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Can everyone benefit from economic integration?

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Abstract

There is no Pareto efficient allocation rule which always encourages economic integration. Further, for any efficient rule treating indistinguishable agents identically in welfare terms, there is an economy in which a third of the agents are hurt upon integration.

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1 Introduction

The market mechanism furnishes gains from trade: all individuals participating in a market can do no worse than they would absent trade. This is because a market permits individuals to abstain from trade. Technically, we say that a market mechanism is *individually rational*.

On the other hand, in a market mechanism, there are in general both winners and losers when separate groups combine into a global market. This can lead to a complicated political dynamic in which various groups compete and negotiate trade treaties.

In this note, we ask whether the presence of winners and losers in trade liberalization is specific to the market mechanism, or whether it persists more broadly.

While globalization affords more opportunities to trade, and more room for comparative advantage in production, unambiguous Pareto improvements are unlikely without some form of redistribution.

Clearly, were there a mechanism whereby economic integration unambiguously benefitted everybody, the politicization of trade-related issues would be mitigated, if not eliminated. The purpose of this note is to investigate whether such a mechanism exists. We do so in a model of pure exchange, absent production. This is not because production is not important–it is–but because even in the benchmark case of exchange, we derive an impossibility. *There is no Pareto efficient mechanism which encourages economic integration.*

This note proceeds as follows. Section 2 presents a simple market where the introduction of a new member unambiguously hurts an agent under market equilibrium. Section 3 presents the model and a few basic propositions. Section 4 establishes the main result. Section 5 establishes a broader result; namely, that any Efficient rule satisfying a basic fairness requirement must violate Integration Monotonicity in a strong way; namely, up to one third of the individuals in a group can be hurt upon integration. Section 6 illustrates the relationship to the existing studies.

2 A simple example

The market mechanism can hurt individuals under economic integration. Here is a simple example involving Cobb-Douglas preferences, which confirms the classic arguments by Mundell [16] and Jones [9].

Example 1 Suppose that there are two consumption goods and three individuals, i, j, k, who have identical Cobb-Douglas preferences represented by

$$u(x_1, x_2) = x_1 x_2.$$

Their endowment vectors are given by

$$\omega_i = (9, 1), \quad \omega_j = (1, 9), \quad \omega_k = (12, 1)\,.$$

Consider the economy consisting of the individuals $\{i, j\}$ alone, say $e_{\{i,j\}}$. In $e_{\{i,j\}}$, the Walrasian solution delivers

$$x_i = x_j = (5, 5), \quad \frac{p_1}{p_2} = 1.$$

On the other hand, when the economy consists of $\{i, j, k\}$, say $e_{\{i,j,k\}}$, the Walrasian solution delivers

$$x_i = \left(\frac{11}{2}, \frac{11}{4}\right), \quad x_j = \left(\frac{19}{2}, \frac{19}{4}\right), \quad x_k = \left(7, \frac{7}{2}\right), \quad \frac{p_1}{p_2} = \frac{1}{2}.$$

Observe that individual i is worse off in the larger economy.

Remark 1 The specific functional form, the specific numbers and the apparent symmetry in the above example are merely for simplicity of presentation, and the example is totally generic. It does not rely on violation of strong monotonicity or strict convexity either, which is often the case in this kind of argument.

This example will be used in the proofs of main theorems, but the proofs do not depend on the exemplified choice of specific functional form or numbers at all.

Motivated by this example, we ask the following question: it is possible to have a rule, which may or may not be market-like, and may include policy intervention/coordination such as compensation and regulation, so that integrating one group with another does not hurt anyone in either group? Let us call this requirement *Integration Monotonicity.*

In the earlier literature, Integration Monotonicity has appeared as *Population Monotonicity* in Sprumont [21], who first considered this concept in the context of TU game. Sprumont gave a characterization of the class of TU games in which a population monotonic rule exists. Yet, it remains unclear if such a rule exists in the current domain of exchange economies (see Section 6 for the details about technical differences).

We show that the answer is negative, if we also require that an allocation prescribed for any given group is efficient. We also extend the negative result to the cases of general structure of political power, while the initial argument presumes that every single person has veto power so that we cannot make anybody worse off compared to the outcomes in the original economies.

3 The model and axioms

There are l goods, where $l \geq 2$. Let $\mathcal{R}$ be the set of strictly convex, strongly monotone preferences and continuous weak orderings over $\mathbb{R}^l_+$.[1]

Let $\mathbb{N}$ be the set of potential individuals, and $\mathcal{I}$ be the set of finite subsets of $\mathbb{N}$. Given $I \in \mathcal{I}$, let $\mathcal{E}_I = (\mathcal{R} \times \mathbb{R}^l_{++})^I$ be the set of *economies* involving individuals in I. Each $e_I = (R_i, \omega_i)_{i \in I} \in \mathcal{E}_I$ consists of $|I|$ preference endowment pairs $(R_i, \omega_i) \in \mathcal{R} \times \mathbb{R}^l_{++}$. Given a weak preference relation R_i, let P_i denote its asymmetric part and I_i denote its indifference part.

Given $I, J \in \mathcal{I}$ with $I \cap J = \emptyset$, $e_I \in \mathcal{E}_I$, and $e_J \in \mathcal{E}_J$, let $e_I \vee e_J = (R_i, \omega_i)_{i \in I \cup J}$ denote the concatenation of e_I and e_J. Also, given $I, J \in \mathcal{I}$ with $J \subset I$ and $e_I \in \mathcal{E}_I$, let $e_I|_J$ denote the restriction of e_I to J.

Given $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$, let

$$F(e_I) = \left\{ x_I \in (\mathbb{R}^l_+)^I : \sum_{i \in I} x_i = \sum_{i \in I} \omega_i \right\}$$

denote the set of feasible allocations in economy e_I.

A *social choice function* is a mapping $\varphi : \bigcup_{I \in \mathcal{I}} \mathcal{E}_I \to \bigcup_{I \in \mathcal{I}} (\mathbb{R}^l_+)^I$ such that for all I and $e_I \in \mathcal{E}_I$, $\varphi(e_I) \in F(e_I)$.

Here, we list the two properties of interest.

Efficiency: For all $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$, there is no $z \in F(e_I)$ such that $z_i \ P_i \ \varphi_i(e_I)$ for all $i \in I$.

Integration Monotonicity: For all $I, J \in \mathcal{I}$ with $I \cap J = \emptyset$, for all $e_I \in \mathcal{E}_I$ and $e_J \in \mathcal{E}_J$,

$$\varphi_i(e_I \vee e_J) \ R_i \ \varphi_i(e_I)$$

for all $i \in I$ and

$$\varphi_j(e_I \vee e_J) \ R_j \ \varphi_j(e_J)$$

for all $j \in J$.

[1] Strongly monotone means that if $x \geq y$ and $x \neq y$, then x is strictly preferred to y.

Observe that Integration Monotonicity equivalently requires that economic disintegration should be harmful to all individuals involved. Nobody should stand to gain from breaking down trade across groups. In this sense, it functions as a kind of stability notion.

Here are some basic implications of Integration Monotonicity and Efficiency. First, it is easy to see that Integration Monotonicity implies Individual Rationality. This is because each individual must do at least as well as they could on their own.

Individual Rationality: For all $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$,

$$\varphi_i(e_I) \; R_i \; \omega_i$$

for all $i \in I$.

Lemma 1 Integration Monotonicity implies Individual Rationality.

Second, Integration Monotonicity and Efficiency imply that we have to select a core allocation for *any* coalition.

Definition 1 Given $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$, a feasible allocation $x_I \in F(e_I)$ is said to be a *core allocation* if there is no $J \subset I$ such that there exists $z_J \in F(e_I|_J)$ such that $z_i \; P_i \; x_i$ for all $i \in J$.

Let $Core(e_I)$ denote the set of core allocations in e_I.[2]

Lemma 2 Suppose φ satisfies Efficiency and Integration Monotonicity. Then it satisfies

$$\varphi(e_I) \in Core(e_I)$$

for all $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$.

Proof. Suppose $\varphi(e_I) \notin Core(e_I)$ for some $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$.

Then there is $J \in \mathcal{I}$ with $J \subset I$ and $z_J \in F(e_I|_J)$ such that

$$z_i \; P_i \; \varphi_i(e_I)$$

for all $i \in J$.

[2] Technically, this is the definition of the *weak core*. However, since preferences are strongly monotone, it coincides with the standard definition of core, whereby there is no $J \subset I$ such that there exists $z_J \in F(e_I|_J)$ such that $z_i \; R_i \; x_i$ for all $i \in I$ and $z_i \; P_i \; x_i$ for some $i \in I$. The weak core definition turns out to be easier to work with for our proof.

Since φ satisfies Efficiency, there is $j \in J$ such that

$$\varphi_j(e_I|_J)\ R_j\ z_j,$$

otherwise we have

$$z_j\ P_j\ \varphi_j(e_I|_J)$$

for all $j \in J$, which violates Efficiency for $e_I|_J$.

Now, for such $j \in J$ we have

$$\varphi_j(e_I|_J)\ R_j\ z_j\ P_j\ \varphi_j(e_I),$$

implying

$$\varphi_j(e_I|_J)\ P_j\ \varphi_j(e_I),$$

which is a violation of Integration Monotonicity. ■

Let us illustrate particular solutions and see if they satisfy or fail the above requirements.

Definition 2 Given $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$, a feasible allocation $x_I \in F(e_I)$ is said to be a Walrasian allocation if there exists $p \in \mathbb{R}^l_+ \setminus \{\mathbf{0}\}$ such that for all $i \in I$ and $z_i \in \mathbb{R}^l_+$ it holds

$$p \cdot z_i \leq p \cdot \omega_i \implies x_i\ R_i\ z_i$$

Let $W(e_I)$ denote the set of Walrasian allocations in e_I. We suppose that the domain $\{\mathcal{E}_I\}_{I \in \mathcal{I}}$ is such that W is taken to be a single-valued function.

Then Walras rule W satisfies Efficiency but violates Integration Monotonicity, as demonstrated in Example 1, while it meets Individual Rationality.

Example 2 Let φ be the SCF given by

$$\varphi(e_I) = \omega_I$$

for all $I \in \mathcal{I}$ and $e_I \in \mathcal{E}_I$.

This satisfies Integration Monotonicity but fails Efficiency.

Let us try to illustrate the difficulty of simultaneously satisfying Efficiency and Integration Monotonicity. Quite obviously, two groups deciding to join could easily consider their prescribed allocations as initial endowments, and then operate according to the Walrasian mechanism. For example, joining $\{i, j\}$ to $\{k\}$, one may treat i's allocation obtained in economy $\{i, j\}$ as his endowment in the integrated economy $\{i, j, k\}$, and similarly for j. This results in a rule which is both efficient, and improves (weakly) the welfare of all three individuals involved when integrating the two economies.[3]

However, this solution is defined only for a *fixed order of integration*: economy $\{i, j, k\}$ here is supposed to have come from integrating $\{i, j\}$ and $\{k\}$. The definition of a rule and of Integration Monotonicity also requires us to specify an allocation whereby $\{i, j, k\}$ could have come from integrating $\{j, k\}$ and $\{i\}$, or from integrating $\{i, k\}$ and $\{j\}$. And all individuals must be made better off, no matter which initial groups lead to $\{i, j, k\}$. We have no guarantee that the allocation recommended for $\{i, j, k\}$ will satisfy this requirement for the other methods of joining groups.

In other words, the sequencing of how economies are joined together matters for the Walrasian allocation; or, more generally, for any rule (this is the content of our theorem below). Observe that allocation rules in our setting do not have a language for discussing past history of allocations. In other words, how economy $\{i, j, k\}$ was arrived at cannot be discussed in the context of a rule. We simply need to recommend an allocation for $\{i, j, k\}$ independently of how we arrived there. In a sense, this property of an allocation rule carries an implicit "path-independence" assumption. One way to read our impossibility is that any method of allocation which is both Efficient and encourages integration is necessarily path-dependent. As a consequence, we can understand the impossibility as implying that politics matter for economic integration; for the Walrasian rule, and for any efficient manner of allocating resources. There is necessarily a complicated game played by groups of economic individuals in terms of trade negotiations. The way in which groups join together will be relevant for every group's final allocation.

4 An impossibility

The following theorem is our main result. It states that there is no rule simultaneously satisfying Efficiency and Integration Monotonicity. In the context of the discussion in the preceding section, one way to interpret this result is that if we always seek to make all individuals weakly better off when integrating economies, we must either eschew Efficiency or end up with *path-dependence*.

[3]This construction may be seen as a reverse version of customs union presented by Kemp and Wan [10].

Theorem 1 There is no φ which satisfies Efficiency and Integration Monotonicity.

Remark 2 Theorem 1 only uses the preference/endowment pairs in Example 1. As such, it could be proved were the domain of the social choice rule much smaller (operating, for example, only on economies containing these preference/endowment pairs). Moreover, because our argument does not rely on a variable-profile axiom with regard to preferences, the impossibility result holds for *vitrually any fixed profile of preferences*. We leave the statement of the result as is for simplicity, though it can obviously be significantly generalized.

Remark 3 The above impossibility extends to the multi-valued case. Let Φ denote a social choice correspondence, and define Integration Monotonicity as follows: for all $I \in \mathcal{I}$, for all $e_I \in \mathcal{E}_I$ and for all $x_I \in \Phi(e_I)$, there exist $J \subset I$, $y_J \in \Phi(e_I|_J)$, and $y_{I \setminus J} \in \Phi(e_I|_{I \setminus J})$ such that $x_i \; R_i \; y_i$ for all $i \in I$. This version of the requirement states that, if a large economy were to disintegrate into two separate economies, we can always choose allocations for the separate economies in which all involved agents are harmed (weakly).

Proof. Consider three individuals, i, j, k as in Example 1.

Then, the Walrasian solution W, which is single-valued here, yields

$$W_i(e_{\{i,j\}}) \; P_i \; W_i(e_{\{i,j,k\}})$$

Let $B(z, \varepsilon)$ denote the open ball with center z and radius ε, and $d(y, z)$ denote the Euclidian distance between y and z.

By continuity, there is $\varepsilon > 0$ such that

$$y_i \; P_i \; z_i$$

for all $y_i \in B(W_i(e_{\{i,j\}}), \varepsilon)$ and $z_i \in B(W_i(e_{\{i,j,k\}}), \varepsilon)$.

By Debreu's theorem of core convergence (Debreu [3]), since Cobb-Douglas preferences are regular, there is an integer r such that for all $y \in Core(r * e_{\{i,j\}})$ and $z \in Core(r * e_{\{i,j,k\}})$ it follows that $d(y_h, W_i(e_{\{i,j\}})) < \varepsilon$ and $d(z_h, W_i(e_{\{i,j,k\}})) < \varepsilon$, where h is any i-type consumer commonly appearing in the two replica economies.[4]

[4]Here, $r * e$ denotes an r-replica of economy e, in the sense of [3].

Since $\varphi(r * e_{\{i,j\}}) \in Core(r * e_{\{i,j\}})$ and $\varphi(r * e_{\{i,j,k\}}) \in Core(r * e_{\{i,j,k\}})$, it follows that

$$\varphi_h(r * e_{\{i,j\}}) \; P_h \; \varphi_h(r * e_{\{i,j,k\}}),$$

which is a violation of Integration Monotonicity. ■

5 Alternative definition of integration monotonicity

Our definition of integration monotonicity supposes that nobody in a group must be harmed under integration. This notion may seem unduly strong. Suppose instead that we require that integration not hurt *too many* people. We show in the following that if "too many" is interpreted as slightly more than one third of the population, an impossibility remains.

Let $\mathcal{P}$ be a map assigning each $I \in \mathcal{I}$ a non-empty family of its non-empty subsets $\mathcal{P}(I)$. We interpret a coalition $J \in \mathcal{P}(I)$ as a coalition which has blocking power; *i.e.* integration should not hurt all members of J. This represents a kind of political structure in which any element $J \in \mathcal{P}(I)$ can "block" the integration with another group.

The following definition weakens integration monotonicity so that it takes a coalition in $\mathcal{P}(I)$ to block.

Integration Monotonicity under $\mathcal{P}$-vetoes: For all $I, J \in \mathcal{I}$ with $I \cap J = \emptyset$, for all $e_I \in \mathcal{E}_I$ and $e_J \in \mathcal{E}_J$, there is no $I' \in \mathcal{P}(I)$ such that

$$\varphi_i(e_I) \; P_i \; \varphi_i(e_I \vee e_J)$$

for all $i \in I'$ and there is no $J' \in \mathcal{P}(J)$ such that

$$\varphi_j(e_J) \; P_j \; \varphi_j(e_I \vee e_J)$$

for all $j \in J'$.

Thus, the condition of Integration Monotonicity coincides with Integration Monotonicity under $\mathcal{P}$-vetoes whenever $\mathcal{P}(I)$ is the set of all nonempty subsets of I for any I.

Integration Monotonicity under $\mathcal{P}$-vetoes tends to be weaker, as it is easier to hurt just one agent via integration than it is to hurt a large group. Note again that Integration Monotonicity is the strongest case in which any single person in a group can veto.

And, in general, Integration Monotonicity under $\mathcal{P}$-vetoes is compatible with Efficiency, depending on the coalition structure $\mathcal{P}$. For example, suppose $\mathcal{P}(I) = \{I\}$ for all $I \in$

$\mathcal{I}$, which means economic integration is approved by a group unless everybody strongly opposes it. Then, it is straightforward to see that Walrasian solution satisfies Efficiency and Integration Monotonicity under $\mathcal{P}$-vetoes.[5]

Consider the following standard property, requiring that indistinguishable agents be treated equally in welfare terms:

Equal Treatment of Equals: For all $I \in \mathcal{I}$, for all $e_I \in \mathcal{E}_I$ and for all $i, j \in I$ with $R_i = R_j$ and $\omega_i = \omega_j$, we have

$$\varphi_i(e_I)\ I_i\ \varphi_j(e_I).$$

Theorem 2 Suppose that for all $I \in \mathcal{I}$, any $J \subseteq I$ with $|J| \geq \lfloor \frac{|I|}{3} \rfloor - 1$ satisfies $J \in \mathcal{P}(I)$. Then there is no φ which satisfies Equal Treatment of Equals, Efficiency and Integration Monotonicity under $\mathcal{P}$-vetoes.

Let us interpret this result before proof. Theorem 2 tells us that any Efficient rule which is "fair" in the sense of satisfying Equal Treatment of Equals must necessarily harm roughly one third of the society in some situations. Thus, if groups of size one third or more hold any political power, we run into the same type of complicated political dynamic described above.

Lemma 3 Suppose that φ satisfies Efficiency and Equal Treatment of Equals. For all $I \in \mathcal{I}$, for all $e_I \in \mathcal{E}_I$ and for all $i, j \in I$ with $R_i = R_j$ and $\omega_i = \omega_j$, we have

$$\varphi_i(e_I) = \varphi_j(e_I).$$

Proof. Follows easily from the standard argument on Efficiency and Equal Treatment of Equals under strict convexity of preference. ■

Lemma 4 Suppose that Efficiency, Equal Treatment of Equals, and Integration Monotonicity under $\mathcal{P}$-vetoes are satisfied. Let $I \in \mathcal{I}$ and let r be an integer. Suppose that for

[5] Suppose not, then we have a situation where $W_i(e_I)\ P_i\ W_i(e_I \vee e_J)$ for all $i \in I$. Then, from optimality of $W_i(e_I \vee e_J)$ under i's equilibrium budget constraint in economy $e_I \vee e_J$ we have $p \cdot W_i(e_I) > p \cdot \omega_i$ for all $i \in I$, where p is the equilibrium price vector in $e_I \vee e_J$. Thus we obtain

$$p \cdot \sum_{i \in I} W_i(e_I) > p \cdot \sum_{i \in I} \omega_i$$

which contradicts feasibility of $W(e_I)$ in economy e_I.

all $J \subseteq r * I$ for which $|J| \geq r - 1$, we have $J \in \mathcal{P}(r * I)$. Then for all $e_I \in \mathcal{E}_I$, $\varphi(r * e_I)$ cannot be blocked by any group of $r|I| - 1$ individuals.

Proof. Without loss of generality, let $I = \{1, \cdots, |I|\}$, and let r be some integer. For its r-replication, let $(i, q) \in r * I$ denote the individual of Type i in the q-th copy out of r-replicas.

Suppose by means of contradiction that $\varphi(r*e_I)$ is blocked by the coalition $(r*I)\backslash\{(i, q)\}$ via allocation $z_{(r*I)\backslash\{(i,q)\}} \in F(e_{r*I}|_{(r*I)\backslash\{(i,q)\}})$. Let

$$\begin{aligned} \overline{z}_i &= \frac{1}{r-1} \sum_{s \neq q}^{r} z_{i,s} \\ \overline{z}_j &= \frac{1}{r} \sum_{s=1}^{r} z_{j,s}, \quad j \neq i. \end{aligned}$$

By convexity we have

$$\overline{z}_i \; P_i \; \varphi_i(e_{r*I})$$

$r - 1$ times and

$$\overline{z}_j \; P_j \; \varphi_j(e_{r*I})$$

r times for each $j \neq i$.

By Efficiency and Equal Treatment of Equals, we must have either

$$\varphi_i(e_{r*I}|_{(r*I)\backslash\{(i,q)\}}) \; R_i \; \overline{z}_i$$

or

$$\varphi_j(e_{r*I}|_{(r*I)\backslash\{(i,q)\}}) \; R_j \; \overline{z}_j$$

for some $j \neq i$.

In the first case, we have a violation of Integration Monotonicity with $\mathcal{P}$-vetoes via $\{(i, s) : s \neq q\}$, which consists of $r - 1$ individuals. In the second case, for some $j \neq i$ we have a violation of Integration Monotonicity with $\mathcal{P}$-vetoes via $\{(j, s) : s = 1, \cdots, r\}$, which consists of r individuals. ■

Lemma 5 Suppose that for any $I \in \mathcal{I}$ and any integer r, if $J \subseteq r * I$ with $|J| \geq r - 1$, we have $J \in \mathcal{P}(r * I)$. Then, for all $I \in \mathcal{I}$, for all $e_I \in \mathcal{E}_I$ in which for all $i \in I$, R_i is Cobb-Douglas, we have that

$$\lim_{r \to \infty} \varphi_i(r * e_I) = W_i(e_I)$$

Proof. The proof of Debreu's convergence theorem (Debreu [3]) requires only that preferences satisfy a regularity condition satisfied by Cobb-Douglas, and that the allocation for $r * e_I$ cannot be blocked by $r|I| - 1$ individuals. The conclusion follows. ■

The proof of Theorem 2 then follows from a similar idea to before, utilizing the structure of Example 1. Consider the three individuals there, $\{i, j, k\}$; call this set I_1, and label the set of agents $\{i, j\}$ as I_2. Because for any I, all J satisfying $|J| \geq \lfloor \frac{|I|}{3} \rfloor - 1$ yields $\mathcal{P}(I)$, we conclude that the antecedent condition for Lemma 5 is satisfied for the set I_1. It also follows for the set I_2. Conclude then that for r large, half of the agents from $r * e_{\{i,j\}}$ are harmed in moving to $r * e_{\{i,j,k\}}$, violating Integration Monotonicity under $\mathcal{P}$ vetoes.

6 Related literature

We conclude by discussing the relationship to existing literature.

There is a long research history in the literature of general equilibrium/international trade/taxation, in which they ask if a government can institute a system of taxes, either non-distortional or distortional, and if there exists a corresponding market equilibrium, so that everybody gains from economic integration, from a rather positive viewpoint that the Walrasian mechanism is being used and a path of economic integration is given and fixed or integration choice is conditioned by a fixed status quo. See Dixit and Norman [5], Kemp and Wan [11], Hammond and Sempere [7, 8] and Weymark [25], among many.

In contrast, we have adopted a variable-population social choice approach, in which we do not take the Walrasian mechanism as given and we do not take a particular path of economic integration as given and fixed. We ask if there is a *rule* which always encourages economic integration. Our study may be seen as a robustness check for the above arguments, as we allow that there are potentially many paths for integration. Our result shows that in the variable population setting, even if we do not commit to the Walrasian mechanism, the conjunction of Pareto efficiency and the requirement that economic integration should hurt nobody reaches impossiblity.

An early result of Aumann and Peleg [1], building off of the ideas of Gale [6] demonstrates that, in Walrasian equilibrium, an individual may destroy some of her endowment, and make herself better off. Postlewaite [17] shows a general impossibility that any social choice rule is manipulable by withholding endowments. On the other hand, he provides a mechanism which is immune to manipulation by destroying endowments. Conceptually, this may be related to our result as the idea of making oneself better off with more endowment seems similar to the idea of making an entire group better off with the introduction of increased trading opportunities.

Population Monotonicity, which Sprumont [21] discussed in the setting of cooperative games with transferable utility, is actually equivalent to our Integration Monotonicity. Population monotonicity states that adding agents to a coalition should affect everybody in the same direction. Unlike in exchange economies, Sprumont showed that the domain of convex games allows population-monotonic allocation rules, and provided a characterization of games allowing population-monotonic rules. Note that in TU games efficiency is a part of the definition of feasible allocations. Our result implies that non-transferable utility games generated by exchange economies requires a different treatment than his.

Sprumont [21] also provides a family of TU games, generated from assignment games, in which the impossibility of Population Monotonicity emerges with four agents. His proof does not require a replication argument like ours. Unfortunately, his proof does not apply to our setting, since assignment games are generated by preferences which do not satisfy the conditions here. We maintain the assumption that preferences are strongly monotonic. All the same, the results are similar in spirit, and conceptually closely related.

In general, in combinatorial matching environments in which preferences are not required to be strongly monotonic, or there may not even be a natural order of quantity, an impossibility may be obtained without replication and with a small number of individuals. To illustrate, consider the housing-market environment due to Shapley and Scarf [19], in which each individual owns just one indivisible object and has a strict preference ranking over the objects. There, one can show in a similar way that (weak) Pareto efficiency and Integration Monotonicity imply that the prescribed allocation must be in the (weak) core for every possible group. It is known that the core allocation is unique in this environment (see [18]). Hence, if there is an integration monotonic and efficient solution, it must be the unique core allocation. However, it is easy to show that the singleton core solution violates Integration Monotonicity: Consider three individuals i, j, k, where i's endowment is denoted by ω_i and similarly for the others, and $\omega_j \ P_i \ \omega_i \ P_i \ \omega_k$, $\omega_k \ P_j \ \omega_i \ P_j \ \omega_j$ and $\omega_j \ P_k \ \omega_i \ P_k \ \omega_k$. Then it follows that $\varphi_i(e_{\{i,j\}}) = \omega_j$, $\varphi_j(e_{\{i,j\}}) = \omega_i$, while $\varphi_i(e_{\{i,j,k\}}) = \omega_i$, $\varphi_j(e_{\{i,j,k\}}) = \omega_k$, $\varphi_k(e_{\{i,j,k\}}) = \omega_j$. Consequently, when k joins the group $\{i, j\}$, i loses.

In our current setting, goods are homogeneous and divisible, preferences are strong monotonic and strictly convex, and we can restrict the domain even to an extremely narrow one, such as the domain of identical Cobb-Douglass preferences. Moreover, because our argument does not rely on a variable-profile axiom with regard to preference, the impossibility result is obtained for *vitrually any fixed profile of preferences.*

In the impossibility arguments like above, it is typically the case that there is no additional gains from trade in the outset which can be distributed to everybody, because of indivisibility and/or lack of strong monotonicity (such as assuming Leontief preferences).

On the other hand, there always exists a potential room for gains from trade in our setting, which can be shared to everybody through redistributions. What we showed is that such room for improving everybody's welfare disappears when the economy becomes sufficiently large, nevertheless.

The following is a brief survey of related ideas and models; see Sprumont [22] for a detailed survey. In the setting of cooperative bargaining, Thomson [23] introduced *Population Monotonicity*, where resources to allocated are taken to be fixed, which requires that everybody should get weakly worse off when there are incoming people. In the setting of allocating private goods with fixed social endowments, Thomson [24] showed that there is a population-monotonic and efficient allocation rule, while Moulin [13] suggested that there is no population-monotonic allocation rule which satisfies envy-freeness as well as efficiency; see [12] for a formal proof. In the setting of allocating fixed amounts of private goods and a fixed amount of numeraire good, where preferences are linear in the numeraire good, Moulin [14] shows that in general there is no population monotonic and efficient allocation rule, while he shows that when preferences exhibit substitutability, the Shapley value is population-monotonic.

The above two definitions of *Population Monotonicity* differ in whether endowments are private and we count on additional resources brought by incoming individuals or we only consider social endowments and take that to be fixed. To avoid confusions between the two versions, we chose the different terminology, Integration Monotonicity.

There are axiomatic studies of solidarity conditions with respect to other kinds of economic changes.

In the setting of allocating private goods when social endowments are given, Moulin and Thomson [15] considered the requirement that having a larger vector of social endowments should not hurt anybody. They showed that this requirement is incompatible with efficiency and individual rationality, and also incompatible with efficiency and the requirement that nobody's consumption vector should not dominate anybody else's consumption vector.

In the setting of exchange economy in which the set of tradable goods may vary, Chambers and Hayashi [2] considered the requirement that expanding the set of tradable goods should not hurt anybody. Together with allocative efficiency and an informational efficiency requirement that only preferences induced over tradable goods should matter, they showed that only one person can extract entire gains from trade and everybody else must end up with the same welfare level as in autarky.

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