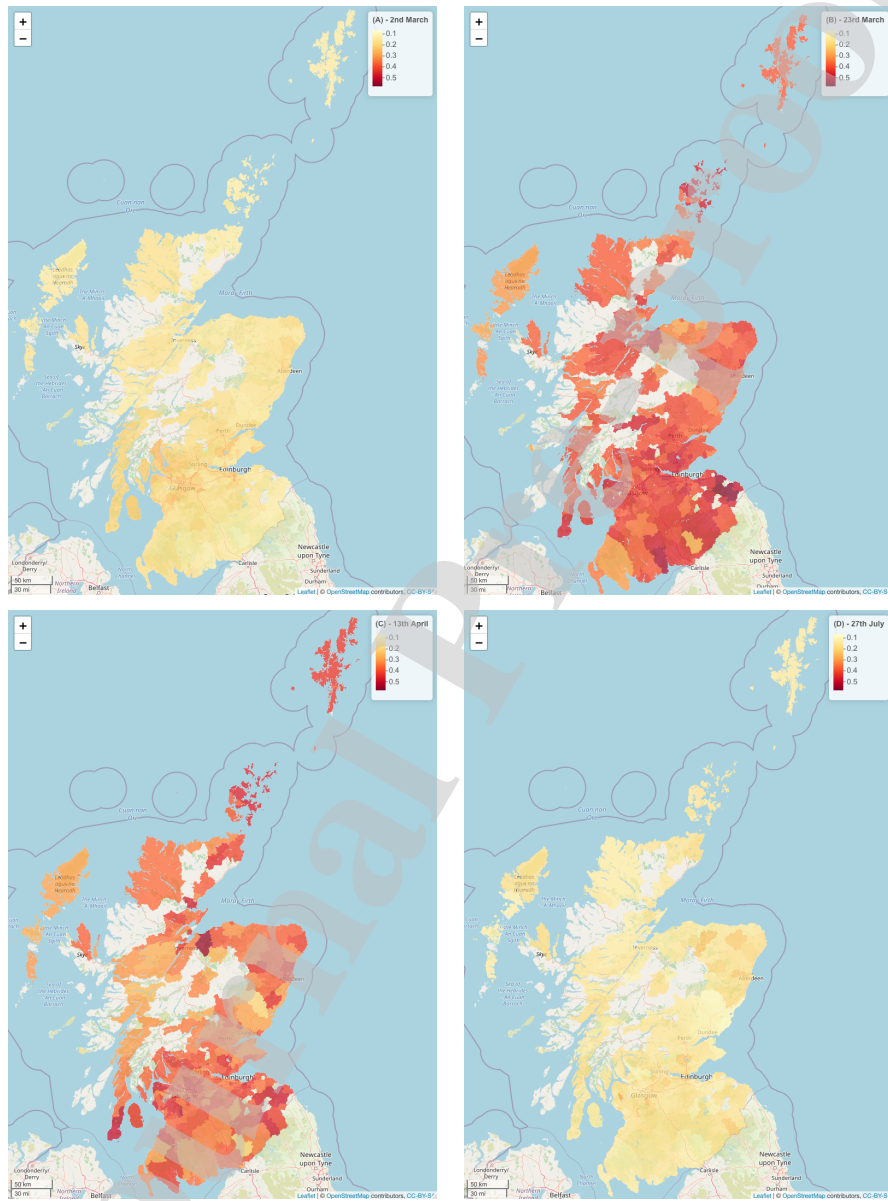


Figure 2: Maps displaying the proportions of NHS 24 calls classified as Covid-19 in four weeks of the pandemic.

500 (coloured red on the map) are mainly rural areas, with 20% of the PDs  
501 peaking in the week beginning 13<sup>th</sup> April. These PDs with later peaks are  
502 mostly in the more remote northern parts of Scotland that are away from  
503 the main cities. The right panel of Figure 3 displays the first week that  
504 the Covid-19 related calls were below their March 2<sup>nd</sup> levels, and a bimodal  
505 pattern is evident with 31% of PDs achieving this by 25<sup>th</sup> May while 40% met  
506 this by 15<sup>th</sup> June. In addition, 4% of the PDs had not seen their Covid-19  
507 related calls drop below the 2<sup>nd</sup> March levels by the end of July, suggesting  
508 that in some areas the first wave of the pandemic had not yet finished by  
509 the end of our study. Finally, there is no clear urban-rural divide in these  
510 approximate end times of the first wave of the pandemic, which suggest that  
511 whilst urban areas were mainly affected first, they did not necessarily see the  
512 end of the wave first.

## 513 5. Discussion

514 This paper has developed a multivariate spatio-temporal model for quantify-  
515 ing the spread of Covid-19 in Scotland during the first wave of the pandemic,  
516 which was a period with limited testing capacity resulting in large numbers of  
517 infected people whose disease status was not confirmed by a diagnostic test.  
518 As a result we quantified the spatio-temporal dynamics of Covid-19 spread  
519 using proxy data from the national telehealth service NHS 24, who members  
520 of the public were advised to call if they experienced symptoms. The model  
521 estimates the joint spatio-temporal trends in the proportions of calls to NHS  
522 24 classified as either Covid-19 directly or as having related symptoms (called  
523 SE1), and a simplification of the model using only the Covid-19 classification  
524 was run on a weekly basis by Public Health Scotland during the first wave of  
525 the pandemic as new data became available to monitor the likely locations  
526 of new outbreaks.

527 Modelling the spatio-temporal dynamics in the NHS 24 data allows us  
528 to study the spread of the pandemic at a small-area scale, albeit with a  
529 proxy measure of infection rates. However, as previously discussed testing  
530 capacity was severely limited in this initial stage of the pandemic, and hence  
531 data on confirmed cases would also only be a proxy measure of the true  
532 infection rates. Additionally, due to the small numbers of positive tests in  
533 this phase of the pandemic, small-area testing data are not available for  
534 confidentiality reasons, making it impossible to study the spread of the virus  
535 at the small-area scale using confirmed case data. Thus while telehealth data

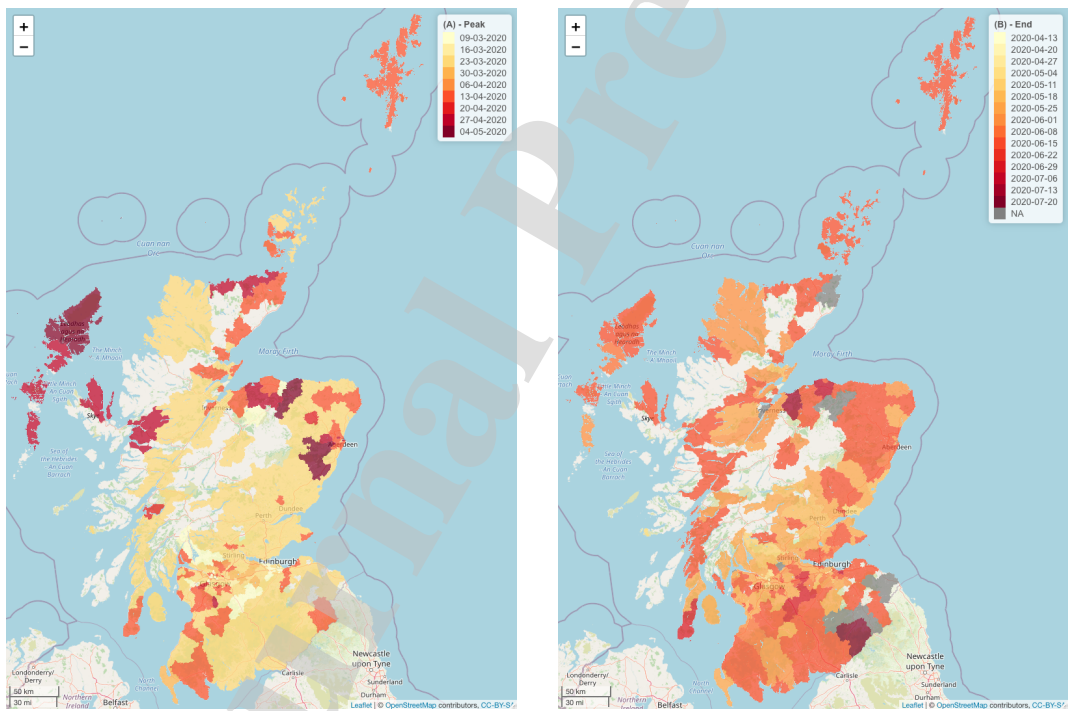


Figure 3: Maps displaying for each PD the weeks when the estimated proportions for the Covid-19 classification: (A) peaked; and (B) were below their 2nd March levels signifying the end of the first wave.

536 are imperfect as discussed above, we have illustrated the value of modelling  
537 them in early stage pandemic situations where reliable confirmed testing data  
538 are not available.

539 The paper has presented a number of findings from our data analysis, the  
540 first being that the first wave of the pandemic peaked in Scotland in the week  
541 beginning 23rd March, with a smaller peak 3 weeks later on 13th April. The  
542 23rd March was the peak of the pandemic for 65% of the PDs, while the 19%  
543 of the PDs that peaked later than 13th April were largely rural areas in the  
544 north and west of Scotland. By the end of July all but 4% of the PDs had  
545 NHS 24 call levels for Covid-19 below the levels observed at the beginning of  
546 March when the first confirmed case was recorded (2nd March) in Scotland,  
547 suggesting that the first wave of the pandemic was coming to an end by this  
548 point.

549 Our other main finding is the differential temporal trends in the Covid-19  
550 and SE1 classifications, with the latter exhibiting a single peak and having a  
551 steeper decline in proportions after the pandemic peak. This steeper descent  
552 in its proportions may be because as the pandemic became more prevalent  
553 from late March onwards people might be more likely to mention Covid-19  
554 directly when they called NHS 24, hence the proportions of calls attributed  
555 to SE1 declined to lower levels than those attributed to Covid-19.

556 The overarching aim of this paper was to estimate the key dynamics of the  
557 Covid-19 pandemic at a high spatio-temporal resolution in a retrospective  
558 manner, which is why no predictive modelling of the proportions of calls  
559 classified as Covid-19 or SE1 into the future was undertaken. However, the  
560 temporally autoregressive nature of the models would make such prediction  
561 straightforward via (6) or (7), and both the proportions  $\{\theta_{k,T+1,j}\}$  and counts  
562  $\{Y_{k,T+1,j}\}$  could be predicted in this way, although for the latter  $\{N_{k,T+1,j}\}$   
563 would also need to be predicted. Thus an area of future work will be to utilise  
564 these MVST models to predict disease burden into the future, to allow NHS  
565 managers to predict the amount of health care resources (e.g. hospital beds)  
566 needed in the future.

567 Another area of future work would be to continue the development of  
568 spatio-temporal modelling tools for telehealth data, because it has clear fu-  
569 ture applications that extend beyond the early stage pandemic setting con-  
570 sidered here. Other examples include the routine monitoring of ordinary  
571 seasonal flu and outbreaks of Norovirus, which would give the NHS better  
572 information on the likely prevalence of these diseases and where and when  
573 outbreaks are likely to occur, thus allowing targeted action to be taken in a



574 timely manner.

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### 581 **Competing Interests**

582 The authors declare they have no competing interests.

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