Han, D. and Barakos, G. N. (2020) Aerodynamic interference model for multi-rotors in forward flight. *Journal of Aircraft*, 57(6), pp. 1220-1223. (doi: 10.2514/1.C035978).

Aerodynamic Interference Model for Multi-Rotors in Forward Flight

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I. Introduction

Multi-rotor aircraft are becoming commonplace and quad-rotors are the most popular configuration for aerial photography. The aerodynamic interference between any two rotors is one of the key factors affecting the aerodynamic performance of these kinds of aircraft. This issue is also an acute problem for tandem, tiltrotor, and coaxial rotor configuration rotorcraft [1].

The aerodynamic interference problems have been recognized by the researchers who focus on multi-rotor aircraft. Luo et al. established a mutual-interference model to consider the aerodynamic interactions for a four rotor system [2], which modeled the rotor system in forward flight as an equivalent circular fixed wing. This model was validated by CFD analysis, and could account for rotor interference reasonably well. Huwang et al. analyzed the aerodynamic interactions for the diamond and square configuration quadrotor aircraft using CFD [3]. The aerodynamic interference could strongly affect the aerodynamic performance of quadrotor aircraft, and the interactional aerodynamics was necessarily to be considered in the design of flight control system. Misiorowski et al. confirmed that the aerodynamic interference could benefit the diamond configuration quadrotor aircraft [4]. A theoretical formula was used to capture the interference effect of the front rotor on the rear rotor [5], and the theoretical predictions for the rotor thrust and drag agreed well with test using a model system. It is obvious that the aerodynamic interference must be considered in the analysis of aerodynamic performance and the design of flight control systems.

For the prediction of flight performance and the design of flight control systems of multi-rotor aircraft, fast predictions and analyses with acceptable precision are required due to the design cycle limit and even real-time demand. For the aerodynamic interference models, it is necessary to balance the efficiency and accuracy. CFD can provide high precision prediction, however, with a lot of computing resources and time. Currently, it is impossible to be directly included in the fast prediction of flight performance and in the design of flight control systems. Omitting the aerodynamic interference is obviously inappropriate, and so, if some analytical models are available, a satisfactory compromise can be found.

The implicit aerodynamic interference model derived by Luo et al. [2] is similar as the explicit model used by Nguyen et al. [5], and both models treat the rotors into equivalent circular wings. The aerodynamic interference between any two rotors is measured by the induced velocity at one rotor generated by the horseshoe vortex trailed from the other. In this work, an explicit expression for the implicit interference model used by Luo et al. [2] is derived, which can significantly reduce the computing time. An general expression for the induced velocity considering the aerodynamic interference between any two rotors is given without any neglect of the effect of one rotor on the other.

II. Analytical Model

For a lifting rotor, the coordinate system is attached to the rotor disk, as shown in Figure 1. The origin of the coordinate system is at the rotor hub center. The x axis is on the rotor disk plane, and its direction is consistent with the incoming flow. The y axis also locates in the rotor disk plane, and is perpendicular to the x axis. The horseshoe vortex is trailed from the retreating $(0, R, 0)$ and the advancing side $(0, -R, 0)$, as shown in Figure 1. R is the radius of the rotor. The wake skew angle γ is the angle between the rotor disk and the trailed horseshoe wake plane. For multi-rotor aircraft, all rotor disk planes are mounted on a plane.

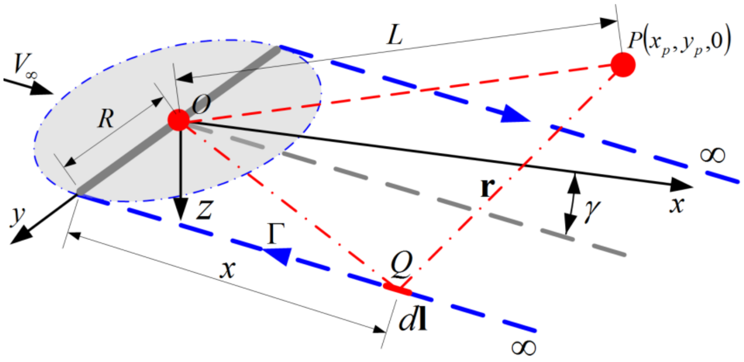


Figure 1 Coordinate system.

At a forward speed (rotor advanced ratio $\mu > 0.1$), the rotor can be modeled as a circular fixed wing [2, 6]. This aerodynamic interference model is based on the Biot-Savart Law, and assumes that the horseshoe vortex trailed from one rotor generates an additional downwash to the other rotor. Considering a vortex segment dl at the Point Q $(x, R, x\tan\gamma)$, the velocity in the z direction induced at Point P by the vortex segment is

$$\left[\frac{\Gamma}{4\pi}\frac{\mathbf{dl}\times\mathbf{r}}{|\mathbf{r}|^3}\right]_z = \frac{\Gamma}{4\pi}\frac{(y_p + R)dx}{\left[(x_p - x)^2 + (y_p + R)^2 + (x\tan\gamma)^2\right]^{\frac{3}{2}}} \tag{1}$$

where, Γ is the strength of the vortex, and

$$\mathbf{dl} = dx\mathbf{i} + dx\tan\gamma\,\mathbf{k}$$

(2)

$$\mathbf{r} = (x_p \mathbf{i} + y_p \mathbf{j}) - (x\mathbf{i} + R\mathbf{j} + x \tan\gamma \, \mathbf{k})$$

(3)

The velocity in the *z* direction at Point P induced by the horseshoe vortex is

$$v_{iP} = \frac{\Gamma}{4\pi}\int_0^\infty \frac{(y_p + R)dx}{\left[(x_p - x)^2 + (y_p + R)^2 + (x\tan\gamma)^2\right]^{\frac{3}{2}}} + \frac{-\Gamma}{4\pi}\int_0^\infty \frac{(y_p - R)dx}{\left[(x_p - x)^2 + (y_p - R)^2 + (x\tan\gamma)^2\right]^{\frac{3}{2}}}$$

$$= \frac{\Gamma\cos\gamma}{4\pi}\cdot\left[\frac{(y_p + R)\left(1 + \frac{x_p\cos\gamma}{\sqrt{x_p^2 + (y_p + R)^2}}\right)}{(y_p + R)^2 + x_p^2\sin^2\gamma} - \frac{(y_p - R)\left(1 + \frac{x_p\cos\gamma}{\sqrt{x_p^2 + (y_p - R)^2}}\right)}{(y_p - R)^2 + x_p^2\sin^2\gamma}\right]$$

(4)

Point P can be considered as a hub center of a rotor. If Point P lies between the two vortices and downstream the incoming flow, both vortices induces downwash velocities. It can degrade the rotor performance, if a rotor lies at this location. If Point P locates outside the vortices and downstream the incoming flow, one vortex induces a downwash velocity, and the other induced a upwash velocity. Since the point lies closer to the vortex inducing the upwash velocity, the resultant velocity is upwash. It can improve the rotor performance, if a rotor lies at this location.

By setting $y_p = 0$, the induced velocity right downstream the rotor can be expressed as

$$v_{iP}|_{y_p=0} = \frac{\Gamma}{2\pi\mathrm{R}}\cdot\left[\frac{\frac{1}{\cos\gamma} + \frac{x_p/R}{\sqrt{\frac{x_p^2}{R^2} + 1}}}{1 + \tan^2\gamma + \frac{x_p^2}{R^2}\sin^2\gamma}\right]$$

(5)

This expression is the same as the downwash for the interference of tandem rotors (wings) [7].

Similar to the method used in Refs. 5 and 7, an dimensionless factor is used to measure the aerodynamic interference. By setting $x_p = y_p = 0$, the induced velocity in the *z* direction at the center of the rotor disk (Point O) is

$$v_{iO} = \frac{\Gamma\cos\gamma}{2\pi R}$$

(6)

The non-dimensional expressions of x_p and y_p are defined as

$$\begin{cases} \bar{x}_p = \dfrac{x_p}{R} \\ \bar{y}_p = \dfrac{y_p}{R} \end{cases}$$

(7)

The aerodynamic interference factor is defined as the ratio of v_{iP} to v_{iO}, and it can be expressed as

$$k_i = \frac{v_{iP}}{v_{io}} = \frac{1}{2}\left[\frac{\left(\bar{y}_p + 1\right)\left(1 + \dfrac{\bar{x}_p \cos\gamma}{\sqrt{\bar{x}_p^2 + \left(\bar{y}_p + 1\right)^2}}\right)}{\left(\bar{y}_p + 1\right)^2 + \sin^2\gamma\, \bar{x}_p^2} - \frac{\left(\bar{y}_p - 1\right)\left(1 + \dfrac{\bar{x}_p \cos\gamma}{\sqrt{\bar{x}_p^2 + \left(\bar{y}_p - 1\right)^2}}\right)}{\left(\bar{y}_p - 1\right)^2 + \sin^2\gamma\, \bar{x}_p^2}\right]$$

(8)

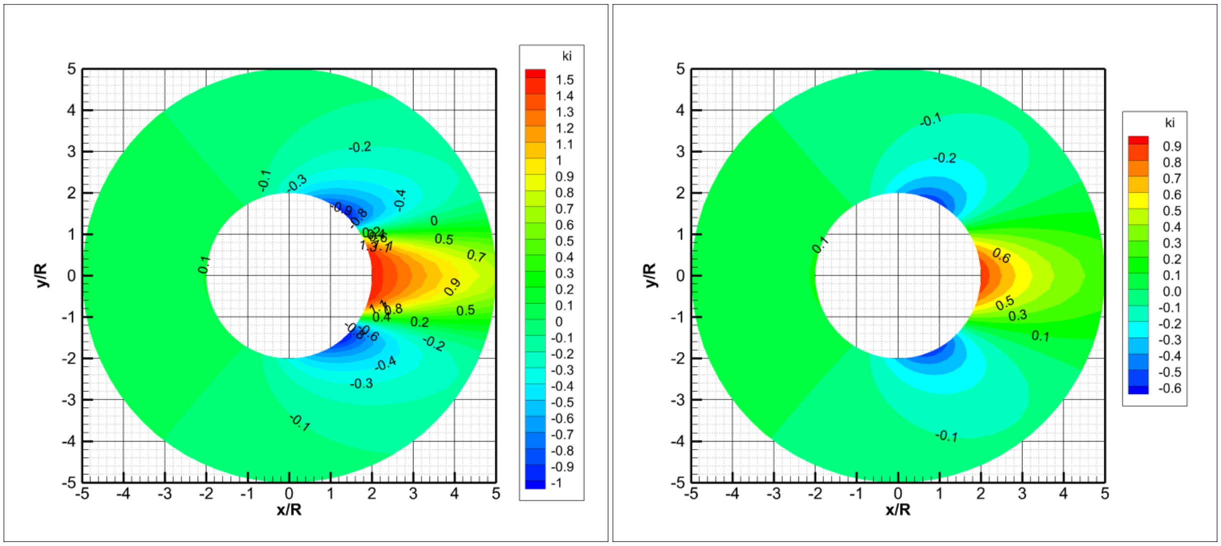
It is assumed that the velocity induced by a rotor on another is uniform. The induced velocity at the rotor hub center can denotes the aerodynamic interaction. For a multi-rotor aircraft, the induced velocity of the rotors can be expressed as

$$\begin{Bmatrix} v_i^{(1)} \\ v_i^{(2)} \\ \vdots \\ v_i^{(n)} \end{Bmatrix} = \begin{bmatrix} \kappa_1 & k_{12} & \cdots & k_{1n} \\ k_{21} & \kappa_2 & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & \kappa_n \end{bmatrix} \begin{Bmatrix} v_{i0}^{(1)} \\ v_{i0}^{(2)} \\ \vdots \\ v_{i0}^{(n)} \end{Bmatrix} = \mathbf{K_i} \begin{Bmatrix} v_{i0}^{(1)} \\ v_{i0}^{(2)} \\ \vdots \\ v_{i0}^{(n)} \end{Bmatrix}$$

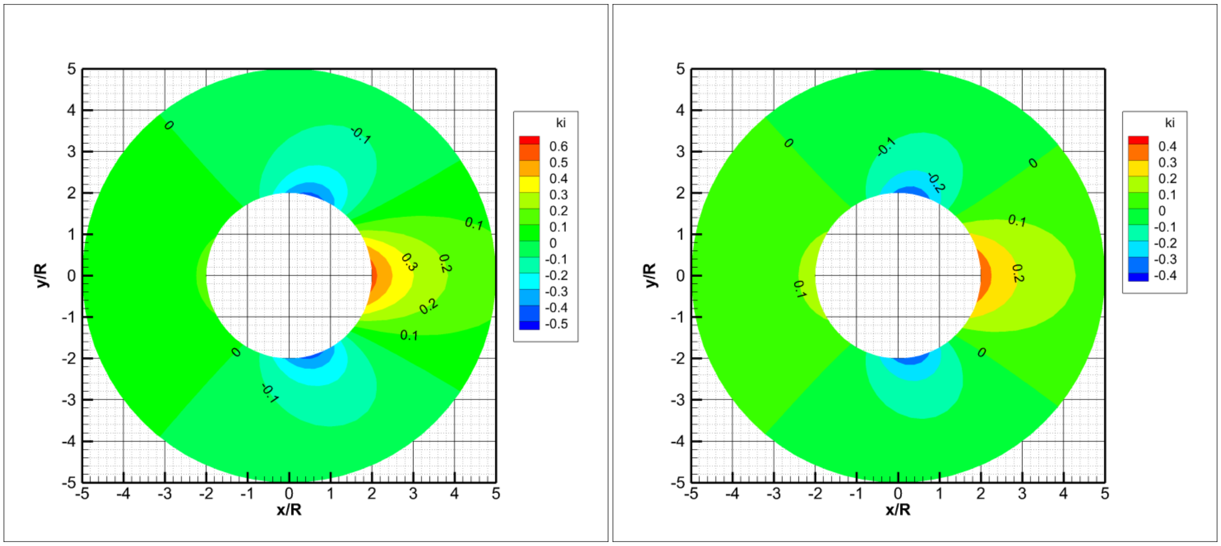
(8)

where, the superscript '*n*' denotes the number of the rotors of a multi-rotor aircraft, and the subscript '0' denotes the velocity of an isolated rotor. k_{ij} denotes the induced velocity factor generated by the *j*th rotor to the *i*th rotor (rotor hub center). κ_i is the correction for the induced losses of a real rotor. $\mathbf{K_i}$ is named as aerodynamic interference matrix. This expression of the induced velocity is consistent with the general formula in Ref. 2. When applying this formula, the effect on the front rotors are also included, and Ref. 2 neglected the effect of the aerodynamic interference on the front rotors.

III. Parametric Analysis



(a) 15^{o} (b) 30^{o}



(c) 45^{o} (d) 60^{o}

Figure 2 Aerodynamic interference factor for different wake skew angle.

Figure 2 shows the aerodynamic interference factor for different wake skew angles. A positive value means an increase in the velocity, and a negative value indicates that the aerodynamic interference is beneficial. The following conclusions can be concluded from the figure:

1) The aerodynamic interference becomes stronger with smaller wake skew angle, when wake approaches closer to the rotor disk. It then becomes weaker, as the distance between the two rotors becomes longer.

2) The relative location of the point to the rotor disk has significant influence on the aerodynamic interference. The strongest interaction appears, as a rotor is right after the rotor. At certain locations, for example, on the right or left of the rotor (relative to the incoming flow), the aerodynamic interaction can provide upwash velocity, which is beneficial for the flight performance of the aircraft.

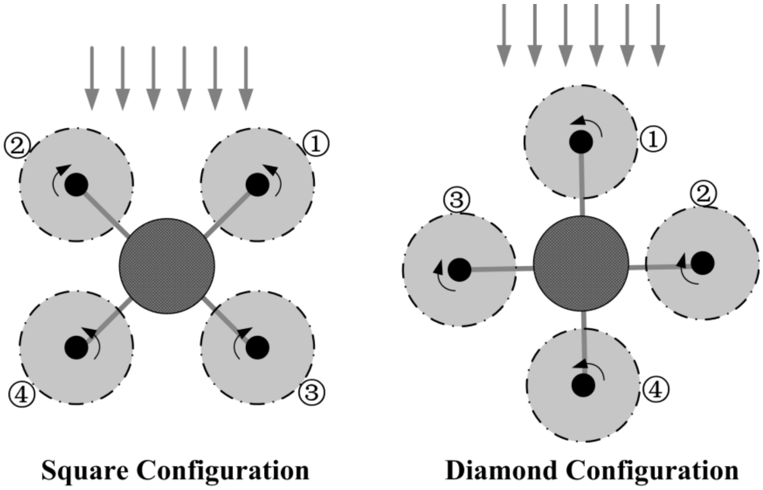


Figure 3 Configurations of quadrotor aircraft.

For a square configuration quadrotor aircraft shown in Figure 3, the distance between the two adjacent rotor shafts is assumed to be 4.0 times the rotor radius, and the wake angle is assumed to be 30^{o}. The correction for the induced losses of a real rotor is assumed to be 1.0. At this state, the interference matrix is

$$\mathbf{K_i} = \begin{bmatrix} 1 & -0.0667 & 0.0320 & 0.0041 \\ -0.0667 & 1 & 0.0041 & 0.0320 \\ 0.3680 & -0.0625 & 1 & -0.0667 \\ -0.0625 & 0.3680 & -0.0667 & 1 \end{bmatrix}$$

It is obvious that the aerodynamic interference is beneficial for the front rotors, and the magnitude is relatively small. The aerodynamic interference can significantly reduce the aerodynamic performance of the rear rotors. The phenomena are consistent with that in Ref. 3 predicted by CFD. It can be concluded that the aerodynamic interference is not beneficial for the flight performance of square formation quadrotor aircraft.

For a diamond configuration quadrotor aircraft (shown in Figure 3) with the same parameters and flight state as the square configuration, the interference matrix is

$$\mathbf{K_i} = \begin{bmatrix} 1 & 0.0091 & 0.0091 & 0.0164 \\ -0.1215 & 1 & -0.0323 & 0.0091 \\ -0.1215 & -0.0323 & 1 & 0.0091 \\ 0.2059 & -0.1215 & -0.1215 & 1 \end{bmatrix}$$

It is obvious that the aerodynamic interference is beneficial for the three rear rotors, especially the left and right ones. It can decrease the upmost rotor performance. The phenomena are consistent with that in Ref. 3 predicted by CFD. It can therefore be concluded that the aerodynamic interference is beneficial for the diamond configuration quadrotor aircraft.

IV. Conclusions

This work presented an analytical expression for the aerodynamic interference between any two lifting rotors of multi-rotor aircraft. A general expression for the induced velocity considering the aerodynamic interference between any two rotors is given without any neglect of the effect of one rotor on another. The aerodynamic interference becomes stronger with smaller wake skew angle, and becomes weaker, as the distance between the two rotors becomes longer. The relative location of the rotor disks has significant influence on the aerodynamic interference. The strongest interaction appears, as a rotor is right downstream another one. At certain locations, for example, on the right or left of the rotor, the interference interaction can be beneficial. From the point of view of flight performance, the aerodynamic interference is overall beneficial for the diamond configuration quadrotor aircraft, and can lower the performance of square configuration quadrotor aircraft.

Acknowledgements

This work is supported from the National Natural Science Foundation of China (11972181), the Six Talent Peaks Project in Jiangsu Province (GDZB-013), and the Open Research Foundation of the Key Rotor Aerodynamics Laboratory (RAL20180201-1).

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