DOI: 10.5840/jphil201611338

THE ANTINOMY OF THE VARIABLE: A TARSKIAN RESOLUTION[*]

The theory of quantification and variable binding developed by Tarski is a fixed point for many debates in metaphysics, formal semantics, and philosophy of logic. However, recent critics—most forcefully, Kit Fine[1]—have posed an intriguing set of challenges to Tarski's account, which re-expose long sublimated anxieties about the variable from the infancy of analytic philosophy.

The problem is a version of a puzzle confronted by Russell, which Fine dubs the "antimony of the variable". This paradox arises from seemingly contradictory things that we wish to say about the variable. On the one hand, there are strong reasons to deny that 'x' and 'y' are synonymous, since they make different contributions when they jointly occur within a sentence. Consider, for instance, the sentence '$\exists x \exists y\ x \leq y$'. One cannot replace the second occurrence of 'x' with 'y' (yielding '$\exists x \exists y\ y \leq y$') without change of meaning. On the other hand, there is a strong temptation to say that distinct variables 'x' and 'y' are synonymous, since sentences differing by the total, proper substitution of 'x' for 'y' always agree in meaning. For instance, '$\forall x\ x \leq x$' and '$\forall y\ y \leq y$' are synonymous in the strongest possible sense. As Fine says, they are mere "notational variants". We suggest that it is best to construe this very strong synonymy as an identity in *structured* meanings: the sentences and their corresponding parts are synonymous all the way down. This suggests that the variables occurring in corresponding positions in these formulas are also synonymous.

One of the innovations of Tarski's semantics is that a variable refers to or designates an object only relative to a *sequence*.[2] One might hope that this goes some way towards resolving the antinomy, since Tarski need not assign any sort

[*] The paper has benefited as a result of comments from Josh Dever, Dilip Ninan, Landon Rabern, Anders Schoubye, and an anonymous referee at this JOURNAL. Previous versions of this paper received useful input from audiences at the Early Analytic Group at the University of Stirling, the Institute of Philosophy in London, the Edinburgh Philosophy of Language Group, and the Formal Metaphysics Workshop at the University of St Andrews. Bryan Pickel acknowledges the help of partial funds from projects CSD2009–00056 and FFI2012-37658 (Spanish Government).

[1] Kit Fine, "The role of variables", this JOURNAL, C, 12 (December 2003): 605-631; Kit Fine, *Semantic Relationism*. (Blackwell Publishing, 2007).

[2] Alfred Tarski, "Der Wahrheitsbegriff in den Formalisierten Sprachen", *Studia Philosophica*, I (1935): 261–405, republished in translation (by J.H. Woodger) as "The concept of truth in formalized languages" in *Logic, Semantics, Metamathematics*. (Oxford, Clarendon Press, 1956): 152–278.

of *referent* to the variable. But this is not enough, since the antinomy concerns whether two variables are *synonymous*. As we formulate the antinomy, it concerns the variable's contribution to the structured meaning of a sentence that contains it. Even on Tarski's sequence-relative semantics, '*x*' and '*y*' may designate different individuals even relative to the same sequence. This suggests that their meanings are different. But this leaves Tarski unable to account for the felt sameness of meaning between distinct but corresponding variables in alphabetic variants.

These challenges would overturn seemingly settled doctrines about the relationship between language and the world. A dramatic reconceptualization of the role of variables in mathematical practice, in natural language semantics, *and even in first-order logic* would be called for. Fine suggests *"semantic relationism",* a radical departure from standard compositional semantics.

However, Tarski's semantics for variables has the resources to resolve the antinomy without abandoning standard compositional semantics. In a neglected passage, Tarski worried about how to determine the value of a variable relative to a sequence. He suggests that, in a given sentence, the first variable should be associated with the first position, the second variable with the second position, and so on. Using a bit of dynamic semantics, we develop this suggestion into a rigorous procedure—which we call *dynamic indexing*—associating each variable with a position in a sequence. The underlying idea is that the semantic contribution of a variable maps a *context* to a position in a sequence. On the semantics we offer, '*x*' and '*y*' will be associated with distinct functions from contexts into positions in sequences. Nonetheless, if '*x*' and '*y*' occur in corresponding positions in sentences that are alphabetic variants, then (in context) they will be correlated with the same position in a sequence. Thus, we offer a sense in which '*x*' and '*y*' have the same semantic role and a sense in which they don't, thereby resolving the antinomy.

I. THE ANTINOMY OF THE VARIABLE

Variables are central to the notation of mathematics and science. Some mathematical and scientific claims are framed using "free" variables. A mathematician might express the claim that an operator such as '+' is commutative using the "open" formula '$x + y = y + x$'. Free variables are of limited use in expressing generality, however, since one cannot express embedded general claims such as negated universal or multiply quantified statements.

For instance, sentence (1) could in principle be rewritten using only free variables. Sentence (2) requires more sophisticated symbolism.

(1) Every number is less than or equal to itself.
(2) Every number is less than or equal to some number.

For this reason, both sentence (1) and (at least one reading of) sentence (2) are regimented using "bound" variables, which fall under the scope of quantifiers such as 'for every' ('∀') and 'for some' ('∃').

(1*) $\forall x\ x \leq x$
(2*) $\exists y \forall x\ x \leq y$

Writing quantified sentences using variables resolves ambiguities and facilitates inference because it wears its compositional structure on its sleeve. In particular, the meaning of the complex expression in this notation is determined by the meanings of its syntactic constituents and their order of combination. That is, meaning is *compositional.* Compositionality helps explain why speakers can grasp the infinitely many sentences of a language. It also constrains the choice of semantic theories, making them more susceptible to empirical disconfirmation.

In contrast to the quantified sentences of formal languages, semanticists commonly derive the semantic features of a quantified sentence from *natural* language—such as (1)—by first regimenting it. Often, they posit a "deeper" level of representation, which captures the "logical form". For example, contemporary linguists provide a syntactic story whereby the quantifier "moves" out front and leaves behind a "trace". The trace is treated as a bound variable.[3] In this way, the syntactic structure—e.g. of (1*)—more directly tracks its semantic evaluation.

But what is the syntactic structure of sentences (1*) and (2*)? The standard answer since Tarski is as follows. In constructing (1*), one starts with the variable 'x' and the two-place predicate '$\leq$' to build the "open sentence" '$x \leq x$'.[4] One then prefixes the quantifier '$\forall x$' yielding '$\forall x\, x \leq x$'. Sentence (2*) is constructed similarly. We start with the variables 'x' and 'y' and the two-place predicate '$\leq$' to build the open sentence '$x \leq y$', then prefix '$\forall x$' to yield '$\forall x\, x \leq y$'. Finally, one attaches '$\exists y$' resulting in '$\exists y \forall x\, x \leq y$'. So, this standard account presupposes what we will call assumption (α): Variables are genuine syntactic constituents of quantified sentences.

Some approaches to the semantics of quantification dispense with assumption (α), and reject the view that variables have an independent semantic role.[5] We will address one of these arising from the Fregean tradition. But we will leave other approaches—such as combinatory logic—for future discussion.

Assuming that a variable is a genuine constituent of a sentence, it must have some meaning or what Fine calls a "semantic role" or "linguistic function".[6] It is the job of semantics to describe this meaning.

The antinomy of the variable concerns whether two variables, 'x' and 'y', agree in meaning. The difficulty is—as Fine (*ibid.*) puts it—"we wish to say contradictory things about their semantic role". The conflict arises because two variables

[3] See Irene Heim and Angelika Kratzer, *Semantics in Generative Grammar*, (Blackwell Publishers, 1998), 178–200.
[4] Tarski was the first to clearly argue that open sentences belong in the same grammatical category as closed sentence. His argument was that the same operators—negation, conjunction, and so on—could attach to both open and closed sentences, see Tarski, *ibid.*, 189-91, and Alfred Tarski, *Introduction to Logic*, (Oxford, Oxford University Press, 1941), 4-5.
[5] In natural language semantics the roles of quantification and variable-binding are sometime separated. The latter job is done by λ-binders, which attach to open sentences that contain variables. It follows that variables are still genuine constituents of a sentence.
[6] Fine 2007, *op. cit.*, p. 7.

occurring *in the same sentence* seem to behave differently, but occurring *in different sentences* their behavior is indistinguishable.[7]

Difference: When variables ‘x’ and ‘y’ jointly occur in a single sentence, they have distinct meanings.

Sameness: In sentences that differ in the total, proper substitution of ‘x’ for ‘y’, these variables have the same meanings.

In what follows, we offer arguments purporting to show that two variables must have these conflicting features, by making explicit the underlying theoretical motivations for ascribing each feature to variables.

I.I. WHY ‘□’ AND ‘□’ MUST NOT AGREE IN MEANING

The argument that ‘x’ and ‘y’ have different meaning is straightforward, since substituting one for the other may fail to preserve meaning. Fine appeals to open sentences containing free variables to make the argument:

> Suppose that we have two variables, say ‘x’ and ‘y’;… [W]hen we consider the semantic role of the variables in the same expression—such as ‘$x > y$’—then it seems…clear that their semantic role is different. Indeed, it is essential to the linguistic function of the expression as a whole that it contains two distinct variables, not two occurrences of the same variable, and presumably this is because the roles of the distinct variables are not the same.[8]

Fine’s crucial premise is that expressions differing only by the substitution of one occurrence of a variable for an occurrence of the other differ in meaning. In Fine’s example, the occurrence of ‘x’ in the open sentence ‘$x > y$’ cannot be substituted with ‘y’—yielding ‘$y > y$’—without change of meaning.

[7] Fine 2007, *op. cit.*, p. 9 does not rest content with this form of the paradox. He asks what it means for ‘x’ and ‘y’ to have different semantic roles in a context. He answers that the pairs of variables ‘x’, ‘y’ and ‘x’, ‘x’ make different contributions to whatever sentences contain them. Since Fine’s opponents don’t offer any semantic characterization of *pairs of expressions,* we will leave the antinomy as it stands.
[8] Fine 2003, *op. cit.*, p. 606.

The argument implicitly appeals to the principle of compositionality, that the semantic features of a whole are determined by the semantic features of their parts and their mode of combination. In particular, if ‘x’ and ‘y’ had exactly the same semantic features, then, replacing the first occurrence of ‘x’ with ‘y’ in ‘$x > y$’ should yield a sentence with the same meaning. But it does not.

While intuitive, some may object to Fine’s appeal to the meanings of *open* sentences. One might doubt whether one has direct access to whether the open sentences ‘$x > y$’ and ‘$y > y$’ agree in meaning. The case could therefore be strengthened if it can be established that replacing ‘x’ with ‘y’ in a closed sentence does not preserve meaning. This can be directly verified by considering the fact that ‘$\exists y \forall x\, y \leq y$’ and ‘$\exists y \forall x\, y \leq x$’ differ in meaning—indeed they may differ in truth-value, though they differ only by replacing an occurrence of ‘x’ with ‘y’. It follows from the principle of compositionality that the variables ‘x’ and ‘y’ differ in meaning.

I.II. WHY ‘□’ AND ‘□’ MUST AGREE IN MEANING

Fine’s argument that ‘x’ and ‘y’ must agree in meaning is elusive:

> Suppose that we have two variables, say ‘x’ and ‘y’; …[W]hen we consider their semantic role in two distinct expressions—such as ‘$x > 0$’ and ‘$y > 0$’, we wish to say that their semantic role is the same. Indeed, this would appear to be as clear a case as any of a mere “conventional” or “notational” difference; the difference is merely in the choice of the symbol and not in its linguistic function.[9]

Undoubtedly, Fine is right that the choice between ‘$x > 0$’ and ‘$y > 0$’ is purely notational, and thus their “meanings” must have something in common. But Fine doesn’t elucidate the theoretical importance of this commonality. The claim that ‘x’ and ‘y’ agree in meaning (in some important sense) is crucial to Fine’s whole project. Without it, there simply is no antinomy. So it is desirable to find some more robust theoretical motivation for the claim.

Such a motivation can be found by appealing to a strong notion of *synonymy* recognized within the formal semantics tradition. This tradition aims at

[9] Fine 2003, *op. cit.*, p. 606.

specifying the truth conditions of a sentence in terms of the *compositional semantic values* of its constituents.[10] The truth conditions of a sentence will be specified as the set of *points of evaluation* (e.g. sets of possible worlds) in which the sentence is true. The problem is that, even within the formal semantics tradition, truth conditions are recognized as rather too *coarse-grained* to serve as the *meanings* of sentences. For instance, assuming all mathematical theorems are necessary, any two theorems (such as 'there are infinitely many prime numbers' and 'two is prime') are truth conditionally equivalent. But any view counting these sentences as wholly synonymous is missing something.[11]

This problem is standardly addressed by identifying the meaning of a sentence not merely with its compositional semantic value, but also with the *procedure* by which the compositional semantic value was derived from the meanings of the sentence's ultimate constituents. Let us call this, the sentence's *structured meaning.* When the antinomy is construed in terms of structured meanings, it derives its force from the conjunction of assumption (α) which states that variables are genuine constituents of sentences which contain them with an additional assumption linking a sentence's syntactic constituents to the constituents of its structured meaning. We call this assumption (β).

(α) Variables are genuine syntactic constituents of quantified and open sentences of the regimented language.

(β) Each syntactic constituent of a sentence of a regimented language must correspond to a constituent of the structured meaning of that sentence.

Assumption (β) traces back to Carnap's strongest notion of synonymy, "intensional isomorphism", which requires that the parts of synonymous sentences agree in meaning.[12]

---

[10] David Lewis, "General semantics", *Synthese* XXII, 1 (1970): 18–67. Richard Montague, "The proper treatment of quantification in ordinary English", in Montague and Richmond Thomason (editor), *Formal Philosophy,* (New Haven: Yale University Press, 1974).

[11] If one thinks that mathematical theorems are contingent—for instance, because one thinks that the existence of mathematical entities is contingent—then the example can easily be altered to our purposes. See the discussion in Heim and Kratzer, *op. cit.*, §12.4.

[12] Rudolph Carnap, *Meaning and Necessity*, (Chicago: University of Chicago Press, 1947). Carnap cites C. I. Lewis as a predecessor, but says that their views developed independently. See C.I.

For similar reasons, Stalnaker speaks of the meaning of a sentence as "the recipe for determining its truth-conditions as a function of the meanings of its components and the compositional rules".[13] As Lewis says,

> Differences in intension, we may say, give us *coarse* differences in meaning. For *fine* differences in meaning we must look to the analysis of a compound into constituents and to the intensions of the several constituents. … For still finer differences in meaning we must look in turn to the intensions of constituents of constituents, and so on. Only when we come to non-compound, lexical constituents can we take sameness of intension as a sufficient condition of synonymy.[14]

In addition to providing a grip on the pre-theoretic notion of *synonymy,* structured meanings have been put to work in developing an account of the *information value* or *belief content* of a sentence, which can solve puzzles associated with propositional attitude ascriptions. Thus, Carnap (*ibid*., §13), followed by many others, argued that belief ascriptions are neither extensional nor intensional since they do not even permit the substitution of intensionally equivalent sentences. Although 'there are infinitely many primes' has the same intension as '2+2=4', the belief ascriptions 'Sam believes that there are infinitely many primes' and 'Sam believes that 2+2=4' may differ in truth-value.

Fine's claim that 'x' and 'y' agree in meaning can be bolstered by framing it in terms of structured meanings*.* One corollary of assumption (β) is that if two sentences are synonymous in the relevant sense, then they must have corresponding constituents which agree in meaning. That is, if ϕ and ψ are synonymous (i.e. have the same structured meaning), then each component α of ϕ must agree in meaning—in the relevant sense—with its counterpart β of ψ.

Lewis, "The Modes of Meaning", *Philosophy and Phenomenological Research*, IV, 2, (1943): 236-250.

[13] Robert Stalnaker, "Impossibilities", p. 65, in *Ways a World Might Be: Metaphysical and Anti Metaphysical Essays*, (Oxford: Claredon Press, 2003): 55-68.

[14] Lewis, *ibid*., p. 31.

The problem now is that formulae that result from the total, proper substitution of one variable 'x' for another 'y' are *synonymous* in the strongest sense.[15] Consider two regimentations of sentence (1). We regimented this sentence as '$\forall x\ x \leq x$', but '$\forall y\ y\ \leq\ y$' is an equally good regimentation of (1). Indeed, one would read both of these aloud as 'every number is less than or equal to itself'. The fact that alphabetic variants such as '$\forall x\ x\ \leq\ x$' and '$\forall y\ y\ \leq\ y$' regiment the same natural language sentence suggests that they are synonymous.[16]

Some semanticists such as Pauline Jacobson have been so gripped by the synonymy of alphabetic variants, that they have abandoned the use of variables in natural language semantics as somehow a "cheat":

> If the variable names such as x_1 and x_2 (or, 1 and 2) are actual model-theoretic objects, then they are of course distinct objects. And yet, when they find themselves in forms which are alphabetic variants, they never make a different semantic contribution… In other words, there is an obvious sense in which x_1 and x_2 really are not different semantic objects—unlike other distinct model-theoretic objects.[17]

To avoid this antinomy, Jacobson herself offers a radical semantic proposal for avoiding variables in her semantics. [18]

---

[15] Alonzo Church, *Introduction to Mathematical Logic*, (Princeton: Princeton University Press, 1956), p. 40 fn. 96 says: "…[an expression] which contains a particular variable as a bound variable is unaltered in meaning by alphabetic change of the variable, at all of its bound occurrences, to a new variable (not previously occurring) which has the same range." See Donald Kalish and Richard Montague, *Logic: Techniques of Formal Reasoning*, (New York: Harcourt, Brace & World, Inc., 1964), Chapter 7 for an explicit definition of uniform substitution and alphabetic variants.

[16] Related arguments occur in Kalish and Montague, *ibid*., p. 165. If, like Quine one thinks that regimentation need not preserve meaning, then one will be inclined to reject this argument (W.V. Quine, *Word and Object*, Cambridge: MIT Press, 1960). See also, e.g. Church, *ibid*., p. 20 and Lewis, *op. cit*., p. 45-46.

[17] Pauline Jacobson, "Towards a variable-free semantics", *Linguistics and Philosophy*, XXII, 2 (1999): 117–185, p. 127.

[18] Related skepticism about variables has also led logicians such as Haskell Curry and Robert Feys, *Combinatory Logic* (Amsterdam: North-Holland Publishing Company, 1958/1968) to develop alternative variable-free systems. For a different approach see the notation developed in N.G. De Bruijn, "Lambda calculus notation with nameless dummies, a tool for automatic formula manipulation, with application to the Church-Rosser theorem," *Indagationes Mathematicae* (Proceedings), LXXV, 5 (1972): 381–392. Simply getting rid of variables in the syntax doesn't automatically get one off the hook with respect to the general problems concerning the structured meaning of quantified sentences. Given that such systems trade variables in the syntax for an array of combinators in the syntax one might worry that analogous problems will re-emerge.

Another manifestation of this synonymy arises because alphabetic variants seem to express exactly the same belief content. This is reflected in the fact that alphabetic variants are intersubstitutable even in propositional attitude reports.[19] Consider sentence (3).

(3) John believes that every number is less than or equal to itself.

(3) is equally well regimented by 'John believes that $\forall x\, x \leq x$' and 'John believes that $\forall y\, y \leq y$'. But, if so, then the structured content expressed by the regimentations of the embedded sentences in context must agree in meaning. Thus, sentences differing only by the total, proper substitution of variables look synonymous in the strongest possible sense. They are mere notational variants, if any sentences are at all.[20] Thus, their corresponding parts must agree in meaning: 'x' and 'y' must agree in meaning.

I.III. THE CHALLENGE OF THE ANTINOMY

We have uncovered that 'x' and 'y' must—in some sense—agree in meaning, but also that they must—in some sense—disagree in meaning. Both Carnap (*ibid.,* 58-59) and Lewis (*ibid*., 45-46) recognized this tension. They wanted to guarantee that alphabetic variants have the same structured meanings. Yet, on their explicit semantics, distinct variables have distinct semantic values due to the constraints of compositionality, giving rise to what Lewis called a "spurious ambiguity" between alphabetic variants such as '$\forall x\, x \leq x$' and '$\forall y\, y \leq y$'.

[19] The total, proper substitution of bound variables preserves sense even on Church's Alternative (0)—his strictest criterion of synonymy, which is meant to model attitude ascriptions. See Alonzo Church, "A Formulation of the Logic of Sense and Denotation", in Paul Henle, *Structure, Method, and Meaning: Essays in Honor of Henry M. Sheffer*. (New York: Liberal Arts Press, 1951): 3-24. See also page 557 of David Kaplan, "Demonstratives", in J. Almog, J. Perry and H. Wettstein (eds), *Themes from Kaplan,* (Oxford University Press, 1989): 481–563.

[20] According to the account in Max Cresswell, *Structured Meanings*, (Cambridge: MIT Press, 1985), a sentence ϕ of arbitrary complexity can be embedded in the that-clause of a belief report. The 'that' operator is polysemous and can operate either on ϕ or on the separate parts of ϕ taken in sequence. In the latter case, the object of belief will be the *structured meaning* of ϕ, which is identified with the ordered n-tuple of the intensions of ϕ's constituents. Cresswell (*ibid*, p. 101) does not actually specify *intensions* for variables, but only intensions relative to an assignment. As a result, his procedure either fails to deliver structured meanings for '$\exists\lambda x F x$' and '$\exists\lambda y F y$' or—if the semantics for variables is naturally extended to provide them intensions—will assign these sentences different structured meanings, delivering the unwelcome result that they may embed differently under belief ascription.

Both Carnap and Lewis introduced artificial, *ad hoc* maneuvers to relieve this tension and thereby identify the structured meanings of alphabetic variants, even though the corresponding constituents had different semantic values. This type of *ad hoc* trick undermines the motivation for appealing to structured meanings in the first place.

Other proponents of structured contents have been at pains to avoid the antinomy. The basic idea behind the structured contents approach is that a sentence has a content which “encodes, or is composed out of, the meanings of [the sentence’s] constituents.”[21] One sort of solution adopted by prominent proponents of structured contents approaches—including both Nathan Salmon and Scott Soames—involves outright denying that the structured meaning of a quantified sentence reflects the meanings of its ultimate components, thereby denying assumption (β).[22] In particular, according to Soames and Salmon the structured proposition expressed by a quantified sentence such as ‘$\exists xFx$’ reflects only the meanings of the quantifier ‘∃’ and the predicate abstract, which we will write as ‘$\hat{x}Fx$’. If we use brackets ‘[…]’ to denote the contribution an expression makes to the structured meaning of a sentence that contains it, then we could display the structured meaning of ‘$\exists xFx$’ as follows:

$$[\exists] \quad [\hat{x}Fx]$$

The semantic contribution of the predicate abstract ‘$\hat{x}Fx$’ is not broken down any further. This has the result that the sentence expresses the same structured meaning as its alphabetic variants such as ‘$\exists yFy$’:

$$[\exists] \quad [\hat{y}Fy]$$

[21] Scott Soames, *Philosophy of Language*, (Princeton: Princeton University Press, 2010), p. 112.
[22] See Nathan Salmon, *Frege’s Puzzle*, (Atascadero: Ridgeview Publishing, 1986), pp. 145-6, theses 27 and 28; and Scott Soames, “Direct reference, propositional attitudes, and semantic content”, p. 224, thesis 28*d* in Nathan Salmon and Scott Soames, *Propositions and Attitudes*, (Oxford: Oxford University Press, 1988).

As a result, this approach might be used to vindicate a broadly Tarskian semantics.

Nonetheless, one might offer three interrelated complaints. First, the approach requires one to intensionalize the contribution of the predicate abstracts so that they reflect the semantic values of some, but not all, of their constituent expressions.[23] This intensionality will be necessary to distinguish the structured content of '$\exists xFx$' from that of, e.g., '$\exists x(Fx \vee x = x)$'.

This leads to the second complaint. Namely, the approach undermines some of the original motivations for structured contents. The structured content of, say, '$\exists x\, Loves(Desdemona, x)$' will not contain the structured content of 'Desdemona', but will *encode* it only in an indirect way.[24] The third complaint is that the solution seemingly entails that the meanings of some constituent expressions are *not even* encoded in the structured contents of sentences that express them. In particular, if the meaning of 'x' is encoded in the structured content of '$\exists xFx$' and the meaning of 'y' is encoded in the meaning of '$\exists yFy$', then the meanings of 'x' and of 'y' would need to be the same. But it is hard to see how this is compatible with the fact that the meaning of 'x' differs from the meaning of 'y' in '$\exists y \forall x\, y \leq x$'. If we take the encoding talk seriously, then the antinomy seems to recur at the level of what is encoded rather than contained in the structured content of a sentence.

More recently, Jeff King's account of structured contents has attempted to do justice to the idea that all of the meaningful syntactic constituents of a sentence are encoded in the structured contents they express. As a result, he has oscillated in trying to adequately capture the distinct contributions of distinct variables in a sentence and at the same time ensure that alphabetic variants have the same

[23] Fine 2007, *op. cit.,* pp. 16-17.

[24] An advocate of structured propositions might worry that this opens the door to a generalization, which accounts for belief content and synonymy in terms of hyperintensional, but unstructured meanings as in Church 1951, *op. cit.*, or George Bealer, *Quality and Concept*, (Oxford: Clarendon Press, 1982).

structured contents. This oscillation perfectly reveals the tensions created by the antinomy of the variable.

In his early work, King suggested that distinct variables must make distinct contributions to the structured contents of sentences that contain them.[25] On his official implementation, the variables contribute *themselves* to the structured meanings, though King allows that the variables may be replaced by suitable proxy objects "so long as each variable contributes a different one".[26] This semantics has the result that alphabetic variants, since they may contain distinct variables, express different structured contents.[27]

In a recent book, King abandons the view that variables contribute anything at all to the structured contents of sentences that contain them.[28] Each variable contributes a mere *gap* or an "empty argument position". But this makes it difficult to see how '$\exists y \forall x\, y \leq y$' and '$\exists y \forall x\, y \leq x$' can have different structured meanings. In particular, the semantic values of the simple constituents of these sentences make the same contributions to their structured meanings.[29]

On our way of viewing the antinomy, it challenges us to articulate a sense in which alphabetic variants are synonymous—have the same structural

[25] Jeffrey King, "Structured propositions and complex predicates", *Noûs* XXIX, 4: (1995): 516-535 (see pp. 533-4, notes 5, 20, and 22); and Jeffrey King, "Structured propositions and sentence structure", *Journal of Philosophical Logic*, XXV (1996): 495-521 (see p. 498).
[26] King 1995, *op. cit*., note 22.
[27] The same problem will plague theories that appeal to "linguistic modes of presentation" such as the *Interpreted Logic Forms* of Richard Larson and Peter Ludlow, "Interpreted logical forms", *Synthese* XCV, 3 (1993): 305-355, since "[ILFs] include complete syntactic phrase-markers, including diacritics (e.g., variables and indices)" (*ibid*., p. 349, note 29).
[28] Jeffery King, *The Nature and Structure of Content*, (Oxford: Oxford University Press, 2007), see pp. 41-2, and pp. 218-222.
[29] King's 2007 view is best understood as embracing a kind of non-compositionality (analogous to that of Fine 2003, *op. cit.*) so that the contributions of non-terminal nodes to structured meanings of sentences that contain them are not determined by the contributions of their simpler components (see footnote 58 below). Juhani Yli-Vakkuri, "Propositions and compositionality", *Philosophical Perspectives* XXVII (2013): 237-274, charges that King's semantics is non-compositional, but in a different sense (see §3.3.2). Yli-Vakkuri argues that King's semantics violates the naïve view that the structured content of a sentence in context is its compositional semantic value, so that substituting two sentences with the same structured meanings results in a sentence with the same structured meaning. We reject this naïve assumption and it plays no part in our argument.

meanings—despite the fact that the variables 'x' and 'y' exhibit different semantic behavior, and thus have different semantic values.

II. ARE THERE VARIABLES?

Problems with variables and "the symbolism of generality" have a long history in analytic philosophy. For instance, in *Principia*, Whitehead and Russell observed that distinct variables make different contributions within the context of a single larger sentence.[30] Thus, 'x is hurt' and 'y is hurt' make distinct contributions in 'x is hurt and y is hurt'. On the other hand, the content of these open sentences express the same "propositional function".

> Accordingly though "x is hurt" and "y is hurt" occurring *in the same context* can be distinguished, "$\hat{x}$ is hurt" and "$\hat{y}$ is hurt" convey no distinction of meaning at all.[31]

Similarly, they hold that quantified sentences that are alphabetic variants express the same proposition, or structured content.

> The symbol '$(x).\,\phi x$' denotes one definite proposition, and there is no distinction in meaning between '$(x).\,\phi x$' and '$(y).\,\phi y$' when they occur in the same context.[32]

Thus, there is an important sense in which 'x' and 'y' agree in meaning, though not when occurring in the same context.[33]

Despite its august roots, we suspect that many philosophers will feel little patience with the antinomy because it rests on assumption (α), that variables are genuine syntactic constituents of sentences. Indeed, there is an important semantic tradition originating with Frege rejecting assumption (α). Frege's

[30] Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, 3 vols., (Cambridge: Cambridge University Press, 1910, 1912, 1913). Russell assumes that variables are "fundamental", which seems to validate assumption (α), in Bertrand Russell, "On denoting", Mind, XIV, 56 (1905): 479-493, (p. 480). For further early discussion of the antinomy see Bertrand Russell, *The Principles of Mathematics*. (Cambridge: At the University Press, 1903): §86-93; and Gottlob Frege "What is a Function?" (1904), published in translation in Peter Geach and Max Black, *Translations from the Philosophical Writings of Gottlob Frege*, (Oxford: Basil Blackwell, 1960): 107-116.

[31] Whitehead and Russell, *ibid*., p. 15.

[32] Ibid., p. 16.

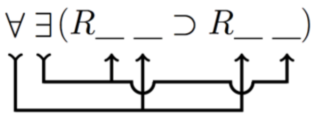
[33] Wittgenstein also grappled with the antimony in the *Tractatus*. See his comments in 4.04—see especially 4.0411—regarding the picture theory and the problem of "mathematical multiplicity". Ludwig Wittgenstein, *Tractatus Logico-Philosophicus*, Pears, D. and McGuinness, B. (trans.), (London: Routledge, 1961).

suspicions about variables issued from considerations resembling the antinomy of the variable. Frege reasons that *if variables are genuine constituents of sentences*, then two variables must have the same reference (and perhaps express the same *sense*).[34]

> We cannot specify what properties *x* has and what differing properties *y* has. If we associate anything with these letters at all, it is the same vague image for both of them.[35]

Since identifying the referents of 'x' and 'y' leads to violations of the principle of compositionality, Frege rejects assumption (α).

Frege's alternative is that bound variables are ultimately typographic parts of the dispersed quantifier sign. Frege has been followed by contemporary logicians such as Kaplan who says:

> Variables serve only to mark places for distant quantifiers to control and to serve as a channel for the placement of values. We need no variables. We could permit gaping formulas (as Frege would have had it) and use wiring diagrams to link the quantifier to its gaps and to channel in values.
>
> 
>
>
> Variables are simply a way of giving the distant quantifiers wireless remote control over the gaps.[36]

On this view, variables are typographic parts of the quantifier sign serving only to link the quantifier to the "open" spaces in predicates.

II.I. THE FREGEAN SEMANTICS

Frege thought of first-level predicates as incomplete, or as containing "gaps" which must be saturated by proper names.[37] Quantifiers, in turn, are

[34] In §28 of the *Grundgesetze* Frege announces as a "leading principle" that every well-formed "name" of his language is to denote something. Gottlob Frege*, The Basic Laws of Arithmetic: Exposition of the System*, Montgomery Furth (trans.), (University of California Press, Berkeley, CA, 1893/1967).

[35] Frege 1904, *op. cit.*, p. 109.

[36] David Kaplan, "Opacity", p. 244, in L. E. Hahn and P. A. Schilpp (eds)., *The Philosophy of W. V. Quine* (La Salle, Ill.: Open Court, 1986): 229-288. Cf. W.V. Quine, *Mathematical Logic, Revised Edition*, (Harvard University Press, 1940/1981): 69-70.

unsaturated, but at a higher level. Their gaps must be saturated by monadic first-level predicates. This renders variables mere typographic parts of the quantifier sign. However, it would be hasty to infer from this that Fregean approaches are immunized against the antinomy of the variable. Russell shows in an appendix to *The Principles of Mathematics* that there is a syntactic variant of the antinomy of the variable that afflicts even Fregean approaches. Because of the prominence of Fregean approaches in the literature, we will briefly rehearse the Fregean view of quantification and Russell's objection.[38]

On Frege's view, predicates result from "removing" occurrences of a name from a sentence. This is the source of their "gaps". For instance, beginning with the sentence '$7 \leq 7$', one can remove the first, the second, or both occurrences of '7' to yield the predicates '( ) ≤ 7', '7 ≤ ( )', and '( ) ≤ ( )', respectively. In the third predicate the gaps must be seen as being filled by the same argument.

Frege's universal quantifier is a second-level predicate of monadic first-level predicates. It includes all occurrences of the "variable" that it binds. Updating for notation, he might write the universal quantifier as '$\forall x \ldots x \ldots$', where '…' can be completed only by the name of a monadic predicate.[39] Quantified formulae such as '$\forall x\ x \leq 7$', '$\forall x\ 7 \leq x$', and '$\forall x\ x \leq x$' result from saturating the quantifier sign '∀x…x…' with the monadic predicates such as '( ) ≤ 7', '7 ≤ ( )', and '( ) ≤ ( )', respectively.

---

[37] Our discussion will focus only on the special case of predicates rather than function names in general.
[38] Fine 2007, *op. cit.*, pp. 16-18 offers his own objections to the Fregean account. The first follows Resnik (Michael Resnik, "Frege's proof of referentiality", in Haaparanta, L., and J. Hintikka (eds.), *Frege Synthesized*, (Boston: Reidel Publishing Company. 1986): 177–195) in arguing that Frege's semantics must be intensional, even at the level of reference. The second charges that Fregean theories of quantification entail that quantified sentences exhibit an unwelcome dependence on their instances.
[39] Frege 1893, *op. cit.*, §8 is explicit that the mark corresponding to the bound variable cannot occur except when prefixed by a quantifier and that a quantifier must attach to an expression containing a mark corresponding to a bound variable on the standard syntax. Although Frege rejects assumption (α) and prefers to avoid talk of "variables", he still has to typographically differentiate various occurrences of the quantifier sign, e.g. '∀x…x…' versus '∀y…y…'. Rule 2 mandates that in forming an expression of generality one must choose a new German letter: Frege remarks, "one German letter is in general as good as any other, with the restriction, however, that the distinctness of these letters can be essential" (*ibid.*). (In Frege's notation German letters adorn the quantifier sign, the concavity, and link the quantifier to the "open" spaces in the relevant predicates.)

We now have enough of a sketch of the Fregean semantics for quantification on the table in order to see why it too is subject to a variant antinomy of the variable. In particular, recall the syntactic derivation of (1*) '$\forall x\ x \leq x$' on the Fregean approach. An expression with two gaps, namely '( ) ≤ ( )', which is a *dyadic* predicate.[40] A single name, such as '7' may saturate both these gaps, resulting in a sentence, '7 ≤ 7'. Then both occurrences of this name may be removed from the sentence to yield a *monadic* predicate '( ) ≤ ( )', which is inserted as an argument into the quantifier '$\forall x \ldots x \ldots$', which takes only monadic predicates.

The problem is immediately apparent. Nothing in the expressions distinguishes the *dyadic* predicate '( ) ≤ ( )' from the *monadic* predicate '( ) ≤ ( )', which results from removing two occurrences of '7' from '7 ≤ 7'. This is the source of Russell's objection, which he frames in terms of function-names.

> Frege wishes to have the empty places where the argument is to be inserted indicated in some way; thus he says that in $2x^3 + x$ the function is $2(\ )^3 + (\ )$. But here his requirement that the two empty places are to be filled by the same letter cannot be indicated: there is no way of distinguishing what we mean from the function involved in $2x^3 + y$.[41]

The worry is that if Frege were to introduce marks capable of typographically distinguishing between these predicates, then that mark would need its own semantic significance, which in this context means designation.[42]

The problem with conceiving of predicates as expressions with gaps is that nothing distinguishes between an expression with one gap and one with two gaps. On Frege's view, gaps are to be conceived of as *omissions* of names from sentences. In the dyadic predicate '( ) ≤ ( )', the two gaps must be capable of

---

[40] This dyadic predicate is required to form sentences such as '7≤12'. The discussion in this section is modeled on Frege's 1893, *op. cit.*, §29 syntactic derivation of '$\forall x\ x = x$'.

[41] Russell 1903, op. cit., §482.

[42] Frege 1893, *op. cit.*, §1 introduces the Greek letters 'ξ' and 'ζ' to mark the argument places of functions. However, he is clear that these are not part of the official symbolism, but occur only in "elucidations". See especially *ibid.*, footnote 10.

being saturated by different proper names. Thus they must have different semantic import. Yet, there is literally nothing corresponding to either gap. There is no sense to be made of the gaps being the same or different. Both gaps are *merely* gaps, there remains no constituent capable of delivering the requisite difference in semantic import. Thus, the antinomy of the variable has been *syntacticized.*

Of course, we don't take these considerations alone to have refuted the Fregean approach.[43] But they do provide sufficient reason to take variables seriously as linguistic units. Indeed, contemporary semanticists, though they take inspiration from Frege, do not follow him in rejecting assumption (α).[44]

III. TARSKI AND THE ANTINOMY

There is good reason to admit variables as constituents of quantified and open sentences. As Tarski showed, quantifiers operate on sentences just as do conjunction and negation. Importantly, they can attach to formulae with arbitrary numbers of free-variables. For this reason, Tarski built a syntax in which variables occur in the same positions as proper names.

III.I. TARSKI'S SEMANTICS

The language he considered includes a set of variables and *n*-ary predicates.[45]

Variables: $x, y, z, \ldots$

Predicates: $F^n, G^n, H^n, \ldots$

Variables combine with predicates to form open sentences. These can be combined with further operators to form more complex sentences.

[43] For further criticism of the Fregean syntax, see Bryan Pickel, "Syntax in Basic Laws §§29-32", *Notre Dame Journal of Formal Logic*, LI, 2 (2010): 253-277.

[44] See, e.g., Montague, *op. cit.,* (p. 250, category $\mathbf{B}_\mathrm{T}$ and p. 258, clause 2) and Heim and Kratzer, *op. cit.*, §5.5.5.

[45] An infinite stock of variables and predicates can be specified by priming: x', x'', …, etc. After this initial presentation, we will allow context to determine the adicity of a predicate rather than by explicit indexing.

Formation rules:

- If π is an n-ary predicate and $\alpha_1, \ldots, \alpha_n$ are variables, then $\pi\alpha_1 \ldots \alpha_n$ is a formula.

- If ϕ and ψ are formulae and α is a variable, then $\phi \wedge \psi$, $\neg\phi$, $\forall\alpha\phi$, and $\exists\alpha\phi$ are formulae.

In contrast to Frege's semantics, quantified sentences result from embedding an open sentence under a quantifier just as they appear to. Tarski's semantics thereby validates assumption (α) in the above argument for the antinomy: variables are genuine constituents of sentences that contain them, occurring in exactly the same positions as proper names.

Tarski's semantics rests on two related insights. One is that only some expressions receive absolute interpretations (relative to a model), while others require something additional, a sequence of individuals, to be interpreted.[46] The other insight is that truth is displaced as the central notion of semantic theory in favor of *satisfaction* by a sequence, represented by the function $[\![\ldots]\!]^{\sigma}$.[47]

To explain the notion of satisfaction Tarski (1935, *op. cit.*) begins with the notion of satisfaction by an object. An open sentence with one variable 'x' may be true or false relative to different assignments to 'x'. Thus, '$x \leq x$' may be true or false relative to each number, depending on whether the number is less than or equal to itself.

[46] Since this paper only concerns truth and not logical consequence we could in principle provide a fully absolute interpretation that does not relativize to a model. The semantics of Tarski 1935, *op. cit.*, is absolute in this way, but in later work such as Tarski and Vaught's 1956 paper the semantics is model-relative in the way we outline below. See Alfred Tarski and Robert Vaught, "Arithmetical extensions of relational systems", *Compositio mathematica*, XIII, (1956): 81-102.

[47] One often hears the remark that Tarski's semantics for first-order logic—in particular the treatment of variable-binding operators—isn't compositional (see, e.g., Scott Soames, "True at," *Analysis* 71, 1 (2011): 124–133). Apparently, Tarksi himself made this remark to Barbara Partee (see Wilfrid Hodges, "Tarski's truth definitions," *The Stanford Encyclopedia of Philosophy* (Spring Edition), Edward N. Zalta (ed.), 2013: § 2.1). But the semantics can easily be made compositional, if the semantic value of a variable is a function from sequences to individuals and the semantic value of a formula is a function from sequences to truth-values. See Brian Rabern, "Monsters in Kaplan's logic of demonstratives", *Philosophical Studies*, CLXIV, 2 (2013): 393-404; and Theo Janssen, "Compositionality", in J. van Benthem and A. ter Meulen (eds.), *Handbook of Logic and Language*, (Amsterdam: Elsevier, 1997): § 2.4.

Satisfaction becomes more complicated for formulae with multiple variables. Consider '$x \leq y$'. An assignment of 3 to 'x' and 7 to 'y' satisfies this formula, but the converse assignment does not satisfy this formula. In general, a formula may have an indefinite number of free variables. If a formula contains *n* free variables, one must speak of the formula as satisfied or not by n-ary sequences of objects.

Further generalizing, Tarski (*ibid.,* p. 191) speaks of an "enumeration" of all the variables of the language $\langle \alpha_1, \dots, \alpha_n, \dots \rangle$ .The variables in this enumeration may be jointly assigned to different sequences of objects. Let the set of these sequences of objects be as follows.

$$‡ \langle \alpha_1, \dots, \alpha_n, \dots \rangle ‡ = \{\langle d_1, \dots, d_n, \dots \rangle : d_1, \dots, d_n \in D \ and \ \alpha_k = \alpha_j \rightarrow d_k = d_j \}$$

This presentation slightly differs from Tarski's since we allow a variable to occur multiple times in an enumeration. The objects in the corresponding positions of the sequences that serve as values for the enumeration, however, must be identical.

One can then define the satisfaction of an atomic open sentence such as '$x \leq y$' by a sequence $\sigma = \langle 3, 7, 3, 9, \dots \rangle$ in terms of whether the entities in the sequence (in positions corresponding to the variables in the enumeration) are in the extension of the predicate '$\leq$'. The recursive semantic clauses can be specified relative to a model $\mathcal{M} = \langle D, I \rangle$, where D is a domain of individuals and I is an interpretation function (which maps an n-ary predicate to a set of n-tuples drawn from *D)* as follows:

Variables: If α is a variable, then

- $[\![\alpha]\!]^\sigma = \sigma(\alpha)$

Sentences: If π is an n-ary predicate and $\alpha_1, \dots \alpha_n$ are variables, then

- $[\![\pi\alpha_1 \dots \alpha_n]\!]^\sigma = 1 \ \ iff \langle [\![\alpha_1]\!]^\sigma, \dots, [\![\alpha_n]\!]^\sigma \rangle \in I(\pi)$

If ϕ and ψ are formulae and α is a variable, then

- $[\![\phi \wedge \psi]\!]^\sigma = 1 \ \ iff [\![\phi]\!]^\sigma = 1$ and $[\![\psi]\!]^\sigma = 1$

- $[\![\neg\phi]\!]^{\sigma} = 1$ *iff* $[\![\phi]\!]^{\sigma} \neq 1$
- $[\![\forall\alpha\phi]\!]^{\sigma} = 1$ *iff* for every $d \in D$, $[\![\phi]\!]^{\sigma[\alpha/d]} = 1$
- $[\![\exists\alpha\phi]\!]^{\sigma} = 1$ *iff* for some $d \in D$, $[\![\phi]\!]^{\sigma[\alpha/d]} = 1$

For any sequence σ, variable α, and $d \in D$, let $\sigma(\alpha) = d$ *iff* $\sigma_j = d$ and α is the j^{th} position of the enumeration $\langle \alpha_1, \dots, \alpha_n \dots \rangle$. To define α-variant sequences let $\sigma[\alpha/d]$ be the sequence $\tau \in \ddagger \langle \alpha_1, \dots, \alpha_n, \dots \rangle \ddagger$ varying from σ at most such that $\tau(\alpha) = d$.

Some of Tarski's remarks might suggest that he avoids the antinomy because he holds that "variables do not possess any meaning by themselves" by which he means that, if variables had referents, then these referents would be "entities of such a kind we do not find in our world at all".[48] In particular, they do not refer to objects. Variables function in a more complicated way. They designate different individuals *relative to different* sequences. But, there is no *absolute* designation of the variable without supplementation by a sequence.

III.II. DOES TARSKI ESCAPE THE ANTINOMY?

Tarski has avoided positing *referents or designata* of the variables. But the antinomy concerns the "meaning" of variables more generally: do variables *agree in meaning*? In Fine's vocabulary, this is equivalent to asking whether they have the same semantic role. In our reconstruction, this is equivalent to asking whether alphabetic variants express the same structured contents. This question may be posed without supposing that a variable refers. Fine formulates an objection to Tarski's semantics, taking as a premise only that the semantic roles of '*x*' and '*y*' can be compared for same-ness or difference.

Yet, Tarski's semantics doesn't directly speak to the "roles" or "meanings" of '*x*' and '*y*'. It merely assigns values to these variables relative to sequences. That is, Tarski offers a *semantic theory* that assigns designata to variables relative to sequences. So although Tarski has avoided assigning absolute referents to

[48] Tarski 1941, *op. cit.*, p. 4. Specifically, Tarski says that a numerical variable would have to denote a variable number, which is neither positive, nor negative, nor zero.

variables, his account leaves open what they "mean". The crucial task then, is to extract a meaning of the variable from Tarski's semantic theory which fulfills the desiderata above: distinct variables in the same sentence contribute differently to the sentence's structured meaning but corresponding variables in alphabetic variants make the same contribution.

Fine sees only two options. The first option is that the *meaning* of a variable is the range of values assigned to it by various sequences. That is, the meaning of '*x*' is the class $\{d: \exists\sigma[\![x]\!]^{\sigma} = d\}$, the domain of the variable. Analogously, the meaning of '*y*' is the class $\{d: \exists\sigma[\![y]\!]^{\sigma} = d\}$. The domains of 'x' and 'y' are the same, thus, Tarski can secure a sense in which '*x*' and '*y*' agree in semantic role.

Yet, Fine rejects this account on the grounds that it doesn't account for the difference between '*x*' and '*y*'.[49] As we saw before, substituting an occurrence of '*x*' for an occurrence of '*y*' in a formula may result in a new formula with different satisfaction conditions. So merely assigning a domain to the variables does not capture their full semantic behavior.[50]

Put in terms of structured meanings, treating the semantic contribution of a variable as its domain would force us to identify the structured meanings of sentences that should remain distinct. This argument requires an assumption, which we will call *structure intrinsicalism*.

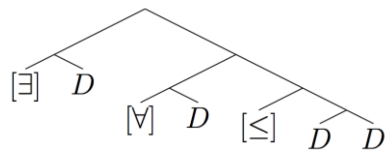
Structure Intrinsicalism: If two sentences have the same syntactic structure and the corresponding terminal constituents of these sentences all agree in meaning, then the sentences agree in meaning.

[49] Fine 2007, *op. cit.*, p. 10. Strictly speaking, Fine objects that Tarski doesn't secure a semantic difference between pairs of variables 'x', 'x' and 'y', 'x'. This charge is a bit hard to interpret within the Tarskian framework, since Tarski's theory offers no instruction for semantically evaluating pairs of variables.

[50] This echoes Church 1956, op. cit., pp. 9-10 who says: "Involved in the meaning of a variable...are the kinds of meaning which belong to a proper name of the range. But a variable must not be identified with a proper name of its range, since there are also differences of meaning between the two."

Structure intrinsicalism can be thought of as a manifestation of the principle of compositionality. Namely, the semantic values assigned to syntactically composite expressions are determined by the semantic values of their components and their mode of combination. So if there is a difference between structured meanings of two expressions, this must derive ultimately from a difference in structure or a difference in semantic values assigned to terminal nodes. We will revisit this assumption when we discuss Fine's own semantics.

Now consider the sentences '$\exists y \forall x\ x \leq\ y$' and '$\exists y \forall x\ x \leq\ x$'. These sentences are not synonymous and so should have different structured meanings. But if the relevant meaning assigned to a variable is its domain, then these two formulas are syntactically isomorphic and their corresponding terminal nodes all agree in meaning. As a result, the formulas themselves agree in structured meanings. If '$\exists$', '$\forall$', and '$\leq$' respectively contribute $[\exists], [\forall]$, and $[\leq]$ to the structured meanings of sentences that contain them, while variables 'x' and 'y' contribute their domain D, then the common structured meaning of '$\exists y \forall x\ x \leq\ y$' and '$\exists y \forall x\ x \leq\ x$' can be presented as follows:



This identity of meaning is obviously unwanted.

Fine offers another suggestion for extracting the meaning of a variable from Tarski's framework. He suggests—in essence—looking at the contribution that a variable makes to formulae that contain it. In particular, the variable contributes an object *relative to any input sequence.* So the meaning of the variable could be construed as just this procedure for taking an input sequence and yielding an object that is the "value" of the variable. This procedure determines a function taking any sequence to the value of the variable relative to that sequence: $\lambda\sigma\ \sigma(x)$. One could frame the semantic theory as assigning this function to a variable as its absolute or sequence-invariant meaning: $[\![x]\!] = \lambda\sigma\ \sigma(x)$. This function is what we might call its "semantic value", since it is sufficient to account for the full *compositional behavior* of a variable.

Fine however, objects that this construal leaves Tarski "unable to account for the fact that the semantic role of the variables x and y is the same in the cross-contextual case[.]"[51] Since distinct variables 'x' and 'y' have distinct semantic values, i.e. $\lambda\sigma\ \sigma(x) \neq \lambda\sigma\ \sigma(y)$, the structured meaning approach discussed above will assign different structured meanings to '$\forall xFx$' and '$\forall yFy$'. But this violates the *desideratum* that alphabetic variants should have the same structured meanings.

Nonetheless, one obvious thought is that the semantic role of a variable somehow combines both these aspects. The variable *possesses* a semantic value, and this distinguishes its meaning from other variables. Yet this semantic value *determines* a domain, which is common among many variables. Fine (*ibid.*) reasons that this in unsatisfactory: "What we have at best is a partial identity of semantic role, in that the range of the two variables is the same. But this is something that holds equally of the cross-contextual and intra-contextual cases." Even though the diverse semantic values of 'x' and 'y' determine that they have a common feature—their domain—it nevertheless remains that these values are *distinct.*

Framed in terms of structured meanings the problem is clear: Either the variables 'x' and 'y' will contribute something *different* to the structured meanings of sentences that contain them or they will not. If they contribute the same thing—say, their domains or gaps or what have you, then the account will over-generate synonyms. If the variables contribute something different—say, themselves or numbers or their semantic values, then the account will distinguish the structured meanings of alphabetic variants. Neither result is desirable. Thus, it seems that the Tarskian approach to variables and quantification cannot meet the challenge posed by the antinomy.

[51] Fine 2007, *op. cit.*, p. 11.

IV. FINE'S ANTI-INTRINSICALISM

Fine takes this to motivate a radical solution to the antinomy that involves distinguishing a variable's intrinsic from its extrinsic semantic features. On this view, the meaning of a variable in isolation cannot explain its semantic behavior. Fine's semantics is informally glossed for a small fragment of the language. We briefly develop its central aspects.

Fine's crucial move to avoid the antimony involves denying the plausible principle of *intrinsicalism*, the doctrine that the semantic behavior of a variable derives from its semantic role, understood as a semantic characterization of that variable in isolation from other expressions. Indeed, even Fine says this principle is "hard to dispute" (*ibid.*, p. 23). His motivation for abandoning it is solely to resolve the antimony.

IV.I. FINE'S SEMANTICS

Fine proposes to semantically evaluate a sentence or complex expression in terms of what values its parts may assume when taken in sequence. Thus, Fine would evaluate the sentence '$x + y = y + x$' for truth or falsity in terms of what values the expressions composing the sequence $\langle x, +, y, =, y, +, x\rangle$ may assume when taken in that sequence.

Fine's idea is that "distinct variables take values independently of one another and that identical variables take the same value".[52] In our example, the sequence $\langle x, +, y, =, y, +, x\rangle$ may assume the value $\langle 7, +, 3, =, 3, +, 7\rangle$, but not the value $\langle 7, +, 3, =, 5, +, 5\rangle$. Fine calls the set of values that a sequence of expressions can assume the *semantic connection* of that sequence, "[t]he aim of relational semantics...is to assign a semantic connection to each sequence of expressions".[53]

We will use '$\ddagger \cdots \ddagger$' to denote the function that takes a sequence of expressions to its semantic connection, the range of values that the constituent expressions are

[52] *Ibid.*, p. 27.
[53] *Ibid.*, p. 25.

capable of taking in that sequence. A sequence consisting of a single variable α is assigned to its domain, *D.*

1-Membered Sequences:

$\ddagger \langle \alpha \rangle \ddagger = \{d : d \in D\}$

The semantic connection on an n-tuple of variables is meant to generalize the notion of a domain for a variable; it is "the set of sequences of values that the [variables] are simultaneously capable of assuming".[54]

2-Membered Sequences:

$\ddagger \langle \alpha_1, \alpha_2 \rangle \ddagger = \{\langle d_1, d_2 \rangle : d_1, d_2 \in D \; and \; \alpha_1 = \alpha_2 \rightarrow d_1 = d_2 \}$

n-Membered Sequences:

$\ddagger \langle \alpha_1, \dots, \alpha_n \rangle \ddagger = \{\langle d_1, \dots, d_n \rangle : d_1, \dots, d_n \in D \; and \; \alpha_k = \alpha_j \rightarrow d_k = d_j \}$

Since variables are the only elements that are coordinated in semantic connections, it will suffice to focus on them.

Fine's semantics derives the truth conditions of a formula ϕ from the semantic connection on its expansion, which enumerates the primitive constituents of ϕ. So the truth conditions of ϕ will be specified as a function of the semantic connection on its expansion. It is worth noting that the recursive procedures are deeply non-compositional. This is partially because the sub-formulae will be evaluated relative to the semantic connection of the whole. But also, the procedure for evaluating a formula consisting of an operator and two sub-formulae such '$(\phi \wedge \psi)$' in terms of these components will not proceed by evaluating each sub-formula in isolation, but in terms of its consequences for the whole.[55]

[54] *Ibid*., p. 27.

[55] As a result, Fine's semantics requires a further twist in order to accommodate the semantics of variable binding: a coordination relation among variables. Though this aspect of Fine's semantics—the coordination scheme—has received the most attention, it ultimately plays little role in the resolution of the antinomy (see Bryan Pickel and Brian Rabern, "Does semantic

We can now ask Fine the question he puts to Tarski: what account, within the framework of the Finean semantics, can be given of the semantic role of the variables? At first pass, Fine's semantics, like Tarski's, doesn't assign any semantic role to the variables. Fine's semantics doesn't traffic in "roles", but merely offers a list of sentences characterizing how a variable '*x*' contributes to the semantic connection of an arbitrary sequence: $\ddagger \langle x \rangle \ddagger = \{a: a \in D\}$; $\ddagger \langle x, y \rangle \ddagger = \{\langle a, b \rangle: a, b \in D\}$; $\ddagger \langle x, x \rangle \ddagger = \{\langle a, a \rangle: a \in D\}$; and so on. This leaves open the question of how to extract semantic roles from Fine's semantics.

Fine suggests that the semantic role of a variable, '*x*', is just the semantic connection on the sequence consisting of just that variable, $\ddagger \langle x \rangle \ddagger = \{a: a \in D\}$. Since $\ddagger \langle x \rangle \ddagger = \{a: a \in D\} = \ddagger \langle y \rangle \ddagger$, '*x*' and '*y*' have the same semantic roles.

But this does not explain the difference in behavior between '*x*' and '*y*', since from their respective semantic roles one cannot derive that $\ddagger \langle x, y \rangle \ddagger$ is $\{\langle a, b \rangle: a, b \in D\}$ while $\ddagger \langle x, x \rangle \ddagger$ is $\{< a, a >: a \in D\}$, which is different. Thus, the semantic connections on pairs of variables are not determined by the semantic connections on those variables taken in isolation.

Fine thinks this is as it should be. The semantic connections on $\langle x, y \rangle$ and $\langle x, x \rangle$ are *primitive* facts about these sequences. To insist on deriving claims about the semantic role of a pair of variables from the semantic roles of the variables themselves, Fine thinks, is to insist on the *intrinsicalist doctrine*, that there is "no difference in semantic relationship without a difference in semantic feature".[56] According to Fine, the behavior of a variable in a sequence of expressions is an *extrinsic* feature of that variable. And the extrinsic features of variables need not derive from the semantic roles of the variables themselves.

relationism solve Frege's puzzle?," (manuscript) for discussion of the coordination schema with an explicit formalization of Fine's relational semantics for first-order logic).

[56] Fine 2007, op. cit., p. 24.

Fine insists that "in asserting that the semantic role of x and y is the same, we only wish to assert that their intrinsic semantic features are the same" (ibid., p. 22). The difference in semantic roles between the pairs $\langle x, y\rangle$ and $\langle x, x\rangle$ are intrinsic features of these pairs, but extrinsic features of the variables themselves. Thus, Fine believes he has secured the difference in semantic role between these sequences. As he says,

> [The relational semantics] embodies a solution to the antinomy: the intrinsic semantic features of x and y (as given by the degenerate semantic connections on those variables) are the same, though the intrinsic semantic features of the pairs x, y and x, x (again, as given by the semantic connections on those pairs) are different.[57]

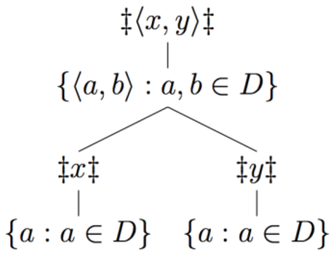
This is Fine's attempted resolution of the antinomy.[58]

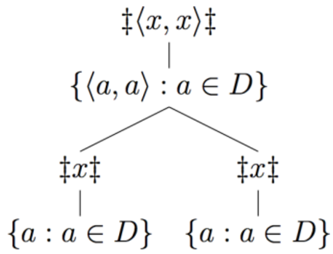
We have re-interpreted Fine's requirement that two variables have the same semantic role as the claim that sentences that are alphabetic variants should have the same structured meanings. Fine can achieve this, since he can think that a variable contributes its domain to the structured meaning of a sentence that contains it. Of course, this move mimics the first option that Fine offers Tarski, according to which the semantic role of a variable is its domain. The difficulty for Tarski, recall, was that this proposal identifies the structured meanings of sentences such as '$\exists y \forall x\ x \leq\ y$' and '$\exists y \forall x\ x \leq\ x$' since these sentences have the same structure and their terminal nodes have the same "meanings".

[57] *Ibid*., pp. 31-2.

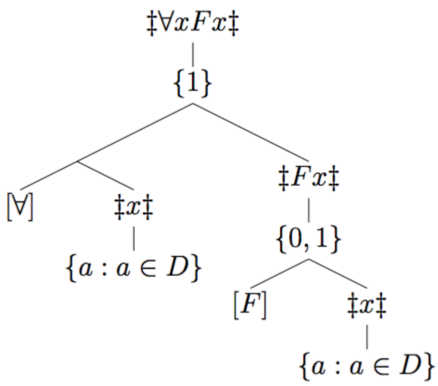
[58] King 2007, *op. cit.,* pp. 218-222 also abandons intrinsicalism, since he thinks that '*x*' and '*y*' contribute the same "thing" to structured contents, namely *nothing*, while 'Fxy' and 'Fxx' nevertheless express different structured contents. In "A theory of bondage", Salmon (see p. 121, footnote 14) likewise gives up intrinsicalism by assigning a semantic role to an occurrence of '*x*' in isolation that cannot be used to predict the behavior of '*x*' in other contexts (Nathan Salmon, "A theory of bondage", in *Content, Cognition, and Communication*, (Oxford: Oxford University Press, 2007): 113-141). Salmon's approach makes use of complicated type-shifting rules, which he assimilates to Frege's "indirect reference". Likewise Aleksandar Kellenberg ("The antinomy of the variable", *dialectica* LXIV, 2 (2010): 225-236) seemingly gives up the project of deriving the semantic role of a sentence from the semantic roles of the expressions it contains: "...it is a mistake to think that the difference in semantic role between 'x > y' and 'x > x' must be due to a difference in semantic role between the pairs of variable types ('x', 'y') and ('x', 'x')... Rather, the expression types 'x > y' and 'x > x' have different semantic roles because the former contains occurrences of two variable types, whereas the latter contains occurrences of only one variable type, although one that occurs twice" (*ibid*., p. 231). Yet, Kellenberg maintains that he's an intrinsicalist.

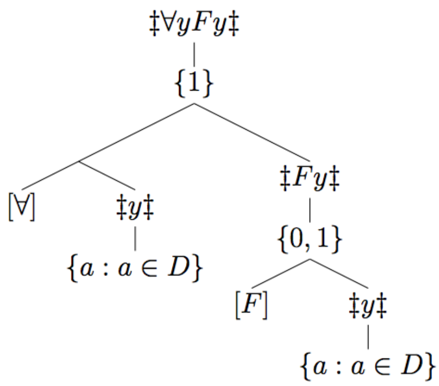
Fine's position, however, is not susceptible to this criticism since he denies *intrinsicalism*. He denies that the "meaning" of a whole follows from the meaning of the parts and their arrangement. So, Fine may say that '$\exists y \forall x\, x \leq y$' and '$\exists y \forall x\, x \leq x$' have different structured meanings, because they have corresponding constituents with different meanings. In particular, '$x \leq y$' and '$x \leq x$' are corresponding constituents of '$\exists y \forall x\, x \leq y$' and '$\exists y \forall x\, x \leq x$', respectively. Fine denies that '$x \leq y$' and '$x \leq x$' have the same "meaning", even though their ultimate constituents have the same meaning. (In the following structured meanings we explicitly display the semantic connection on an expression immediately below the expression's node in the tree.)





On the other hand, Fine may say that all corresponding constituents of alphabetic variants such as '$\forall xFx$' and '$\forall yFy$' agree in meaning:





Thus, Fine's denial of intrinsicalism may also be taken as a denial of what we call *structure intrinsicalism*: composite expressions with the same syntactic structure may have distinct "meanings" even though their corresponding constituent expressions agree in meaning.

Our puzzle is that 'x' cannot be substituted for 'y' in '$\exists y \forall x\, x \leq y$', yielding '$\exists y \forall x\, x \leq x$', without change of meaning. Yet, alphabetic variants are completely synonymous and so must have the same structured meanings. Fine's resolution to this puzzle involves denying both *intrinsicalism* and, more generally, *compositionality*.

In order to respect intrinsicalism, the semantic axioms governing variables 'x' and 'y' must assign them to distinct semantic values. In Tarski's explicit semantics the variables are ordered in a context-invariant way. The semantic value of the n^{th} variable in the ordering is a function that takes a sequence to the n^{th} position in the sequence. We represent that ordering as a function mapping variables into numbers. Let c be a function from the set of variables $\{x, y, z, \dots\}$ to the set of natural numbers $\mathbb{N}$. Then Tarski can be viewed as assigning the following "meanings" to 'x' and 'y' given the ordering function c:

$$[\![x]\!] = \lambda\sigma\ \sigma_{c(x)}$$

$$[\![y]\!] = \lambda\sigma\ \sigma_{c(y)}$$

On this semantics, 'x' and 'y' are assigned to different objects, since they occupy different positions in the ordering and so $c(x) \neq c(y)$. The variables do have their domains in common. Yet, as we have seen, the domain is not a suitable candidate to secure the semantic sameness between 'x' and 'y'.

In assigning different semantic values to the variables 'x' and 'y', Tarski—and his followers in standard formal semantics—seem unable to account for the fact that alphabetic variants are synonymous. Moreover, the context invariant enumeration is—as Tarski (1935, *op. cit.,* 191, note 1) himself admits—"purely technical". Related approaches popular in formal semantics such as Heim and Kratzer (*op. cit.*, see §5.3.3) likewise face this difficulty, since they simply lexicalize the semantic difference between variables.[59]

[59] Indeed, in her 1982 dissertation Heim proposes to identify the objects playing the roles of variables ("discourse referents") simply with numbers (p. 166): Irene Heim, *The Semantics of Definite and Indefinite Noun Phrases*, PhD. Dissertation, (University of Massachusetts at Amherst, 1982).

But Tarski is sensitive to these challenges. He suggests that a less stipulative enumeration of the variables could be given in terms of the order of the variables in a formula.

> [We could number all the variables of every given expression] on the basis of the natural order in which they follow one another in the expression: the sign standing on the extreme left could be called the first, the next the second, and so on. In this way we could again set up a certain correlation between the free variables of a given function and the terms of the sequence. This correlation…would obviously vary with the form of the function in question.[60]

Tarski is proposing that we replace the stipulative, and pre-established, enumeration of the variables—whereby e.g. x is associated with the 1st member of a sequence and y is associated with the 2nd, etc.—with an enumeration c that is sensitive to the formula being evaluated. In particular, Tarski proposes that if 'x' is the *nth* variable *in a formula*, then 'x' will be associated with the *nth* position in a sequence. Tarski's brief suggestion, we claim, contains the resources needed to resolve the antinomy.[61]

On this proposal, each position in any sentence ϕ induces a context c which associates the variables in ϕ with positions in sequences. Namely, the value of the variable 'x' for an input sequence will be the object in the $c(x)^{th}$ position of that sequence. If sentences ϕ and ϕ^* are alphabetic variants and 'x' and 'y' occupy corresponding positions, then 'x' and 'y' will be associated with positions in sequences by c and c^* so that $c(x) = c^*(y)$ and so $\lambda\sigma\ \sigma_{c(x)} = \lambda\sigma\ \sigma_{c^*(y)}$. Thus, '$x$' in the context induced by its position in the sentence '$\forall xFx$' will have the same semantic value as 'y' in the context induced by '$\forall yFy$'.

---

[60] Tarski 1935, op. cit., p. 191, note 1.

[61] The semantic axioms for the variables 'x' and 'y' can now be written as $[\![x]\!] = \lambda c\lambda\sigma\ \sigma_{c(x)}$ and $[\![y]\!] = \lambda c\lambda\sigma\ \sigma_{c(y)}$. This has an interesting pay-off. Although according to the semantics $[\![x]\!] \neq [\![y]\!]$, it nonetheless assigns them a common semantic property, namely $\lambda\alpha\ ([\![\alpha]\!] = \lambda c\lambda\sigma\ \sigma_{c(\alpha)})$. If one thinks of the "semantic role" of an expression as its "representational function", then it is natural to think of the semantic role of an expression as the property attributed to it by its canonical axiom in the semantic theory. It follows that these two semantic axioms deliver the result that 'x' and 'y' have the same semantic roles.

Structured meanings should be specified in terms of the *context*-saturated meanings of the variables not in terms of the context-unsaturated meanings. In practical terms, this means that if a variable 'x' in sentence ϕ is to be evaluated at context c, then the 'x' will contribute its semantic value at c, namely $\lambda\sigma\ \sigma_{c(x)}$. This is analogous to Kaplan's idea that the belief content of a sentence is given by its *content* (context-saturated meaning) and not its *character* (context-unsaturated meaning), though with a crucial difference, since our discourse contexts evolve as the sentence is processed.[62]

We will provide recursive procedures specifying the truth conditions and the structured meaning of each sentence. Given this semantics, alphabetic variants will have the same structured meanings. On the other hand, if 'x' and 'y' occur in a single sentence ϕ, then they will be evaluated at contexts c and c^* such that $c(x) \neq c^*(y)$ and so $\lambda\sigma\ \sigma_{c(x)} \neq \lambda\sigma\ \sigma_{c^*(y)}$. As a result, '$x$' and '$y$' will have different semantic contents in the contexts induced by their respective positions in '$\exists y \forall x\ x \leq\ y$'. As a result, replacing 'y' by 'x', yielding '$\exists y \forall x\ x \leq\ x$', will not preserve meaning. This, in essence, resolves the antinomy of the variable.

But significant explanatory burdens remain. First, we must explain how to evaluate a variable in a context. We must then explain the formal procedure by which the context evolves as a sentence is processed. As Fine remarks, the appeal to contexts "does not really solve the puzzle but merely pushes it back a step."[63] He asks, "why do we say that the variables x and y have a different semantic role in" '$\exists y \forall x\ x \leq\ y$' (*ibid*.)? Our answer is that the difference in meaning of 'x' and 'y' in '$\exists y \forall x\ x \leq\ y$' can be explained by a theory describing the evolution of the enumeration in a sentence. This same theory will predict that

[62] Kaplan 1989, op. cit., proposes two layers of meaning for an expression, the character and the content. These two layers of meaning play different roles in Kaplan's semantic theory: the content is the information asserted in a particular context, whereas, the character of an expression encodes what content the expressions would have in any context. If in context c, A says to B 'I am hungry' and, in c*, B says to A 'You are hungry', then they have said the same thing relative to their respective contexts. We likewise propose two layers of meaning, one captures how the value of a variable depends on the discourse context, and the other captures the information value of the variable relative to a discourse context. Variables 'x' and 'y' have different context-unsaturated meanings, but in the context of alphabetic variants, such as $\forall xFx$ and $\forall yFy$, their "content" is the same.

[63] Fine 2007, op. cit., p. 8.

corresponding variables in alphabetic variants have the same meaning in their respective contexts.

V.I THE EVOLUTION OF CONTEXT

In order to implement Tarski's idea compositionally, we assess a variable against a sequence of individuals σ, and also a discourse context c, which enumerates the variables.

$$[\![x]\!] = \lambda c \lambda \sigma\ \sigma_{c(x)}$$

$$[\![y]\!] = \lambda c \lambda \sigma\ \sigma_{c(y)}$$

Following Tarski we assume that each sequence of individuals σ is of infinite length.[64] Effectively a sequence σ maps numbers onto objects in the domain. In Tarksi's semantics every variable has a value relative to a sequence because Tarski stipulates an initial, static mapping c from variables to numbers. The value of a variable 'x' relative to a sequence σ and the stipulated enumeration c is $\sigma_{c(x)}$.

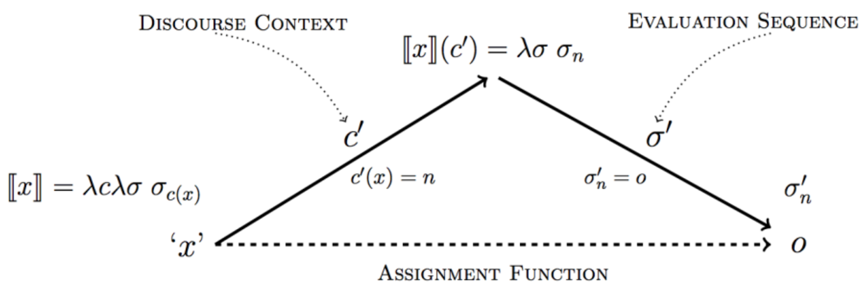
By way of contrast in our semantics the initial discourse context c—the initial mapping from variables to numbers—does not associate any variables with numbers. There is no pre-established ordering of the variables. Rather the enumeration is determined dynamically as a sentence is processed—newly introduced variables are associated with new positions in sequences.[65] The relevant context will be supplied by the variable's position in the sentence (as we explain below). Collectively, the discourse context and the sequence define an assignment function on the variables in the sentence.[66]

---

[64] Tarski 1935, *op. cit.*, p. 195, note 1 also considers the possibility of employing finite sequences. Dekker (1994) develops the idea in a different direction more in line with the proposal in the text. A formula is initially evaluated against a finite or null sequence, which grows as quantifiers are processed. This idea could easily be integrated into the proposal of the text.

[65] Our approach integrates ideas from Heim's 1982, *op. cit.,* file-change semantics and Vermeulen's model of variables as stacks: C.F.M. Vermeulen, "Variables as stacks", *Journal of Logic, Language and Information* IX, 2 (2000): 143–167. This strategy is also hinted at in Bryan Pickel, "Variables and attitudes", Noûs XLIX, 2: 333–356, §5.3.

[66] Strictly speaking, c, will be a relation, not a function, since a context might evolve by adding multiple indices to 'x', e.g. $\{\langle x,1\rangle,\langle x,2\rangle\}$, as in the formula '$\exists x \exists x Fx$'. But since it is only the highest index that will matter we now define $c(\alpha)$ as the highest n such that there is a variable α and $\langle \alpha, n\rangle \in c$.



We can then assign a *structured meaning* to a sentence by assessing the semantic values of its constituents at the context induced by their position in the sentence.

On the resulting semantic theory, the variables 'x' and 'y' contribute functions from sequences into objects to the structured meanings of sentences that contain them. These functions are in no way typographic. They lie purely on the "non-conventional side" of language.[67]

How are variables introduced into discourse? Since we focus on the semantics of first-order logic, we assume that they are introduced by quantifiers. A quantifier '$\exists x$' attached to a formula ϕ is processed first by updating the context so that 'x' is assigned to a new position in the sequence and then by evaluating ϕ with respect to this new context.

In our model, we need to keep track of the highest number assigned to a variable in discourse context c. Represent this by $Num(c) = Max(\{n: \exists\alpha\ \langle\alpha, n\rangle \in c\})$. The basic idea, then, is that one evaluates a quantified formula $\exists x\phi$ relative to c by evaluating its sub-formula, ϕ, relative to the extension of c that assigns 'x' to the next position in a sequence, to $Num(c) + 1$.

[67] Cf. Fine 2007, *op. cit.*, p. 11. There may be a residual worry about absolute or context-unsaturated semantic value of the variable. In particular, contexts are modeled as functions from variables into the natural numbers, representing positions in sequences. So the domain of this function may include linguistic items. The worry is that the very appeal to such functions makes the semantics objectionably typographic on the grounds that it has to "incorporate the variables themselves […] into the very identity of" their semantic values (*ibid.*, 32). We find this worry misguided. We don't see how the appeal to such a set could be objectionably typographic. There is nothing objectionable about the claim that the semantic role of a predicate is to map entities to truth-values. But many predicates will map themselves to a truth-value (e.g. 'is a predicate'). For us, a context is just a set, that *may* include ordered pairs of variables and numbers. Perhaps the worry is that the set will have the variables themselves in its transitive closure. But this will also be true of the extensions of many predicates.

Variables: If α is a variable, then

- $[\![\alpha]\!] = \lambda c \lambda \sigma\ \sigma_{c(\alpha)}$

Sentences: If π is an n-ary predicate and $\alpha_1, \dots \alpha_n$ are variables, then

- $[\![\pi\alpha_1, \dots, \alpha_n]\!]^{c,\sigma} = 1$ iff $\langle [\![\alpha_1]\!](c,\sigma), \dots, [\![\alpha_n]\!](c,\sigma)\rangle \in I(\pi)$

If ϕ and ψ are formulae and α is a variable, then

- $[\![\neg\phi]\!]^{c,\sigma} = 1$ iff $[\![\phi]\!]^{c,\sigma} \neq 1$
- $[\![\phi \wedge \psi]\!]^{c,\sigma} = 1$ iff $[\![\phi]\!]^{c,\sigma} = 1$ and $[\![\psi]\!]^{c,\sigma} = 1$
- $[\![\forall\alpha\phi]\!]^{c,\sigma} = 1$ iff for all $d \in D$, $[\![\phi]\!]^{c^*,\sigma[c^*(\alpha)/d]} = 1$,
  where $c^* = c \cup \{\langle \alpha, Num(c) + 1\rangle\}$
- $[\![\exists\alpha\phi]\!]^{c,\sigma} = 1$ iff for some $d \in D$, $[\![\phi]\!]^{c^*,\sigma[c^*(\alpha)/d]} = 1$,
  where $c^* = c \cup \{\langle \alpha, Num(c) + 1\rangle\}$

On our semantics, a formula is evaluated against two parameters, a discourse context c and a sequence of individuals σ. We can think of the closed formulae of our language as regimenting *sentences* of natural language. As Tarski observed in defining truth relative to satisfaction only, if a closed formula is satisfied by one sequence, then it will be satisfied by any sequence. This led him to define truth for a sentence in terms of satisfaction by every sequence. We similarly observe that, for a closed formula ϕ and context c, if $[\![\phi]\!](c,\sigma) = 1$, then $[\![\phi]\!](c^*,\sigma) = 1$ for any c^*. This leads us to a slightly different definition of truth *simpliciter* than might be offered on the Tarskian model. For us, a sentence ϕ will be true *simpliciter* if $[\![\phi]\!](c,\sigma) = 1$ for the empty discourse context $c = \emptyset$ and every sequence σ. In particular, we find it more natural to think of a sentence as evaluated at an empty discourse context, which evolves as the formula is processed.[68]

Unlike Fine's semantics, this semantics is strongly compositional insofar as if $[\![\alpha]\!] = [\![\beta]\!]$ and ϕ_α differs from ϕ_β only in that α is substituted for β, then $[\![\phi_\alpha]\!] = [\![\phi_\beta]\!]$. The only plausible candidates for failures of substitution—and the only expressions sensitive to the dynamic enumeration—are variables. But for any

[68] We do not mean to foreclose the possibility of treating *open formulae* as representing sentences of natural language as well, such as those containing demonstrative or anaphoric pronouns. Doing so, of course, would require that truth be identified with satisfaction, not by every sequence, but by some contextually salient sequence or sequences.

variables α and β, if $[\![\alpha]\!] = [\![\beta]\!]$, then $\alpha = \beta$. This just reflects the fact that the context-invariant semantic values of any two variables differ.[69]

Nonetheless, the semantic value of distinct variables α and β may nonetheless coincide in their appropriate respective discourse contexts. In particular, the variable 'x' in '$\forall xFx$' has the same context saturated meaning as 'y' in '$\forall yFy$'. In evaluating each sentence relative to the null context, one must evaluate the sub-formulas 'Fx' and 'Fy' relative to an updated context in each case. One evaluates 'Fx' in an updated context $c*$ such that $c^*(x) = 1$. Similarly, one evaluates 'Fy' in an updated context $c**$ such that $c^{**}(y) = 1$. Thus, both variables have the same meanings in context: $[\![x]\!](c^*) = [\![y]\!](c^{**}) = \lambda\sigma\, \sigma_1$. We will use the meanings in contexts to define structured meanings. On the other hand, consider the variables 'x' and 'y' in the open sentence '$x \leq y$' embedded in '$\exists y \forall x\, x \leq y$'. These variables will be assessed relative to a single context c such that $c(x) = 1$ and $c(y) = 2$. As a result, 'x' and 'y' will have different context sensitive meanings in this sentence: namely $[\![x]\!](c) = \lambda\sigma\, \sigma_1 \neq [\![y]\!](c) = \lambda\sigma\, \sigma_2$.

V.II STRUCTURED MEANINGS

We have now given a truth conditional semantics. We now show how the structured meanings for formulae can be read off the semantic values of their constituents *in contexts.* We recursively assign structured meanings to every sentence ϕ of the first-order language at any context c, written $[\phi]^c$. (Let $Num(c) = Max(\{n : \exists\beta\, \langle\beta, n\rangle \in c\})$. And, let $c + \alpha = c \cup \{\langle\alpha, Num(c) + 1\rangle\}$. For any logical connective $\gamma \in \{\neg, \wedge, \forall, \exists\}$ let its contribution $[\gamma]$ be its semantic value.)

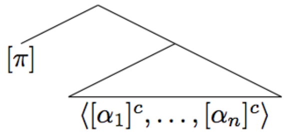
---

[69] Some ways of construing compositionality are more demanding. They might require for instance that every expression is provided a semantic value, and that the semantic value of every complex expression is determined by the semantic values of its immediate constituents and their mode of combination. The semantics offered here is syncategorematic in the case of sentential connectives, since we haven't assigned them semantic values in isolation but have only provided truth-conditions for constructions that contain them. But we could easily extract a non-syncategorematic semantics for instance the semantic value of the existential quantifier can be provided as follows: $[\![\exists\alpha]\!] = \lambda p. \lambda c \lambda\sigma. \exists d\; p(c^*, \sigma[c^*(\alpha)/d]) = 1$, where $c^* = c \cup \{\langle\alpha, Num(c) + 1\rangle\}$. (In the preceding lexical entry the variables p, c, σ, d are meant to be of designated semantic types as follows: p ranges over functions from discourse contexts and sequences of individuals to $\{0,1\}$; c ranges over discourse contexts; σ ranges over sequences of individuals drawn from the domain; and d ranges over individuals in the domain D.)

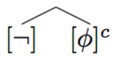
If α is a variable $[\alpha]^c = [\![\alpha]\!](c)$

If π is an n-place predicate, then $[\pi] = I(\pi)$

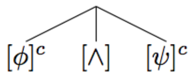
$[\pi\alpha_1 \ldots \alpha_n]^c =$



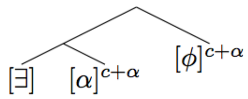
$[\neg\phi]^c =$



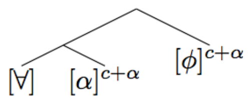
$[\phi \wedge \psi]^c =$



$[\exists\alpha\phi]^c =$



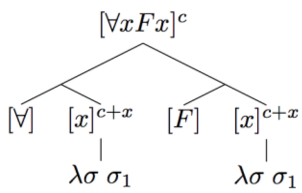
$[\forall\alpha\phi]^c =$

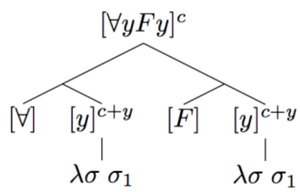


This procedure recursively specifies a structured meaning for every sentence of the language of first-order logic in terms of the semantic values of the basic constituents of the sentence.

Moreover, the structured meanings are appropriate in that the semantics delivers the same structured meanings to pre-theoretically synonymous sentences and distinct structured meanings for sentences that are not synonymous, in the sense of expressing distinct belief contents. Return to our problematic pairs: '$\forall xFx$' and '$\forall yFy$' should have the same structured meanings while '$\exists y\forall x\ x \leq y$' and '$\exists y\forall x\ x \leq x$' should have distinct structured meanings.

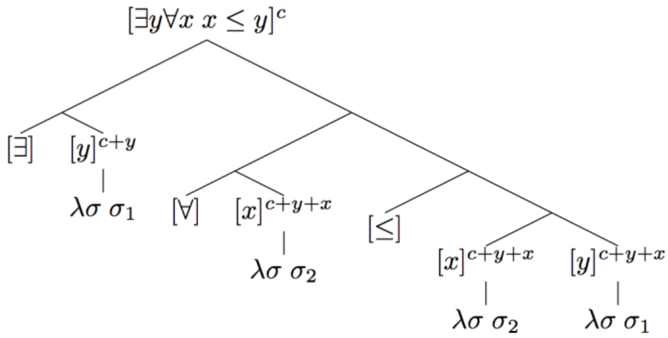
This result is delivered by our semantics. The structured meanings of '$\forall xFx$' and '$\forall yFy$' will be the same, namely:



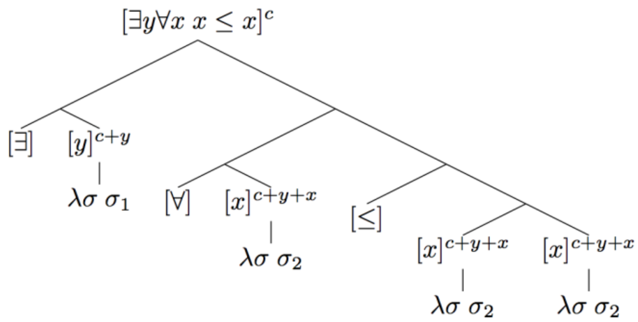


In their respective formulas, 'x' and 'y' are evaluated at contexts that assign them to the same function from sequences into objects.

On the other hand, the variables ‘x’ and ‘y’ in ‘$\exists y \forall x\, x \leq y$’ will be associated with different functions from sequences to objects. They will thereby make different truth conditional contributions, which will figure into the structured meaning of this sentence, namely:



As desired, the structured meaning of ‘$\exists y \forall x\, x \leq y$’ is different from the structured meaning of ‘$\exists y \forall x\, x \leq x$’, namely:



As a result we have resolved our puzzle. The variable ‘x’ cannot be substituted for ‘y’ in ‘$\exists y \forall x\, x \leq y$’, yielding ‘$\exists y \forall x\, x \leq x$’, without change of meaning. Yet, alphabetic variants are completely synonymous and so must have the same structured meanings.

Finally, our view preserves intrinsicalism in two senses. First, we’ve assigned distinct semantic values to distinct variables. These semantic values predict the truth conditional contribution of a variable ‘x’ in a sentential context c, namely $[\![x]\!](c)$. The truth conditional contribution of the variable in a sentential context corresponds to its value in the structured meaning assigned to that sentence. Second, the truth conditional contributions of complex expressions and sentences in wider contexts—and thus their contributions to the structured meanings of sentences that contain them—are a function of the truth conditional contributions of their components in context. It is for this reason that the structured meaning of a sentence can be described in terms of the semantic contributions of its terminal nodes. We don't need to assign emergent semantic contributions to non-terminal nodes, as do anti-intrinsicalists such as Fine and King. Thus, our Tarskian resolution to the antinomy preserves intrinsicalism.

BRYAN PICKEL

BRIAN RABERN

University of Edinburgh