Seismic Risk Evaluation by Fragility Curves using Metamodel Methods

H.Z. Yang[1†] and C.G. Koh[2]

[1†]*School of Engineering, University of Glasgow, Glasgow, United Kingdom.*
[†]*E-mail: Hezhen.yang@Glasgow.ac.uk*

[2] *Department of Civil and Environmental Engineering, National University of Singapore, Singapore.*

Abstract

This study proposed an efficient and reliable fragility estimation by metamodel methods. To evaluate seismic risk, it is important to account for ground motions and structural parameter uncertainties. Nevertheless, seismic fragility analysis with uncertainties is impractical since it requires many time consuming nonlinear dynamic simulations. To this end, an efficient approach based on metamodel in conjunction with Monte Carlo simulation is proposed to develop fragility curves. Several metamodel methods such as Kriging, Response Surface Method (RSM), and Radial Basis Function (RBF) are investigated for this purpose. Optimum Latin Hypercube Design is used to generate "space filling" samples for nonlinear dynamic analysis under seismic excitations A metamodel is constructed based on these Design of Experiment samples which include the input parameters and output fragility results. Kriging and RBF are better than the RSM metamodel. The fragility curves can be generated based on metamodel by considering the randomness of earthquake ground motions and uncertainties in material properties. Finally, seisimic risk are evaluated by fragilty curves. The computation time is significantly reduced by applying the metamodel method with acceptable accuracy. The proposed methodology also avoids nonlinear dynamic non-convergent problems. Kriging and RBF methods complement each other and are able to accurately evaluate fragility curves.

Keywords: Fragility curves; Earthquake; Metamodel; Probabilistic seismic assessment; Monte Carlo Simulation

1 Introduction

Consideration of uncertainty in seismic analysis is important. Primarily because earthquake is a random natural event and no two earthquakes are alike [1]. In addition to uncertainties in seismic ground motions, seismic risk evaluation should consider uncertainties in structural and material parameters. Probabilistic methods provide a quantitative means to account for these inherent uncertainties in structural safety assessment [2]. In this context, fragility curves are very useful in presenting the probability of failure (in terms of life safety, for example) with respect to an input parameter such as peak ground acceleration (PGA) [3]. They can be developed by employing analytical, empirical and hybrid methods, and expert judgment in some cases. Analytical fragility methods have attracted significant attention from researchers because of their theoretical basis over other methods. However, these methods involve solving nonlinear dynamic equations and the computational effort is high for the multiple simulations needed in considering seismic uncertainties.

Fragility analysis has been widely applied for seismic evaluation of bridges [4], frame buildings[5], and masonry buildings[1]. These methods have the potential to overcome the excessive computation time problem for seismic probabilistic evaluation. However, there is a scarcity of advanced metamodel methods such as Kriging and Radial Basis Function (RBF) that evaluate the seismic probability fragility analysis.

The main purpose of this study is to develop a methodology to obtain fragility curves with high efficiency and accuracy. Typical random variables are chosed to describe uncertainties affecting the structural behavior. There are two challenges involved in this task. The first challenge is to reduce the huge computation time. The second challenge is to overcome non-convergence problem in numerous nonlinear dynamic analyses. A four story building in Italy [6] is selected as an illustration case study. Three metamodel methods are compared, such as RSM, Kriging and RBF metamodels.

2 Theories and Methodology

2.1 Fragility function

A fragility function is defined as:

$$\mathrm{P}(\mathrm{C}|IM = x) = \Phi\left(\frac{\ln(x/\theta)}{\beta}\right) \tag{1}$$

where $\mathrm{P}(\mathrm{C}|IM = x)$ is the probability of structure to collapse, $IM = x$ is a ground motion with intensity measure, Φ is the standard normal cumulative distribution, θ is the median of the fragility function, and β is the standard deviation of ln(*IM*).

2.2 Analytical Fragility Curves development methods

Vamvatsikos and Cornell (2002) predicted nonlinear structural dynamic properties under seismic ground motions by the Incremental Dynamic Analysis (IDA) [7]. IDA utilizes scaled ground motions to find different structural damage states till the structure collapses. IDA aims to develop analytical fragility curves. Multiple Stripe Analysis (MSA) can also establish accurate fragility curves with a minimal number of nonlinear dynamic analyses. We assume that the observation of collapse from each ground motion is independent records. The probability that a ground motion with $IM = x_j$ is given by the binomial distribution as follow:

$$\mathrm{p}(Z_j \text{ collapses in } n_j \text{ ground motions}) = \binom{\mathrm{n_j}}{\mathrm{z_j}} \mathrm{p_j}^{\mathrm{z_j}} (1 - \mathrm{p_j})^{\mathrm{n_j} - \mathrm{z_j}} \tag{2}$$

Equation (1) is substituted for pj, so that the fragility parameters are explicit as follows:

$$\text{Likelihood} = \prod_{\mathrm{j}=1}^{\mathrm{m}} \binom{\mathrm{n_j}}{\mathrm{z_j}} \Phi\left(\frac{\ln(x_\mathrm{j}/\theta)}{\beta}\right)\left(1 - \Phi\left(\frac{\ln(x_\mathrm{j}/\theta)}{\beta}\right)\right) \tag{3}$$

Estimates of the fragility function parameters are obtained by maximizing this likelihood function as shown in the following equation.

$$\{\hat{\theta}, \hat{\beta}\} = \arg\max_{\theta,\beta} \sum_{\mathrm{j}=1}^{\mathrm{m}} \left\{\ln\binom{\mathrm{n_j}}{\mathrm{z_j}} + \mathrm{z_j}\ln\Phi\left(\frac{\ln(x_\mathrm{j}/\theta)}{\beta}\right) + (\mathrm{n_j} - \mathrm{z_j})\ln\left(1 - \Phi\left(\frac{\ln(x_\mathrm{j}/\theta)}{\beta}\right)\right)\right\} \tag{4}$$

2.3 Metamodel Methods

Metamodel methods are applied for predicting the relationship between the inputs and outputs with high efficiency. As shown in Figure 1, the basic principle of metamodel methods can be described as follows. The original system solution space includes input and output information. First, the input design parameters are generated with Design of Experiment. Second, the output response is calculated from the original system solution space. Third, the input and output relationships are constructed by metamodel. If the metamodel accuracy is sufficiently high, the metamodel can be used instead of the original system without requiring time consuming calculations to obtain the output responses. Convergent problem in nonlinear dynamic analysis process is unavoidable. During DOE process, selecting sample points which are near the convergent problem point can smooth calculation samples. In this way, the metamodel method can overcome convergent problem or failure cases during the seismic risk analysis for buildings.

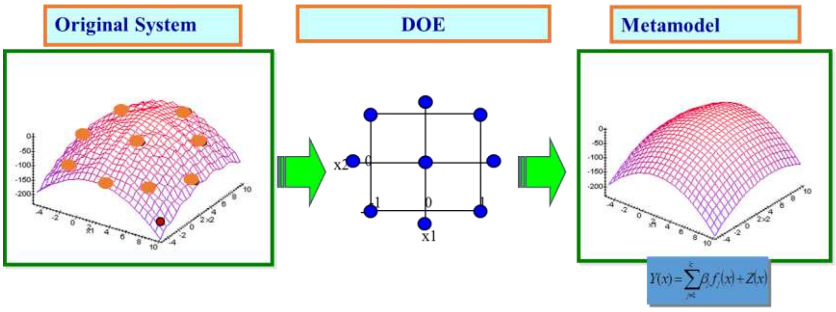

Figure 1: Basic principle of metamodel methods

2.4 Metamodel Accuracy Evaluation

The accuracy of the metamodel is evaluated by several statistical indicators. The root-mean-square error (RMSE) and the R^2 for the data points used to construct the metamodel are calculated. Furthermore, the average error (AveErr), RMSE and the maximum error (MaxErr) for all of the data points which not used in generating the models are also checked. These statistical indicators may be defined as follows:

$$R^2 = 1 - \frac{SS_E}{S_{yy}} \tag{5}$$

$$\text{with } SS_E = \sum_{i=1}^{n} \left(y_i - \bar{y}_i\right)^2 \quad S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n} \tag{6}$$

$$\text{AvgErr} = \frac{1}{n} \sum_{i=1}^{n} \frac{\left|y_i - \bar{y}_i\right|}{y_i} \tag{7}$$

$$\text{RMSE} = \frac{\sqrt{\frac{1}{n} \sum_{t=1}^{n} \left(y_i - \bar{y}_i\right)^2}}{\frac{1}{n} \sum_{i=1}^{n} y_i} \tag{8}$$

$$\mathrm{MaxErr} = \underset{i}{\mathrm{Max}}\left[\frac{|y_i - \bar{y}_i|}{y_i}\right] \tag{9}$$

where n is the number of data, y_i is the measured response and $\bar{y}_i$ is the predicted response for each data point.

3. Numerical Cases

3.1 Four story building model

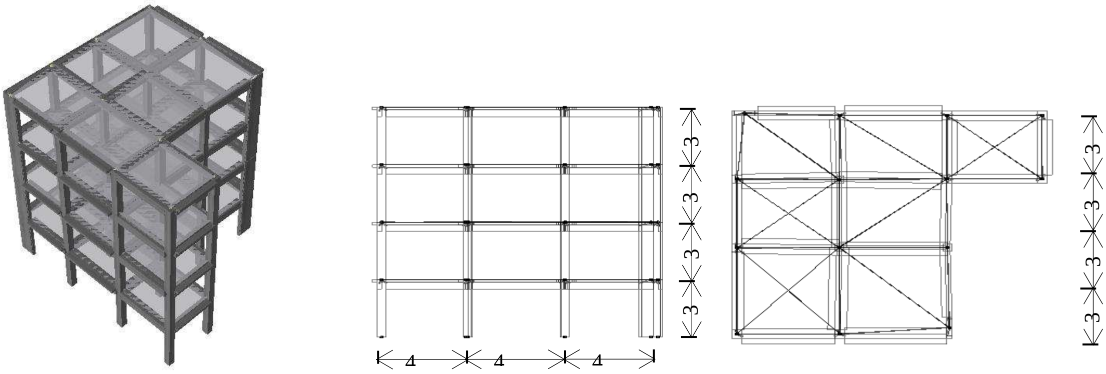

Figure 2: Finite element model of the four-story building and Layout for the building

Fragility analysis is applied to a four-story building model and the layout, as shown in figure 2. The model describes a typical four-story building constructed in the 1960s in Italy and strengthened with reinforced concrete jackets. The detailed finite element method of the four-story building models are created by Seismsoft [6].To illustrate the effectiveness of the proposed methodology, fragility curves are evaluated by the metamodel. First, the Optimum Latin Hypercube Simulation method simulated design points for the seismic dynamic analysis. Second, RSM, RBF and Kriging methods are employed for constructing a metamodel. Thirdly, Monte Carlo Simulations based on the metamodel are employed for the fragility analysis of the four-story building.

3.2 Uncertainties

Building performance under future earthquakes is largely unknown and cannot be predicted with certainty. In this study, the compressive and tensile strengths of concrete, as well as steel yield strengths are chosen as principal random variables. PGA is assumed to have a uniform distribution with a range 0.6g to 1.2 g. The distribution characteristics and the values are given in Table 1.

Table 1. Uncertainties of random variables

Variable	Mean (MPa)	COV(%)	Probabilistic distribution
Concrete yield strength (fc)	24	21	Normal
Young's modulus of concrete (Ec)	23,025	20.6	Normal
Steel yield strength (fy)	400	10	Normal

Twenty uncorrelated earthquake acceleration histories consistent with the design requirements are artificially generated and used as input ground motion. Input variables include the uncertainties in construction material properties and uncertainties in ground motions. Uncertainties in ground motions use a suite of artificial accelerograms [8, 9].Three performance levels defined in FEMA 356 are used, as shown in table 2.

Table 2. Limits associated with various structural performance levels

Structural performance level	Permissible top drift ratio (%)
Immediate Occupancy (IO)	1%
Life Safety (LS)	2%
Collapse Prevention (CP)	4%

3.3 Comparison of Multiple Stripes Analysis and Incremental Dynamic Analysis

The incremental dynamic analysis (IDA) can be used to obtain seismic capacity of the structure. The different damage states could be obtained from the capacity curves. The fragility curves described the probability of the defined limit state of a structure as a function of ground motion intensity measures, IM.

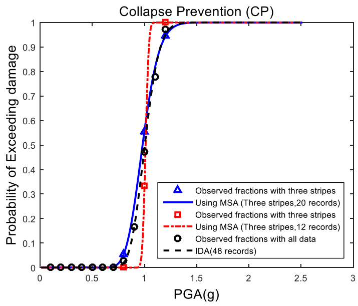

Figure 3: The fragility curves developed by MSA and IDA with different ground motion records

Multiple-stripe analysis (MSA) is a collection of single-stripe analyses performed at multiple levels of the spectral acceleration [10]. This will provide estimates of fragility curves parameters (θ, β) for each curve. Ground motion selection needs to be both “sufficient” and “efficient”. This work adopts the spectral representation based simulation methodology used in stochastic engineering .For low-rise buildings, approximately10 to 20 ground motion records are usually sufficient to provide adequate accuracy in the estimation of seismic demand. Here, we compared12,20, and 48 records. As shown in figure 3, the accuracy of 20 ground motions is similar to 48 ground motions of the IDA developed fragility curves. There is a significant error for 12 ground motions records compared with 48 records. Hence, 20 ground motions records are selected for fragility analysis in this study.

3.4 Metamodel Accuracy Evaluation

In the present study, the Optimal Latin Hypercube method is used as Design of Experiment (DOE) method. Several metamodels are constructed to perform the comparison. A good fit is achieved if the coefficient of determination R^2 is close to 1. This coefficient is nevertheless often insufficient for accuracy evaluation since it increases with the number of data points used. Hence validating the model using additional data points which are not used in the training set is essential. For the purpose of these statistical tests, 45 additional combinations of input variables are generated randomly. The resulting statistical measures are computed and displayed in Table 3.

Table 3. Metamodel accuracy evaluation indicators for the prediction of the fragility

Data	Statistical criteria	Fragility curves parameter (θ)			Fragility curves parameter (β)		
		RSM	RBF	Kriging	RSM	RBF	Kriging
Set I (30 points)	R2	0.94	0.95	0.95	0.34	0.773	0.67
	AveErr(%)	1.66	1.49	1.53	14.68	8.577	10.39
	RMSE (%)	1.40	0.83	0.79	10.59	3.769	4.68
	MaxErr (%)	3.97	4.06	5.54	50.76	36.29	43.26
Set II (18 points)	R2	0.92	0.91	0.91	0.16	0.252	0.22
	AveErr (%)	1.83	1.95	1.95	16.53	15.573	15.92
	RMSE (%)	1.39	1.26	1.24	10.75	8.791	8.74
	MaxErr (%)	4.45	4.74	5.46	68.53	64.04	61.72
Set III (12 points)	R2	0.71	0.90	0.87	-1.94	-0.26	-0.43
	AveErr (%)	3.55	2.04	2.40	30.86	20.22	21.51
	RMSE (%)	2.89	1.64	2.00	24.49	15.04	16.63
	MaxErr (%)	8.63	4.34	5.92	97.28	66.59	70.79

The R^2 value is also reduced when few sample data are used, as shown in data set II and III. If there is a set of data samples with a significant difference between the measured and predicted values, the R^2approaches zero or negative. In Table 3, dataset II and data set III are used to conclude that there is a significant amount of error in the metamodel. The maximum absolute error for β is larger than 50%. The root mean square error for β is larger than 10%. In data set I, the 30 samples RSM is able to predict θ with good accuracy. However, it is unable to predict β with acceptable accuracy. In data set I, the R^2 for the RSM predicting $\beta$is smaller than 0.5, maximum absolute error for the RSM predicting β is larger than 50%. For RBF and Kriging model, the R^2 is larger than 0.5 and the maximum absolute error is smaller than 50%. Both the 30 samples for RBF and the 30 samples for Kriging are able to predict (θ, β) with acceptable accuracy. According to the above analysis, data set I is used to construct the metamodel for the fragility analysis by considering uncertainties.

Kriging and RBF metamodels are more appropriate than RSM metamodel for the relationship between the input probabilistic seismic analysis and output fragility analysis results which have higher order nonlinearity and sometime even discontinuous properties. Kriging and RBF metamodels are more appropriate than RSM metamodel for simulating such relationships. It is very important to use a suitable metamodel to represent the most significant properties.

3.5 Development of fragility curves by considering uncertainties

The proposed approach utilizes the metamodel method for obtaining fragility curves considering uncertainties. The performance comparison of the fragility analysis time with and without the use of metamodel is shown in Figure 4. The computation time is based on a desktop computer with a single 3.4-GHz processor. Monte Carlo simulations of 10,000 samples are used for developing fragility curves considering uncertainties. In the case of twenty ground motions, the computation time is 16.3 days with applying the metamodel but would be increased by more than two orders of magnitude without applying the metamodel. The strategy for developing fragility curves is based on metamodel which does not require a large number of nonlinear dynamic analyses. Hence the metamodel method can reduce the computation time significantly and help in avoiding nonlinear dynamic non-convergent problem.

The metamodel method is used in calculating the fragility curves for the building by considering ground motions and material properties uncertainties. Tens of thousands of fragility curves are developed based on constructed Kriging and RBF, as shown in Figure 5. But in order to avoid underestimation of the building fragility, a more robust means of probabilistic seismic risk evaluation is necessary. To obtain the fragility curves, 10,000 simulations are sufficient [11]. Using the metamodel, Monte Carlo simulations of 10,000 samples are used to estimate the mean and standard deviation. Since simulations are conducted based on the metamodel, the fragility analysis only requires several minutes (on a single 3.4 GHz processor). Fragility curves for the three limit states (Immediate Occupancy, Life Safety and Collapse Prevention) are determined by considering uncertainties that are scattered within some ranges.

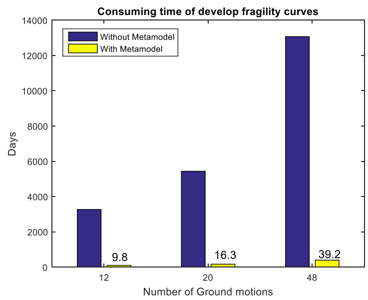

Figure 4: Comparison of consuming time of develop fragility curves considering uncertainties

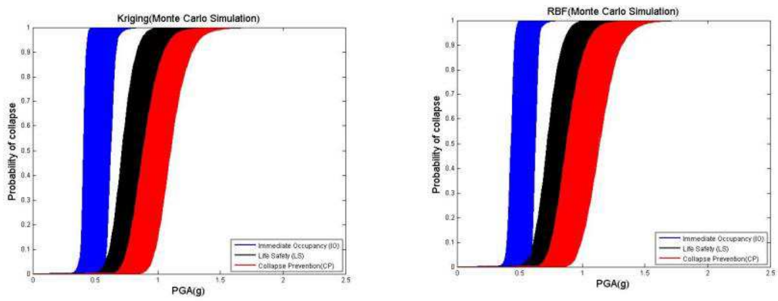

Figure 5: Fragility curves generated by Monte Carlo Simulation based on the Metamodel

4. Conclusions

The proposed approach utilizes the metamodel in combination with Monte Carlo simulations (MCS) for obtaining fragility curves. This strategy makes it practical to carry out probabilistic seismic risk evaluation by using MCS which would otherwise be very computationally demanding. The randomness of the ground motions and material uncertainties are considered in the fragility analysis. Among the three metamodel methods studied, Kriging and RBF are able to accurately predict the fragility curves when compared with response surface method (RSM). Fragility curves considering uncertainties can be predicted directly based on metamodels for seismic risk assessment of buildings. By this way, the fragility analysis can avoid requiring the repetition of seismic nolinear dynamics. The advantages of the proposed methodology are as follows.

1) Uncertainties in material properties and ground motion randomness are considered in seismic risk assessment.

2) The computational time is significantly reduced by two orders of magnitude.

3) The metamodel method could smooth calculation samples. Hence, the nonlinear dynamic analysis convergent problem could be avoided by using the MSA methodology.

4) The main strength of this method lies in its versatility. Uncertainties in both the structure model and ground motion model can be considered easily.

Acknowledgements

This work was financially supported by the Ministry of Education of Singapore (Academic Research Fund R-302-000-095-112).

References

[1] M. Rota, A. Penna, and G. Magenes, "A methodology for deriving analytical fragility curves for masonry buildings based on stochastic nonlinear analyses," *Engineering Structures,* vol. 32, pp. 1312-1323, May 2010.

[2] J. E. Padgett and R. DesRoches, "Sensitivity of seismic response and fragility to parameter uncertainty," *Journal of Structural Engineering-Asce,* vol. 133, pp. 1710-1718, Dec 2007.

[3] A. H. M. M. Billah and M. S. Alam, "Seismic fragility assessment of highway bridges: a state-of-the-art review," *Structure and Infrastructure Engineering,* vol. 11, pp. 804-832, 2015/06/03 2015.

[4] E. S. Choi, R. DesRoches, and B. Nielson, "Seismic fragility of typical bridges in moderate seismic zones," *Engineering Structures,* vol. 26, pp. 187-199, Jan 2004.

[5] M. S. Kircil and Z. Polat, "Fragility analysis of mid-rise R/C frame buildings," *Engineering Structures,* vol. 28, pp. 1335-1345, Jul 2006.

[6] Seismosoft, "SeismoStruct – A computer program for static and dynamic nonlinear analysis of framed structures," p. available online from http://www.seismosoft.com., 2015.

[7] D. Vamvatsikos and C. A. Cornell, "Incremental dynamic analysis," *Earthquake Engineering & Structural Dynamics,* vol. 31, pp. 491-514, 2002.

[8] V. Saxena, G. Deodatis, M. Shinozuka, and M. Q. Feng, "Development of fragility curves for multi-span reinforced concrete bridges," *Proceedings of International Conference on Monte Carlo Simulation,* pp. 499-504, 2000.

[9] G. Deodatis, "Non-stationary stochastic vector processes: seismic ground motion applications," *Probabilistic Engineering Mechanics,* vol. 11, pp. 149-167, 7// 1996.

[10] J. W. Baker, "Efficient analytical fragility function fitting using dynamic structural analysis," *Earthquake Spectra,* vol. 31, pp. 579-599, 2015.

[11] S. K. Saha, V. Matsagar, and S. Chakraborty, "Uncertainty quantification and seismic fragility of base-isolated liquid storage tanks using response surface models," *Probabilistic Engineering Mechanics,* vol. 43, pp. 20-35, 1// 2016.