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A Cognitive and Quantitative Approach to Mathematical Concretization

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Abstract

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This chapter analyses the linguistic role of analogy as a strategy of concretizing abstract concepts addressed in popular science focused on mathematics. A rich analogy used in a text popularising number theory is explored through firstly a quantitative method and then using conceptual blending, a theory taken from cognitive linguistics. The chapter demonstrates the use of corpus-based and cognitive approaches to language in the analysis of the ways in which popular science texts aim to give non-experts a sense of understanding of complex mathematical concepts.

We have suggested that … we tend to structure the less concrete and inherently vaguer concepts (like those for the emotions) in terms of more concrete concepts, which are more clearly delineated in our experience.

(Lakoff and Johnson 1980, p. 112)

as soon as one gets away from concrete physical experience and starts talking about abstractions or emotions, metaphorical understanding is the norm.

(Lakoff 1993, p. 205)

Mathematical texts for non-expert readers, whether they are presented as literature or as popular science, need to engage with the fundamental problem of how to describe and represent abstract mathematical entities using a natural language (one developed and evolved through usage) such as English. Human beings, as embodied minds perceiving and construing the world around them, generally linguistically encode abstractions in terms of either concrete entities or events readily perceived by the physical senses. This process—which is variously called metaphor, analogy, reification, or concretization—is a key one for books popularising mathematics as a means of using natural language to describe what would otherwise be encoded in challenging notation. However, such a process has to draw carefully on its concrete 'sources' in order to render on a reader an accurate and coherent impression of a given mathematical concept.

AQ1

This chapter analyses at close range a rich example of this sort of concretization, and does so using both quantitative and cognitive methods. I am particularly interested in identifying how, within a large text, rich analogies can be reliably found, and also in describing the cognitive process of constructing one such analogy. To do this, there are two approaches which will be used in turn: firstly, quantitative methods are necessary in order to examine long texts for areas of particularly significant or dense concretizations for more detailed analysis, and secondly, a linguistic approach known as *conceptual blending* will be used to describe the cognitive 'construction' of such a dense concretization in the mind of the reader.

In doing this, in addition to undertaking a close reading of a mathematical analogy, I hope to show the potential use of a new method of finding analogical material in electronic collections of text, and apply for the first time a conceptual blending analysis to a piece of popular science focusing on mathematics, and demonstrate where its utility may be found. I also propose some new methods and refinements to existing methods for stylistic analysis.

Literature and Quantitative Analysis

In order to find, with minimal human intervention, an area of concretization-dense popular mathematics, the first part of this chapter uses some new computational methods. Such a quantitative approach to literature is 'founded on principles of maximal attainable objectivity, procedures that are as replicable as possible' (Sinclair 2007, p. 6). Such an approach traces its origins to pre-computational quantitative work, of which T. C. Mendenhall's article in 1887 is the most widely cited. For Mendenhall's studies (1887, on the frequency of different word lengths in major authors, and 1901 on the Shakespeare authorship question), two women undertaking 'very exhausting' work for several months took the place of what would be done on a modern computer in an instant (Mendenhall 1901, p. 102). With the increasing availability of electronic texts, the field of linguistic stylistics was prominent in embracing the additional toolset that quantitative approaches to language provided to literary analysis. For example, Leech and Short in 1981 stated:

> The more a critic wishes to substantiate what he says about style, the more he will need to point to the linguistic evidence of texts; and linguistic evidence, to be firm, must be couched in terms of numerical frequency …. If challenged, I ought to be, and can be, in a position to support my claim with quantitative evidence. (Leech and Short 1981, pp. 46–47)

This approach has become standard within stylistics—including the study of authorship—and after a long period of having little to no impact on the study of literature elsewhere, has cross-fertilised in recent years with the well-established field of broad-scale literary history (as outlined by Underwood 2017) and other experimental and quantitative approaches to literature which are often placed under the umbrella of digital humanities (although note that this umbrella also includes regular and somewhat tiresome debates about its own definition, approach, coverage, and aims; more of this can be found in Terras et al. 2013). Despite the growing use of quantitative methods, John Burrows' observation that in the humanities '[c]omputer-based evidence, especially when it incorporates statistical analysis, is too often regarded with special deference or special scepticism' (1992, p. 91) remains valid today.

Deference aside, for the present study computational and quantitative methods provide something which in the words of Scott Selisker (2012) can 'humbly supplement – usually by just a little, and perhaps one day by a great deal – what we know about literature' (Selisker 2012, n.p.). It is most likely that the most significant impact on literature of such quantitative methods is that they can be used to provide, as they do in this chapter, a starting-point for the close reading which is so essential to the work of the humanities.

Methodology

The quantitative methodology developed for the work in this chapter aims to join a detailed stylistic analysis of a dense mathematical analogy to a desire to find—as objectively as is reasonable—sufficiently dense areas of mathematical concretization to warrant analysis. The reason for this is that there can be a danger when constructing an argument about texts that one relies on incomplete evidence, often as an unintentional result of selective attention. This problem is particularly acute in linguistics (and linguistic stylistics), where any given native speaker's intuitions about the language they speak are often neither consistent nor accurate (Gibbs 2006, p. 4). It is not always easy to find evidence which is not gained through selective attention—the process of a person reading a book is not a purely mechanical one to be made automatic—and so I aim in this section to develop and employ some quantitative methods to find, from a bottom-up perspective driven by the data itself, an area to which an analyst could usefully pay most attention. The methodology is new, and tailored to this particular genre and research question, but improvements can likely be made, and an adapted version of it would be usable for any study of metaphor or analogy.

I apply this methodology to a popular science text which is particularly full of concrete metaphors for mathematical concepts: Marcus du Sautoy's *The Music of the Primes* (2003; hereafter *MP*). Of the increasing number of popular mathematics books, it fulfils some technical criteria (substantial length, highly ranked in book review sites, available as an electronic text, and written by a professional mathematician, not a journalist) as well as being a well-regarded and popular book. Even the title of the book indicates its enthusiastic engagement with metaphor and analogy, in contrast to the generally more analogy-free material found in textbooks. *MP* focuses on the highly abstract field of prime number theory, and so du Sautoy's analogies are likely to have a strong contrast between abstract and concrete. It is approximately 125,000 words long, which allows, as suggested above, for a computational 'distant-reading' approach to the content designed to identify a text's highly analogical sections. I will also briefly analyse a technical mathematical textbook, which is very low in analogical and metaphorical context, as a comparison to the popular style of *MP*—this means that if the work I describe below to identify analogical concretizations finds some in *MP* and none in the comparison text, we can be more confident that the process is actually finding analogies in the text, rather than producing haphazard results. The comparison text is W. A. Coppel's (2009) *Number Theory* textbook.

With these two books in hand, the general outline of what we need to do with them is firstly to manipulate the texts into a way in which the *meaning* of the words in them can be analysed (that is, to allow us to work with the words and their content), and then to find where these meanings are analogical (that is, where the text is discussing something not to do with mathematics in order to illustrate mathematics). Finally, I then want to identify areas particularly dense in that analogical material. My assertion is that the 'real' content of both books is mathematical, with some biographical and historical background, and that any other text in *MP* could potentially be there for the purposes of analogy.

For the first part, manipulating the texts ready for us to look at their meaning, the work was accomplished by a series of custom Python programs I produced using a series of routines known as the Natural Language Toolkit (NLTK; Bird et al. 2009). The programs firstly separate each text from one long-running document into 400-word files which represent smaller 'chunks' of each book. Within each file, the words in that text were then reduced to their lower-case lemmas (sometimes known as a word's dictionary form, so that *run, Runs, ran, running* are all reduced to *run*, while *Cat* and *cats* are reduced to *cat*; see also Alexander and Dallachy 2019). Next, frequently occurring function words were removed from the files on the basis of a standard pre-existing 'stop list' of these words (thus eliminating, for example, *the, of, and, to, a, in, is,* and so on). Thus far, these are all straightforward digital humanities approaches to counting words. Finally, the number of times each remaining lemma occurs in each 400-word chunk of text is then counted and summed in a database.

In the second stage, all these words (and their frequencies) need to be connected to the concepts they encode. To connect words to meanings, the database of *The Historical Thesaurus of English* (Kay et al. 2019; hereafter *HT*) is ideal. The *HT* is a meaning-based rearrangement of the complete contents of the first edition of the *Oxford English Dictionary*, together with the later supplements and the 1990s' three-volume *Additions* series (being approximately equivalent to the second edition of the *OED* in 1989 with the later *Additions*). It therefore lists every meaning recorded in English along with every word which has been used to realise that meaning. Despite the word *Historical* in its title, it should be noted that when filtered to the present day, the *HT* forms the largest thesaurus of contemporary English currently available. In a database program, I took the lists of frequencies (for each 400-word chunk of text) of both books and connected the words to their corresponding *HT* categories. The data now consists, for each chunk of each text, of the words, their frequencies, and each *HT* category which is 'activated' by each word. Note that no disambiguation is performed—unlike in Alexander et al. (2015)—because a simple statistic in the next stage will deal with the issue of multiple meanings for one word (such as when *cosh,* as an abbreviation for the hyperbolic cosine, overlaps with identical word forms of *cosh* meaning a hovel and a bludgeon).

This simple statistic is known as log-likelihood (Dunning 1993), which lets us look at where frequencies of words or meanings are statistically significant in a text. For example, if a book uses the words *garden* and *flowers* and *lawn* so often that, by comparison with other books, all those words are statistically significant, it probably means the book is 'about' gardening, or at least has a decent amount of its content about gardens. Of course, even though the word meanings of a text may be 'about' gardening, the text itself may actually be saying something about personal growth, or loss, or environmental sustainability; the computer can tell the first, and the reader the second. For our purpose, we want to find where in *MP* there are particularly significant occurrences of meanings 'about' something other than mathematics, as in the next section I want to analyse how they are used to help a reader understand prime number theory. The log-likelihood statistic itself identifies those meanings in our two texts which are activated unusually frequently in comparison with the meanings in a reference corpus, which gives a norm against which our mathematical texts are compared. The reference corpus used in this essay is a set of one million random sentences taken from Wikipedia, as a source of a very large amount of expository text. The log-likelihood figure for each of the *HT* categories activated in the texts indicates whether that meaning category occurs significantly more often than one might expect given the reference corpus baseline. For example, if the *Number Theory* textbook used as the comparison text has 5858 references to *HT* category *01.06.04.07 Algebra*,[1] the first step in looking at log-likelihood is to provisionally assume that both *Number Theory* and the reference corpus should have identical proportions of words activating that category (this is known as the null hypothesis; it is the default position one might expect should the category *not* be key to the text, and so exists to be disproved by the statistical test). The *expected* frequency of that category in the text is calculated using the reference corpus, which in this case gives a prediction of 911 occurrences in *Number Theory*, much smaller than the actual occurrence of 5858. The log-likelihood formula then takes the difference between the prediction and the actuality and returns a score of significance; as McIntyre and Walker put it, ‘the higher the log-likelihood value the more key or statistically significant an item is or the higher the likelihood that the unusually high or low occurrence of an item is not due to chance’ (2010, p. 517). In the case of this example, the log-likelihood value is 12,111, which is an extraordinarily high figure. Rayson et al. (2004) suggest figures above 15.13 to be significant, while figures above 37.3 mean there is a probability of only one in one billion that the difference is not statistically significant. The log-likelihood value in this example suggests very strongly that the semantic category of *01.06.04.07 Algebra* is significant in *Number Theory*, which should be no surprise given the topic of the textbook. The log-likelihood test is an established one (for a range of users see, *inter alia*, Baayen 2008; Baker 2010; Larose 2006; Mendoza-Denton et al. 2003), and in the case of *Number Theory* gives us just a very large list of meanings—as represented by *HT* categories—all to do with mathematics (with one exception, discussed in the following section). This means *Number Theory* does its job well as a comparison text; there are no analogies here, no meanings on our list about anything other than numbers and mathematics. *MP*, however, has a much wider range of entries—some about mathematics, and some not.

Thus far, the method is providing what it was intended to produce.

Reading the frequency lists of *MP*, there are four categories of entry: relevant material (about mathematics), supporting material, analogical material, and accidental ‘noise’. The next task is to remove everything but analogical material from our lists. Firstly, ‘noise’ are those semantic categories which are present due to the interference of homonymous or polysemous terms. As an example, in the *Number Theory* comparison text the irrelevant meaning category *01.02.08.01.22.06.06 Preparation of bread* comes out as supposedly statistically significant. This can be discounted because the category is entirely activated by one polysemous word (*proof*, with its multiple meanings of leaving bread to rise and also its mathematical sense), and so the category’s ‘lexical spread’ (how many words are used to represent that meaning) is low. By comparison, a category of *Gardening* in a gardening text would have a huge range of words used to represent meanings and concepts about gardening. Therefore, an entire semantic domain—such as *Clothing, Law*, or *Music*—which is potentially key but consists only of one polysemous word used over and over again can be discounted for this analysis as irrelevant ‘noise’. Fortunately, the log-likelihood process also removes much other ‘noise’ by focusing only on the significant concepts.

Putting all of the *MP* chunks back together to see the text as a whole, those hundred meaning categories with the highest log-likelihood score (the ‘key categories’) after removing the ‘noise’ as above, give us a list of categories either about mathematics (and related concepts) or potentially analogical. Our final step is to contrast those categories which might be analogical with those which are consistent with the *literal* topic of the text. This judgement is a decision which needs to be made by the analyst, and so for this essay I followed some basic guidelines to categorise the key categories. Firstly, as *MP* is about mathematics, I categorised as relevant material any category within the *HT* structure of mathematics, number, and computer science (which the *HT*, with its historical perspective, currently has as an adjunct of mathematics). Such categories came from *HT* sections including *01.06.04.x Mathematics* and *02.01.x Mental capacity* (describing processes of thought, understanding, and knowledge), as well as the more specific domains *Infiniteness* and *Logical syllogism.* Secondly, areas of supporting material which are of importance to the context of the text were also considered 'relevant' to the literal topic; for example, as the text is academically related, the high log-likelihood of *03.06.06.02 College/university* is to be expected, as is *01.01.06.10.01 United States* and *01.02.07.08.04 Person,* where the author describes the academics involved in research and their locations. Elsewhere, categories of *Determination, Persistence,* and *City* (describing where mathematicians interviewed or discussed came from or worked) were also relevant. In terms of noise, the polysemy of *prime* was the source of most of the domains removed for a low lexical spread (including senses in fencing, preparing a boiler, and the 6 am canonical hour). All these relevant and noisy domains were therefore set aside.

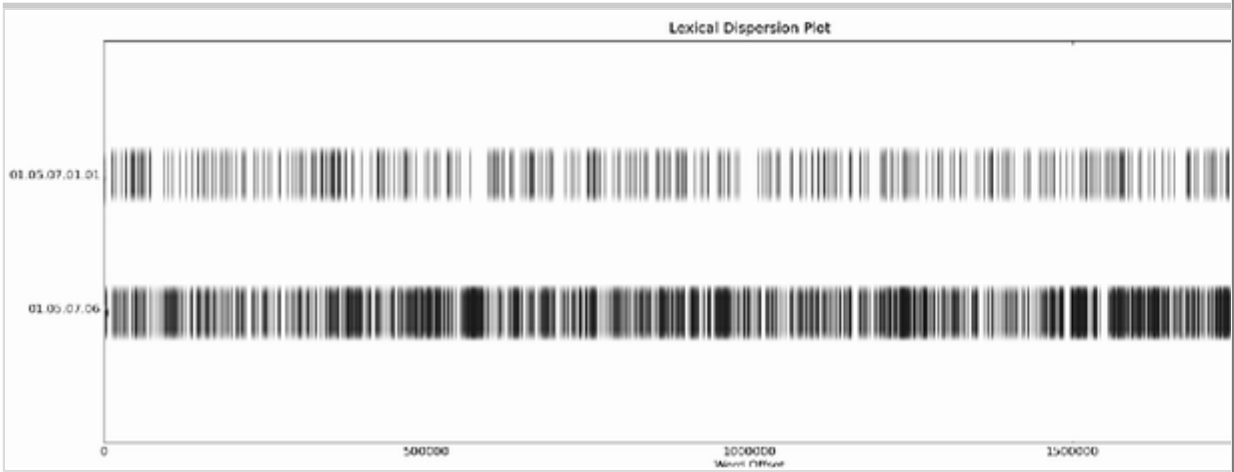
We now have a list of semantic categories which are statistically 'key' to the text when compared to general expository discourse, and which are not immediately relevant to the literal topic of the text, and which are not irrelevant 'noise'. It is here that our sought-after concretized analogies are to be found.

The Music of the Primes

Taking these *HT* categories which represent potential analogies, there are two with a very high log-likelihood score: *01.05.07.01.01 Distance/farness* and *01.05.07.06 Direction*. Figure 32.1 below shows the dispersion of these categories throughout the text, indicating that they are not clustered in only some parts of the book but rather appear often and repeatedly across the whole book.

Fig. 32.1

A dispersion plot of key analogical categories in *MP*; the plot represents the book from its start on the left to its end on the right, and each small vertical line indicates a point in *MP* where either *HT* category is used



Looking back at the database which stores not just the *HT* categories used, but also the words from the text which activate those categories, there are words such as *way, far, line, level,* and *point* all used to encode the semantic categories *Distance/farness* and *Direction*. It is clear that these words are used often as analogies in *MP*. A random set of concordance lines (in Fig. 32.2) shows an extended analogy of a landscape or topography.

Fig. 32.2

Twenty random concordance lines of *way* and *far* from MP, output using the author's programs. Concordances such as these are read by running one's eye down the central column, in bold, and reading just enough context as is necessary to establish the sense of each word. Only a set amount of characters on either side of the central word are reproduced in a concordance, and concordances should not be read like normal sentences

```
blem As mathematicians navigate their way across the mathematical te
0 bc they had evolved a very physical way of understanding what it i
thor Not only do they finally see the way to the peak but also they
new mathematics encountered along the way The problems allow for exp
ling of exhilaration at discovering a way to reach the summit of som
ily rigid geometry There was only one way that the landscape could b
r case mathematicians finally found a way to avoid having to cross t
dy and Littlewood were fighting their way across Riemann s strange l
 height of the Riemann landscape This way he might be able to find a
important breakthrough He had found a way to show that points at sea
othesis for primes but he had found a way to show that points at sea
 be seen until one had travelled very far north in the landscape As
al test By the time you had gone this far north if the graph still d
 primes never run dry Or perhaps this far off peak is just a mathema
further you count Gauss had seen this far off mountain peak but it w
os in his imaginary map Remarkably so far his analysis had worked wi
s finally powerful enough to navigate far enough north in Riemann s
ician Hugh Montgomery about the zeros far up Riemann s ley line Odly
ng the largest percentage of zeros so far to be found on Riemann s l
up in a straight line He couldn t see far enough across his landscap
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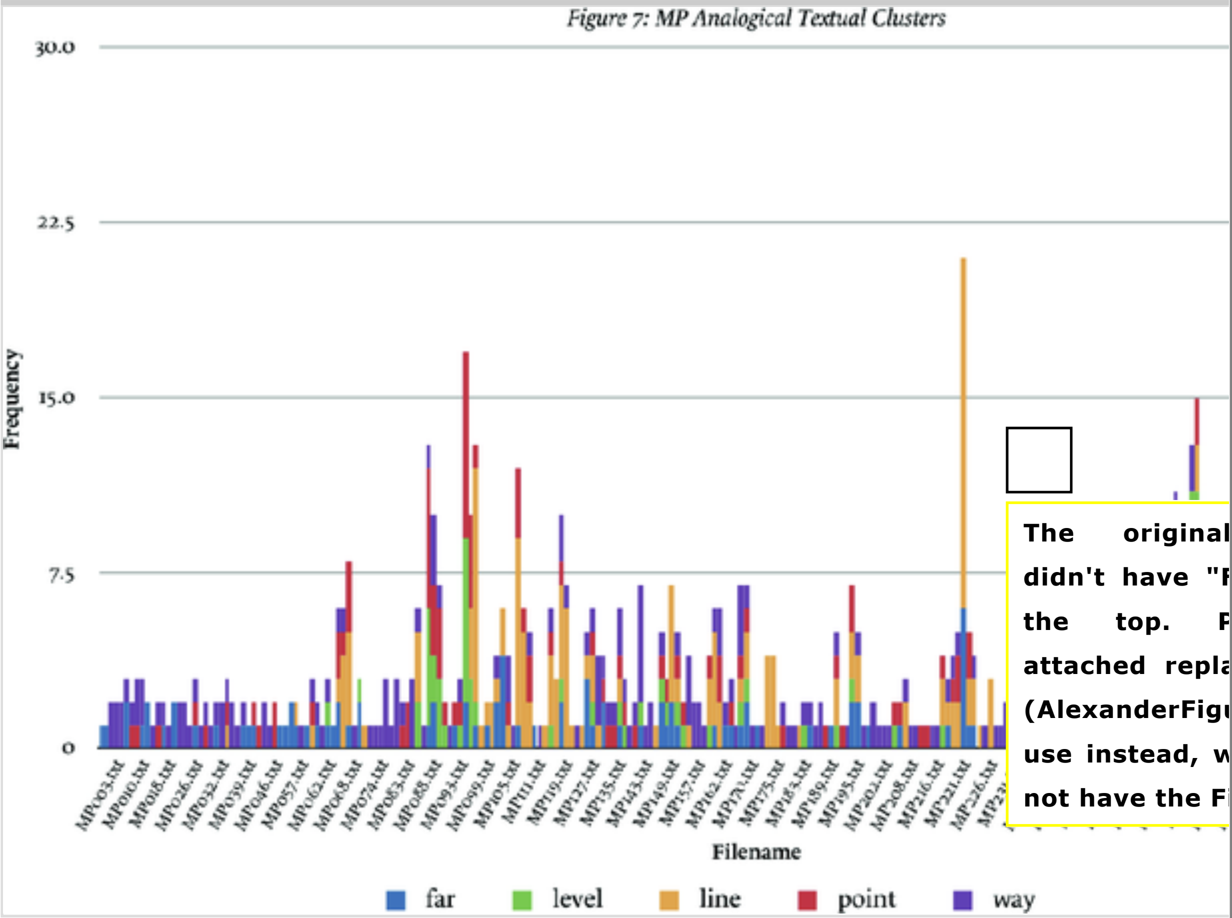
While some uses of *way* here are not analogical, there are clear indications that references to a landscape—*terrain, a very physical way of understanding, peak, encountering on the way, summit, landscape, sea level*, *imaginary map, his landscape,* and so on—are used to explicate abstractions around prime number theory. A brief examination of *line* (used both in the entrenched metaphor of *number line*, see Mazur (2004), and in lines on a landscape) and *level* (used primarily in the phrase *sea level*) can support that point.

Using these indicative words, it is possible to plot where in *MP* they occur (or, more accurately, in how many of *MP*'s 327 chunked-up files of 400 words each), in order to find dense analogical areas for analysis. A graph of this is given in Fig. 32.3. There is a high spike representing the word *line* in file 224, and two clusters, centring around files 97 and 277. From the graph, it is apparent that the cluster at 97 is more mixed with different words than the one at 277, as the latter consists mainly of the lexeme *level*, whereas the 97 cluster has a wider range of analogical items. As I aimed to identify a complex analogy, I chose the material centring around file 97 to analyse, as it is both a dense cluster of analogy and consists of the most mixed terms.

Fig. 32.3

Frequency of key terms in *MP*, arranged by the 400-word ‘chunks’ the book was split into for analysis



Combining Concepts

The second half of this chapter will engage in a close reading of this cluster of concretized analogy. Many different approaches could be taken, but as a linguist I am primarily concerned with how the language of a passage gives rise to meaning in the mind of a reader. The approach below comes from the framework of conceptual blending theory,[2] a relatively new linguistic approach to the construction of meaning in the same general approach as George Lakoff and Mark Johnson’s classic *Metaphors We Live By* (1980) and subsequent work.[3] Blending is ‘a theoretical framework for exploring human information integration’ (Coulson and Oakley 2000, p. 176), where analysis involves looking at the ‘processes in which the imaginative capacities of meaning construction are invoked to produce emergent structure’ (Coulson 2001, p. 115). In short, blending is a basic mental operation where two sets of information held in the mind are integrated in order to gain a 'compressed' and insightful meaning. It is fundamental to the construction of meaning, especially of meanings that we have to 'simulate' as readers or hearers, and has even been suggested as the most crucial element of linguistic meaning.

Blending works by taking more than one mental 'input' and blending them to create a resulting concept: a sentence like *University Avenue runs through our campus*, an example of what linguists call fictive motion, blends the concept of firstly an absolutely static paved avenue, secondly that avenue's static context of a campus, and thirdly the active concept of 'running', consisting of a mobile element travelling along a path. The resulting blended idea combines all of these elements, applying a motion verb to an immobile object and giving a 'compressed', graspable idea which simulates motion where no motion is placed and takes advantage of the bias of human perception towards motion, which is more easily understood than a stative concept. For our purposes with regards to popular mathematics, the analogous concretizations looked at in this chapter take advantage of our easy comprehension of physical situations as opposed to the challenge of understanding abstract mathematical concepts. Blending can therefore describe, in an empirically supported way, our ability as readers and hearers to combine two different sorts of information to arrive at an integrated understanding. It is ideally suited for an analysis of analogy. More detail is given below about the theory and its aims, before I apply it to a passage from *MP*.

Blending's leading theorists, Gilles Fauconnier and Mark Turner, describe the wider implications of the theory:

[Conceptual blending] serves a variety of cognitive purposes. It is dynamic, supple, and active in the moment of thinking. It yields products that frequently become entrenched in conceptual structure and grammar, and it often performs new work on its previously entrenched products as inputs. Blending is easy to detect in spectacular cases but it is for the most part a routine, workaday process that escapes detection except on technical analysis. It is not reserved for special purposes, and is not costly.[4]

In blending, structure from input mental spaces is projected to a separate, 'blended' mental space. The projection is selective. Through completion and elaboration, the blend develops structure not provided by the inputs. Inferences, arguments, and ideas developed in the blend can have effect in cognition, leading us to modify the initial inputs and to change our view of the corresponding situations.

(Fauconnier and Turner 1998, p. 133)

The theory has developed from its origins in the 1990s into an extensive cognitive approach to the construction of meaning in natural language. Crucially, while blending is used to describe the integration of different concepts into a unified whole (for the purposes of this chapter, the blending of mathematical concepts with concrete analogical material), it also describes a process of *compression* of complex concepts towards what is called human scale. Ungerer and Schmid believe compression towards human scale to be ‘the ultimate goal of the whole blending process’ (2006, p. 260). They continue:

The crucial effect of compression is that the conceptual complexity of the inputs from several sources is reduced considerably. A newly integrated and unified conceptual structure emerges that is cognitively manageable and thus has … ‘human scale’. (2006, p. 260)

Human scale can be characterised as a situation which is easily processed, being based on simple, familiar, and embodied concepts. Fauconnier and Turner describe this as follows:

The most obvious human-scale situations have direct perception and action in familiar frames that are easily apprehended by human beings: An object falls, someone lifts an object, two people converse, one person goes somewhere. They typically have very few participants, direct intentionality, and immediate bodily effect and are immediately apprehended as coherent. (2002, p. 312)

They further state that human scale ‘is the level at which is it natural for us to have the impression that we have direct, reliable, and comprehensive understanding’ (2002, p. 323). The obvious consequence of this for this essay is that the reduction of complex situations to ones which are at human scale, and thus more easily apprehended, is a clear example of the sort of explication and concretization which characterises popular writing on mathematics.

An analysis of an instance of conceptual blending—such as the one below from *MP*—is an attempt to represent a process going on in the mind when a reader or hearer attempts to comprehend the meaning of a text or utterance. *Blending* is both the name for the theory and for the overall process of comprehension, which takes 'mental space' inputs (knowledge structures in the mind which contain the various information we have about a thing or concept) and when these input spaces are selectively blended together we have another mental space, the *blend*, which should be at human scale, and compressed in the process of blending into something we can easily grasp. This work goes on in parallel (including alternative interpretations), and takes only milliseconds.

For example, the metaphorical blend *to be in the dark about something* contains two 'input spaces' (that of darkness, and that of comprehension—the first from the words used and the second from the context), and they have in common the goal of perception. From the mental space of darkness we take knowledge about the difficulty of perceiving objects in the dark and the potential danger of doing so, and from the mental space of comprehension we take information about comprehension being a desired goal of many activities, a necessary part of experience, and a goal which gives rise to frustration if being blocked. The entrenched COMPREHENSION IS TOUCHING (*I've grasped your idea, hold onto that thought,* etc.) and UNDERSTANDING IS LIGHT (*she enlightened me, a bright idea, seen in this light I don't agree*, etc.) metaphors may also form an additional input space here, as they are standard (and even default) manners of thinking about perception. All this information in the mind is activated by the words used in the expression, and the mind generates the final blend, which combines a goal of comprehension with a difficulty (or impossibility) of perception in the present environment, the danger of attempting to continue without that knowledge, and the frustration of being in that environment. The technical process of the mind blending all of this—which is too complex to discuss here, but is gone into in detail in either Fauconnier and Turner (2002) or Coulson (2001)—involves taking all of this information and producing the most relevant understanding, which is the output blend. Finally, such a blend can have *emergent structure* by the juxtaposition of elements;

darkness is not in itself a negative thing (and can have pleasant connotations, for example during sleep or relaxation) but when put alongside the comprehension mental space and the touching/light entrenched metaphors, the mind can connect a goal-frustration scenario to darkness and so convey the frustration intended to be communicated by the *in the dark* expression.[5]

To analyse the long 'chunk' of *MP* which has been identified as highly analogical, at the level of detail of the above example, would be both tedious and unreasonably lengthy. Instead, an analysis is best formed by examining the mental spaces evoked by the passage (by the words used in the text) and describing the output blends created by the process of blending the mind goes through when encountering these mental spaces. Mental spaces are traditionally very hard to describe, but here we can name them according to their *HT* category name for that concept, principally through the headword nouns which occur in the passage, and this is an advantage of this chapter's new methodology. As the *HT* is a meaning-based organisation of every recorded English concept which can be expressed by a word or phrase, it is ideal to be used as a supplementary aid to describing conceptual blending.

The large analogical passage from *MP* (pages 84−86 in the original text, at the start of the fourth chapter) is too long to reproduce here but key quotations are given below. It comes as part of a description of work done in the distribution of the prime numbers. The ability to predict the distribution of primes is a highly sought-after goal in mathematics, and in 1859 Bernhard Riemann hypothesised that primes can be partly understood by looking at where the graph of a particular function (known as the Riemann zeta function) crosses the x-axis of the graph (that is, where it equals zero). Du Sautoy's primary challenge in the passage under analysis is that 'To illustrate his graph, Riemann needed to work in four dimensions' (*MP* p. 85; all following quotes are pp. 84−86). Each input is a complex number a + b*i* and each output is a complex number c + d*i*, so that its graph exists in two-dimensional complex space, which can be viewed as a four-dimensional real space.

The passage can be analysed in stages, taking each input space in turn as it occurs throughout the extract in order to show the creation of this complex analogical blend. The passage's goal is to generate an understanding of this four-dimensional space for non-mathematicians (who do not naturally relate to higher-dimensional mathematics easily, as any reader of popular science from Edwin A. Abbott's *Flatland* (1884) onwards can understand from the energy many authors have put into its explication). The last stage of the quantitative analysis above gave us an indication that the extract in question focuses on the language of a landscape to discuss the prime numbers (which du Sautoy does by analogizing a landscape to the four-dimensional graph mentioned above). From the passage the idea of dimensional reduction (a 4-dimension graph to a 3-dimension landscape) is formed from a number of intersecting blends of knowledge about graphs, shadows, and landscapes. There is a total of twenty-seven input spaces in the passage, in four major sections. I work through each in turn below.

AQ2

Mathematics (4 inputs)

The extract begins at the start of a chapter, and opens by reintroducing Riemann and his work combining the mathematics of prime and imaginary numbers in the Riemann zeta function. (Input spaces: mathematics, prime numbers, imaginary numbers, and the zeta function).

Alchemy (4 inputs)

Du Sautoy then compares Riemann's work to discovering, 'like some mathematical alchemist', a mixture of elements like a sought-after treasure. These four input spaces (treasure, alchemy, admixture, and elements) are not used again, and form an illustrative comparison which does not link to the key landscape analogy, but does connect the zeta function to the idea of treasure. The idea of treasure does not reoccur in the extract, but rather is a brief digression to emphasise the important nature of Riemann's work. With this done, the opening discussion then moves to the key analogy.

Landscape (14 inputs)

In the main part of the extract, the zeta function is compared to a vista, and then a large number of additional input spaces are brought in. A reader is taken by du Sautoy from the Reimann zeta function (the same mental space we have already encountered) to new elements, introducing mathematical functions in general, then equations, then graphs of equations. After discussing cosmological analogies for four-dimensional mathematics (where time is the fourth dimension) as well as an economic analogy, du Sautoy describes three-dimensional economic graphs as a 'landscape' with peaks and valleys; this description brings the reader to the idea of a dimension-as-landscape, which will be du Sautoy's major analogy for the Riemann zeta function.

Shadow Side-Analogy (5 inputs)

At this point, du Sautoy pauses his landscape analogy to argue that a reduction of a complex idea to a simpler one can still help you understand that complex idea. He argues this point through an analogy between reducing a three-dimensional face to a two-dimensional shadow or silhouette, and reducing Riemann's four dimensions to a 'three-dimensional shadow' of them:

Looking at shadows is one of the best ways to understand them. Our shadow is a two-dimensional picture of our three-dimensional body. From some perspectives the shadow provides little information, but from side-on, for example, a silhouette can give us enough information about the person in three dimensions for us to recognise their face. In a similar way, we can construct a three-dimensional shadow of the four-dimensional landscape that Riemann built using the zeta function which retains enough information for us to understand Riemann's ideas. (*MP*, p. 85)

This creates the idea of reduction of four dimensions to three dimensions so that du Sautoy can then complete and return to his analogy of a landscape; as a landscape exists in three dimensions, the analogy can't begin until we are introduced to the idea that dimensions can be reduced in a representation like a shadow.

Landscape Continued

There are only a few remaining input spaces. The landscape analogy is returned to with a section which combines the language of a graph with the language of a map:

Gauss's two-dimensional map of imaginary numbers[6] charts the numbers that we shall feed into the zeta function. The north-south axis keeps track of how many steps we take in the imaginary direction, whilst the east-west axis charts the real numbers. We can lay this map out flat on a table. What we want to do is to create a physical landscape situated in the space above this map. The shadow of the zeta function will then turn into a physical object whose peaks and valleys we can explore. (*MP*, p. 85)

The vocabulary of landscape is broadened to an immersive three-dimensional world, with peaks, hills, and sea level:

As Riemann began to explore this landscape, he came across several key features. Standing in the landscape and looking towards the east, the zeta landscape levelled out to a smooth plane 1 unit high above sea level. If Riemann turned round and started walking west, he saw a ridge of undulating hills running from north to south. The peaks of these hills were all located above the line that crossed the east-west axis through the number 1. Above this intersection at the number 1 there was a towering peak which climbed into the heavens. It was, in fact, infinitely high. As Euler had learned, feeding the number 1 into the zeta function gives an output which spirals off to infinity. Heading north or south from this infinite peak, Riemann encountered other peaks. None of these peaks, however, were infinitely high. (*MP*, p. 86)

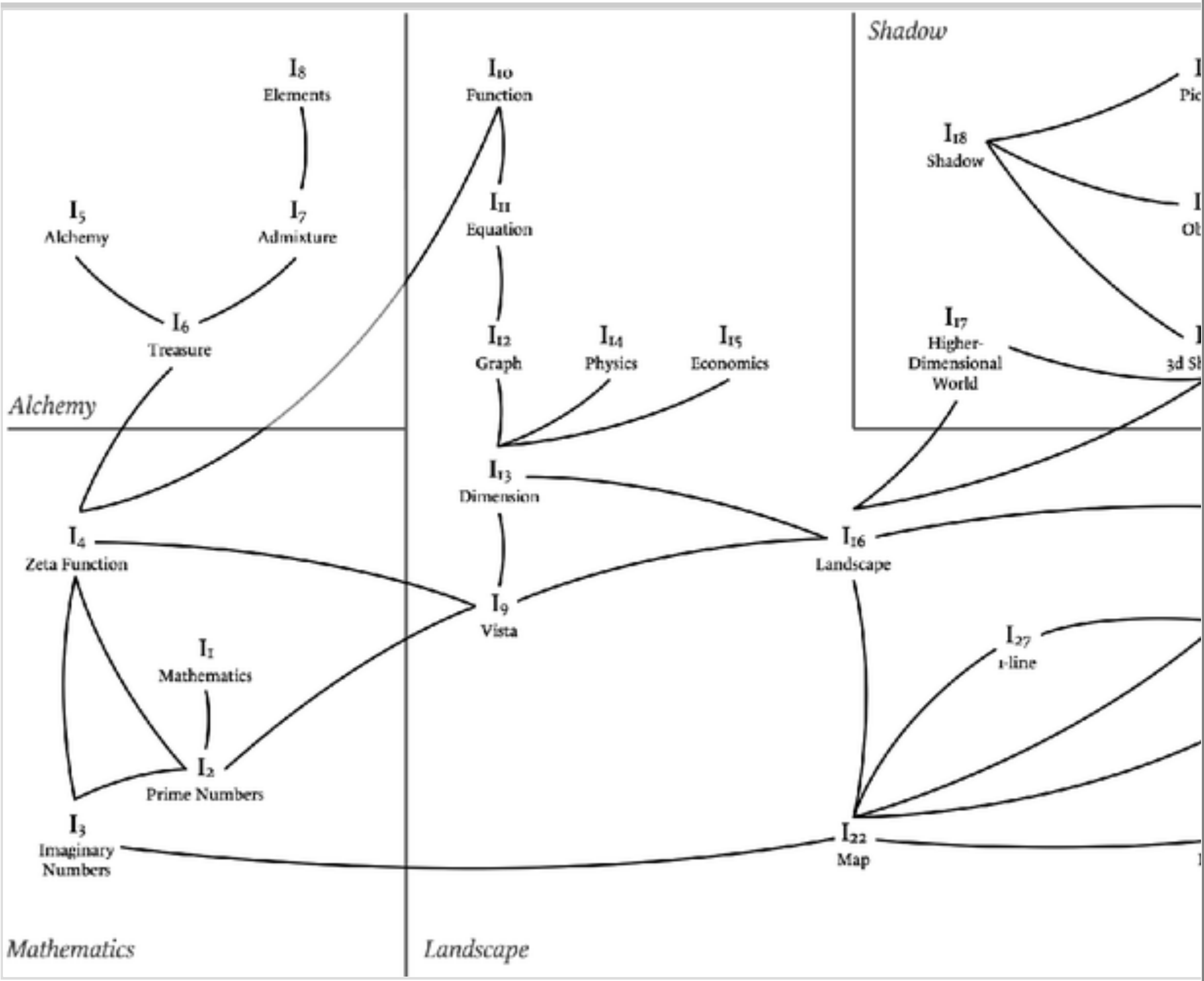
This is where the extract identified by the computational analysis above ends, although the analogy continues for some time. The chapter continues, still using figurative material (as can be seen by the dispersion plot in Fig. 32.1), but in a less concentrated manner.

A Map of the Riemann Landscape

This network of input spaces in this passage from du Sautoy's book, with their textual connections to each other shown, is illustrated in Fig. 32.4. The subscript numbers indicate the order in which the input spaces appear in the text (starting from bottom left), and the spaces have had to be arranged in an order which tries as much as possible to correspond to the order of the spaces in the text while also allowing for their display. Each of the twenty-seven input spaces is listed, and the lines between them indicate which spaces are connected to each other in the text (so when du Sautoy discusses a shadow as a two-dimensional picture of a three-dimensional object, the connective verbs and prepositions connect *shadow* to *picture* and *object*, which is what the lines between spaces in the top right of Fig. 32.4 represent).

Fig. 32.4

A ‘map’ of the input spaces in the analysed extract from *MP*



Our final step in this analysis is to describe the blends created through the complex interaction of these spaces.

The Blends

There are four blends which are created in the course of the extract, coming together to form the final fifth blend, which itself represents the full set of concretized human-scale knowledge created by the passage.

As discussed above, the first blend straightforwardly uses the alchemy/treasure comparison to make explicit that the zeta function is a valuable result. The other three blends are ones which redefine mathematical concepts introduced and refined through analogy. Blend two concludes the 'shadow' textual group by integrating information from that group into a single conceptual blend, which profiles the nature of a three-dimensional shadow of a four-dimensional entity; the third blend builds on this and integrates it with the landscape analogy to redefine the landscape mental space into one which takes account of the higher-order dimensionality it describes (the four-dimensional space of which the three-dimensional landscape is a shadow); and the fourth blend integrates the measures of the map's axes with the concept of height above the map.

These combine with each other into the full arch-blend, which is the final Riemann landscape. It explicitly combines the second blend above (a 'shadow' of a four-dimensional object, which gives a perspective on the object it is a shadow of, but loses some information) with the third blend (a three-dimensional thing can be thought of as a landscape, with hills, valleys, a sea level, and so on) and the fourth blend (the 'height' of an item, as with du Sautoy's infinitely high peak, which relates to the output of the Riemann zeta function) to come up with the final blended concept of the Riemann landscape. This landscape has hills, troughs, directions, sea level, and a particular line marking the number one (all brought in from input spaces). It tracks three-dimensional elements of interest which correspond to these hills and troughs and infinite peaks, all of which represent axes of a 'shadow' or dimensional reduction of four-dimensional space, which is the blended input space of the valuable zeta function. It also has emergent structure (in conceptual blending terms) which is the nature of the core analogy of the whole passage; that this landscape of the zeta function may have three axes but as we travel through this space, the human-scale experience of the varied shape of the land, with its valleys and infinite peaks, gives a reader a sense of comprehension of the more complex four-dimensional zeta function itself.

Conclusion

As the analysis above illustrates, the concretization of mathematical concepts in a non-technical text (with significant narrative elements) is a complex and recursive situation. When authors make their choices of how to represent mathematics for readers other than experts in a shared common discourse, the natural option is to take established, concrete physical concepts—a map, a shadow, or a landscape—in order to relate the technical material to graspable ideas at 'human scale'.

Such mappings, realised here in the cognitive linguistics paradigm as input spaces, are not entirely unrestricted—in the same way that we can say a theory has *strong foundations* and is *well buttressed* and is *a towering approach* but can't quite say the theory has *good French windows* or *a typical wooden theoretical staircase*, our conceptual structure does not permit us to concretize popular mathematics with any old analogy (the restrictions here are known as mapping scope; see Lakoff 1993). The approach described here, when applied to a sufficiently large corpus of mathematical texts, could in a large-scale way identify the *Historical Thesaurus* categories which are statistically prevalent in the collection of texts and so describe the areas of our conceptual 'inventory' which are available as analogies for popular or literary mathematics. These sorts of big data projects, enabled for the *Historical Thesaurus* by other computational advances such as those described in Piao et al. (2017), will still need to be matched with a close reading—whether of the type above or in another style entirely—of the texts themselves in order to determine not just what can and cannot be analogically mapped to mathematics, but why those mapping scope restrictions exist. Similarly, I hope the conceptual blending analysis presented here—although much abbreviated, especially when considering the richness of blending theory—illustrates the potential power of blending for a close analysis of the construction of meaning in dense and tightly packed passages of analogy. In parallel with my aim of selecting a passage for analysis as objectively as is reasonably possible, the blending approach requires acknowledging every mental space activated in a section of text and accounting for each space's links both to its neighbours and to the human scale blends to which they contribute.

The study of literature and mathematics—taking literature in its broadest sense—can gain to an extent from an engagement with cognitive and quantitative approaches, even if the details of such approaches may appear rather technical and overly narrow for some purposes. Quantitative approaches cannot and should not replace an analyst's reading of a text, but they can supplement our existing methods for finding areas worth studying. Similarly, cognitive approaches should not be used for their own sake, but can focus our attention on the fundamental methods by which texts give rise to meaning in the mind—from the shortest passage to the broadest genre—and so enact the fundamental processes of language.

Notes

1. The long number is the *HT* hierarchical reference code for this concept.

2. The theory has also been known as *conceptual integration theory* (which is treated as synonymous with *conceptual blending* in the literature).

3. Recently, Woźny (2018) has applied conceptual blending theory to fundamental mathematics (not popularised or literary representations), and the theory is congruent with Lakoff and Nuñez's (2000) approach to mathematics (see also Fauconnier and Lakoff [2010]).

4. 'Costly' here appears to be used in the sense of requiring extra mental processing time. [My note].

5. For many more examples and worked demonstrations, including psychological justifications for conceptual blending's approach, see Fauconnier and Turner (2002) and Coulson (2001).

6. Correctly speaking, Gauss refers to a two-dimensional map of *complex* numbers. Complex numbers are expressed in the form a + b*i*. The real part a is indicated on the horizontal number line (stretching west-east in du Sautoy's terms) and the imaginary part indicated on the perpendicular number line (running north-south). See further Rozenfeld (1988). [My note].

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