Optimal Time-Consistent Monetary, Fiscal and Debt Maturity Policy*

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Abstract

The textbook optimal policy response to an increase in government debt is simple—monetary policy should actively target inflation, and fiscal policy should smooth taxes while ensuring debt sustainability. Such policy prescriptions presuppose an ability to commit. Without that ability, the temptation to use inflation surprises to offset monopoly and tax distortions, as well as to reduce the real value of government debt, creates a state-dependent inflationary bias problem. High debt levels and short-term debt exacerbate the inflation bias. But this produces a debt stabilization bias because the policy maker wishes to deviate from the tax smoothing policies typically pursued under commitment, by returning government debt to steady-state. As a result, the response to shocks in New Keynesian models can be radically different, particularly when government debt levels are high and maturity short.

Keywords:
New Keynesian Model, Government Debt, Monetary Policy, Fiscal Policy, Time Consistency Policy, Maturity Structure
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1. Introduction

Conventional monetary-fiscal policy analysis assigns monetary policy the task of controlling demand and inflation and fiscal policy the job of ensuring fiscal sustainability. Optimal policy analyses support this policy assignment. In sticky price New Keynesian models with one-period government debt, Schmitt-Grohe and Uribe (2004b) show that even a mild degree of price stickiness implies negligible use of inflation surprises to stabilize debt and near random walk behavior in government debt and tax rates when policy makers can commit to time-inconsistent monetary and fiscal policies, in response to shocks. In other words, monetary policy should be used to stabilize inflation, not debt, while a tax smoothing fiscal policy ensures fiscal sustainability. Although Sims (2013) questions the robustness of this result when government can issue long-term nominal bonds since this implies variations in bond prices can be used as a device to stabilize debt, Faraglia et al. (2013), Leeper and Leith (2016), Leeper and Zhou (2013) and Sheedy (2014) find that, as part of a Ramsey problem, the consensus policy assignment remains largely optimal - the use of inflation to stabilize debt is negligible.

Relaxing the assumption that the policy maker can commit in a model with single-period debt, Niemann et al. (2013) find that the desire to inflate away the debt burden leads to large and persistent movements in inflation which are absent under commitment. The current paper also assumes time-consistent policy making and assesses the importance of both debt maturity and the level of debt for the resulting equilibrium. Three key findings emerge:

1. The temptation to use inflation surprises to stabilize debt grows with the level of debt and shrinks with the average maturity of that debt. As a result the equilibrium inflationary bias problem can be significantly lower with longer maturity debt.
2. The response to shocks is radically different under discretion vs. commitment and depends crucially on the level and maturity of government debt. Under commitment (regardless of the level and maturity of debt) the policy maker sustains debt at a new steady-state level after effectively eliminating the inflationary consequences of any

shock. Under discretion, the policy response is radically different - the debt-dependent inflationary bias leads the policy maker to more than offset the fiscal consequences of the shock to avoid exacerbating these biases. This perverse policy response is heightened for higher debt levels or shorter maturity.

3. Allowing the policy maker to choose the relative proportions of short- versus long-term debt as part of the time-consistent policy problem provides the current policy maker with a means to influence the pace at which a given stock of debt is reduced in the future. When the inflationary bias problem bites less (when prices are more flexible and markups lower) the policy maker will seek to issue less short-term debt, thereby increasing average debt maturity. Since the debt stabilization bias rises in debt levels, but falls in maturity, this helps ensure future policy makers stabilize debt more slowly and at a lower inflationary cost. Conversely, when the inflation bias problem is greater, issuing more short-term debt helps ensure future policy makers stabilize debt more rapidly.

Aside from the key papers cited above which analyze the policy problem in the context of New Keynesian models, monetary frictions have been used to generate a cost for inflation and generate trade-offs between the use of monetary and fiscal policy. For example, Schmitt-Grohe and Uribe (2004a) study Ramsey policy in a flexible-price model with a cash-in-advance constraint, while Martin (2009) studies the time-consistency problems that arise from the interaction between debt and monetary policy, since inflation surprises reduce the real value of nominal liabilities. Martin (2011, 2013, 2014) examine time-consistent policies in variants of the monetary search model of Lagos and Wright (2005). Niemann et al. (2013) combines both a cash-in-advance constraint and sticky prices in the context of time-consistent policy with single-period debt - the monetary friction helps ensure the model can sustain a positive debt-to-GDP ratio in steady-state. Monetary frictions are considered in Appendix I, but most of the analysis abstracts from such frictions and emphasizes nominal price stickiness as the conventional approach to generating sizable real effects from monetary

policy.

The paper proceeds as follows. The benchmark model is described in section 2 and the optimal time-consistent policy problem is contrasted with Ramsey policy in section 3. Section 4 describes the solution method and 5 presents the numerical results. Section 6 concludes.

2. The Model

The model is a standard New Keynesian model, but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or long-term borrowing.[1]

2.1. Households

The utility function of the representative household takes the specific form

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right) \tag{1}$$

Households appreciate private consumption, C_t, as well as the provision of public goods, G_t, and dislike supplying labor, N_t. Private consumption is made up of a basket of goods defined by,

$$C_t \equiv \left(\int_0^1 C_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{2}$$

where j denotes the good's variety and $\epsilon_t > 1$ is the elasticity of substitution between varieties. This is assumed to be time-varying, following the AR(1) process,

$$\ln(\epsilon_t) = (1-\rho_\epsilon)\ln(\overline{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t, \varepsilon_t \sim N(0,1) \tag{3}$$

[1]Most countries issue long-term nominal debt such that even modest changes in inflation and interest rates can have substantial impact on the market value of debt - see Hall and Sargent (2011) and Sims (2013) for the empirical findings on the contribution of this kind of fiscal financing to the decline in the U.S. debt-to-GDP ratio from 1945 to 1974.

as a device for introducing mark-up shocks.

The households' optimal allocation of consumption across individual goods implies their demand for good j,

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} C_t$$

where $P_t(j)$ is the price of good j and the aggregate price level is defined as, $P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon_t} dj\right)^{\frac{1}{1-\epsilon_t}}$.

The budget constraint at time t is given by

$$P_t^M B_t^M \leq \Xi_t + (1 + \rho P_t^M) B_{t-1}^M + W_t N_t (1 - \tau_t) - P_t C_t + Tr_t \tag{4}$$

where $\int_0^1 P_t(j) C_t(j) dj = P_t C_t$, Ξ_t is the representative household's share of profits in the imperfectly competitive firms producing these goods, W_t are wages, and τ_t is an wage income tax rate. There is also an exogenous fiscal transfer to the household, $Tr_t = P_t tr$, which is introduced to ensure the model reflects the data in terms of the breakdown of fiscal expenditures into public consumption and transfers.[2] In period t households buy government bonds, B_t^M, at price P_t^M, which, following Woodford (2001), are actually a portfolio of many bonds which pay a declining coupon of ρ^j dollars $j+1$ periods after they were issued, where $0 < \rho \leq \beta^{-1}$. A measure of the duration of the bond is given by $(1 - \beta\rho)^{-1}$, which allows calibration of ρ to capture the observed maturity structure of government debt.[3] Households bring nominal wealth of $(1 + \rho P_t^M) B_{t-1}^M$ into period t.

Households maximize utility subject to the budget constraint (4) to obtain the optimal

[2]It is important to note that real transfers are an exogenously given constant and are not considered to be a policy instrument. Allowing transfers to be chosen optimally would enable the policy maker to levy a lump-sum tax in order to finance a negative distortionary labor income tax and offset the distortion arising from monopolistic competition,. This is a common, but unrealistic, assumption in linear-quadratic analyses of optimal fiscal and monetary policy in New Keynesian models.

[3]In the special case where $\rho = 0$, the bonds reduce to the familiar single period bonds typically studied in the literature.

allocation of consumption across time and price the declining payoff consols,

$$\beta E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(1 + \rho P_{t+1}^M \right) \right\} = P_t^M \tag{5}$$

It is convenient to define the stochastic discount factor (for nominal payoffs) for later use, $Q_{t,t+1} \equiv \beta \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right)$where $E_t Q_{t,t+1} = R_t^{-1}$ is the inverse the short-term interest rate which is the policy instrument of the monetary authority.

The second first order condition (FOC) relates to their labor supply decision and is given by,

$$(1 - \tau_t) \left(\frac{W_t}{P_t} \right) = N_t^{\varphi} C_t^{\sigma} \tag{6}$$

That is, the marginal rate of substitution between consumption and leisure equals the after-tax wage rate.

Besides these FOCs, necessary and sufficient conditions for household optimization also require the households' budget constraints to bind with equality. Defining $D_t \equiv (1 + \rho P_t^M) B_{t-1}^M$, after imposing the no-arbitrage conditions and the no-Ponzi-game condition, the transversality condition can be written as,

$$\lim_{T \to \infty} E_t \left[Q_{t,T} D_T \right] = 0 \tag{7}$$

2.2. Government

Aggregate public consumption takes the same form as private consumption,[4]

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{8}$$

[4]An alternative modeling approach would be to introduce an 'aggregator' firm which converts the individual goods to a final output which is purchased by households and the government. The model implies, equivalently, that households and the government perform this aggregation themselves.

such that government demand for individual goods is given by,

$$G_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} G_t$$

Government expenditures, consisting of transfers, Tr_t, and consumption, G_t, are financed by levying labor income taxes at the rate τ_t, and by issuing long-term bonds B_t^M. The government's sequential budget constraint is then given, in real terms, by

$$P_t^M b_t = (1 + \rho P_t^M)\frac{b_{t-1}}{\Pi_t} - w_t N_t \tau_t + G_t + tr \tag{9}$$

where $w_t = W_t/P_t$ is the real wage and $\Pi_t \equiv P_t/P_{t-1}$ the gross rate of inflation. Transfers $tr = Tr_t/P_t$ are fixed at a data-consistent average. It is important to note that the state variable, $b_t \equiv B_t^M/P_t$, which deflates the number of nominal bonds by the price level does not capture the real value of government debt. That is given by, $P_t^M b_t$. Instead, introducing the variable b_t enables the policy problem to be written solely in terms of this state variable without the need to account for B_t^M and P_t.[5]

2.3. Firms

Firm j faces three constraints, firstly a linear production function,

$$Y_t(j) = N_t(j) \tag{10}$$

where the real marginal cost of production is defined as $mc_t \equiv W_t/P_t = (1 - \tau_t)N_t^{\varphi}C_t^{\sigma}$. Secondly, a demand curve for their product,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t} Y_t$$

[5]Retaining B_t^M and P_t as separate states would be impossible to solve using our solution algorithm since both variables would be non-stationary in an equilibrium with positive inflation.

which is the sum of private and public demand, where $Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj\right]^{\frac{\epsilon_t}{\epsilon_t - 1}}$. Finally, quadratic adjustment costs in changing prices, as in Rotemberg (1982), defined for firm j as,

$$\eta_t(j) \equiv \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t \tag{11}$$

where $\phi \geq 0$ measures the degree of nominal price rigidity. The adjustment cost, which accounts for the negative effects of price changes on the customer–firm relationship, increases in magnitude with the size of the price change and with the overall scale of economic activity Y_t.

The problem facing firm j is to maximize the discounted value of nominal profits,

$$\max_{P_t(j)} E_t \sum_{z=0}^{\infty} Q_{t,t+z} \Xi_{t+z}(j)$$

subject to these constraints above, where nominal profits are defined as,

$$\Xi_t(j) \equiv P_t(j) Y_t(j) - mc_t Y_t(j) P_t - \frac{\phi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 P_t Y_t$$

The FOCs imply the following non-linear Phillips curve relationship,

$$\Pi_t(\Pi_t - 1) = \beta E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1}(\Pi_{t+1} - 1)\right] + \phi^{-1}((1-\epsilon_t) + \epsilon_t(1-\tau_t) N_t^{\varphi} C_t^{\sigma}) \tag{12}$$

2.4. Market Clearing

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j) + G_t(j) + \eta_t(j)$$

such that, in a symmetrical equilibrium,

$$Y_t\left[1 - \frac{\phi}{2}(\Pi_t - 1)^2\right] = C_t + G_t \tag{13}$$

There is also market clearing in the bonds market where the portfolio of long-term bonds held by households evolves according to the government's budget constraint.

That completes the description of the model, which is summarized in Appendix D. Before analyzing the optimal policy problem the competitive equilibrium is defined as follows.

Definition 1 (Competitive equilibrium). *A competitive equilibrium consists of government fiscal policies,* $\{G_t, \tau_t, b_t\}_{t=0}^{\infty}$*, prices,*$\left\{R_t, w_t, P_t^M, \Pi_t\right\}_{t=0}^{\infty}$*, and private sector allocations,* $\{C_t, N_t, Y_t, \Xi_t\}_{t=0}^{\infty}$*, satisfying* $\forall \{\epsilon_t\}_{t=0}^{\infty}$*, (i) the private sector optimization taking government policies and prices as given, that is, the household budget constraint (4), the production function* $Y_t = N_t$*, and the optimality conditions, (5), (6) and (12); (ii) the market clearing condition (13); (iii) the government's budget constraint (9); and (iv) the transversality condition (7), for a given initial level of government debt* b_{-1}*.*

3. Optimal Policy Under Commitment and Discretion

This section outlines the policy problems under both commitment and discretion, before contrasting the resultant FOCs to gain insight into the time-consistency problems generated under discretion. In both cases the policy problem amounts to choosing a set of government policies, $\{R_t, G_t, \tau_t, b_t\}_{t=0}^{\infty}$, in order to maximize the utility of the representative household, (1), subject to the constraints implied by the competitive equilibrium defined above. The difference between commitment and discretion lies in whether or not the policy maker is able to commit to future policies

3.1. Commitment Policy

Following Leeper and Leith (2016), Ramsey policy is derived to serve as a benchmark against which to contrast time-consistent policy. The Lagrangian for the policy problem is

given by

$$
\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \{ & \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta C_t^{\sigma} Y_t^{-1} E_t (C_{t+1})^{-\sigma} Y_{t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \end{array} \right] \\
& + \lambda_{3t} \left[P_t^M b_t - (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma} - G_t - tr \right] \\
& + \lambda_{4t} \left[P_t^M - \beta E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \Pi_{t+1}^{-1} \left(1 + \rho P_{t+1}^M \right) \right\} \right] \}
\end{aligned}
\tag{14}
$$

Here the consumption Euler equation has been used to eliminate the short-run nominal interest rate, R_t, from the remaining constraints. By committing to an entire path of policy instruments, the policy maker is able to influence expectations in order to improve the policy trade-offs they face.

The resultant set of FOCs are given by,

$$
\begin{aligned}
C_t \quad & C_t^{-\sigma} - \lambda_{1t} + \lambda_{2t} \left[\begin{array}{c} \sigma\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} \\ +\sigma\phi\beta C_t^{\sigma-1} Y_t^{-1} E_t \left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right) \end{array} \right] \\
& +\lambda_{3t}\left[\sigma\left(\tfrac{\tau_t}{1-\tau_t}\right)\left(Y_t\right)^{1+\varphi} C_t^{\sigma-1}\right] \\
& -\lambda_{4t}[\sigma\beta E_t\left\{\left(C_t\right)^{\sigma-1}\left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}^{-1}\left(1+\rho P_{t+1}^M\right)\right\}] \\
& -\lambda_{2t-1}[\sigma\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}\left(C_t\right)^{-\sigma-1} Y_t\Pi_t\left(\Pi_t-1\right)] \\
& +\lambda_{4t-1}\left[\sigma(C_{t-1})^{\sigma}\left(C_t\right)^{-\sigma-1}\Pi_t^{-1}\left(1+\rho P_t^M\right)\right] = 0 \\
Y_t \quad & -Y_t^{\varphi} + \lambda_{1t}\left[1-\tfrac{\phi}{2}\left(\Pi_t-1\right)^2\right] + \lambda_{3t}\left[(1+\varphi)Y_t^{\varphi} C_t^{\sigma}\left(\tfrac{\tau_t}{1-\tau_t}\right)\right] \\
& +\lambda_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi\beta C_t^{\sigma} Y_t^{-2} E_t\left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right] \\
& +\lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}\left(C_t\right)^{-\sigma}\Pi_t\left(\Pi_t-1\right)] = 0 \\
\tau_t \quad & \epsilon_t\lambda_{2t} + \lambda_{3t} Y_t = 0 \\
G_t \quad & \chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \\
P_t^M \quad & \lambda_{3t}[b_t - \rho\tfrac{b_{t-1}}{\Pi_t}] + \lambda_{4t} \\
& -\lambda_{4t-1}\left[\rho(C_{t-1})^{\sigma}\left(C_t\right)^{-\sigma}\Pi_t^{-1} P_t^M\right] = 0 \\
\Pi_t \quad & -\lambda_{1t}\left[Y_t\phi\left(\Pi_t-1\right)\right] - \lambda_{2t}\left[\phi\left(2\Pi_t-1\right)\right] + \lambda_{3t}\left[\tfrac{b_{t-1}}{\Pi_t^2}\left(1+\rho P_t^m\right)\right] \\
& +\lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}\left(C_t\right)^{-\sigma} Y_t(2\Pi_t-1)] \\
& +\lambda_{4t-1}\left[(C_{t-1})^{\sigma}\left(C_t\right)^{-\sigma}\Pi_t^{-2}(1+\rho P_t^M)\right] = 0 \\
b_t \quad & \lambda_{3t} P_t^M - \beta E_t\left[\lambda_{3t+1}\tfrac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] = 0
\end{aligned}
$$

To obtain a global solution using the algorithm described in Leeper and Leith (2016) define the following state variables, $\widetilde{\lambda}_{2t}$ and $\widetilde{\lambda}_{4t}$ such that $\lambda_{2t} = \widetilde{\lambda}_{2t} C_t^{-\sigma} Y_t$ and $\lambda_{4t} = \widetilde{\lambda}_{4t}(C_t)^{-\sigma}$ which allows us to rewrite the FOCs as shown in Appendix F.

The commitment equilibrium is determined by the system given by the FOCs, the constraints in (F.1), and the exogenous process for the markup shock, (3). The solution to this system is a set of time-invariant equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \epsilon_t, \widetilde{\lambda}_{2t-1}, \widetilde{\lambda}_{4t-1}\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \widetilde{\lambda}_{2t}, \lambda_{3t}, \widetilde{\lambda}_{4t}\}$ for all $t \geq 0$. It is the expansion in the set of state variables to include $\widetilde{\lambda}_{2t-1}$ and $\widetilde{\lambda}_{4t-1}$which captures the commitments made under Ramsey policy.

3.2. Discretionary Policy

The policy under discretion seeks to maximize the value function,

$$V(b_{t-1},\epsilon_t) = \max_{C_t,G_t,Y_t,\Pi_t,\tau_t,b_t} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \widetilde{\beta} E_t \left[V(b_t,\epsilon_{t+1}) \right] \right\}$$

subject to the resource constraint (13), the New Keynesian Phillips curve (12), and the government's budget constraint,(9) after using labor supply, (6), the production function, (10) and bond price, (5), equations to eliminate N_t, w_t and P_t^M from the constraints. The possibility that the policy maker suffers from a degree of myopia is captured by assuming they may discount the future more heavily than households, $\widetilde{\beta} \leq \beta$. A plausible degree of myopia is necessary to ensure the steady-state level of debt under discretion matches the data - this is discussed below.[6]

In conducting this optimization the policy maker is constrained to act in a time-consistent manner. In other words the policy maker cannot make time-inconsistent promises as to how they will behave in the future in order to have a beneficial impact on current policy trade-offs through expectations as they would under Ramsey policy. Instead economic agents anticipate the incentives facing the policy maker in each period and form expectations accordingly. However, the current policy can still influence those expectations by affecting the states the next period's policy maker inherits. To capture this future expectations are replaced by the following state-dependent auxiliary functions,

$$M(b_t,\epsilon_{t+1}) \equiv (C_{t+1})^{-\sigma} Y_{t+1} \Pi_{t+1} (\Pi_{t+1} - 1) \tag{15}$$

$$L(b_t,\epsilon_{t+1}) \equiv (C_{t+1})^{-\sigma} (\Pi_{t+1})^{-1} (1 + \rho P_{t+1}^M) \tag{16}$$

in the NKPC and bond pricing equations, respectively. These functions reflect the fact

[6] An alternative device for delivering positive steady-state debt levels, which has been used in the literature (see for example Schmitt-Grohe and Uribe (2004b) and Niemann et al. (2013)), is to introduce a monetary friction. We consider this approach in online Appendix I.

that, in equilibrium, we can map endogenous variables to the state-space and expectations are formed rationally based on that mapping. The current policy maker, in turn, takes account of this in setting policy. Before deriving the FOCs, it is helpful to define $X_1(b_t, \epsilon_{t+1}) \equiv \partial X(b_t, \epsilon_{t+1})/\partial b_t$ for $X = \{L, M\}$ which captures the impact of changing debt on expectations. The Lagrangian for the policy problem can be written as,

$$\begin{aligned}
\mathcal{L} = & \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \widetilde{\beta} E_t[V(b_t, \epsilon_{t+1})] \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta C_t^{\sigma} Y_t^{-1} E_t [M(b_t, \epsilon_{t+1})] \end{array} \right] \\
& + \lambda_{3t} \left[\begin{array}{c} \beta b_t C_t^{\sigma} E_t [L(b_t, \epsilon_{t+1})] - \frac{b_{t-1}}{\Pi_t} (1 + \rho \beta C_t^{\sigma} E_t [L(b_t, \epsilon_{t+1})]) \\ + \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma} - G_t - tr \end{array} \right]
\end{aligned} \tag{17}$$

where the model equilibrium also requires us to define bond prices, $P_t^M = \beta C_t^{\sigma} E_t [L(b_t, \epsilon_{t+1})]$ since these are embedded in the auxiliary function $L(b_t, \epsilon_{t+1})$. The policy maker optimizes (17) by choosing C_t, G_t, Y_t, Π_t, τ_t, b_t and the multipliers, λ_{1t}, λ_{2t}, λ_{3t}. It should be noted that even though the policy maker optimizes with respect to all endogenous variables, they are not acting as a social planner. Instead, they are choosing standard policy instruments in order to influence the decentralized equilibrium in a manner which maximizes their objective function subject to the time-consistency constraint. The FOCs for the policy problem are detailed below.

The discretionary equilibrium is determined by the system given by the FOCs, the constraints in (17), the auxiliary equations, (15) and (16), bond prices, $P_t^M = \beta C_t^{\sigma} E_t [L(b_t, \epsilon_{t+1})]$, and finally the exogenous process for the markup shock, (3). The solution to this system is a set of time-invariant Markov-perfect equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \epsilon_t\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$ for all $t \geq 0$.

Further insight into the trade-offs facing the policy maker can be generated by considering

$$
\begin{array}{ll}
C_t & C_t^{-\sigma} - \lambda_{1t} + \lambda_{2t}\left[\sigma\epsilon_t(1-\tau_t)^{-1}Y_t^{\varphi}C_t^{\sigma-1} + \sigma\phi\beta C_t^{\sigma-1}Y_t^{-1}E_t\left[M(b_t,\epsilon_{t+1})\right]\right] \\
 & +\lambda_{3t}\left[\sigma\beta b_t C_t^{\sigma-1}E_t\left[L(b_t,\epsilon_{t+1})\right] - \rho\sigma\beta\frac{b_{t-1}}{\Pi_t}C_t^{\sigma-1}E_t\left[L(b_t,\epsilon_{t+1})\right] + \sigma\left(\frac{\tau_t}{1-\tau_t}\right)(Y_t)^{1+\varphi}C_t^{\sigma-1}\right] = 0 \\
Y_t & -Y_t^{\varphi} + \lambda_{1t}\left[1-\frac{\phi}{2}\left(\Pi_t-1\right)^2\right] + \lambda_{3t}\left[(1+\varphi)Y_t^{\varphi}C_t^{\sigma}\left(\frac{\tau_t}{1-\tau_t}\right)\right] \\
 & +\lambda_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma} - \phi\beta C_t^{\sigma}Y_t^{-2}E_t\left[M(b_t,\epsilon_{t+1})\right]\right] = 0 \\
\tau_t & \epsilon_t\lambda_{2t} + \lambda_{3t}Y_t = 0 \\
G_t & \chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \\
\Pi_t & -\lambda_{1t}\left[Y_t\phi\left(\Pi_t-1\right)\right] - \lambda_{2t}\left[\phi\left(2\Pi_t-1\right)\right] + \lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}\left(1+\rho\beta C_t^{\sigma}E_t\left[L(b_t,\epsilon_{t+1})\right]\right)\right] = 0 \\
b_t & -\widetilde{\beta}E_t\left[\lambda_{3t+1}\frac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] + \lambda_{2t}\left[\phi\beta C_t^{\sigma}Y_t^{-1}E_t\left[M_1(b_t,\epsilon_{t+1})\right]\right] \\
 & +\beta\lambda_{3t}\left[C_t^{\sigma}E_t\left[L(b_t,\epsilon_{t+1})\right] + b_tC_t^{\sigma}E_t\left[L_1(b_t,\epsilon_{t+1})\right] - \rho\frac{b_{t-1}}{\Pi_t}C_t^{\sigma}E_t\left[L_1(b_t,\epsilon_{t+1})\right]\right] = 0
\end{array}
$$

specific FOCs, which can also be contrasted with those implied by commitment outlined above. The FOC for taxation,

$$\epsilon_t\lambda_{2t} + \lambda_{3t}Y_t = 0$$

identical under both commitment and discretion, reveals a key feature of the underlying policy problem. In the absence of a need to satisfy the budget constraint through distortionary taxation, $\lambda_{3t} = 0$, the tax instrument would be used to eliminate the costs associated with the output-inflation trade-off implicit in the NKPC, $\lambda_{2t} = 0$. In other words, if it were not for the need to raise tax revenues to satisfy the government's budget constraint, taxes could be adjusted to eliminate any undesired movements in inflation arising from mark-up shocks.

Similarly, the FOC for inflation highlights the nature of the inflationary bias contained in the model,

$$
\begin{aligned}
0 \;=\; & -\lambda_{1t}\left[Y_t\phi\left(\Pi_t-1\right)\right] - \lambda_{2t}\left[\phi\left(2\Pi_t-1\right)\right] \\
& +\lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}\left(1+\rho P_t^M\right)\right]
\end{aligned}
\tag{18}
$$

The first two terms of the FOC capture the standard inflationary bias problem. The first term measures the costs of raising inflation, and the second term the output benefits of doing so

(given inflationary expectations) which are evaluated positively when the economy operates at a suboptimally low level due to tax and monopolistic competition distortions. However, in the presence of debt the third term in the FOC for inflation captures an additional reason for wanting to raise inflation relative to expectations - the erosion of the real value of debt. Economic agents will anticipate that higher debt increases the government's desire to introduce inflation surprises, implying that inflationary expectations in the NKPC are increasing in the level of government debt, $E_t\left[M_1(b_t, \epsilon_{t+1})\right] > 0$ until inflation is sufficiently high to eliminate policy surprises (in the absence of further shocks). The state dependence of the inflationary bias will be key in driving the policy maker's desire to reduce debt relative to what would be observed under a time-inconsistent Ramsey policy - a tendency we label the "debt stabilization bias".

The three terms in (18) are common to the FOC for inflation under both discretion and commitment, but where the latter contains the following additional terms,

$$+\lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1} \left(C_t\right)^{-\sigma} Y_t(2\Pi_t - 1)]$$
$$+\lambda_{4t-1}\left[\left(C_{t-1}\right)^{\sigma}\left(C_t\right)^{-\sigma}\Pi_t^{-2}(1+\rho P_t^M)\right]$$

The first captures the extent of the policy maker's commitment not to raise inflation in an attempt to raise output; this commitment reduces inflationary expectations. The second their commitment not to use inflation to reduce bond prices. The numerical analysis below suggests that the state-dependent inflationary bias is significant, but that, if able, the policy maker would largely commit to not using inflation as a device to stabilize debt.

The remaining key FOC is for government debt which highlights the "debt stabilization bias". This bias can be understood by considering the FOC for debt, which can be simplified as,

$$\underbrace{P_t^M \lambda_{3t} - \widetilde{\beta} E_t\left[\frac{\lambda_{3t+1}}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right]}_{\text{tax smoothing}}$$

$$\underbrace{-\lambda_{3t}C_t^{\sigma}\left(\phi\epsilon^{-1}\beta E_t\left[M_1(b_t,\epsilon_{t+1})\right]-\left[(b_t-\rho\frac{b_{t-1}}{\Pi_t})E_tL_1(b_t,\epsilon_{t+1})\right]\right)}_{\text{debt stabilization bias}}=0 \tag{19}$$

Equation (19) describes the policy maker's optimal debt policy which can be decomposed into two elements. The first line gives the optimal trade-off between current and future distortions associated with the need to satisfy the government's intertemporal budget constraint - tax smoothing. These terms are present under both commitment and discretion. The second line, captures wedges which are introduced when the policy maker is unable to commit, defining the debt stabilization bias.[7] It is helpful to discuss the implications of 'tax smoothing', before assessing how that policy is impacted by the 'debt stabilization bias' generated by an inability to commit.

The tax-smoothing argument in Barro (1979), requiring that the marginal costs of taxation are smoothed over time, is reflected in the relationship between λ_{3t} and λ_{3t+1} in the first line of (19). Initially assume the policy maker is not myopic, so $\widetilde{\beta}=\beta$. In this case, when the return (adjusted for any covariance with the future costs of satisfying the government's intertemporal budget constraint, λ_{3t+1}) on holding the government bonds is equal to the household's rate of time preference, the distortions associated with satisfying the budget constraint are constant in steady-state and steady-state debt will follow a random walk. Effectively, under tax smoothing, the policy maker trades-off the short-run costs of reducing the stock of debt against the discounted value of the long-term benefits of lower debt. When debt service costs are consistent with the household/government's rate of time preference, as they are in steady-state, these will be exactly balanced at a debt level which depends upon the history of the shocks hitting the economy.

Reintroducing myopia, such that $\widetilde{\beta}<\beta$, implies that when real interest rates differ from the policy maker's rate of time preference, then the policy maker will choose to tilt these

[7] The remaining FOCs determine the policy mix employed to achieve the debt dynamics implied by the debt stabilization bias, (19) which, in turn, is driven by the state-dependent inflationary bias problem, (18).

distortions backwards (forwards) in time depending on whether debt service costs are below (above) the policy maker's rate of time preference. For example, when the real rate of return on debt, $r_t^b \equiv E_t\left[\frac{1}{\Pi_{t+1}}\frac{(1+\rho P_{t+1}^M)}{P_t^M}\right] = \beta^{-1}$, this implies $E_t\left[\frac{\lambda_{3t+1}}{\lambda_{3t}}\frac{1}{\Pi_{t+1}}\frac{(1+\rho P_{t+1}^M)}{P_t^M}\right] = \widetilde{\beta}^{-1} > \beta^{-1}$ such that λ_{3t} is rising over time. The myopic policy maker would allow debt to rise.

However under discretion, (19) is a generalized Euler equation, which, in the second line, includes partial derivatives of policy functions with respect to debt due to the time-consistency requirement. In general the form of these auxiliary functions is unknown, which is why the policy problem needs to be solved numerically. However, that numerical solution robustly gives clear signs for these derivatives, $M_1(b_t, \epsilon_{t+1}) > 0$ and $L_1(b_t, \epsilon_{t+1}) < 0$ which have an intuitive interpretation.

The first term on the second line of (19) reflects the fact that inflation expectations rise with debt levels (through the inflation biases discussed above - see the FOC for inflation), $M_1(b_t, \epsilon_{t+1}) > 0$, and since this is costly in the presence of nominal inertia, there is a desire to deviate from tax smoothing, in order to reduce debt and the associated increase in inflation. This is the first reason for wanting to reduce debt relative to the level that would be supported by a benevolent Ramsey planner.

The second term in square brackets in the second line captures the impact of higher debt on bond prices. Since higher debt raises inflation, which in turn reduces bond prices, $L_1(b_t, \epsilon_{t+1}) < 0$, this term also serves to encourage a reduction in debt levels, when debt is relatively short-term. Why? High, but falling debt levels imply an upward trend in bond prices which makes it cheaper to issue new debt, but more costly to buy-back the existing debt stock. As debt maturity is increased, the latter effect rises relative to the former, and hence the desire to reduce debt levels is reduced, ceteris paribus. This trade-off between tax-smoothing and time-consistency determines the equilibrium level of debt and inflation, where inflation is expected to be closer to zero as debt maturity rises, for a given level of debt.[8]

[8]For completeness the opposite cases $M_1(b_t, \epsilon_{t+1}) < 0$ and $L_1(b_t, \epsilon_{t+1}) > 0$, should be considered. However

4. Solution Method and Calibration

For the model described in the previous section, the equilibrium policy functions cannot be computed in closed form and local approximation methods are not applicable, as the model's steady state around which local dynamics should be approximated is endogenously determined as part of the model solution and thus is a priori unknown. This necessitates the use of global solution methods. Specifically, the Chebyshev collocation method with time iteration. The detailed algorithm is presented in Appendix G. In general, optimal discretionary policy problems can be characterized as a dynamic game between the private sector and successive governments. Multiplicity of equilibria is a common problem in dynamic games of this kind. Since the solution algorithm uses polynomial approximations, it is, in effect, searching only for continuous Markov-perfect equilibria where agents condition their strategies on payoff-relevant state variables, see Judd (2004) for a discussion.

Before solving the model numerically, the benchmark values of structural parameters must be specified. The calibration of parameters is summarized in Table A.1. We set $\beta = (1/1.02)^{1/4} = 0.995$, which implies a 2% annual real interest rate. The intertemporal elasticity of substitution is set to one half ($\sigma = \sigma^g = 2$) which is in the middle of standard estimates.[9] The Frisch labor supply elasticity is set to $\varphi^{-1} = 1/3$. The steady-state elasticity of substitution between intermediate goods is chosen as $\bar{\epsilon} = 14.33$, which implies a monopolistic markup of approximately 7.5%, similar to Siu (2004), and in the middle of conventional estimates.

The fiscal variables are calibrated to ensure the benchmark model mimics the key ratios in U.S. data over the period 1954-2008 as discussed in Appendix C and reported in the first column of Table A.2. Parameter $\chi = 0.0076$ ensures government consumption is 7.8%

this would imply that higher debt reduced inflationary expectations and raised bond prices. This in turn would encourage the policy maker to deviate from the policy of tax smoothing by raising rather than lowering debt. This non-intuitive case is not something ever observed in the numerical analysis.

[9] In the robustness exercises conducted in the online Appendix N the elasticity for public spending is lowered in line with the evidence in Debortoli and Nunes (2013). However, this does not affect the key results.

of GDP, transfers are set to be 9% and the myopia of the policy maker is set to $\widetilde{\beta} = 0.982$ (an effective time horizon of just under 20 years) which supports an annualized steady-state debt-to-GDP ratio of 31%. The coupon decay parameter, $\rho = 0.95$, corresponds to around 5 years of debt maturity, consistent with U.S. data. The implied ratio of tax revenues to GDP in steady-state is slightly higher than the data average of 17.5% reflecting the fact that actual policy has often run a deficit in recent decades.

The price adjustment cost parameter, $\phi = 50$, implies, given the equivalence between the linearized NKPCs under Rotemberg and Calvo pricing (see Leith and Liu, 2016), that on average firms re-optimize prices every six months - in line with empirical evidence. Finally, the cost-push shock process is parameterized as $\rho_\epsilon = 0.939$ and $\sigma_\epsilon = 0.052$ in line with estimates in Chen et al. (2017) and Smets and Wouters (2003).

With this benchmark parameterization, the model solution generates a maximum Euler equation error over the full range of the grid is of the order of 10^{-6}. We plot these errors in Appendix H. As suggested by Judd (1998) , this order of accuracy is reasonable.[10]

5. Numerical Results

This section explores the properties of the equilibrium under optimal time-consistent policy. Subsection 5.1 considers the steady-state under a series of alternative parameterizations. Subsection 5.2 contrasts the policy response to shocks under commitment and discretion, and how debt maturity affects those differences. Subsection 5.3 does the same for the level of debt and the debt-maturity decision is endogenized in subsection 5.4.

5.1. Steady State

Table A.2 summarizes the steady state values for a variety of parameterizations, contrasting them with the data averages contained in column 1. The analysis begins with the benchmark calibration after temporarily removing policy maker myopia such that $\widetilde{\beta} = \beta$ -

[10] All other model variants considered are equally well approximated - these results are available upon request.

column 3 of Table A.2. The key trade-offs underpinning this steady-state equilibrium can been seen by considering the (deterministic) steady-state value of FOC for debt, equation (19),

$$b(1-\rho\frac{1}{\Pi})L_1(b,\overline{\epsilon}) = \phi\overline{\epsilon}^{-1}\beta M_1(b,\overline{\epsilon}) \tag{20}$$

As noted above, the numerical solution of the policy problem implies $L_1(b,\overline{\epsilon}) < 0$ and $M_1(b,\overline{\epsilon}) > 0$. Assuming $\rho < \Pi$, this equation can only hold with a negative debt stock.[11] This is indeed what happens with $\frac{bP^M}{4Y} = -153\%$ and a steady-state inflation rate of -1.1%. The reason for the deflation can be seen from the FOC for inflation, equation (18),

$$\lambda_1\left[Y\phi\left(\Pi-1\right)\right] = \lambda_3\left[Y\phi\epsilon^{-1}\left(2\Pi-1\right)+\frac{b}{\Pi^2}\left(1+\rho\frac{\beta}{\Pi-\rho\beta}\right)\right] \tag{21}$$

which equates the resource costs of a marginal increase in inflation with the marginal benefits in terms of higher output and reduced debt of an inflation surprise. For a positive value of debt and suboptimally low level of equilibrium output, the inflation bias will be positive. However, as debt turns negative the marginal benefits of inflation surprises fall as this reduces the value of the government's assets. If debt turns sufficiently negative, the equilibrium supports a steady-state deflation which ensures the policy maker is not tempted to introduce any further surprise deflation to increase the value of the assets she has accumulated. As a result the accumulated assets fall short of the war chest level needed to support the first best allocation.[12]

Introducing policy maker myopia can overturn this result - see the second column of Table A.2, labeled "benchmark". The intuition is as follows - within line one of (19) the myopic policy maker weighs the costs of debt reduction more than the long-term benefits, thereby tilting the tax smoothing element of optimal debt policy towards rising debt levels. This is

[11] No parameter permutations have been found which imply $\rho > \Pi$ such that the model without myopia can sustain a positive steady-state debt stock. Intuitively, unless debt stocks are negative, the economy remains sufficiently distorted that the inflationary bias problem ensures $\Pi > \rho$.

[12] The war chest asset stock would be 4,636% of GDP - see Appendix E.

then balanced against the existing debt stabilization bias to deliver a higher equilibrium level of debt, cet. par.. By introducing myopia, the benchmark has been calibrated to replicate a positive debt-to-GDP ratio of 31% and government consumption to output of 7.8%. The steady-state rate of inflation this implies is 3%. The key equation defining this steady-state is the FOC for debt given by

$$b(1-\rho\frac{1}{\Pi})L_1(b,\bar{\epsilon}) = \phi\bar{\epsilon}^{-1}\beta M_1(b,\bar{\epsilon}) - C^{-\sigma}P^M(1-\frac{\widetilde{\beta}}{\beta}) \tag{22}$$

where the myopia can turn the RHS of this condition negative, thereby supporting a positive steady-state debt-to-GDP ratio. It is notable that this change does little to affect the other key fiscal ratios of government consumption and taxation relative to GDP. Column 4 increases policy maker myopia further to $\widetilde{\beta} = 0.975$, which is equivalent to reducing the policy maker's time horizon from 20 to 12 years. This more than doubles the steady-state debt-to-GDP ratio to 75.6% and inflation rises to 4.5%.

Increasing the flexibility of prices means both that the costs of inflation are lower and that monetary policy has smaller effects on the real economy - see the first two terms in the FOC for inflation, (18), respectively. This has the effect of making the inflationary bias problem less costly which reduces the debt-stabilization bias. As a result the government is able to sustain a higher debt-to-GDP ratio which rises by 5.5%, as they are less driven to reduce the state-dependent inflationary bias problem. This leads to a larger steady-state rate of inflation of 3.8%, but it should be remembered that inflation is now less costly, so that moderates the inflationary bias problem.

Finally, reducing the mark-up (from 7.5% to 5% in the final column of Table A.2) is important since it implies the inflationary bias problem is lower for a given level of debt. (The gains to a surprise inflation are lower, when the economy is less distorted - see the impact of a higher value of ϵ in the second term of the FOC for inflation, (18)). As a result the desire to influence the state-dependent inflationary bias problem by reducing debt is less

- the debt stabilization bias has been reduced. This substantially increases the steady-state debt-to-GDP ratio to almost 90% and the steady-state rate of inflation to 3.7%.

Table A.3, considers the impact of changes in the maturity structure of debt. Column 1 adopts the common assumption that debt is only of a single period's duration (one quarter in the context of the model parameterization). In this case the steady-state debt-to-GDP ratio turns negative, -11% and inflation is 3.5%. Increasing debt maturity to 30 years leads to a significant increase in the debt-to-GDP ratio to over 102% of GDP and inflation to over 5%. This reflects the discussion above - longer maturity debt reduces the debt stabilization bias allowing the government to sustain a higher steady-state debt-to-GDP ratio.

In summary, myopia, monopolistic competition distortions and debt maturity are the key drivers of the equilibrium rate of inflation and debt-to-GDP ratio, while other endogenously determined steady-state fiscal ratios are largely unaffected by these changes. This highlights the importance of the state-dependent inflationary bias and the associated debt stabilization bias in jointly determining the equilibrium outcomes for inflation and debt.

5.2. Responding to Shocks - Debt Maturity

This subsection contrasts how the policy maker responds to shocks, under both commitment and discretion with either single-period or long-term debt.[13] Figure B.1 plots the outcomes for key variables following a rise in the markup $\frac{\varepsilon_t}{\varepsilon_t - 1}$ by 0.5%. Under commitment the policy maker cuts taxes to largely offset the shock, but in the long-run slightly raises taxation in order to sustain (but not reverse) the higher stock of debt that emerges as a result. There is a very limited use of surprise inflation in the short-run to reduce the need to increase taxes in the long-run, but this is small. Debt maturity has a negligible impact, only facilitating a more gradual use of inflation to stabilize debt, but barely noticeably.[14]

[13]The time-consistent policy problem with single-period debt has a raised degree of myopia to ensure it shares the same steady-state debt-to-GDP ratio as the benchmark model.

[14]See Leeper and Leith (2016) for a discussion of how surprise inflation can contribute to the stabilization of debt of different maturities.

The policy outcome under discretion is radically different. The case of single period debt is represented by the red dash-dotted line in Figure B.1.[15] Although tax cuts could in theory offset the inflationary consequences of the mark-up shock as under commitment, this would exacerbate the increase in debt which drives the inflationary bias problem discussed above. As a result the policy maker *raises* tax rates to ensure that debt falls as a more effective way of mitigating the inflationary consequences of the mark-up shock. Nevertheless the higher tax rates and mark-up shock do increase inflation and monetary policy is tightened to help offset that.[16] The end result is that the response to the mark-up shock is overwhelmingly driven by the desire to reduce debt through tax increases and thereby mitigate the state-dependent inflationary bias problem. Government spending largely moves in line with output such that there is negligible variation in the ratio of G/Y- government consumption is hardly used as an instrument of either macroeconomic or fiscal stabilization.[17]

Although debt maturity has little impact on policy outcomes under commitment it matters a lot under discretion. Longer-term debt significantly reduces the debt-stabilization bias such that the steady-state rate of inflation is significantly lower when debt is of longer maturity (falling by 3%). Moreover, the reduction in the debt-stabilization bias with longer maturity debt also reduces the desire of the policy maker to offset the fiscal repercussions of the mark-up shock, resulting in much more moderate tax increases and monetary policy tightening. As a result debt falls by less than in the case of single period debt.[18]

[15]To ensure comparability, myopia has been increased in the case of single period debt to ensure the steady-state debt-to-GDP ratio is the same as the benchmark model.

[16]The gross quarterly real interest rate is defined as $E_t[R_t/\Pi_{t+1}]$ and is plotted in this, and subsequent figures, as a net annualized percentage.

[17]If the intertemporal elasticity of substitution for government consumption is reduced to $\sigma_g = 1$ in line with the evidence summarized in Debortoli and Nunes (2013), the standard deviation of the G/Y ratio rises from 0.8% to 1.4%, which is closer to the data average of 1.9%. However, this does not have a significant impact on any of the experiments conducted in the paper other than to marginally enhance the role played by government consumption. See Appendix N.

[18]Appendix J considers the impact of a government spending shock. This shows a similar pattern of response - under commitment a slight rise in taxation is sufficient to stabilize the debt stock at a permanently higher level, while monetary policy tightens to effectively eliminate inflation. In contrast, discretionary policy acts to reduce the inflationary impact of the shock by reducing debt (and the associated inflationary bias problem) through substantial tax increases, while moderating the tightening of monetary policy.

5.3. Responding to Shocks - Level of Debt

This subsection explores the impact of the level of debt on policy outcomes under commitment and discretion, examining the same mark-up shock as above, but with steady-state levels of debt-to-GDP of 51.5% and 15.8%, respectively. These levels capture the peaks and troughs of the US debt-to-GDP ratio following World War II - see Figure K.1 in Appendix K. This appendix introduces switches in the degree of policy-maker myopia which enables us to track these movements.[19] However, here the focus is on how debt levels affect the policy response to shocks under commitment and discretion.

Since, under commitment, steady-state debt follows a random walk, when analyzing commitment initial steady-state conditions consistent with these two debt levels are adopted. In contrast, under discretion different steady-state debt-to-GDP ratios can be considered by adopting a high or low myopia regime. Therefore, four scenarios are considered - the impact of a rise in the markup $\frac{\varepsilon_t}{\varepsilon_t - 1}$ of 0.5% under commitment and discretion starting from steady-states with either high or low levels of debt.

The policy response under commitment is essentially the same as before, regardless of the level of debt - see Figure B.2. In the short to medium term, taxes fall to offset the mark-up shock, but eventually rise to sustain the higher stock of debt that emerges as a result. Since there is a negligible tightening of monetary policy, debt dynamics across high and low debt levels are largely unaffected and the rise in the debt-to-GDP ratio as a result of the shock is similar across debt levels.

Under discretion we again find that taxes actually rise following the mark-up shock generating a sustained fall in government debt as a more effective way of moderating inflation than cutting taxes and offsetting the cost-push shock directly. Here, the fact that high debt levels worsen the inflationary bias problem means that the desire to reduce debt is more

[19] Conventional economic shocks cannot mimic the data in this respect. The standard deviation of the annualized debt-to-GDP ratio is only 0.7% under the benchmark calibration despite the equivalent volatility in the data being 9%. Even allowing for temporarily unstable paths for transfers as in Bi et al. (2013) cannot generate data-consistent movements in debt-to-GDP ratios.

pronounced the higher the level of debt. Therefore, higher debt levels and, as shown above, shorter debt maturity are more likely to give rise to perverse tax and debt policy responses which move in the opposite direction to those observed under commitment.

5.4. Debt Management and the Debt Stabilization Bias

Up until this point, the level of debt maturity has been held fixed by parameterizing ρ. This subsection allows for the policy maker to have some control over the maturity structure as part of the time-consistent optimal policy problem, by allowing them to issue a mixture of short and long-maturity debt, possibly of opposite signs (i.e. one can be held as an asset and the other a liability). Before adding this extra element to the benchmark model, in order to identify the trade-offs facing the policy maker in such an environment a simple three period perfect foresight model is analyzed where the policy maker chooses the mix of one and two period bonds issued in the first period.

5.4.1. 3 Period Model

Appendix L derives the model and policy problem in full. In period $t = 0$ the government can issue a mixture of one and two period bonds following the budget constraint,

$$Q_{0,1}b_{0,1} + Q_{0,2}b_{0,2} = -\tau_0 + \zeta_0 + b_{-1,0}\nu_0 + Q_{0,1}b_{-1,1}\nu_0 \tag{23}$$

where $\nu_t = \Pi_t^{-1}$ is the inverse of the gross rate of inflation, $Q_{t,t+j}$ is the price of zero coupon debt in period t, which matures in period $t + j$ and the state variables are defined as, $b_{j,k} \equiv B_{j,k}/P_j$ reflecting the quantity of zero coupon nominal bonds issued in period j which mature in period k, deflated by the price level in period j. The perfect foresight equilibrium path follows an initial perturbation generated by transfers in period 0 being $\zeta_0 > 0$, relative to their value of zero in all other periods. Taxes are τ_0 and there is no government consumption.

The economy is an endowment economy with no government consumption such that private consumption always equals its endowment and bond pricing equations are given by,

$Q_{t,t+1} = \beta\nu_{t+1}$ and $Q_{t,t+2} = \beta^2\nu_{t+1}\nu_{t+2}$, for one and two-period bonds respectively. The government budget constraints in periods $t = 1$ and 2, are given, respectively, by,

$$Q_{1,2}b_{1,2} = -\tau_1 + \nu_1 b_{0,1} + Q_{1,2}\nu_1 b_{0,2} \tag{24}$$

and,

$$\tau_2 = \nu_2 b_{1,2} \tag{25}$$

Therefore in the second period, $t = 1$, the government can only issue one period bonds and in the final period the government must repay all outstanding debt. Combining the flow budget constraints and bond pricing equations yields the government's intertemporal budget constraint,

$$\zeta_0 + b_{-1,0}\nu_0 + b_{-1,1}\nu_0\nu_1 = \tau_0 + \beta\tau_1 + \beta^2\tau_2$$

which shows that inflation in periods $t = 0$ and 1 acts upon the value of one- and two-period bonds inherited from period $t = -1$, but that the transfers shock ζ_0 needs to be fully funded by taxation.

Following Leeper and Leith (2016) it is assumed that inflation and taxation are costly, such that social welfare is given by,

$$-E_0 \sum_{t=0}^{2} \beta^t \left(\tau_t^2 + \theta(\nu_t - 1)^2\right) \tag{26}$$

The parameter θ captures the relative cost of inflation - a lower value of θ would map to a reduced inflationary bias problem in our benchmark model through more myopia, less price stickiness or lower markups.

Commitment

Appendix L derives the optimal policy under commitment which implies perfect tax smoothing,

$$\tau_0 = \tau_j \text{ for } j = 1, 2$$

and a pattern of inflation across each period given by,

$$\nu_2 = 1$$

$$-\theta(\nu_1 - 1) = \tau_0 \nu_0 b_{-1,1}$$

and

$$-\theta(\nu_0 - 1) = \tau_0(b_{-1,0} + \beta \nu_1 b_{-1,1})$$

The policy maker commits to zero net inflation in period $t = 2$, and only introduces inflation ($\nu_t < 1$) in periods $t = 0$ and 1 to the extent that she inherits a debt stock which matures in those periods. In the absence of an initial debt stock, $b_{-1,0} = b_{-1,1} = 0$, there would be no net inflation and the policy maker would finance the transfers perturbation solely through taxation, $\zeta_0 = \tau(1+\beta+\beta^2)$ where τ is the tax-smoothing tax rate applied in each of the three periods. This outcome can be contrasted with that which emerges under the time-consistent policy.

Discretion

The time-consistent policy is solved in Appendix L by backward induction. In period $t = 2$ the policy maker maximizes period 2 welfare subject to the budget constraint, implying that the optimal policy mix is given by,

$$-\theta \nu_2(\nu_2 - 1) = \tau_2 b_{1,2} \nu_2 \tag{27}$$

This describes the debt-driven inflationary bias - higher levels of debt inherited in period $t = 2$ raise inflation, more so for lower values of θ which imply a reduction in the relative costs of inflation. It should be noted that this inflation does not serve to reduce the real value of debt as it will already have been factored into bond prices when the debt was issued in period 1. Instead the taxes needed to pay off the debt are given by, $\tau_2 = \nu_2 b_{1,2}$.

In period $t = 1$ the policy maker conducts a similar optimization, but treats the period

$t = 2$ policy mix, equation (27), as an Incentive Compatibility Constraint (ICC) in their optimization. The resultant FOCs are,

$$-\theta\nu_1(\nu_1 - 1) = \tau_1(b_{0,1}\nu_1 + \beta\nu_1\nu_2 b_{0,2}) \tag{28}$$

$$\tau_2 = \tau_1(2\nu_2 - 1) + \frac{2\tau_1\tau_2\nu_1 b_{0,2}}{\theta} \tag{29}$$

The first has the same interpretation as above - the higher the level of debt inherited the greater the inflation and taxation. The second expression guides the period 1 policy maker's optimal rate of debt reduction in order to achieve the desired balance between the current and next period policy mix. If, $b_{0,2} = 0$ and the period $t = 0$ policy maker only issued single-period debt, the period $t = 1$ policy maker would tax more today than the policy maker is required to tax tomorrow, $\tau_1 > \tau_2$ since $\nu_2 < 1$, that is, the debt-stabilization bias causes the policy maker to reduce debt more quickly than tax-smoothing would imply. This rate of correction will be higher as debt levels rise. However, the period $t = 0$ policy maker can influence that behavior by changing the quantity of two period debt they issue. Again, these FOCs will serve as additional ICCs on the period $t = 0$ policy maker.

Now consider the period $t = 0$ policy maker who maximizes the welfare objective (26) subject to the series of budget constraints (23)-(25) and the three ICCs (27)-(29) generated by the policy makers' choices in periods $t = 1$ and $t = 2$. The set of FOCs this implies is detailed in Appendix L. The policy maker will deliver inflation in period $t = 0$ in a similar way to the subsequent policy makers,

$$-\theta\nu_0(\nu_0 - 1) = \tau_0(\nu_0 b_{-1,0} + \beta\nu_0\nu_1 b_{-1,1}) \tag{30}$$

It is convenient to simplify this and the other FOCs by considering the case where there is no initial stock of debt, $b_{-1,0} = b_{-1,1} = 0$, but the policy maker has to finance a transfers shock, $\zeta_0 > 0$. In this case there would be no inflation in period $t = 0$, $\nu_0 = 1$ and the transfers shock must be entirely financed through taxation, $\zeta_0 = \tau_0 + \beta\tau_1 + \beta^2\tau_2$. In subsequent

periods the inflation generated depends upon the quantity of debt remaining, as described by equations (27) and (28), but this does not actually contribute to the financing of that debt as it simply reflects the inflationary bias problem generated by a desire to reduce debt levels through inflation surprises.

The period $t = 0$ policy maker has an additional policy instrument with which to influence the future - the maturity structure of the debt they leave to the future. Appendix L shows they will choose $b_{0,2}$ to ensure the second ICC generated by the $t = 1$ policy maker's choices (29), does not constrain the time 0 policy problem. Instead, Appendix L demonstrates that the first period policy maker will achieve the following pattern of taxation over time,

$$\begin{aligned} \tau_0 &= \frac{1}{2\nu_1 - 1}\tau_1 + \frac{\beta(1-\nu_1)}{2\nu_1 - 1}\tau_2 \\ \tau_0 &= \frac{(1-\nu_1)}{2\nu_1 - 1}\tau_1 + \frac{1}{2\nu_2 - 1}\tau_2 \end{aligned}$$

which can be equated to yield,

$$\tau_2 = (2\nu_1 - 1)\tau_1 + (1-\nu_1)(1+\beta(2\nu_2 - 1))\nu_1^{-1}\tau_2 \tag{31}$$

Despite an inability to commit, the period $t = 0$ policy maker can achieve this desired evolution of policy by issuing an appropriate amount of two-period debt such that ICC (29) is isomorphic to this expression. How debt maturity is used by the period $t = 0$ policy maker can be seen by contrasting (31) with what would be chosen by the period $t = 1$ policy maker in an environment with only single period debt, $\tau_2 = \tau_1(2\nu_2 - 1)$. This implies that the first policy maker wishes the period $t = 1$ policy maker to reduce debt by less, delaying some of the fiscal adjustment to period $t = 2$. By issuing two-period bonds they, therefore, reduce the debt-stabilization bias in period $t = 1$, levy less taxation and, likely, mitigate the inflation bias too - see equation (28).

The complete equilibrium is shown in Figure B.3 which contrasts outcomes under com-

mitment and discretion (as well as the case of time-consistent policy with only short-term debt) as a function of θ. For all values of θ considered, an inability to commit means that the reduction in debt is front loaded in period $t = 0$ as a result of the debt-stabilization bias. As described above, with only single period debt, debt continues to be stabilized aggressively with tax rates falling over time. In contrast the ability to issue two period debt reduces the inflationary and debt-stabilization biases in period $t = 1$, allowing the policy maker to reduce taxes in that period without being adversely impacted by higher inflation. This slowing of debt stabilization in period $t = 1$ then results in higher taxes and inflation in the final period. As the costs of inflation increase, the desire to lengthen debt maturity to slow the pace of debt reduction in period $t = 1$ is reduced and the stock of short-term debt switches from negative to positive, thereby reducing overall debt maturity.

5.4.2. Full Model

In order to assess whether or not the benchmark model exhibits the same properties we augment it to include single period debt alongside the longer-maturity debt enabling the policy maker to adjust the average maturity of the debt stock. The wealth of the existing bondholders entering period t is now $D_t \equiv (1 + \rho P_t^M)B_{t-1}^M + B_{t-1}^S$, the household then buys bonds, $P_t^M B_t^M + P_t^S B_t^S$ and as a result the government's budget constraint becomes,

$$P_t^M b_t^M + P_t^S b_t^S = \frac{b_{t-1}^S}{\Pi_t} + (1 + \rho P_t^M)\frac{b_{t-1}^M}{\Pi_t} - \frac{W_t}{P_t} N_t \tau_t + G_t + tr$$

The remainder of the policy problem is unchanged, except for the fact that policy functions now have three arguments, the elasticity of substitution between goods, ϵ_t, and the levels of both maturities of bond, b_{t-1}^S and b_{t-1}^M. Appendix M derives the resultant FOCs. The implication of the analysis above is that in the extended benchmark model any parameter change which reduces the inflationary bias problem is likely to result in an increase in the proportion of long-term debt if the policy maker is given the opportunity to issue both short

and long-term debt. Therefore, we conjecture that reduced price stickiness and markups and greater myopia should all lead to a greater reliance on long-term debt, and may even result in the policy maker accumulating a short-term asset in order to leverage the benefits of issuing long-term debt.

Table A.4 contains steady-state debt-to-GDP ratios for the benchmark model alongside variants which allow the policy maker to simultaneously issue short-term debt (possibly in negative quantities). Comparing the first two columns it can be seen that for the benchmark calibration the costs of inflation are sufficiently high that the policy chooses to shorten maturity in line with the results for high values of θ above. Instead if we reduce the degree of price stickiness in column three, the desire to leverage long-term debt as a means of reducing the debt stabilization bias becomes apparent and the government accumulates short-term assets in order to raise the proportion of longer-term debt. The next column reduces the mark-up which, by reducing the inflationary bias problem, ceteris paribus, also allows the policy maker to lengthen maturity by reducing the proportion of short-term debt in overall debt. Finally, increasing the myopia of the policy maker reduces the debt-stabilization bias and encourages the policy maker to issue more long-term debt.

The use of debt maturity in this way also occurs in response to shocks. Figure B.4 considers the response to a markup shock when the policy maker can issue short-term debt. The increase in the inflation bias caused by the rise in the markup reduces the current policy maker's desire to delay future debt reduction and so they issue relatively more short-term debt. Outside of the initial period, this causes the policy maker to moderate the rise in taxation (which is otherwise inflationary) resulting in a medium term increase in the debt-to-GDP ratio in contrast to the benchmark model. In the first period the policy maker undertakes a sharp tightening of monetary policy which induces a fall in bond prices, making it cheaper for the government to retire those bonds. The fiscal consequences of this are offset by an associated rise in taxation in the initial period.

6. Conclusions

The existence of nominal debt induces a state-dependent inflation bias problem as the policy maker wishes to utilize inflation surprises to offset monopolistic competition and tax distortions and reduce the real value of debt. This temptation is greater with higher debt levels and shorter debt maturity, resulting in a debt stabilization bias as the policy maker deviates from Ramsey policy by returning debt to steady-state to mitigate the associated inflation biases.

The response to shocks in such an environment seeks to avoid exacerbating these biases, and is radically different from policy under commitment as a result. Endogenizing the debt maturity decision gives the current policy maker an additional tool through which to influence the pace of future debt stabilization - lengthening debt maturity when the underlying costs associated with the inflation bias are reduced and vice versa.

The dependence of the inflationary bias on both the level and maturity of government debt highlighted by the paper, implies an obvious area for future research would be to explore how monetary policy institutions can be insulated from such effects, given the apparent inability of central banks to commit (see Chen et al. (2017)) means that even an independent monetary authority has entered into a strategic game with the fiscal policy maker.

References

Barro, R. J. (1979). On the determination of the public debt. *The Journal of Political Economy*, 940–971.

Bi, H., E. M. Leeper, and C. Leith (2013). Uncertain fiscal consolidations. *The Economic Journal 123*(566), F31–F63.

Chen, X., T. Kirsanova, and C. Leith (2017). How optimal is US monetary policy? *Journal of Monetary Economics 92*, 96 – 111.

Debortoli, D. and R. Nunes (2013). Lack of Commitment and the Level of Debt. *Journal of the European Economic Association 11*(5), 1053–1078.

Faraglia, E., A. Marcet, R. Oikonomou, and A. Scott (2013). The impact of debt levels and debt maturity on inflation. *The Economic Journal 123*(566), F164–F192.

Hall, G. J. and T. J. Sargent (2011). Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics. *American Economic Journal: Macroeconomics 3*(3), 192–214.

Judd, K. L. (1998). *Numerical Methods in Economics*. MIT press.

Judd, K. L. (2004). Existence, uniqueness, and computational theory for time consistent equilibria: A hyperbolic discounting example. *mimeo, Stanford University*.

Lagos, R. and R. Wright (2005). A Unified Framework for Monetary Theory and Policy Analysis. *Journal of Political Economy 113*(3), pp. 463–484.

Leeper, E. and C. Leith (2016). Understanding inflation as a joint monetary-fiscal phenomenon. *Taylor, J. and H. Uhlig (eds), the Handbook of Macroeconomics, Volume 2*.

Leeper, E. M. and X. Zhou (2013). Inflation's Role in Optimal Monetary-Fiscal Policy. *NBER Working Paper No. 19686*.

Leith, C. and D. Liu (2016). The inflation bias under Calvo and Rotemberg pricing. *Journal of Economic Dynamics & Control 73*(2), 283–297.

Martin, F. M. (2009). A positive theory of government debt. *Review of economic Dynamics 12*(4), 608–631.

Martin, F. M. (2011). On the joint determination of fiscal and monetary policy. *Journal of Monetary Economics 58*(2), 132–145.

Martin, F. M. (2013). Government Policy in Monetary Economies. *International Economic Review 54*(1), 185–217.

Martin, F. M. (2014). The Value of Constraints on Discretionary Government Policy. *Mimeo*.

Niemann, S., P. Pichler, and G. Sorger (2013). Public debt, discretionary policy, and inflation persistence. *Journal of Economic Dynamics and Control 37*(6), 1097–1109.

Rotemberg, J. J. (1982). Sticky Prices in the United States. *Journal of Political Economy 90*(6), 1187–1211.

Schmitt-Grohe, S. and M. Uribe (2004a). Optimal fiscal and monetary policy under imperfect competition. *Journal of Macroeconomics 26*(2), 183–209.

Schmitt-Grohe, S. and M. Uribe (2004b). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory 114*(2), 198–230.

Sheedy, K. D. (2014). Debt and incomplete financial markets: a case for nominal GDP targeting. *Brookings Papers on Economic Activity 2014*(1), 301–373.

Sims, C. A. (2013). Paper money. *The American Economic Review 103*(2), 563–584.

Siu, H. E. (2004). Optimal fiscal and monetary policy with sticky prices. *Journal of Monetary Economics 51*(3), 575–607.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*(5), 1123–1175.

Woodford, M. (2001). Fiscal Requirements for Price Stability. *Journal of Money, Credit and Banking 33*(3), 669–728.

Appendix A. Tables

Table A.1: Parameterization

Parameter	Value	Definition
β	0.995	Quarterly discount factor, household.
$\widetilde{\beta}$	0.982	Quarterly discount factor, policy maker.
σ	2	Relative risk aversion coefficient
σ^g	2	Relative risk aversion coefficient for government spending
φ	3	Inverse Frish elasticity of labor supply
$\bar{\epsilon}$	14.33	Elasticity of substitution between varieties
ρ	0.95	Debt maturity structure (5 years)
χ	0.0076	Scaling parameter associated with government spending
ρ_ϵ	0.939	AR-coefficient of cost-push shock
σ_ϵ	0.052	Standard deviation of cost-push shock
ϕ	50	Rotemberg adjustment cost coefficient

Table A.2: Steady-state: myopia, price flexibility and monopolistic competition

Variable	Data	Benchmark	No myopia	Myopia $\widetilde{\beta} = 0.975$	Price flexibility $\phi = 30$	Markup $\frac{\bar{\epsilon}}{\bar{\epsilon}-1} = 5\%$
$\frac{bP^M}{4Y}$	31.2%	31.2%	-152.9%	75.6%	36.7%	89.8%
$(\Pi^4 - 1)$	3.5%/2.4%[20]	3.0%	-1.1%	4.5%	3.8%	3.7%
$(R^4 - 1)$	5.66%/4.9%	5.1%	0.9%	6.7%	5.9%	5.8%
Y	N.A.	0.977	0.985	0.975	0.977	0.980
G/Y	7.84%	7.82%	7.93%	7.76%	7.81%	7.75%
τ	17.5%	18.9%	15.3%	19.8%	19.0%	19.5%

Table A.3: Steady-state: maturity

Variable	Benchmark	1 Qtr Maturity $\rho = 0$	1 Yr Maturity $\rho = 0.7538$	10 Yr Maturity $\rho = 0.9799$	30 Yr Maturity $\rho = 0.9966$
$\frac{bP^M}{4Y}$	31.2%	-11.1%	12.8%	53.6%	102.0%
$(\Pi^4 - 1)$	3.0%	1.5%	2.5%	3.62%	5.1%
$(R^4 - 1)$	5.1%	3.5%	4.6%	5.7%	7.2%
Y	0.977	0.979	0.978	0.976	0.973
G/Y	7.82%	8.01%	7.80%	7.83%	7.82%
τ	18.9%	18.2%	18.4%	19.4%	20.4%

[20] Over the full sample the average inflation rate was 3.5% (with a standard deviation of 2.3%), while following the Great Moderation (post 1985) the average inflation rate falls to 2.4% with a standard deviation of 0.76%.

Table A.4: Steady-state debt-to-GDP ratios: myopia, price flexibility and monopolistic competition

	Benchmark	Benchmark Endog. Maturity	Price flexibility $\phi = 10$	Lower Markup $\frac{\bar{\epsilon}}{\bar{\epsilon}-1} = 5\%$	Myopia $\widetilde{\beta} = 0.975$
Debt to GDP Ratio(%)	31.2%	31.2%	88.2%	24.7%	74.7%
Share of Single Period Debt(%)	0	4.4%	0.9%	-0.7%	2.4%

Appendix B. Figures

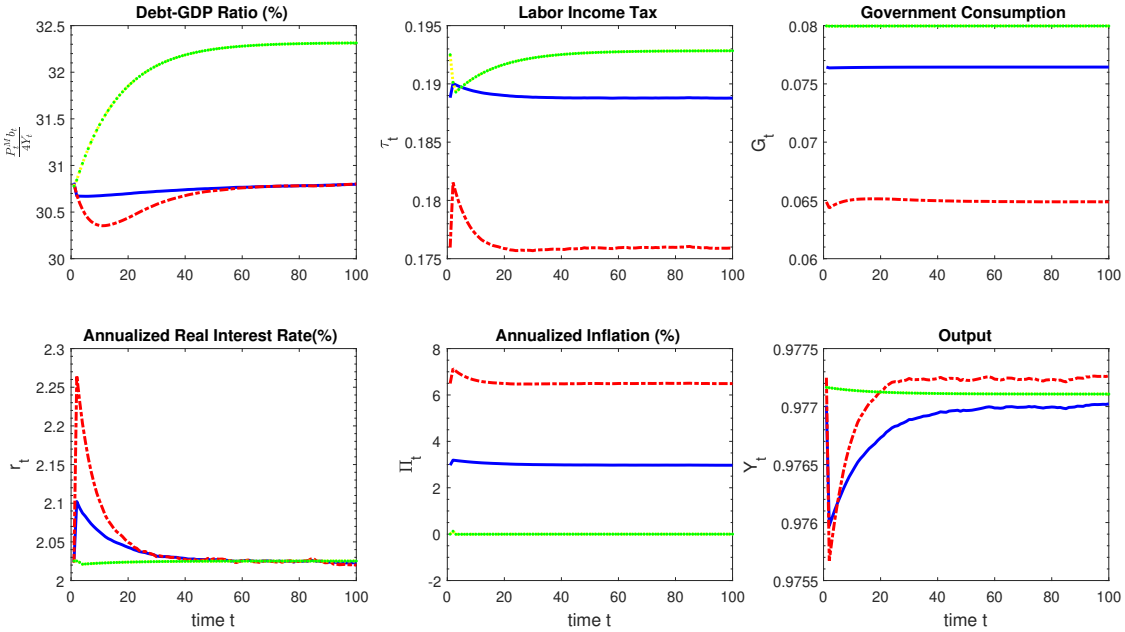

Figure B.1: Markup shock - commitment vs. discretion - debt maturity
Note: Yellow dotted line represents outcomes under commitment with long-term debt, and green points commitment with single period debt. These largely overlap. Solid blue line presents discretion with long-term debt, and red dash-dotted line discretion with single period debt. Myopia has been increased in the case of single period debt to ensure the steady-state debt-to-GDP ratio is the same as the other model variants considered.

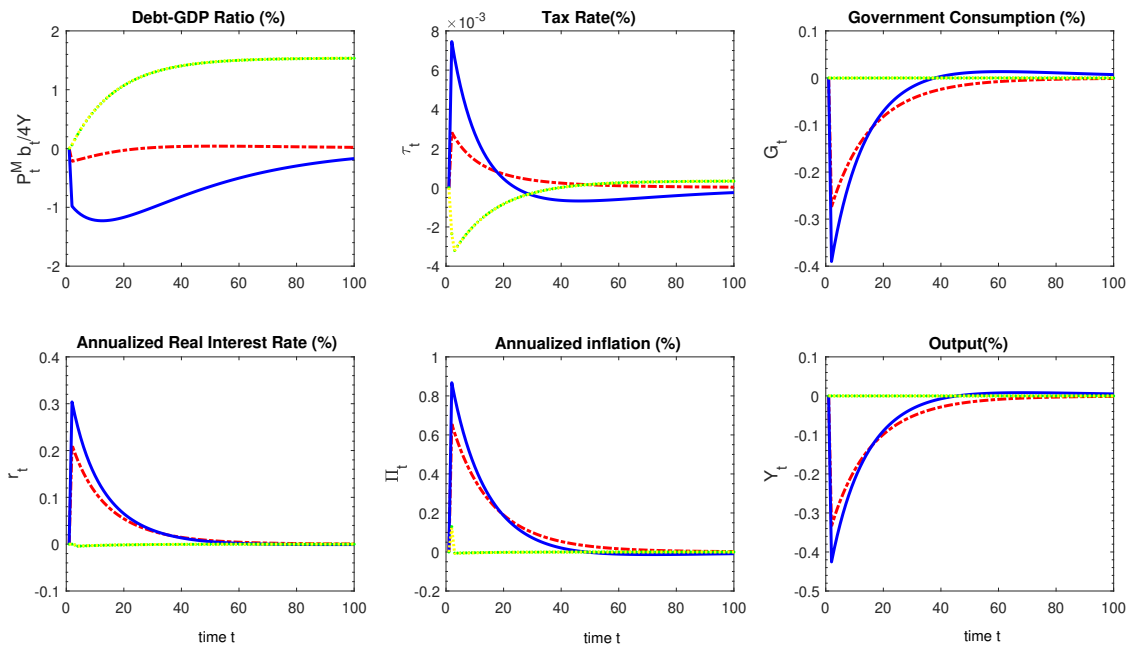

Figure B.2: Markup shock - commitment vs. discretion - level of debt
Note: Yellow dotted line represents outcomes under commitment with high (52% of GDP) levels of debt, and green points commitment with low (16% of GDP) levels of debt. These largely overlap. Solid blue line presents discretion with high levels of steady-state debt, and red dash-dotted line discretion with low levels of steady-state debt. Debt, taxes, interest rates and inflation are expressed in deviations from stochastic steady-state. Government consumption and output in percentage deviations from stochastic steady-state.

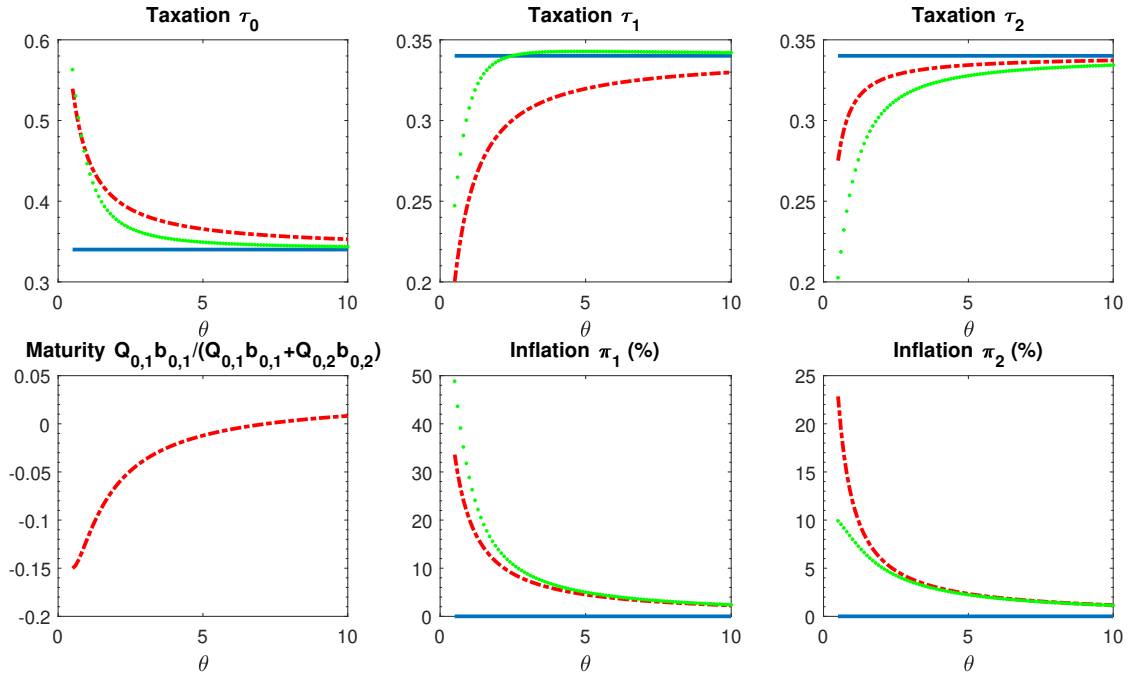

Figure B.3: Equilibrium outcomes for the three-period model
Note: The figure gives the equilibrium outcomes for the three-period model as a function of the cost of inflation, θ . Three cases are considered - commitment (solid blue line), discretion with single period debt only (green dots) and discretion with one and two-period bonds (red dot-dashed line).

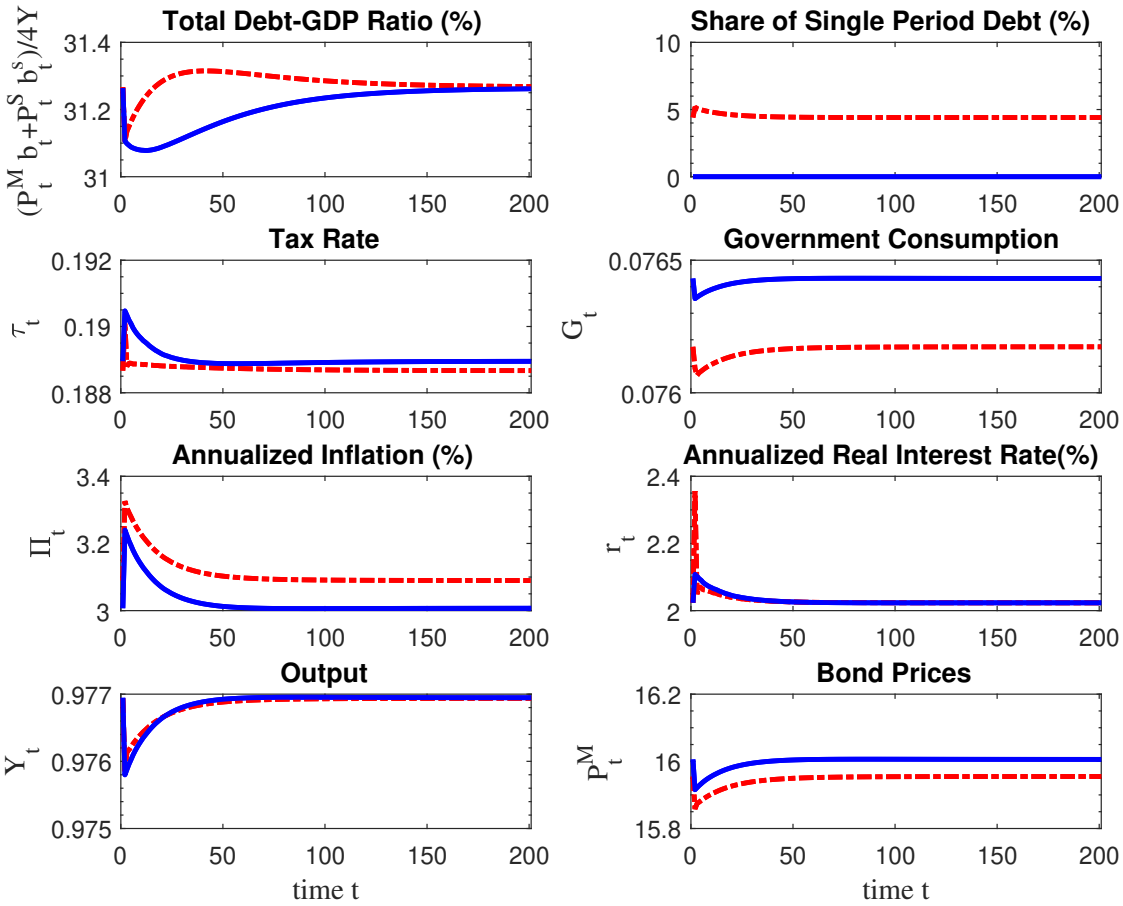

Figure B.4: Endogenous debt maturity and fiscal consolidation
Note: Endogenous debt maturity - red dash-dotted line. Benchmark case of exogenous debt maturity - solid blue line.

Technical Appendix (Not for Publication)

Appendix C. Data Appendix

We follow Chen et al. (2018) and Bianchi and Ilut (2017) in constructing our fiscal variables. The data for government spending, tax revenues and transfers, are taken from National Income and Product Accounts (NIPA) Table 3.2 (Federal Government Current Receipts and Expenditures) released by the Bureau of Economics Analysis. These data series are nominal and in levels.

Government Spending. Government spending is defined as the sum of consumption expenditure (line 21), gross government investment (line 42), net purchases of nonproduced assets (line 44), minus consumption of fixed capital (line 45), minus wage accruals less disbursements (line 33).

Total tax revenues. Total tax revenues are constructed as the difference between current receipts (line 38) and current transfer receipts (line 16).

Transfers. Transfers is defined as current transfer payments (line 22) minus current transfer receipts (line 16) plus capital transfers payments (line 43) minus capital transfer receipts (line 39) plus subsidies (line 32).

Federal government debt. Federal government debt is the market value of privately held gross Federal debt, which is downloaded from Dallas Fed web-site

The above three fiscal variables are normalized with respect to Nominal GDP. **Nominal GDP** is taken from NIPA Table 1.1.5 (Gross Domestic Product).

Real GDP. Real GDP is take download from NIPA Table 1.1.6 (Real Gross Domestic Product, Chained Dollars)

The GDP deflator. The GDP deflator is obtained from NIPA Table 1.1.5 (Gross Domestic Product).

Effective Federal Funds Rate. Effective Federal Funds Rate is taken from the St. Louis Fed website.

The implied ratios are presented in the first column of Table A.2.

Appendix D. Summary of Model

We now summarize the model and its steady state before turning to the time-consistent policy problem. Consumption Euler equation,

$$\beta R_t E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{D.1}$$

Pricing of longer-term bonds,

$$\beta E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \left(\frac{P_t}{P_{t+1}} \right) \left(1 + \rho P_{t+1}^M \right) \right\} = P_t^M \tag{D.2}$$

Labour supply,

$$N_t^{\varphi} C_t^{\sigma} = (1 - \tau_t) \left(\frac{W_t}{P_t} \right) \equiv (1 - \tau_t) w_t$$

Resource constraint,

$$Y_t \left[1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right] = C_t + G_t \tag{D.3}$$

Phillips curve,

$$\begin{aligned} 0 &= (1 - \epsilon_t) + \epsilon_t mc_t - \phi \Pi_t \left(\Pi_t - 1 \right) \\ &+ \phi \beta E_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \right] \end{aligned} \tag{D.4}$$

Government budget constraint,

$$P_t^M b_t = (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} - \left(\frac{\tau_t}{1 - \tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma} + G_t + tr \tag{D.5}$$

Technology,

$$Y_t = N_t \tag{D.6}$$

Marginal costs,

$$mc_t = W_t/P_t = (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma}$$

The objective function for social welfare is given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} \right) \tag{D.7}$$

There are two state variables, real debt b_t and the elasticity of substitution between good varieties, ϵ_t.

Appendix D.1. The Deterministic Steady State

Given the system of non-linear equations, the corresponding steady-state system can be written as follows:

$$\frac{\beta R}{\Pi} = 1$$

$$\frac{\beta}{\Pi}\left(1+\rho P^M\right) = P^M$$

$$(1-\tau)w = N^{\varphi} C^{\sigma}$$

$$Y\left[1 - \frac{\phi}{2}\left(\Pi - 1\right)^2\right] = C + G$$

$$(1-\epsilon) + \epsilon mc + \phi\left(\beta - 1\right)\left[\Pi\left(\Pi - 1\right)\right] = 0$$

$$P^M b = (1+\rho P^M)\frac{b}{\Pi} - \left(\frac{\tau}{1-\tau}\right) Y^{1+\varphi} C^{\sigma} + G + tr$$

$$Y = N$$

$$mc = w = (1-\tau)^{-1} Y^{\varphi} C^{\sigma}$$

$$P^M = \frac{\beta}{\Pi - \beta\rho}$$

$$mc = w = \frac{\epsilon - 1}{\epsilon}$$

$$\frac{C}{Y} = \left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{1/\sigma} Y^{-\frac{\varphi+\sigma}{\sigma}}$$

$$\frac{G}{Y} = 1 - \frac{C}{Y} = 1 - \left[(1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right)\right]^{1/\sigma} Y^{-\frac{\varphi+\sigma}{\sigma}}$$

$$P^M b = \frac{\beta}{1-\beta}\left[\tau\left(\frac{\epsilon-1}{\epsilon}\right) - \frac{G}{Y}\right] Y$$

Note that,

$$Y^{\varphi+\sigma}\left(1-\frac{G}{Y}\right)^{\sigma} = (1-\tau)\left(\frac{\epsilon-1}{\epsilon}\right) \tag{D.8}$$

which will be used to contrast with the allocation that would be chosen by a social planner.

Appendix E. First-Best Allocation

In some analyses of optimal fiscal policy (e.g., Aiyagari et al., 2002), it is desirable for the policy maker to accumulate a 'war chest' which pays for government consumption and/or fiscal subsidies to correct for other market imperfections. In order to assess to what extent our optimal, but time-consistent policy attempts to do so, it is helpful to define the level of government accumulated assets that would be necessary to mimic the social planner's allocation under the decentralized solution. The first step in doing so is defining the first-best allocation that would be implemented by the social planner. The social planner ignores the nominal inertia and all other inefficiencies, and chooses real allocations that maximize the representative consumer's utility, subject to the aggregate resource constraint and the aggregate production function. That is, the first-best allocation $\{C_t^*,\, N_t^*,\, G_t^*\}$ is the one that maximizes utility (D.7), subject to the technology constraint (D.6), and aggregate resource constraint $Y_t = C_t + G_t$.

The first order conditions imply that

$$(C_t^*)^{-\sigma} = \chi\,(G_t^*)^{-\sigma_g} = (N_t{}^*)^{\varphi} = (Y_t{}^*)^{\varphi}$$

That is, given the resource constraints, it is optimal to equate the marginal utility of private and public consumption to the marginal disutility of labor effort and the optimal share of government consumption in output is

$$\frac{G_t^*}{Y_t^*} = \chi^{\frac{1}{\sigma_g}} \left(Y_t^*\right)^{-\frac{\varphi+\sigma_g}{\sigma_g}}$$

In a deterministic steady state and assuming $\sigma = \sigma_g$, this implies the optimal share of government consumption in output is

$$\frac{G^*}{Y^*} = \left(1 + \chi^{-\frac{1}{\sigma}}\right)^{-1}$$

and the first-best level of steady-state output is given by,

$$(Y^*)^{\varphi+\sigma} \left(1 - \frac{G^*}{Y^*}\right)^{\sigma} = 1 \tag{E.1}$$

It is illuminating to contrast the allocation achieved in the steady state of the decentralized equilibrium with this first best allocation. We do this by finding policies and prices that make the first-best allocation and the decentralized equilibrium coincide. Appendix D shows that the steady-state level of output in the decentralized economy is given by,

$$Y^{\varphi+\sigma} \left(1 - \frac{G}{Y}\right)^{\sigma} = (1-\tau) \left(\frac{\epsilon - 1}{\epsilon}\right) \tag{E.2}$$

Comparing (E.2) and (E.1), and assuming the steady state share of government consumption is the same, then the two allocations will be identical when the labor income tax rate is set optimally to be,

$$\tau^* = 1 - \frac{\epsilon}{\epsilon - 1} = \frac{-1}{\epsilon - 1} \tag{E.3}$$

Notice that the optimal tax rate is negative, that is, it is effectively a subsidy which offsets the monopolistic competition distortion. This, in turn, requires that the government has

accumulated a stock of assets defined as,

$$\frac{P^{M*}b^*}{4Y^*} = \frac{\beta}{4\left(1-\beta\right)}\left[\frac{-1}{\epsilon} - \left(1+\chi^{-\frac{1}{\sigma}}\right)^{-1} - \frac{tr}{Y}\right]$$

Using our benchmark calibration below, this would imply that a stock of assets of 4636% of GDP would be required to generate sufficient income to pay for government expenditure (consumption and fiscal transfers) and a labor income subsidy which completely offsets the effects of the monopolistic competition distortion. In the absence of policy maker myopia, the steady-state level of debt in our optimal policy problem while negative, falls far short of this 'war chest' value.

It is also interesting to note the implied optimal share of government spending in GDP that would be chosen by the social planner is 7.7% which is very close to that chosen by the policy maker in our decentralized (7.82%) distorted economy implying that G is 3.9% lower than the first best while GDP is 5.4% smaller than it would be under the social planner's allocation.

Appendix F. Policy Problem under Commitment

In this section we outline the problem under commitment and contrast that with the focs under discretion.

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{ & \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} \left(\Pi_t - 1 \right)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t \left(\Pi_t - 1 \right) \\ + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left(C_{t+1} \right)^{-\sigma} Y_{t+1} \Pi_{t+1} \left(\Pi_{t+1} - 1 \right) \end{array} \right] \\
& + \lambda_{3t} \left[P_t^M b_t - (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma} - G_t - tr \right] \\
& + \lambda_{4t} \left[P_t^M - \beta E_t \left\{ \left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \Pi_{t+1}^{-1} \left(1 + \rho P_{t+1}^M \right) \right\} \right] \}
\end{aligned}
\tag{F.1}
$$

The resultant set of focs are given by,

$$
\begin{aligned}
C_t \quad & C_t^{-\sigma} - \lambda_{1t} + \lambda_{2t} \left[\begin{array}{c} \sigma\epsilon_t(1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} \\ +\sigma\phi\beta C_t^{\sigma-1} Y_t^{-1} E_t \left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right) \end{array} \right] \\
& +\lambda_{3t}\left[\sigma\left(\tfrac{\tau_t}{1-\tau_t}\right)\left(Y_t\right)^{1+\varphi} C_t^{\sigma-1}\right] \\
& -\lambda_{4t}[\sigma\beta E_t\left\{\left(C_t\right)^{\sigma-1}\left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}^{-1}\left(1+\rho P_{t+1}^M\right)\right\}] \\
& -\lambda_{2t-1}[\sigma\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}\left(C_t\right)^{-\sigma-1} Y_t\Pi_t\left(\Pi_t-1\right)] \\
& +\lambda_{4t-1}\left[\sigma(C_{t-1})^{\sigma}\left(C_t\right)^{-\sigma-1}\Pi_t^{-1}\left(1+\rho P_t^M\right)\right] = 0 \\
Y_t \quad & -Y_t^{\varphi} + \lambda_{1t}\left[1-\tfrac{\phi}{2}\left(\Pi_t-1\right)^2\right] + \lambda_{3t}\left[(1+\varphi)Y_t^{\varphi} C_t^{\sigma}\left(\tfrac{\tau_t}{1-\tau_t}\right)\right] \\
& +\lambda_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1} Y_t^{\varphi-1} C_t^{\sigma} - \phi\beta C_t^{\sigma} Y_t^{-2} E_t\left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right] \\
& +\lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}\left(C_t\right)^{-\sigma}\Pi_t\left(\Pi_t-1\right)] = 0 \\
\tau_t \quad & \epsilon_t\lambda_{2t} + \lambda_{3t} Y_t = 0 \\
G_t \quad & \chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \\
P_t^M \quad & \lambda_{3t}[b_t - \rho\tfrac{b_{t-1}}{\Pi_t}] + \lambda_{4t} \\
& -\lambda_{4t-1}\left[\rho(C_{t-1})^{\sigma}\left(C_t\right)^{-\sigma}\Pi_t^{-1} P_t^M\right] = 0 \\
\Pi_t \quad & -\lambda_{1t}\left[Y_t\phi\left(\Pi_t-1\right)\right] - \lambda_{2t}\left[\phi\left(2\Pi_t-1\right)\right] + \lambda_{3t}\left[\tfrac{b_{t-1}}{\Pi_t^2}\left(1+\rho P_t^m\right)\right] \\
& +\lambda_{2t-1}[\phi C_{t-1}^{\sigma} Y_{t-1}^{-1}\left(C_t\right)^{-\sigma} Y_t(2\Pi_t-1)] \\
& +\lambda_{4t-1}\left[(C_{t-1})^{\sigma}\left(C_t\right)^{-\sigma}\Pi_t^{-2}(1+\rho P_t^M)\right] = 0 \\
b_t \quad & \lambda_{3t} P_t^M - \beta E_t\left[\lambda_{3t+1}\tfrac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] = 0
\end{aligned}
$$

These are the same as those under discretion except for the lagged terms in the LMs which represent the impact of past commitments on current behavior. Additionally, the FOC for debt no longer includes the impact of changing that state on expectations, as expectations are now driven by the credible promises made by the policy maker. As a result the FOC for debt is a policy of pure tax smoothing implying that steady-state debt follows a random walk.

To obtain a global solution using the algorithm described in Leeper and Leith(2017) define the following state variables, $\widetilde{\lambda}_{2t}$ and $\widetilde{\lambda}_{4t}$ such that $\lambda_{2t} = \widetilde{\lambda}_{2t} C_t^{-\sigma} Y_t$ and $\lambda_{4t} = \widetilde{\lambda}_{4t}(C_t)^{-\sigma}$ and the set of FOCs can be rewritten as,

$$
\begin{aligned}
C_t \quad & C_t^{-\sigma} - \lambda_{1t} + \widetilde{\lambda}_{2t}\left[\sigma\epsilon_t(1-\tau_t)^{-1}Y_t^{1+\varphi}C_t^{-1} + \sigma\phi\beta C_t^{-1}E_t\left(C_{t+1}\right)^{-\sigma}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right] \\
& +\lambda_{3t}\left[\sigma\left(\tfrac{\tau_t}{1-\tau_t}\right)\left(Y_t\right)^{1+\varphi}C_t^{\sigma-1}\right] \\
& -\widetilde{\lambda}_{4t}[\sigma\beta E_t\left\{\left(C_t\right)^{-1}\left(C_{t+1}\right)^{-\sigma}\Pi_{t+1}^{-1}\left(1+\rho P_{t+1}^M\right)\right\}] \\
& -\widetilde{\lambda}_{2t-1}[\sigma\phi\left(C_t\right)^{-\sigma-1}Y_t\Pi_t\left(\Pi_t-1\right)] \\
& +\widetilde{\lambda}_{4t-1}\left[\sigma\left(C_t\right)^{-\sigma-1}\Pi_t^{-1}\left(1+\rho P_t^M\right)\right] = 0 \\
Y_t \quad & -Y_t^{\varphi} + \lambda_{1t}\left[1-\tfrac{\phi}{2}\left(\Pi_t-1\right)^2\right] + \lambda_{3t}\left[(1+\varphi)Y_t^{\varphi}C_t^{\sigma}\left(\tfrac{\tau_t}{1-\tau_t}\right)\right] \\
& +\widetilde{\lambda}_{2t}\left[\epsilon_t\varphi(1-\tau_t)^{-1}Y_t^{\varphi} - \phi\beta Y_t^{-1}E_t\left(C_{t+1}\right)^{-\sigma}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right] \\
& +\widetilde{\lambda}_{2t-1}[\phi\left(C_t\right)^{-\sigma}\Pi_t\left(\Pi_t-1\right)] = 0 \\
\tau_t \quad & \epsilon_t\widetilde{\lambda}_{2t}C_t^{-\sigma} + \lambda_{3t} = 0 \\
G_t \quad & \chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \\
P_t^M \quad & \lambda_{3t}[b_t - \rho\tfrac{b_{t-1}}{\Pi_t}] + \widetilde{\lambda}_{4t}(C_t)^{-\sigma} \\
& -\widetilde{\lambda}_{4t-1}\left[\rho\left(C_t\right)^{-\sigma}\Pi_t^{-1}\right] = 0 \\
\Pi_t \quad & -\lambda_{1t}\left[Y_t\phi\left(\Pi_t-1\right)\right] - \widetilde{\lambda}_{2t}C_t^{-\sigma}Y_t\left[\phi\left(2\Pi_t-1\right)\right] + \lambda_{3t}\left[\tfrac{b_{t-1}}{\Pi_t^2}\left(1+\rho P_t^m\right)\right] \\
& +\widetilde{\lambda}_{2t-1}[\phi\left(C_t\right)^{-\sigma}Y_t(2\Pi_t-1)] \\
& +\widetilde{\lambda}_{4t-1}\left[\left(C_t\right)^{-\sigma}\Pi_t^{-2}(1+\rho P_t^M)\right] = 0 \\
b_t \quad & \lambda_{3t}P_t^M - \beta E_t\left[\lambda_{3t+1}\tfrac{1}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] = 0
\end{aligned}
$$

The commitment equilibrium is determined by the system given by the FOCs, the constraints in (F.1), and the exogenous process for the markup shock, (3). The solution to this system is a set of time-invariant equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \epsilon_t, \widetilde{\lambda}_{2t-1}, \widetilde{\lambda}_{4t-1}\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \widetilde{\lambda}_{2t}, \lambda_{3t}, \widetilde{\lambda}_{4t}\}$ for all $t \geq 0$. It is the expansion in the set of state variables which captures the commitments made under Ramsey policy.

Appendix G. Numerical Algorithm

This section describes the Chebyshev collocation method with time iteration used in the paper. See Judd (1998) for a textbook treatment of the numerical techniques involved.

Let $s_t = (b_{t-1}, \epsilon_t)$ denote the state vector at time t, where real stock of debt b_{t-1} is

endogenous and elasticity of substitution between goods ϵ_t is exogenous and respectively, with the following laws of motion:

$$P_t^M b_t = (1 + \rho P_t^M)\frac{b_{t-1}}{\Pi_t} - w_t N_t \tau_t + G_t + tr$$

$$\ln(\epsilon_t) = (1 - \rho_\epsilon)\ln(\bar{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t,\ \varepsilon_t \sim N(0,1)$$

where $0 \leq \rho_\epsilon < 1$.

There are 7 endogenous variables and 3 Lagrangian multipliers. Correspondingly, there are 10 functional equations associated with the 10 variables $\left\{C_t, Y_t, \Pi_t, b_t, \tau_t, P_t^M, G_t, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}\right\}$. Defining a new function $X : \mathbb{R}^2 \to \mathbb{R}^{10}$, in order to collect the policy functions of endogenous variables as follows:

$$X(s_t) = \left(C_t(s_t), Y_t(s_t), \Pi_t(s_t), b_t(s_t), \tau_t(s_t), P_t^M(s_t), G_t(s_t), \lambda_{1t}(s_t), \lambda_{2t}(s_t), \lambda_{3t}(s_t)\right)$$

Given the specification of the function X, the equilibrium conditions can be written more compactly as,

$$\Gamma\left(s_t, X(s_t), E_t\left[Z\left(X(s_{t+1})\right)\right], E_t\left[Z_b\left(X(s_{t+1})\right)\right]\right) = 0$$

where $\Gamma : \mathbb{R}^{2+10+3+3} \to \mathbb{R}^{10}$ summarizes the full set of dynamic equilibrium relationships, and

$$Z\left(X(s_{t+1})\right) = \begin{bmatrix} Z_1\left(X(s_{t+1})\right) \\ Z_2\left(X(s_{t+1})\right) \\ Z_3\left(X(s_{t+1})\right) \end{bmatrix} \equiv \begin{bmatrix} M(b_t, \epsilon_{t+1}) \\ L(b_t, \epsilon_{t+1}) \\ \left(\Pi_{t+1}\right)^{-1}\left(1 + \rho P_{t+1}^M\right)\lambda_{3t+1} \end{bmatrix}$$

with

$$M(b_t, \epsilon_{t+1}) = \left(C_{t+1}\right)^{-\sigma} Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1} - 1\right)$$

$$L(b_t, \epsilon_{t+1}) = (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1 + \rho P_{t+1}^M)$$

and

$$Z_b\left(X(s_{t+1})\right) = \begin{bmatrix} \frac{\partial Z_1(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_2(X(s_{t+1}))}{\partial b_t} \\ \frac{\partial Z_3(X(s_{t+1}))}{\partial b_t} \end{bmatrix} \equiv \begin{bmatrix} \frac{\partial M(b_t,\epsilon_{t+1})}{\partial b_t} \\ \frac{\partial L(b_t,\epsilon_{t+1})}{\partial b_t} \\ \frac{\partial\left[(\Pi_{t+1})^{-1}\left(1+\rho P^M_{t+1}\right)\lambda_{3t+1}\right]}{\partial b_t} \end{bmatrix}$$

More specifically,

$$L_1(b_t,\epsilon_{t+1}) = \frac{\partial\left[(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}(1+\rho P^M_{t+1})\right]}{\partial b_t}$$

$$\begin{aligned} &= -\sigma(C_{t+1})^{-\sigma-1}(\Pi_{t+1})^{-1}(1+\rho P^M_{t+1})\frac{\partial C_{t+1}}{\partial b_t} \\ &- (C_{t+1})^{-\sigma}(\Pi_{t+1})^{-2}(1+\rho P^M_{t+1})\frac{\partial \Pi_{t+1}}{\partial b_t} + \rho(C_{t+1})^{-\sigma}(\Pi_{t+1})^{-1}\frac{\partial P^M_{t+1}}{\partial b_t} \end{aligned}$$

and

$$M_1(b_t,\epsilon_{t+1}) = \frac{\partial\left[(C_{t+1})^{-\sigma}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right]}{\partial b_t}$$

$$\begin{aligned} &= -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial C_{t+1}}{\partial b_t} + (C_{t+1})^{-\sigma}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial Y_{t+1}}{\partial b_t} \\ &+ (C_{t+1})^{-\sigma}Y_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t} + (C_{t+1})^{-\sigma}Y_{t+1}\Pi_{t+1}\frac{\partial \Pi_{t+1}}{\partial b_t} \end{aligned}$$

$$\begin{aligned} &= -\sigma(C_{t+1})^{-\sigma-1}Y_{t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial C_{t+1}}{\partial b_t} + (C_{t+1})^{-\sigma}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{\partial Y_{t+1}}{\partial b_t} \\ &+ (C_{t+1})^{-\sigma}Y_{t+1}\left(2\Pi_{t+1}-1\right)\frac{\partial \Pi_{t+1}}{\partial b_t} \end{aligned}$$

Note we are assuming $E_t\left[Z_b\left(X(s_{t+1})\right)\right] = \partial E_t\left[Z\left(X(s_{t+1})\right)\right]/b_t$, which is valid due to the Interchange of Integration and Differentiation Theorem. Then the problem is to find a vector-valued function X that Γ maps to the zero function. Projection methods can be used.

Following the notation convention in the literature, we simply use $s = (b,\epsilon)$ to denote the current state of the economy $s_t = (b_{t-1},\epsilon_t)$, and s' to represent next period's state that

evolves according to the law of motion specified above. The Chebyshev collocation method with time iteration, which we use to solve this nonlinear system, can be described as follows:

1. Define the collocation nodes and the space of the approximating functions:

   - Choose an order of approximation (i.e., the polynomial degrees) n_b and n_ϵ for each dimension of the state space $s = (b, \epsilon)$, then there are $N_s = (n_b + 1) \times (n_\epsilon + 1)$ nodes in the state space. Let $S = (S_1, S_2, ..., S_{N_s})$ denote the set of collocation nodes.

   - Compute the $n_b + 1$ and $n_\epsilon + 1$ roots of the Chebychev polynomial of order $n_b + 1$ and $n_\epsilon + 1$ as

$$z_b^i = \cos\left(\frac{(2i-1)\pi}{2(n_b+1)}\right), \text{ for } i = 1, 2, ..., n_b + 1.$$
$$z_\epsilon^i = \cos\left(\frac{(2i-1)\pi}{2(n_\epsilon+1)}\right), \text{ for } i = 1, 2, ..., n_\epsilon + 1.$$

   - Compute collocation points ϵ_i as

$$\epsilon_i = \frac{\epsilon_{max} + \epsilon_{min}}{2} + \frac{\epsilon_{max} - \epsilon_{min}}{2} z_\epsilon^i = \frac{\epsilon_{max} - \epsilon_{min}}{2}\left(z_\epsilon^i + 1\right) + \epsilon_{min}$$

   for $i = 1, 2, ..., n_\epsilon + 1$, which map $[-1, 1]$ into $[\epsilon_{min}, \epsilon_{max}]$. Note that the number of collocation nodes is $n_\epsilon + 1$. Similarly, compute collocation points b_i as

$$b_i = \frac{b_{max} + b_{min}}{2} + \frac{b_{max} - b_{min}}{2} z_b^i = \frac{b_{max} - b_{min}}{2}\left(z_b^i + 1\right) + b_{min}$$

   for $i = 1, 2, ..., n_b + 1$, which map $[-1, 1]$ into $[b_{min}, b_{max}]$. Note that

$$S = \{(b_i, \epsilon_j) \mid i = 1, 2, ..., n_b + 1,\ j = 1, 2, ..., n_\epsilon + 1\}$$

   that is, the tensor grids, with $S_1 = (b_1, \epsilon_1)$, $S_2 = (b_1, \epsilon_2)$, ..., $S_{N_s} = (b_{n_b+1}, \epsilon_{n_\epsilon+1})$.

- The space of the approximating functions, denoted as Ω, is a matrix of two-dimensional Chebyshev polynomials. More specifically,

$$\Omega(S) = \begin{bmatrix} \Omega(S_1) \\ \Omega(S_2) \\ \vdots \\ \Omega(S_{n_\epsilon+1}) \\ \vdots \\ \Omega(S_{N_s}) \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_1)) & T_0(\xi(b_1)T_2(\xi(\epsilon_1)) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_1)) \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_2)) & T_0(\xi(b_1)T_2(\xi(\epsilon_2)) & \cdots & T_{n_b}(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_2)) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & T_0(\xi(b_1)T_1(\xi(\epsilon_{n_\epsilon+1})) & T_0(\xi(b_1)T_2(\xi(\epsilon_{n_\epsilon+1})) & \cdots & T_0(\xi(b_1)T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1})) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & T_0(\xi(b_{n_b+1})T_1(\xi(\epsilon_{n_\epsilon+1})) & T_0(\xi(b_{n_b+1})T_2(\xi(\epsilon_{n_\epsilon+1})) & \cdots & T_0(\xi(b_{n_b+1})T_{n_\epsilon}(\xi(\epsilon_{n_\epsilon+1})) \end{bmatrix}_{N_s \times N_s}$$

  where $\xi(x) = 2(x - x_{min}) / (x_{max} - x_{min}) - 1$ maps the domain of $x \in [x_{min}, x_{max}]$ into $[-1, 1]$.

- Then, at each node $s \in S$, policy functions $X(s)$ are approximated by $X(s) = \Omega(s)\Theta_X$,

  where

$$\Theta_X = \left[\theta^c, \theta^Y, \theta^\Pi, \theta^b, \theta^\tau, \theta^{\widetilde{p}}, \theta^G, \theta^{\lambda_1}, \theta^{\lambda_2}, \theta^{\lambda_3}\right]$$

  is a $N_s \times 10$ matrix of the approximating coefficients.

2. Formulate an initial guess for the approximating coefficients, Θ_X^0, and specify the stopping rule ϵ_{tol}, say, 10^{-6}.
3. At each iteration j, we can get an updated Θ_X^j by implement the following time iteration step:

- At each collocation node $s \in S$, compute the possible values of future policy functions $X(s')$ for $k = 1, ..., q$. That is,

$$X(s') = \Omega(s')\Theta_X^{j-1}$$

where q is the number of Gauss-Hermite quadrature nodes. Note that

$$\Omega(s') = T_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon'))$$

is a $q \times N_s$ matrix, with $b' = \widehat{b}(s;\theta^b)$, $\ln(\epsilon') = (1-\rho_\epsilon)\ln(\overline{\epsilon}) + \rho_\epsilon \ln(\epsilon) + z_k\sqrt{2\sigma_\epsilon^2}$, $j_b = 0, ..., n_b$, and $j_\epsilon = 0, ..., n_\epsilon$. The hat symbol indicates the corresponding approximate policy functions, so $\widehat{b}$ is the approximate policy for real debt, for example. Similarly, the two auxiliary functions can be calculated as follows:

$$M(s') \approx \left(\widehat{C}(s';\theta^c)\right)^{-\sigma} \widehat{Y}(s';\theta^y)\widehat{\Pi}(s';\theta^\pi)\left(\widehat{\Pi}(s';\theta^\pi) - 1\right)$$

and,

$$L(s') \approx \left(\widehat{C}(s';\theta^c)\right)^{-\sigma} \left(\widehat{\Pi}(s';\theta^\pi)\right)^{-1} \left(1 + \frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}\right)}{1-\rho\beta}\right)$$

Note that we use $\widetilde{P}_t^M = (1-\rho\beta)\,P_t^M$ rather than P_t^M in numerical analysis, since the former is far less sensitive to maturity structure variations.

- Now calculate the expectation terms $E\left[Z\left(X(s')\right)\right]$ at each node s. Let ω_k denote the weights for the quadrature, then

$$E\left[M(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{C}(s';\theta^c)\right)^{-\sigma}\widehat{Y}(s';\theta^y)\widehat{\Pi}(s';\theta^\pi)\left(\widehat{\Pi}(s';\theta^\pi) - 1\right) \equiv \overline{M}\left(s', q\right)$$

$$E\left[L(s')\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\widehat{C}(s';\theta^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^\pi)\right)^{-1}\left(1 + \frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}\right)}{1-\rho\beta}\right) \equiv \overline{L}\left(s', q\right)$$

and

$$E_t\left[\left(\frac{1+\rho P_{t+1}^M}{\Pi_{t+1}}\right)\lambda_{3t+1}\right] \approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\left(\frac{1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}\right)}{1-\rho\beta}}{\widehat{\Pi}(s';\theta^{\pi})}\right)\widehat{\lambda}_3(s';\theta^{\lambda_3}) \equiv \Lambda\left(s',q\right).$$

Hence,

$$E\left[Z\left(X(s')\right)\right] \approx E\left[\widehat{Z}\left(X(s')\right)\right] = \begin{bmatrix} \overline{M}\left(s',q\right) \\ \overline{L}\left(s',q\right) \\ \Lambda\left(s',q\right) \end{bmatrix}$$

- Next calculate the partial derivatives under expectation $E\left[Z_b\left(X(s')\right)\right]$.

- Note that we only need to compute $\partial C_{t+1}/\partial b_t$, $\partial Y_{t+1}/\partial b_t$, $\partial \Pi_{t+1}/\partial b_t$ and $\partial P_{t+1}^M/\partial b_t$, which are given as follows:

$$\frac{\partial C_{t+1}}{\partial b} \approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^c_{j_b j_\epsilon}}{b_{max}-b_{min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{C}_b\left(s'\right)$$

$$\frac{\partial Y_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^y_{j_b j_\epsilon}}{b_{max}-b_{min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{Y}_b\left(s'\right)$$

$$\frac{\partial \Pi_{t+1}}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^\pi_{j_b j_\epsilon}}{b_{max}-b_{min}}T'_{j_b}(\xi(b'))T_{j_\epsilon}(\xi(\epsilon')) \equiv \widehat{\Pi}_b\left(s'\right)$$

$$\frac{\partial P_{t+1}^M}{\partial b_t} \approx \sum_{j_b=0}^{n_b}\sum_{j_\epsilon=0}^{n_\epsilon}\frac{2\theta^{\widetilde{p}}_{j_b j_\epsilon}}{\left(b_{max}-b_{min}\right)\left(1-\rho\beta\right)}T'_{j_b}(\xi(b_i))T_{j_\epsilon}(\xi(\epsilon_j)) \equiv \widehat{P}_b^M\left(s'\right)$$

Hence, we can approximate the two partial derivatives under expectation

$$\frac{\partial E\left[M(s')\right]}{\partial b}$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\begin{bmatrix} -\sigma\left(\widehat{C}(s';\theta^c)\right)^{-\sigma-1}\widehat{Y}(s';\theta^y)\widehat{\Pi}(s';\theta^\pi)\left(\widehat{\Pi}(s';\theta^\pi)-1\right)\widehat{C}_b\left(s'\right) \\ +\left(\widehat{C}(s';\theta^c)\right)^{-\sigma}\widehat{\Pi}(s';\theta^\pi)\left(\widehat{\Pi}(s';\theta^\pi)-1\right)\widehat{Y}_b\left(s'\right) \\ +\left(\widehat{C}(s';\theta^c)\right)^{-\sigma}\widehat{\Pi}(s';\theta^\pi)\left(2\widehat{\Pi}(s';\theta^\pi)-1\right)\widehat{\Pi}_b\left(s'\right) \end{bmatrix}$$

$$\equiv \widehat{M}_b\left(s',q\right),$$

$$\frac{\partial E\left[L(s')\right]}{\partial b}$$

$$\approx \frac{1}{\sqrt{\pi}}\sum_{k=1}^{q}\omega_k\begin{bmatrix} -\sigma\left(\widehat{C}(s';\theta^c)\right)^{-\sigma-1}\left(\widehat{\Pi}(s';\theta^\pi)\right)^{-1}(1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}\right)}{1-\rho\beta})\widehat{C}_b\left(s'\right) \\ -\left(\widehat{C}(s';\theta^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^\pi)\right)^{-2}(1+\frac{\rho\widehat{P^M}\left(s';\theta^{\widetilde{p}}\right)}{1-\rho\beta})\widehat{\Pi}_b\left(s'\right) \\ +\rho\left(\widehat{C}(s';\theta^c)\right)^{-\sigma}\left(\widehat{\Pi}(s';\theta^\pi)\right)^{-1}\widehat{P}_b^M\left(s'\right)\end{bmatrix}$$

$$\equiv \widehat{L}_b\left(s',q\right).$$

That is,

$$E\left[Z_b\left(X(s')\right)\right]\approx E\left[\widehat{Z}_b\left(X(s')\right)\right]=\begin{bmatrix}\widehat{M}_b\left(s',q\right)\\ \widehat{L}_b\left(s',q\right)\end{bmatrix}$$

4. At each collocation node s, solve for $X(s)$ such that

$$\Gamma\left(s,X(s),E\left[\widehat{Z}\left(X(s')\right)\right],E\left[\widehat{Z}_b\left(X(s')\right)\right]\right)=0$$

The equation solver *csolve* written by Christopher A. Sims is employed to solve the resulted system of nonlinear equations. With $X(s)$ at hand, we can get the corresponding coefficient

$$\widehat{\Theta}_X^j=\left(\Omega\left(S\right)^T\Omega\left(S\right)\right)^{-1}\Omega\left(S\right)^T X(s)$$

5. Update the approximating coefficients, $\Theta_X^j=\eta\widehat{\Theta}_X^j+\left(1-\eta\right)\Theta_X^{j-1}$, where $0\leq\eta\leq 1$ is some dampening parameter used for improving convergence.
6. Check the stopping rules. If $\left\|\Theta_X^j-\Theta_X^{j-1}\right\|<\epsilon_{tol}$, then stop, else update the approximation coefficients and go back to step 3.

When implementing the above algorithm, we start from lower order Chebyshev polynomials and some reasonable initial guess. Then, we increase the order of approximation and take as starting value the solution from the previous lower order approximation. This informal homotopy continuation idea facilitates obtaining the solution.

Remark. Given the fact that the price P_t^M fluctuates significantly for larger ρ, in numerical analysis, the rule for P_t^M is scaled by $(1-\rho\beta)$, that is, $\widetilde{P}_t^M = (1-\rho\beta)P_t^M$. In this way, the steady state of $\widetilde{P}_t^M$ is very close to β, and $\widetilde{P}_t^M$ does not differ hugely as we change the maturity structure.

Appendix H. Euler Equation Errors

To assess the accuracy of solutions, we calculate the Euler equation errors on an evenly-spaced grid that consists of 40 points of b_t and 40 points of $log(\epsilon_t)$. The results are similar on a finer grid.

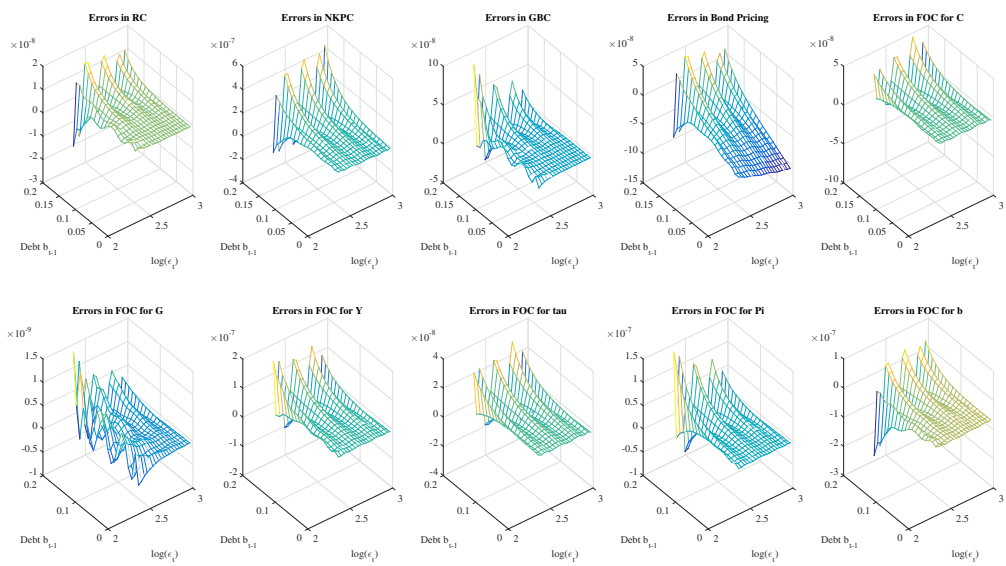

Figure H.1: Euler equation errors in the state space used to the solve the benchmark model. This figure plots the Euler equation errors on an evenly-spaced grid.
Note: Euler equation errors for other model variants are available upon request.

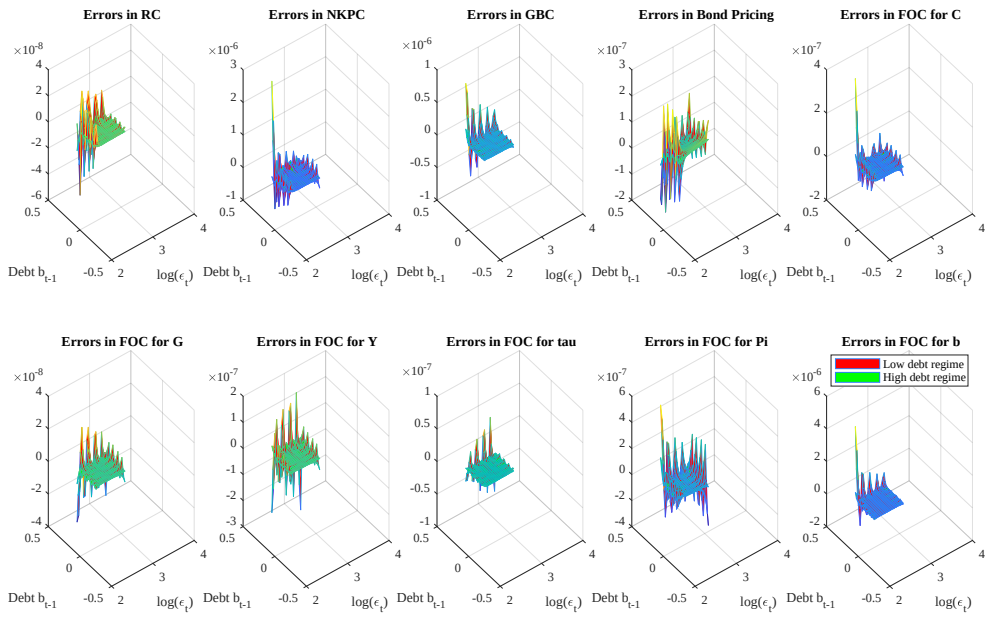

Figure H.2: Euler equation errors in the state space used to the solve the benchmark model augmented with switching in the degree of policy maker myopia. This figure plots the Euler equation errors on an evenly-spaced grid.
Note: Euler equation errors for other model variants are available upon request.

Appendix I. Model with Money

In this Section we introduce a monetary friction which has been used as a device to achieve a positive steady-state debt-to-GDP ratio in (near) flexible price models.

Appendix I.1. Households' Problem

The budget constraint at time t is given by

$$\int_0^1 P_t(j)C_t(j)dj(1+s(v_t)) + P_t^M B_t^M + M_t \leq \Xi_t + (1+\rho P_t^M)B_{t-1}^M + M_{t-1} + W_t N_t(1-\tau_t)$$

where $P_t(j)$ is the price of variety j , Ξ is the representative household's share of profits in the imperfectly competitive firms, W are wages, and τ is an wage income tax rate. Money, M_t,

facilitates consumption purchases since consumption purchases are subject to a proportional transaction cost $s(v_t)$, which depends on consumption-based money velocity,

$$v_t = \frac{\int_0^1 P_t(j)C_t(j)dj}{M_{t-1}}$$

The transaction cost function satisfies the same assumptions as in Schmitt-Grohe and Uribe (2004). Specifically, $s(v)$ satisfies:(i) $s(v)$ is non-negative and twice continuously differentiable; (ii)there is a satiation level of velocity, $\underline{v}$, such that $s(\underline{v}) = s'(\underline{v}) = 0$; (iii) $(v - \underline{v})s'(v) > 0$ for $v \neq \underline{v}$; and (iv)$2s'(v) + vs''(v) > 0$ for all $v \geq \underline{v}$. Note, however, following Niemann et al. (2013), that we have changed the timing assumption of Schmitt-Grohe and Uribe (2004) to make this more akin to a cash-in-advance constraint. This ensures that unanticipated inflation is costly in the absence of sticky prices, just as anticipated inflation is.

As a result of introducing this transactions cost, the households' first order conditions become,

$$\beta R_t E_t \left\{ \frac{\mu_{t+1}}{\mu_t} \left(\frac{P_t}{P_{t+1}} \right) \right\} = 1 \tag{I.1}$$

where

$$\mu_t \equiv \frac{C_t^{-\sigma}}{(1 + s(v_t) + s'(v_t)v_t)}$$

and the declining payoff consols,

$$\beta E_t \left\{ \frac{\mu_{t+1}}{\mu_t} \left(\frac{P_t}{P_{t+1}} \right) \left(1 + \rho P_{t+1}^M\right) \right\} = P_t^M \tag{I.2}$$

Their second FOC relates to their demand for money,

$$1 = \beta E_t \left(\frac{\mu_{t+1}}{\mu_t} \right) \left(\frac{P_t}{P_{t+1}} \right) (1 + s'(v_{t+1})v_{t+1}^2)$$

The final FOC relates to their labour supply decision and is given by,

$$(1-\tau_t)\left(\frac{W_t}{P_t}\right) = N_t^{\varphi}\mu_t^{-1}$$

That is, the marginal rate of substitution between consumption and leisure equals the after-tax wage rate.

Appendix I.2. Firms' Problem

The problem facing firms is the same as previously except the stochastic discount factor used to discount future profits is now given by, $Q_{t,t+1} = \beta\left(\frac{\mu_{t+1}}{\mu_t}\right)\Pi_{t+1}^{-1}$, such that the NKPC becomes,

$$\begin{aligned} 0 &= (1-\epsilon) + \epsilon mc_t - \phi\Pi_t\left(\Pi_t - 1\right) \\ &+ \phi\beta E_t\left[\left(\frac{\mu_{t+1}}{\mu_t}\right)\frac{Y_{t+1}}{Y_t}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\right] \end{aligned} \tag{I.3}$$

Appendix I.3. Market Clearing

Goods market clearing requires, for each good j,

$$Y_t(j) = C_t(j)(1+s(v_t)) + G_t(j) + \eta_t(j)$$

which allows us to write,

$$Y_t = C_t(1+s(v_t)) + G_t + \eta_t$$

with $\eta_t = \int_0^1 \eta_t\left(j\right)dj$. In a symmetrical equilibrium,

$$Y_t\left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right] = C_t(1+s(v_t)) + G_t$$

Appendix I.4. The Government

The government's sequential budget constraint is adjusted to account for the seigniorage revenues,

$$P_t^M B_t^M + P_t^S B_t^S + M_t + \tau_t W_t N_t = P_t G_t + B_{t-1}^S + (1 + \rho P_t^M) B_{t-1}^M + M_{t-1}$$

which can be rewritten in real terms

$$P_t^M b_t + m_t = (1 + \rho P_t^M) \frac{b_{t-1}}{\Pi_t} + \frac{m_{t-1}}{\Pi_t} - \frac{W_t}{P_t} N_t \tau_t + G_t \tag{I.4}$$

where real debt is defined as, $b_t \equiv B_t^M / P_t$, and real money balances, $m_t = \frac{M_t}{P_t}$.

Appendix I.5. The Discretionary Policy Problem

The policy under discretion can be described as a set of decision rules for $\{C_t, Y_t, \Pi_t, b_t, \tau_t, G_t\}$ which maximize the following Lagrangian,

$$\begin{aligned}
\mathcal{L} = & \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \beta E_t [V(b_t, m_t, \epsilon_{t+1})] \right\} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t (1 + s(v_t)) - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon) + \epsilon (1-\tau_t)^{-1} Y_t^{\varphi} \mu_t^{-1} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta \mu_t^{-1} Y_t^{-1} E_t [M(b_t, m_t, \epsilon_{t+1})] \end{array} \right] \\
& + \lambda_{3t} \left[\begin{array}{c} \beta b_t \mu_t^{-1} E_t [L(b_t, m_t, \epsilon_{t+1})] - \frac{b_{t-1}}{\Pi_t} \left(1 + \rho \beta \mu_t^{-1} E_t [L(b_t, m_t, \epsilon_{t+1})] \right) \\ + \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} \mu_t^{-1} - G_t + m_t - \frac{m_{t-1}}{\Pi_t} \end{array} \right] \\
& + \lambda_{4t} \left[1 - \beta \mu_t^{-1} E_t N(b_t, m_t, \epsilon_{t+1}) \right] \\
& + \lambda_{5t} \left[\mu_t - \frac{C_t^{-\sigma}}{(1 + s(v_t) + s'(v_t) v_t)} \right] \\
& + \lambda_{6t} [v_t - \frac{C_t \Pi_t}{m_{t-1}}]
\end{aligned} \tag{I.5}$$

where the auxiliary functions are defined as,

$$M(b_t, m_t, \epsilon_{t+1}) = \mu_{t+1} Y_{t+1} \Pi_{t+1} \left(\Pi_{t+1} - 1\right) \tag{I.6}$$

$$L(b_t, m_t, \epsilon_{t+1}) = \mu_{t+1} (\Pi_{t+1})^{-1} (1 + \rho P^M_{t+1}) \tag{I.7}$$

$$N(b_t, m_t, \epsilon_{t+1}) = \mu_{t+1} (\Pi_{t+1})^{-1} (1 + s'(v_{t+1}) \left(v_{t+1}\right)^2) \tag{I.8}$$

We can write the first order conditions (FOCs) for the policy problem as follows:

The FOC for consumption,

$$C_t^{-\sigma} - \lambda_{1t} \left[1 + s(v_t)\right] - \lambda_{6t} \frac{v_t}{C_t}$$

$$+ \lambda_{5t} \mu_t \sigma C_t^{-1} = 0 \tag{I.9}$$

output,

$$-Y_t^{\varphi} + \lambda_{1t} \left[1 - \frac{\phi}{2} \left(\Pi_t - 1\right)^2\right]$$

$$+\lambda_{2t} \left[\epsilon \varphi (1 - \tau_t)^{-1} Y_t^{\varphi - 1} \mu_t^{-1} - \phi \beta \mu_t^{-1} Y_t^{-2} E_t \left[M(b_t, m_t, \epsilon_{t+1})\right]\right]$$

$$+ \lambda_{3t} \left[(1 + \varphi) Y_t^{\varphi} \mu_t^{-1} \left(\frac{\tau_t}{1 - \tau_t}\right)\right] = 0 \tag{I.10}$$

taxation,

$$\epsilon \lambda_{2t} + \lambda_{3t} Y_t = 0 \tag{I.11}$$

government consumption,

$$\chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0 \tag{I.12}$$

inflation

$$-\lambda_{1t} \left[Y_t \phi \left(\Pi_t - 1\right)\right] - \lambda_{2t} \left[\phi \left(2\Pi_t - 1\right)\right]$$

$$+ \lambda_{3t} \left[\frac{b_{t-1}}{\Pi_t^2} \left(1 + \rho \beta \mu_t^{-1} E_t \left[L(b_t, m_t, \epsilon_{t+1})\right]\right) + \frac{m_{t-1}}{\Pi_t^2}\right] - \lambda_{6t} \frac{v_t}{\Pi_t} = 0 \tag{I.13}$$

marginal utility, μ_t,

$$\lambda_{2t}\mu_t^{-2}[-\epsilon(1-\tau_t)^{-1}Y_t^{\varphi} - \phi\beta Y_t^{-1}E_t\left[M(b_t, m_t, \epsilon_{t+1})\right]]$$

$$+\lambda_{3t}\mu_t^{-2}[-\beta b_t E_t\left[L(b_t, m_t, \epsilon_{t+1})\right] + \frac{b_{t-1}}{\Pi_t}\rho\beta E_t\left[L(b_t, m_t, \epsilon_{t+1})\right] - \frac{\tau_t}{1-\tau_t}\left(Y_t\right)^{1+\varphi}] \tag{I.14}$$

$$+\lambda_{4t}\left[\beta\mu_t^{-2}E_t\left[N(b_t, m_t, \epsilon_{t+1})\right]\right] + \lambda_{5t} = 0$$

and velocity,

$$-\lambda_{1t}C_t s'(v_t) + \lambda_{6t}$$

$$+\lambda_{5t}\mu_t\left[\frac{(2s'(v_t) + s''(v_t)v_t)}{(1 + s(v_t) + s'(v_t)v_t)}\right] = 0 \tag{I.15}$$

The remaining FOCs are for government debt,

$$0 = -\beta E_t\left[\frac{\lambda_{3t+1}}{\Pi_{t+1}}(1+\rho P_{t+1}^m)\right] + \lambda_{2t}\left[\phi\beta\mu_t^{-1}Y_t^{-1}E_t\left[M_1(b_t, m_t, \epsilon_{t+1})\right]\right]$$

$$+\beta\lambda_{3t}\left[\mu_t^{-1}E_t\left[L(b_t, m_t\epsilon_{t+1})\right] + b_t\mu_t^{-1}E_t\left[L_1(b_t, m_t, \epsilon_{t+1})\right] - \rho\frac{b_{t-1}}{\Pi_t}\mu_t^{-1}E_t\left[L_1(b_t, m_t, \epsilon_{t+1})\right]\right] \tag{I.16}$$

$$-\lambda_{4t}\left[\beta\mu_t^{-1}E_t\left[N_1(b_t, m_t, \epsilon_{t+1})\right]\right]$$

and money balances,

$$\beta E_t[-\lambda_{3t+1}\frac{1}{\Pi_{t+1}} + \lambda_{6t+1}\frac{v_{t+1}}{m_t}]$$

$$+\lambda_{2t}\left[\phi\beta\mu_t^{-1}Y_t^{-1}E_t\left[M_2(b_t, m_t, \epsilon_{t+1})\right]\right]$$

$$+\beta\lambda_{3t}\left[\beta^{-1} + b_t\mu_t^{-1}E_t\left[L_2(b_t, m_t, \epsilon_{t+1})\right] - \rho\frac{b_{t-1}}{\Pi_t}\mu_t^{-1}E_t\left[L_2(b_t, m_t, \epsilon_{t+1})\right]\right]$$

$$-\lambda_{4t}\left[\beta\mu_t^{-1}E_t\left[N_2(b_t, m_t, \epsilon_{t+1})\right]\right] = 0 \tag{I.17}$$

The discretionary equilibrium is determined by the system given by the FOCs, (I.9),

- (I.17), the constraints in (I.5) , the auxiliary equations, (I.6)-(I.8), bond prices, $P_t^M = \beta C_t^\sigma E_t \left[L(b_t, \epsilon_{t+1})\right]$, and the exogenous process for the markup shock,

$$\ln(\epsilon_t) = (1-\rho_\epsilon)\ln(\overline{\epsilon}) + \rho_\epsilon \ln(\epsilon_{t-1}) + \sigma_\epsilon \varepsilon_t,\ \varepsilon_t \sim N(0,1)$$

The solution to this system is a set of time-invariant Markov-perfect equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, m_{t-1}, \epsilon_t\}$ to the optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, m_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}, \lambda_{4t}, \lambda_{5t}, \lambda_{6t}\}$ for all $t \geq 0$.

Solving this model can generate a positive steady-state debt-to-GDP ratio. However, it is only when price stickiness is reduced to implausibly low levels ($\phi < 4$, an effective average price duration of less than 4 months) that the debt-to-GDP ratio can be turned mildly positive. For example with $\phi = 2.5$ (equivalent to a Calvo probability of no price change of 0.14 and an average price duration of just under 3.5 months), the steady-state debt-to-GDP ratio is 13.3%, but this implies very large inflation response to shocks alongside negligible movements in the debt-to-GDP ratio. This suggests that the mild myopia adopted in this paper is a more data-consistent motivation for existence of a positive steady-state debt and the observed fluctuations in debt relative to that steady-state, which also facilitates a comparison with the commonly used cashless economy framework of much New Keynesian analysis of optimal policy.

Appendix J. Government Spending Shock

In this Appendix we drop the assumption that government spending is a fiscal instrument available to the optimizing policy maker and assume that the share of government consumption in GDP follows an exogenous AR(1) process as in Chen et al(2018).

$$\ln(G_t/Y_t) = (1-\rho_g)\ln(\overline{G}/\overline{Y}) + \rho_g \ln(G_{t-1}/Y_{t-1}) + \sigma_g \varepsilon_{g,t},$$

where $\varepsilon_{g,t}$ is standard normally distributed. Figure J.1 plots the response to a positive shock to this process under commitment and discretion with different debt maturities.

Under commitment there is a small step increase in taxes to sustain the debt issued to fund the prolonged rise in government consumption. There is a tightening of monetary policy to largely offset the rise in inflation which would otherwise emerge as a result of the increase in government consumption. The short-term debt rises by slightly more than longer term debt as inflation is used, sparingly, to reduce bond prices. However, this is so mild that it cannot be seen in the plot of inflation.

The policy response under discretion is radically different. Tax rates rise substantially, reducing debt in the face of an increase in government consumption. This reduction in debt serves to reduce the debt-dependent inflationary biases and helps mitigate the inflationary consequences of the government consumption shock. This difference relative to the Ramsey policy is heightened with shorter-maturity debt, as was the case under an inflationary mark-up shock. The figure also shows that while inflation has been effectively eliminated under commitment, it is substantial under discretion especially as debt maturity falls. This is reflected in the differences in the level of output across policies which captures the resource costs of inflation.

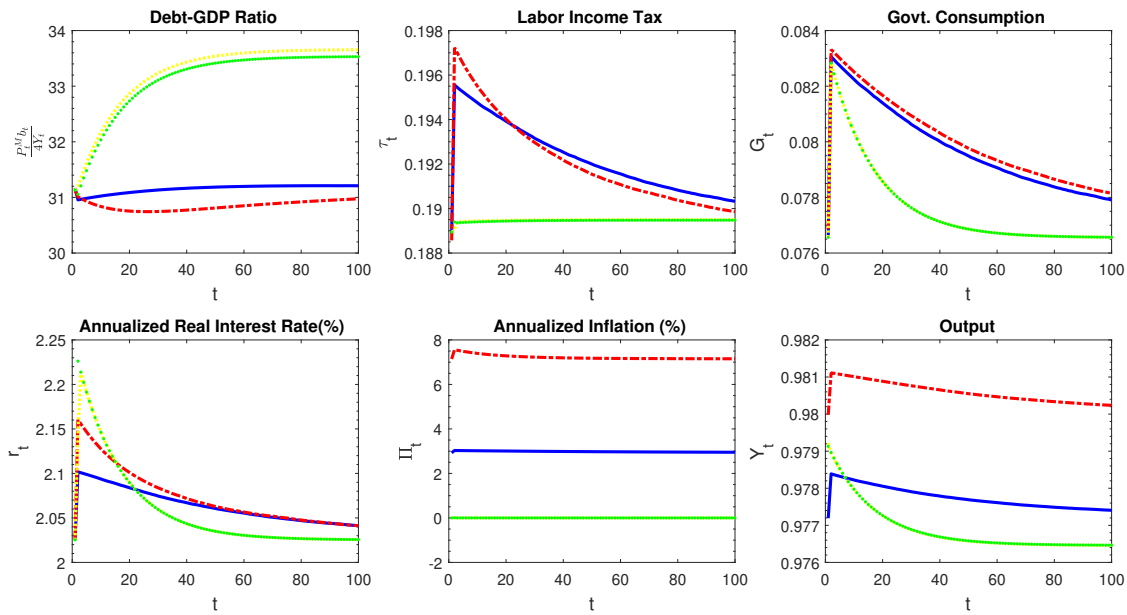

Figure J.1: Government spending shock - commitment vs. discretion - debt maturity
Note: Yellow dotted line represents outcomes under commitment with long-term debt, and green points commitment with single period debt. These largely overlap. Solid blue line presents discretion with long-term debt, and red dash-dotted line discretion with single period debt. Myopia has been increased in the case of single period debt to ensure the steady-state debt-to-GDP ratio is the same as the other model variants considered.

Appendix K. Switches in Policy Maker Myopia

The model, despite matching key fiscal data averages, cannot capture the key trends in the debt-to-GDP ratio seen in the data. Therefore, in order to generate plausible movements in the debt-to-GDP ratio, there is a need to go beyond standard economic shocks and consider political frictions.

Specifically, the degree of policy maker myopia is assumed switch between two regimes, $\{\widetilde{\beta}_L, \widetilde{\beta}_H\}$ where $\widetilde{\beta}_L > \widetilde{\beta}_H$ where the former 'L' regime has a low degree of myopia and correspondingly supports a lower level of debt. Conversely, the high myopia regime is consistent with a higher debt level. There is an associated transition probability matrix governing the evolution of this two-state Markov process, $\begin{bmatrix} p_L & 1-p_L \\ 1-p_H & p_H \end{bmatrix}$ where p_i is the probability of remaining in regime i $(i = H, L)$ given we are currently regime i and $1-p_i$ is the probability of exit to the other regime j, $, j = (H, L), j \neq i$. The policy maker is assumed to not be in conflict with their future selves but to discount the future in line with whatever degree

of myopia is in place at the time. As a result, the degree of myopia becomes an additional state variable so that the value function is defined as,

$$V(b_{t-1}, \epsilon_t, \widetilde{\beta}_{i,t}) = \max \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t/A_t)^{1+\varphi}}{1+\varphi} + \widetilde{\beta}_{i,t} E_t \left[V(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1}) \right] \right\}$$

subject to the same constraints as before but where all auxiliary functions are based on this expanded state-space where $\widetilde{\beta}_{it} = \widetilde{\beta}_L$ or $\widetilde{\beta}_H$.

As a result of this change the policy problem is reformulated as,

$$\begin{aligned}
\mathcal{L} = & \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \widetilde{\beta}_{i,t} E_t [V(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1})] \right\} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left[M(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1}) \right] \end{array} \right] \\
& + \lambda_{3t} \left[\begin{array}{c} \beta b_t C_t^{\sigma} E_t \left[L(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1}) \right] - \frac{b_{t-1}}{\Pi_t} \left(1 + \rho \beta C_t^{\sigma} E_t \left[L(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1}) \right] \right) \\ + \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma} - G_t - tr \end{array} \right]
\end{aligned} \tag{K.1}$$

The policy maker optimizes this Lagrangian by choosing C_t, G_t, Y_t, Π_t, τ_t, b_t and the multipliers, λ_{1t}, λ_{2t}, λ_{3t}. The only difference between these FOCs and those in the benchmark model are that the FOC for debt now depends upon the myopia of the current policy maker, $\widetilde{\beta}_{i,t}$, such that

$$\underbrace{P_t^M \lambda_{3t} - \widetilde{\beta}_{i,t} E_t \left[\frac{\lambda_{3t+1}}{\Pi_{t+1}} (1 + \rho P_{t+1}^M) \right]}_{\text{tax smoothing}}$$

$$\underbrace{-\lambda_{3t} C_t^{\sigma} \left(\phi \epsilon^{-1} \beta E_t \left[M_1(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1}) \right] - \left[(b_t - \rho \frac{b_{t-1}}{\Pi_t}) E_t L_1(b_t, \epsilon_{t+1}, \widetilde{\beta}_{i,t+1}) \right] \right)}_{\text{debt stabilization bias}} = 0 \tag{K.2}$$

The solution to the resultant system of FOCs is a set of time-invariant Markov-perfect equilibrium policy rules $y_t = H(s_{t-1})$ mapping the vector of states $s_{t-1} = \{b_{t-1}, \epsilon_t, \widetilde{\beta}_{i,t}\}$ to the

optimal decisions for $y_t = \{C_t, G_t, Y_t, \Pi_t, \tau_t, b_t, P_t^M, \lambda_{1t}, \lambda_{2t}, \lambda_{3t}\}$ for all $t \geq 0$. In formulating the policy problem in this way the policy maker does not try to tie the hands of their future selves. They simply accept that there are periods in which they will be relatively more or less patient. To allow for conflict between two policy makers of different degrees of myopia, it would be necessary for each policy maker to evaluate the anticipated policy outcomes when their opponent was in power using their own discount factor and adjust policies in influence their opponent's behavior. It would be interesting to consider these strategic interactions in future work.

The calibration of this Markov switching process follows Chen et al. (2018) in identifying key shifts in the trend of the U.S. debt-to-GDP ratio - see Figure K.1. Between 1954 and the first budget of the Reagan presidency in 1981 debt is on a downward trend. Similarly following Clinton's first budget until the first budget of the George W. Bush there is a sustained reduction in the debt-to-GDP ratio. We label these episodes as being periods of low myopia. In contrast the periods of rising debt-to-GDP ratios covering all other periods are labeled as high myopia. Given this labeling the implied transition matrices between the two regimes can be estimated as, $\begin{bmatrix} 0.9859 & 1-0.9859 \\ 1-0.9868 & 0.9868 \end{bmatrix}$. $\widetilde{\beta}_L = 0.9866$ and $\widetilde{\beta}_H = 0.9759$ are chosen to replicate the peaks and troughs of the debt-to-GDP ratio found in the data, while the remainder of the benchmark calibration is retained. The success of this exercise can be seen in Figure K.1 where the model implied dynamics of the debt-to-GDP ratio track the data both in the sense of ensuring the model can achieve the highs and lows seen in the data, but also the pace at which debt increases or decreases over time.

Figure K.1 also allows us to examine the transitions between high and low debt regimes. Consider the reduction in debt between the end of WWII and the Reagan budget of 1981. The transitional dynamics are strikingly different from the ultimate steady-state (conditional on remaining in the particular myopia regime). A relatively patient government inheriting a large debt stock faces, for a given rate of inflation, greater incentives to induce inflation

surprises to reduce that debt burden. This induces them to raise taxation sharply and to a lesser extent reduce government consumption to facilitate the reduction in debt. At the same time monetary policy is tightened to partially offset the increase in inflation. In the longer term, the successful reduction in the debt-to-GDP ratio allows the economy to sustain lower taxes, higher government consumption and lower inflation.

The rise in debt from 1981 until the first Clinton budget is explained by a switch to a relatively myopia policy maker. Since they care less about the future costs of servicing debt, their incentives to reduce a given level of debt through surprise inflation are lower. This enables them to dramatically reduce taxes and, to a lesser extent, increase government consumption in the short-run, while simultaneously enjoying relatively low inflation. However, ultimately the myopic policy maker suffers from higher taxation, lower government consumption and higher inflation as a result of the debt they accumulate in the long-run.

Aside from capturing the trends in the debt-to-GDP ratio, does this mimic other key macroeconomic data? Prior to 1980 there is no obvious link between model implied fiscal variables and the data, since the extremely high inflation observed in the late 60s and 1970s, are likely to drive the downward trend in the debt-to-GDP ratio. Subsequently, however, the switches in policy maker myopia imply sharp tax cuts under Reagan and the second George Bush and similarly sharp increases under Clinton which loosely correlate with the shifts in the ratio of tax revenues to GDP seen in the data, although these are far more gradual. Similarly the shifts in government spending implied by the model post 1981 are not obviously inconsistent with the detrended data plotted in Figure K.1.

When it comes to monetary policy and inflation, the switch to a myopic policy maker in 1981 leads to a tightening of monetary policy and a fall in inflation, qualitatively consistent with the Volcker disinflation, but quantitatively far smaller than seen in the data. Overall, the simple model does surprisingly well in fitting the data using only relatively infrequent switches in policy maker myopia as a means of doing so, although there is clearly still much left to explain.

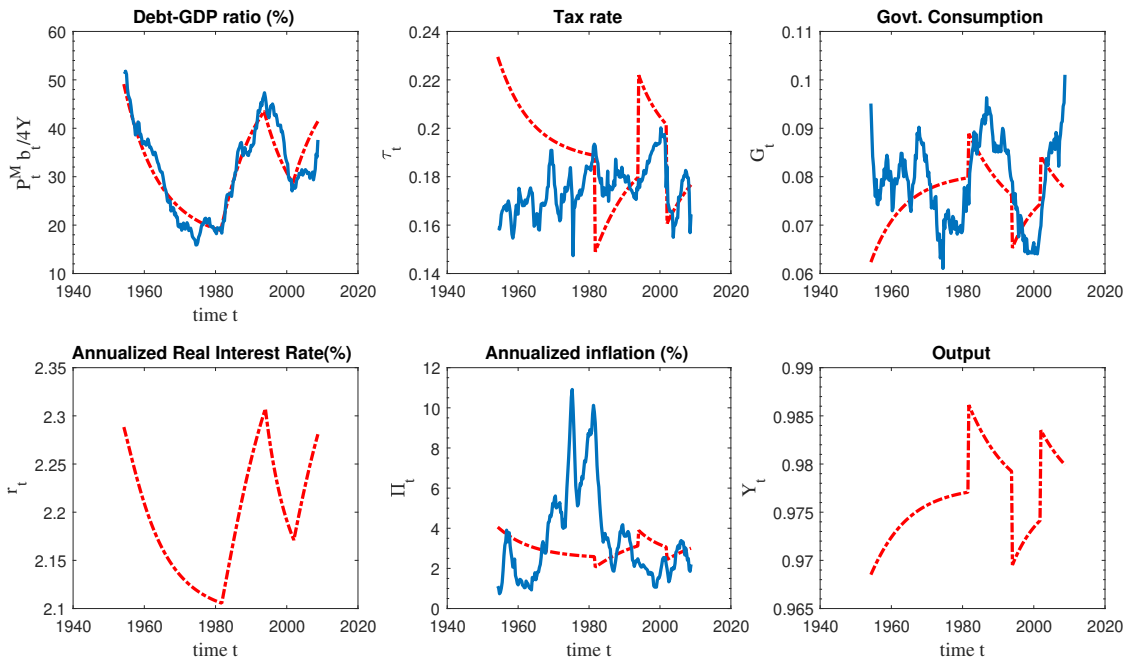

Figure K.1: Matching data with regime switches
Note: Solid-line reflects the data as described in Appendix C. The data for G has been detrended. The red dash-dotted line is the simulated outcome from the benchmark model assuming higher policy maker myopia between the first Reagan and Clinton budgets, and following the George W. Bush administrations' first budget.

Appendix L. Endogenous Short-Term Debt - 3 Period Model

In this section we consider a far simpler model adapted from Leeper and Leith (2016) in order to highlight the trade-offs facing the policy-maker who can issue both long and short-term bonds. The economy is a perfect-foresight endowment economy with no government consumption such that consumption always equals its endowment (which is assumed constant at γ). Households can save in the form of one and two period bonds, such that their three budget constraints are given by, in period $t = 0$,

$$Q_{0,1}b_{0,1} + Q_{0,2}b_{0,2} = \gamma - c_0 + \zeta_0 - \tau_0 + b_{-1,0}\nu_0 + Q_{0,1}b_{-1,1}\nu_0 \tag{L.1}$$

where the state variables are defined as, $b_{j,k} \equiv B_{j,k}/P_j$ reflecting the quantity of zero coupon nominal bonds issued in period j which mature in period k, deflated by the price level in period j, $\nu_t = \Pi_t^{-1}$ is the inverse of the gross rate of inflation and $Q_{t,t+j}$ is the price of zero coupon debt in period t, which matures in period $t + j$. There is an endowment, γ, in each

period, which finances consumption c_0, taxation τ_0 and net savings. A transfer shock in period 0, ζ_0, serves as surprise for the policy maker beyond the predetermined level of debt they inherit. Therefore the household inherits a stock of one and two period bonds which were issued in period $t = -1$, $b_{-1,0}$ and $b_{-1,1}$, respectively, and decides how much to consume, c_0, alongside the quantity of one and two period bonds to purchase in period $t = 0$, $b_{0,1}$ and $b_{0,2}$.

The corresponding period $t = 1$ constraint is,

$$Q_{1,2}b_{1,2} = \gamma - c_1 - \tau_1 + \nu_1 b_{0,1} + Q_{1,2}\nu_1 b_{0,2} \tag{L.2}$$

where it is no longer possible to purchase two period bonds as the economy ceases to exist at the end of period t=2. The final period $t = 2$,

$$\tau_2 = \gamma - c_1 + \nu_2 b_{1,2} \tag{L.3}$$

The household maximizes utility,

$$\sum_{t=0}^{2} \beta^t u(c_t) \tag{L.4}$$

subject to the series of budget constraints. Given the resource constraint implies consumption in each period is constant and equal to the household endowment, $c_t = \gamma$, the bond pricing equations reduce to,

$$\beta\nu_{t+1} = Q_{t,t+1} \tag{L.5}$$

$$\beta^2\nu_{t+1}\nu_{t+2} = Q_{t,t+1}Q_{t+1,t+2} = Q_{t,t+2} \tag{L.6}$$

The government's budget constraints then mirror those of the household, in period $t = 0$,

$$\beta\nu_1 b_{0,1} + \beta^2\nu_1\nu_2 b_{0,2} = -\tau_0 + \zeta_0 + b_{-1,0}\nu_0 + Q_{0,1}b_{-1,1}\nu_0 \tag{L.7}$$

period $t = 1$,

$$\beta\nu_2 b_{1,2} = -\tau_1 + \nu_1 b_{0,1} + \beta\nu_2\nu_1 b_{0,2} \tag{L.8}$$

and the final period $t = 2$,

$$\tau_2 = \nu_2 b_{1,2} \tag{L.9}$$

Following Leeper and Leith (2016) it is assumed that inflation and taxation are costly, such that social welfare is given by,

$$-\sum_{t=0}^{2} \beta^t \left(\tau_t^2 + \theta(\nu_t - 1)^2\right) \tag{L.10}$$

where τ_t is the tax rate and $\nu_t = \Pi_t^{-1}$ is the inverse of the gross rate of inflation. The parameter θ captures the relative cost of inflation and can be, more generally, thought of as the welfare cost of the inflation bias problem which in this simple model will be associated with the desire to reduce the real value of debt rather than boosting the size of the real economy.

We now consider optimal policy under both commitment and discretion in this simple economy.

Appendix L.1. Commitment

The commitment policy is simple to characterize using the following Lagrangian,

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{t=0}^{2} \beta^t[-\frac{1}{2}\left(\tau_t^2 + \theta(\nu_t - 1)^2\right)] \\ & +\lambda\left[b_{-1,0}\nu_0 + \beta\nu_0\nu_1 b_{-1,1} + \zeta_0 - \tau_0 - \beta\tau_1 - \beta^2\tau_2\right]\end{aligned}$$

with FOCs for taxation of,

$$\tau_t = -\lambda \text{ for } t = 0, 1, 2$$

and deflation,

$$-\beta^2\theta(\nu_2 - 1) = 0 \text{ i.e. } \nu_2 = 1$$

$$-\beta\theta(\nu_1 - 1) = \tau_0\beta\nu_0 b_{-1,1}$$

and

$$-\theta(\nu_0 - 1) = \tau_0(b_{-1,0} + \beta\nu_1 b_{-1,1})$$

These imply that under commitment pure tax smoothing is applied. Inflation is only generated to the extent that the time $t = 0$ policy maker inherits debt from the previous period. In the absence of such debt, the policy maker would commit to zero net inflation and the tax rate would be set to satisfy the intertemporal budget constraint, $\zeta_0 = \tau(1 + \beta + \beta^2)$ where $\tau_t = \tau$ for $t = 0, 1, 2$. This outcome can be contrasted with that under discretion.

Appendix L.2. Discretion

By focusing on a three period model, we can tractably analyze the time-consistent policy problem by backward induction. To do so, we solve the period $t = 2$ problem and use the resultant FOC as an incentive compatibility constraint (ICC) for the problem in period t=1 which can be analyzed as a standard Ramsey problem subject to this additional constraint. The FOCs from this problem now become the ICCs for the period $t = 0$ problem. This bypasses the need to solve for the policy functions for each endogenous variable as a function of the states.

Appendix L.2.1. Period $t = 2$ Problem

Consider the period $t = 2$ problem which maximizes social welfare in the final period, subject to the budget constraint,

$$\mathcal{L} = -\frac{1}{2}\left(\tau_2^2 + \theta(\nu_2 - 1)^2\right)$$
$$+ \lambda_2 \left[-b_{1,2}\nu_2 + \tau_2\right]$$

with FOCs for taxation,

$$-\tau_2 + \lambda_2 = 0$$

and deflation,

$$-\theta(\nu_2 - 1) - b_{1,2}\lambda_2 = 0$$

Appendix L.2.2. Period $t = 1$ Problem

The two FOCs for the $t = 2$ problem can be combined as,

$$-\theta\nu_2(\nu_2 - 1) = \tau_2^2$$

This captures the balance between inflation and taxation used by the policy maker in period $t = 2$. The policy maker implementing policy in period $t = 1$ will take into account that the period $t = 2$ government will behave in this way. The intertemporal budget constraint facing the period $t = 1$ policy maker is given by,

$$b_{0,1}\nu_1 + \beta\nu_1\nu_2 b_{0,2} = \tau_1 + \beta\tau_1$$

Therefore the period t=1 problem becomes,

$$\begin{aligned}\mathcal{L} &= \sum_{t=1}^{2} \beta^{t-1}[-\frac{1}{2}\left(\tau_t^2 + \theta(\nu_t - 1)^2\right)] \\ &\quad +\mu\left[b_{0,1}\nu_1 + \beta\nu_1\nu_2 b_{0,2} - \tau_1 - \beta\tau_2\right] \\ &\quad +\lambda\left[-\theta\nu_2(\nu_2 - 1) - \tau_2^2\right]\end{aligned}$$

The associated FOCs are as follows, firstly for taxation τ_1,

$$-\tau_1 - \mu = 0$$

and in period $t = 2$, τ_2,

$$-\beta\tau_2 - \beta\mu - 2\tau_2\lambda = 0$$

deflation,

$$-\theta(\nu_1 - 1) + \mu(b_{0,1} + \beta\nu_2 b_{0,2}) = 0$$

or, equivalently,

$$-\theta\nu_1(\nu_1 - 1) + \mu(\tau_1 + \beta\tau_2) = 0$$

and,

$$-\beta\theta(\nu_2 - 1) + \mu(\beta\nu_1 b_{0,2}) - \lambda\theta(2\nu_2 - 1) = 0$$

From the taxation FOCs we obtain,

$$\mu = -\tau_1$$

and,

$$\lambda = \frac{\beta(\tau_1 - \tau_2)}{2\tau_2}$$

such that the FOCs reduce to,

$$\begin{aligned} -\theta\nu_1(\nu_1 - 1) &= \tau_1(\tau_1 + \beta\tau_2) \\ &= \tau_1(b_{0,1}\nu_1 + \beta\nu_1\nu_2 b_{0,2}) \end{aligned}$$

and,

$$-\beta\theta(\nu_2 - 1) = \tau_1\beta\nu_1 b_{0,2} + \frac{\beta(\tau_1 - \tau_2)}{2\tau_2}\theta(2\nu_2 - 1)$$

which simplifies as,

$$\tau_2 = \tau_1(2\nu_2 - 1) + \frac{2\tau_1\tau_2\nu_1 b_{0,2}}{\theta}$$

Appendix L.2.3. Period $t = 0$ Problem

Now the policy maker in period t=0 faces three ICCs. Those from period $t = 1$,

$$-\theta\nu_1(\nu_1 - 1) = \tau_1(\tau_1 + \beta\tau_2) \tag{L.11}$$

and

$$-\theta(\nu_2 - 1) = \tau_1\nu_1 b_{0,2} + \frac{(\tau_1 - \tau_2)}{2\tau_2}\theta(2\nu_2 - 1) \tag{L.12}$$

and the constraint describing the behavior of the period $t = 2$ policy maker,

$$-\theta\nu_2(\nu_2 - 1) = \tau_2^2 \tag{L.13}$$

We also have the intertemporal budget constraint in period 0,

$$b_{-1,0}\nu_0 + \beta\nu_0\nu_1 b_{-1,1} = \tau_0 + \beta\tau_1 + \beta^2\tau_2^2 \tag{L.14}$$

Since $b_{0,2}$ only appears in one constraint (and not in the objective function) this constraint will not bite. The period $t = 0$ policy maker will ensure that debt maturity $b_{0,2}$ is chosen to ensure this constraint doesn't bite. As a result equation (L.12) will not act as a constraint

on the period 0 policy maker's behavior, but it will determine the equilibrium value of two period debt issued in period 0, $b_{0,2}$. Therefore, the policy problem becomes,

$$\begin{aligned}\mathcal{L} &= \sum_{t=0}^{2}\beta^{t-1}[-\frac{1}{2}\left(\tau_t^2+\theta(\nu_t-1)^2\right)] \\ &+\mu\left[b_{-1,0}\nu_0+\beta\nu_0\nu_1 b_{-1,1}+\zeta_0-\tau_0-\beta\tau_1-\beta^2\tau_2\right] \\ &+\lambda\left[-\theta\nu_2(\nu_2-1)-\tau_2^2\right] \\ &+\gamma[-\theta\nu_1(\nu_1-1)-\tau_1(\tau_1+\beta\tau_2)]\end{aligned}$$

The FOCs for taxation in the three periods are as follows,

$$-\tau_0-\mu=0$$

$$-\beta\tau_1-\beta\mu-\gamma(2\tau_1+\beta\tau_2)=0$$

and,

$$-\beta^2\tau_2-\beta^2\mu-\gamma\beta\tau_1-2\tau_2\lambda=0$$

These can be used to define the three Lagrange multipliers,

$$\mu=-\tau_0 \tag{L.15}$$

$$\gamma=\beta\frac{\tau_0-\tau_1}{2\tau_1+\beta\tau_2} \tag{L.16}$$

995 and

$$\lambda=\frac{\beta^2}{2\tau_2}[\tau_0-\tau_2-\tau_1\frac{\tau_0-\tau_1}{2\tau_1+\beta\tau_2}] \tag{L.17}$$

The remaining three FOCs are for deflation,

$$-\theta(\nu_0-1)+\mu(b_{-1,0}+\beta\nu_1 b_{-1,1})=0 \tag{L.18}$$

$$-\beta\theta(\nu_1 - 1) + \mu\beta\nu_0 b_{-1,1} - \gamma\theta(2\nu_1 - 1) = 0 \tag{L.19}$$

and

$$-\beta^2\theta(\nu_2 - 1) - \lambda\theta(2\nu_2 - 1) = 0 \tag{L.20}$$

Therefore the system to solve are the FOCs (L.18)-(L.20) and the constraints (L.11)-(L.14) given the definitions of the LMs (L.15)-(L.17) and the unknown variables are, $\{\tau_0, \tau_1, \tau_2, \nu_0, \nu_1, \nu_2\}$ conditional on the initial level of debt, $b_{-1,0}$ and $b_{-1,1}$. These equations can all be solved using standard numerical solvers without the need to take numerical derivatives in contrast to the benchmark infinite horizon model used in the main paper. Applying the solution to the flow budget constraints can then track the evolution of debt.

If we consider the special case where there is no initial debt, $b_{-1,0} = b_{-1,1} = 0$ prior to the transfers shock, $\zeta_0 > 0$, then the FOCs reduce to,

$$\nu_0 = 1$$

and the transfer shock is entirely financed by taxation,

$$\zeta_0 = \tau_0 + \beta\tau_1 + \beta^2\tau_2$$

While the LMs can be solved as,

$$\gamma = \frac{\beta(1-\nu_1)}{2\nu_1 - 1} \text{and } \lambda = \frac{\beta^2(1-\nu_2)}{2\nu_2 - 1}$$

Substituting into the FOCs for taxation yields,

$$\tau_0 = \frac{1}{2\nu_1 - 1}\tau_1 + \frac{\beta(1-\nu_1)}{2\nu_1 - 1}\tau_2$$

and

$$\tau_0 = \frac{(1-\nu_1)}{2\nu_1 - 1}\tau_1 + \frac{1}{2\nu_2 - 1}\tau_2$$

These can be equated to give the following relationship between taxes/inflation in periods 1 and 2,

$$\tau_1 + \beta(1-\nu_1)\tau_2 = (1-\nu_1)\tau_1 + \frac{2\nu_1 - 1}{2\nu_2 - 1}\tau_2$$

which can be written as,

$$\tau_2 = (2\nu_1 - 1)\tau_1 + (1-\nu_1)(1+\beta(2\nu_2 - 1))\nu_1^{-1}\tau_2$$

Therefore this is the pattern of taxes and deflation the policy maker in period 0 will choose debt maturity to achieve in period 1 through the ICC from period 1,

$$\tau_2 = \tau_1(2\nu_2 - 1) + \frac{2\tau_1\tau_2\nu_1 b_{0,2}}{\theta} \tag{L.21}$$

i.e. the period 0 policy maker chooses $b_{0,2}$ to affect period 1 policy so as to achieve their desired pattern of taxes/deflation. Comparing the two expressions implies,

$$\frac{2\tau_1\tau_2\nu_1 b_{0,2}}{\theta} = (1-\nu_1)(1+\beta(2\nu_2 - 1))\nu_1^{-1}\tau_2$$

which simplifies as,

$$\nu_1 b_{0,2} = \frac{\theta}{2\tau_1}(1-\nu_1)(1+\beta(2\nu_2 - 1))\nu_1^{-1}$$

Appendix M. Optimal Policy Under Discretion With Endogenous Short-Term Debt

The Lagrangian for the policy problem can be written as,

$$\begin{aligned}
\mathcal{L} = & \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma_g}}{1-\sigma_g} - \frac{(Y_t)^{1+\varphi}}{1+\varphi} + \widetilde{\beta} E_t[V(b_t^M, \epsilon_{t+1}, b_t^S)] \right\} \\
& + \lambda_{1t} \left[Y_t \left(1 - \frac{\phi}{2} (\Pi_t - 1)^2 \right) - C_t - G_t \right] \\
& + \lambda_{2t} \left[\begin{array}{c} (1-\epsilon_t) + \epsilon_t (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma} - \phi \Pi_t (\Pi_t - 1) \\ + \phi \beta C_t^{\sigma} Y_t^{-1} E_t \left[M(b_t^M, \epsilon_{t+1}, b_t^S) \right] \end{array} \right] \\
& + \lambda_{3t} \left[\begin{array}{c} \beta b_t C_t^{\sigma} E_t \left[L(b_t^M, \epsilon_{t+1}, b_t^S) \right] + \beta b_t^S C_t^{\sigma} E_t \left[K\left(b_t^M, \epsilon_{t+1}, b_t^S\right) \right] \\ - \frac{b_{t-1}}{\Pi_t} \left(1 + \rho \beta C_t^{\sigma} E_t \left[L(b_t^M, \epsilon_{t+1}, b_t^S) \right] \right) \\ - \frac{b_{t-1}^S}{\Pi_t} + \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma} - G_t - tr \end{array} \right]
\end{aligned}$$

where

$$M(b_t^M, \epsilon_{t+1}, b_t^S) = (C_{t+1})^{-\sigma} Y_{t+1} \Pi_{t+1} (\Pi_{t+1} - 1)$$

$$L(b_t^M, \epsilon_{t+1}, b_t^S) = (C_{t+1})^{-\sigma} (\Pi_{t+1})^{-1} (1 + \rho P_{t+1}^M)$$

$$K\left(b_t^M, \epsilon_{t+1}, b_t^S\right) = C_{t+1}^{-\sigma} \Pi_{t+1}^{-1}$$

We can write the first order conditions for the policy problem as follows: consumption,

$$C_t^{-\sigma} - \lambda_{1t} + \lambda_{2t} \left[\sigma \epsilon (1-\tau_t)^{-1} Y_t^{\varphi} C_t^{\sigma-1} + \sigma \phi \beta C_t^{\sigma-1} Y_t^{-1} E_t \left[M(b_t, \epsilon_{t+1}, \widetilde{\beta}_{it+1}, b_t^S) \right] \right]$$

$$+ \lambda_{3t} \left[\begin{array}{c} \sigma \beta b_t C_t^{\sigma-1} E_t \left[L(b_t, \epsilon_{t+1}, \widetilde{\beta}_{it+1}, b_t^S) \right] + \sigma \beta b_t^S C_t^{\sigma-1} E_t \left[K\left(b_t, \epsilon_{t+1}, \widetilde{\beta}_{it+1}, b_t^S\right) \right] \\ - \rho \sigma \beta \frac{b_{t-1}}{\Pi_t} C_t^{\sigma-1} E_t \left[L(b_t, \epsilon_{t+1}, \widetilde{\beta}_{it+1}, b_t^S) \right] + \sigma \left(\frac{\tau_t}{1-\tau_t} \right) (Y_t)^{1+\varphi} C_t^{\sigma-1} \end{array} \right] = 0$$

government spending,

$$\chi G_t^{-\sigma_g} - \lambda_{1t} - \lambda_{3t} = 0$$

output,

$$-Y_t^{\varphi} + \lambda_{1t}\left[1 - \frac{\phi}{2}\left(\Pi_t - 1\right)^2\right]$$

$$+\lambda_{2t}\left[\epsilon\varphi(1-\tau_t)^{-1}Y_t^{\varphi-1}C_t^{\sigma} - \phi\beta C_t^{\sigma}Y_t^{-2}E_t\left[M(b_t^M, \epsilon_{t+1}, b_t^S)\right]\right]$$

$$+\lambda_{3t}\left[(1+\varphi)Y_t^{\varphi}C_t^{\sigma}\left(\frac{\tau_t}{1-\tau_t}\right)\right] = 0$$

taxation,

$$\epsilon\lambda_{2t} + \lambda_{3t}Y_t = 0$$

inflation,

$$-\lambda_{1t}\left[Y_t\phi\left(\Pi_t - 1\right)\right] - \lambda_{2t}\left[\phi\left(2\Pi_t - 1\right)\right]$$

$$+\lambda_{3t}\left[\frac{b_{t-1}}{\Pi_t^2}\left(1 + \rho\beta C_t^{\sigma}E_t\left[L(b_t^M, \epsilon_{t+1}, b_t^S)\right]\right) + \frac{b_{t-1}^S}{\Pi_t^2}\right] = 0$$

and the FOCs for government debt b_t^M and b_t^S, respectively,

$$-\widetilde{\beta}E_t\left[\frac{\lambda_{3t+1}}{\Pi_{t+1}}(1+\rho P_{t+1}^M)\right] + \lambda_{2t}\phi\beta C_t^{\sigma}Y_t^{-1}E_t\left[M_1(b_t^M, \epsilon_{t+1}, b_t^S)\right]$$

$$+\beta C_t^{\sigma}\lambda_{3t}\left[\begin{array}{c} E_t\left[L(b_t^M, \epsilon_{t+1}, b_t^S)\right] + b_tE_t\left[L_1(b_t^M, \epsilon_{t+1}, b_t^S)\right] + b_t^SE_t\left[K_1\left(b_t^M, \epsilon_{t+1}, b_t^S\right)\right] \\ -\rho\frac{b_{t-1}}{\Pi_t}E_t\left[L_1(b_t^M, \epsilon_{t+1}, b_t^S)\right] \end{array}\right] = 0$$

and

$$-\widetilde{\beta}E_t[\frac{\lambda_{3t+1}}{\Pi_{t+1}}] + \lambda_{2t}\phi\beta C_t^{\sigma}Y_t^{-1}E_t\left[M_3(b_t^M, \epsilon_{t+1}, b_t^S)\right]$$

$$+\beta C_t^{\sigma}\lambda_{3t}\left[\begin{array}{c} b_tE_t\left[L_3(b_t^M, \epsilon_{t+1}, b_t^S)\right] + E_t\left[K\left(b_t^M, \epsilon_{t+1}, b_t^S\right)\right] + b_t^SE_t\left[K_3\left(b_t^M, \epsilon_{t+1}, b_t^S\right)\right] \\ -\rho\frac{b_{t-1}}{\Pi_t}E_t\left[L_3(b_t^M, \epsilon_{t+1}, b_t^S)\right] \end{array}\right] = 0$$

Appendix N. Lower Intertemporal Elasticity of Substitution for Government Consumption

This section recreates Figure B.1 after reducing the inverse of the elasticity of substitution for government consumption in utility to $\sigma_g = 1$. This increases the use of government consumption as a fiscal policy instrument, but without changing any of the conclusions of the main paper. See Figure N.1 below.

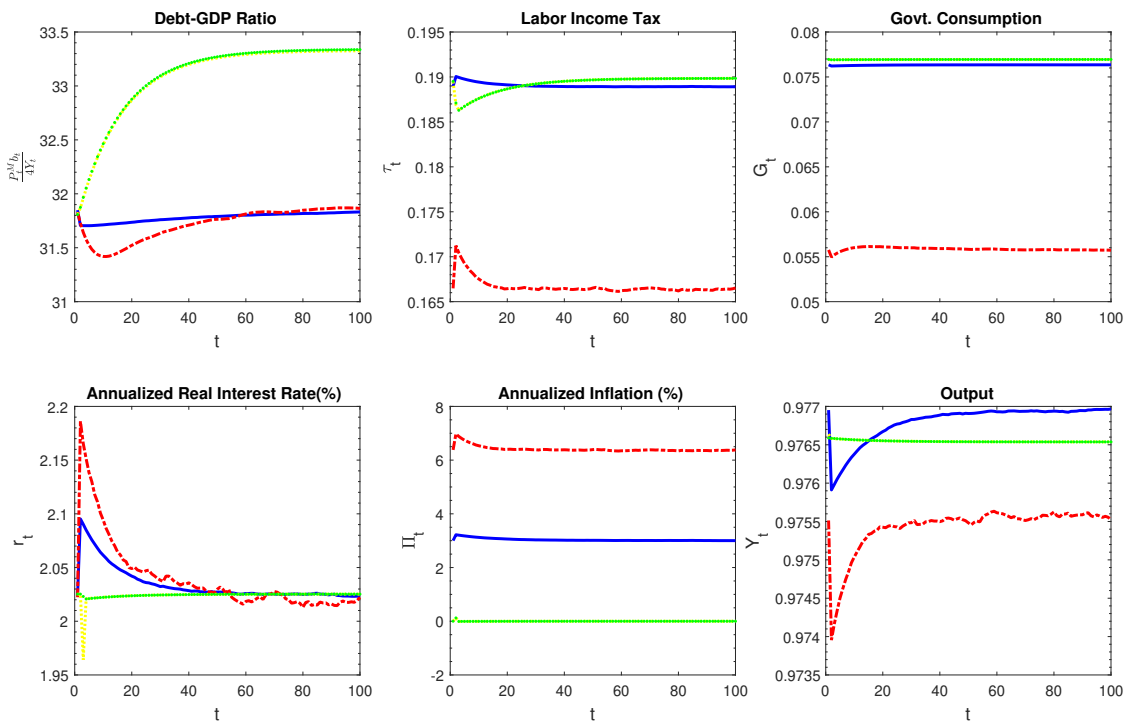

Figure N.1: Impulse response to markup shock - commitment vs. discretion $\sigma_g = 1$
Note: Yellow dotted line represents outcomes under commitment with long-term debt, and green points commitment with single period debt. These largely overlap. Solid blue line presents discretion with long-term debt, and red dash-dotted line discretion with single period debt. Myopia has been increased in the case of single period debt to ensure the steady-state debt-to-GDP ratio is the same as the other model variants considered.

References

Aiyagari, S. R., A. Marcet, T. J. Sargent, and J. Seppala (2002). Optimal Taxation without State-Contingent Debt. *Journal of Political Economy 110*(6), 1220–1254.

Bianchi, F. and C. Ilut (2017). Monetary/fiscal policy mix and agent's beliefs. *Review of Economic Dynamics*.

Chen, X., E. M. Leeper, and C. Leith (2018). US monetary and fiscal policies-conflict or cooperation? *mimeograph, University of Glasgow*.

Judd, K. L. (1998). *Numerical Methods in Economics*. MIT press.

Leeper, E. and C. Leith (2016). Understanding inflation as a joint monetary-fiscal phenomenon. *Taylor, J. and H. Uhlig (eds), the Handbook of Macroeconomics, Volume 2*.

Niemann, S., P. Pichler, and G. Sorger (2013). Public debt, discretionary policy, and inflation persistence. *Journal of Economic Dynamics and Control 37*(6), 1097–1109.

Schmitt-Grohe, S. and M. Uribe (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory 114*(2), 198–230.