Forecasting the Term Structure of Government Bond Yields in Unstable Environments*

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Abstract

In this paper we model and predict the term structure of US interest rates in a data-rich and unstable environment. The dynamic Nelson-Siegel factor model is extended to allow the model dimension and the parameters to change over time, in order to account for both model uncertainty and sudden structural changes, in one setting. The proposed specification performs better than several alternatives, since it incorporates additional macro-finance information during hard times, while it allows for more parsimonious models to be relevant during normal periods. A dynamic variance decomposition measure constructed from our model shows that parameter uncertainty and model uncertainty regarding different choices of predictors explain a large proportion of the predictive variance of bond yields.

Keywords: Term Structure of Interest Rates; Nelson-Siegel; Dynamic Model Averaging; Bayesian Methods; Term Premia.

JEL Classification Codes: C32; C52; E43; E47; G17.

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1 Introduction

Modeling the term structure of interest rates using risk factors is a vast and expanding research frontier in financial economics.[1] Diebold and Li (2006) propose a dynamic Nelson-Siegel (NS) model in order to predict the yield curve.[2] However, Altavilla, Giacomini and Ragusa (2014) indicate that the original version of the dynamic NS model without macro information has weaker predictive power in the past twenty years. Various attempts have been made to tackle the weakening predictive power of term structure models. While Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006) stress the importance of key macro variables for the yield curve modeling, Dewachter and Iania (2012) and Dewachter, Iania and Lyrio (2014) show financial factors to be more prominent for explaining yields. This evidence is supported by other researchers, such as Moench (2008), who show that a term structure model augmented with a broad macro-finance information set can provide superior forecasts.

That said, the majority of term structure studies rely on fixed information sets, that is preselection of all possible predictors, and they rarely question whether the introduction of different predictors *per se* can become a source of forecast uncertainty as suggested in Dangl and Halling (2012). Moreover, it is well documented by many econometricians that economic and financial predictors have short-lived predictive content and econometric relationships are unstable; see the general review by Rossi (2013), as well as Dangl and Halling (2012), Koop and Korobilis (2012) and Byrne, Korobilis and Ribeiro (2018) for examples pertaining to stock return, inflation and exchange rate predictability, respectively. In this paper, therefore, we raise and attempt to answer the following questions: To what extent does the consideration of various predictors proliferate forecast uncertainty of term structure models? How do we improve bond yield forecasts when predictors are possibly unstable and short-lived? To answer these questions, our paper builds upon previous work and proposes a modeling framework for term structure forecasting that has several salient features.

Firstly, we incorporate financial information in addition to traditional macro variables, as the global financial crisis highlighted the importance of the financial market for macroeconomic activity and bond yields more generally. We incorporate a substantial range of macro-finance risk factors with model combination techniques that distil large datasets.[3] Estimating a large vector autoregressive model (VAR) with macroeconomic

[1]See Piazzesi (2010), Gürkaynak and Wright (2012), Duffee (2013) and Diebold and Rudebusch (2013) for extensive reviews.

[2]They use three pricing factors to capture most of the variation in bond yield data, which has been well established in Nelson and Siegel (1987) and Litterman and Scheinkman (1991).

[3]Allowing for a large macro-finance information set fully accounts for, and extends, the point of

and financial factors is a non-trivial task due to the proliferation of parameters in large dimensions, see Carriero, Kapetanios and Marcellino (2012) and Coroneo, Giannone and Modugno (2015). Therefore, following Koop and Korobilis (2013) a Bayesian shrinkage estimation methodology is adopted, in order to estimate our large system with many variables. Secondly, we employ time-varying parameter (TVP) VARs to fully capture different degrees of structural change and both coefficient instability and stochastic volatility are taken into account. Van Dijk et al. (2014) suggest that a term structure model considering time-varying parameters can significantly improve its predictive power. In the same spirit, our forecasting exercises are conducted in a TVP dynamic Nelson-Siegel setup that extends Bianchi, Mumtaz and Surico (2009).

Our key methodological contribution is to adopt a model averaging methodology that can potentially mitigate forecast uncertainty originating from different choices of predictors. The proposed Dynamic Model Averaging (DMA) method can determine in a data-based manner which macroeconomic or financial risks are relevant for the yield curve at each point in time. That is, unlike traditional model averaging approaches that select important predictors that are relevant during the whole data sample[4], DMA is a probabilistic framework that allows at each point in time different predictors to be relevant for forecasting. Our application of DMA extends Koop and Korobilis (2013) from the VAR setting to a more general factor-augmented system for term structure modeling, which is new to the literature. We choose at different points in time between three candidate models: i) one with three pricing factors only; ii) pricing factors plus three key macroeconomic indicators; and iii) pricing factors augmented using up to fifteen macro and financial factors. The third macro-finance model is like a 'kitchen sink' model allowing for much more macro-finance information to be incorporated in the spirit of Moench (2008). Model probabilities are assigned to each of the models at each point in time and, thus, averaging is dynamically implemented. When compared to alternative time-varying parameter estimation methodologies, this method is more robust as it encompasses moderate to sudden changes in economic conditions. DMA allows agents to flexibly shift to a more plausible model specification conditional on the most recent information, and Elliott and Timmermann (2008) indicate that model averaging methods in general can reduce the total forecast risk associated with using only a single 'best' model.

Moreover, our setup allows a quantitative evaluation of various sources of forecast uncertainty, by employing an informative variance decomposition following Dangl and Halling (2012). We quantify the relative importance of parameter uncertainty and model uncertainty in terms of different predictors, and show that both parameter uncertainty

Ludvigson and Ng (2009) that large datasets can improve forecasting power.

[4]See Bauer (2016) for an example of Bayesian model averaging in a static setup.

and model uncertainty are important and in total account for one third of predictive variance. Therefore, when choice of predictors is uncertain, DMA that builds upon TVP models is promising in assimilating macro-finance information dynamically.

We examine empirically U.S. term structure dynamics using over fourty years of monthly observations. The proposed approach has useful empirical properties in yield forecasting, as it considers parameter and model uncertainty and is robust to potential structural breaks. We compare the forecast performance of DMA to a basic dynamic Nelson-Siegel model and several variants, and show that substantial gains in yield predictability are due to the ensemble of salient features – time-varying parameters and dynamic model averaging. The contribution of each feature we incorporate in our model is significant and time-varying, and we find macro-finance information is important during recessions. The superior out-of-sample forecasting performance of DMA, especially for short rates, reveals plausible expectations of market participants in real time.[5] Using only conditional information, DMA provides successful term premium alternatives to full-sample estimates produced by the no-arbitrage term structure models of Kim and Wright (2005), Wright (2011) and Bauer, Rudebusch and Wu (2014).

This paper is structured as follows. Section 2 describes the framework and the estimation method for modeling bond yield dynamics. Section 3 describes the data and discusses the results. Specifically, the first two subsections test the parameter instability and elaborate on the usefulness of employing DMA. Section 3.4 displays the point and density forecasting performance of our term structure model. Section 3.6 shows the model-implied term premia has informative economic implications. Section 4 concludes.

2 Methods

2.1 Cross-Sectional Restrictions and Yield Factor Dynamics

Following Nelson and Siegel (1987) and Diebold and Li (2006) we assume that three factors summarize most of the information in the term structure of interest rates. The Nelson and Siegel (1987) (NS) approach has an appealing structure that is parsimonious, flexible, and allows for an easy interpretation of the estimated factors. Let $y_t(\tau)$ denote

[5] The indicators of real activity and the stock market are helpful in explaining the movements, see Appendix C.5. This is consistent with Kurmann and Otrok (2013) and Bansal, Connolly and Stivers (2014), who relate the changes in the term structure to news shocks on total factor productivity and asset-class risk, respectively.

yields at maturity τ, then the factor model we use is of the form:[6]

$$y_t(\tau) = L_t^{NS} + \frac{1 - e^{-\tau\lambda^{NS}}}{\tau\lambda^{NS}} S_t^{NS} + \left(\frac{1 - e^{-\tau\lambda^{NS}}}{\tau\lambda^{NS}} - e^{-\tau\lambda^{NS}} \right) C_t^{NS} + \varepsilon_t(\tau), \tag{2.1}$$

where L_t^{NS} is the "Level" factor, S_t^{NS} is the "Slope" factor, C_t^{NS} is the "Curvature" factor and $\varepsilon_t(\tau)$ is the error term. In the formulation above, λ^{NS} is a parameter that controls the shapes of loadings for the NS factors. For estimation purposes, we can rewrite equation (2.1) in the equivalent form,

$$y_t(\tau) = \mathbf{B}(\tau) F_t^{NS} + \varepsilon_t(\tau),$$

where $F_t^{NS} = \left[L_t^{NS}, S_t^{NS}, C_t^{NS} \right]'$ is the vector of three NS factors, $\mathbf{B}(\tau)$ is the loading vector and $\varepsilon_t(\tau)$ is the error term.

The above Nelson-Siegel restrictions on loadings are restrictions that apply on the cross-section of yields. We use simple ordinary least squares (OLS) to extract three NS factors, and following Diebold and Li (2006), Bianchi, Mumtaz and Surico (2009) and Van Dijk et al. (2014), we set $\lambda^{NS} = 0.0609$. We assume these factors are observed without errors, which is a standard assumption in term structure modeling. The interpretation of the Nelson-Siegel factors is of considerable empirical importance. The Level factor L_t^{NS} loads on all maturities evenly. The Slope factor S_t^{NS} approximates the long-short spread, and its movements are captured by placing more weights on shorter maturities. The Curvature factor C_t^{NS} captures changes that have their largest impact on medium-term maturities, and therefore medium-term maturities load more heavily on this factor. In particular, using the setting $\lambda^{NS} = 0.0609$, the C_t^{NS} has the largest impact on the bond at 30-month maturity, see Diebold and Li (2006).[7]

Time-series dynamics An important and novel aspect of our methodology is in modeling the factor dynamics. Following Bianchi, Mumtaz and Surico (2009), the extracted Nelson-Siegel factors augmented with macroeconomic variables follow a time-varying parameter vector autoregression (TVP-VAR) of order p of the form

$$\begin{bmatrix} F_t^{NS} \\ M_t \end{bmatrix} = c_t + B_{1t} \begin{bmatrix} F_{t-1}^{NS} \\ M_{t-1} \end{bmatrix} + \cdots + B_{pt} \begin{bmatrix} F_{t-p}^{NS} \\ M_{t-p} \end{bmatrix} + v_t, \tag{2.2}$$

[6]This is an asymptotically flat approximating function, and Siegel and Nelson (1988) demonstrate that this property is appropriate if forward rates have finite limiting values.

[7]Further discussion of these factors can be found in Appendix B.

where c_t are time-varying intercepts, $B_{1t}, ..., B_{pt}$ are time-varying autoregressive coefficients, M_t is a vector of macro-finance risk factors, and v_t is the error term. Following Coroneo, Giannone and Modugno (2015) and Joslin, Priebsch and Singleton (2014), we do not impose additional restrictions on the VAR system above. In our framework instead macro-finance variables only affect the unobserved NS factors and do not interact contemporaneously with the observed yields, so that they are unspanned by the yields. In other words, a 'knife-edge' restriction is imposed on the coefficients of macro-finance variables in the cross section, while the time-series dynamics (VAR model) are left unconstrained.[8]

For the purpose of econometric estimation, we work with a more compact form of Eq. (2.2). We can show that the p-lag TVP-VAR can be written as

$$z_t = X_t \beta_t + v_t, \tag{2.3}$$

where $z_t = \left[L_t^{NS}, S_t^{NS}, C_t^{NS}, M_t'\right]'$, M_t is a $q \times 1$ vector of macro-finance factors, $X_t = I_n \otimes \left[z_{t-1}', ..., z_{t-p}'\right]$ for $n = q+3$, $\beta_t = \left[c_t, vec\left(B_{1t}\right)', \cdots, vec\left(B_{pt}\right)'\right]'$ is a vector summarizing all VAR coefficients, $v_t \sim N\left(0, \Sigma_t\right)$ with Σ_t an $n \times n$ covariance matrix. This regression-type equation is completed by describing the law of motion of the time-varying parameters β_t and Σ_t. For β_t we follow the standard practice in the literature from Bianchi, Mumtaz and Surico (2009) and consider random walk evolution for the VAR coefficients,

$$\beta_{t+1} = \beta_t + \mu_t, \tag{2.4}$$

based upon a prior β_0 discussed below, and $\mu_t \sim N\left(0, Q_t\right)$. Following Koop and Korobilis (2013) we set $Q_t = \left(\Lambda^{-1} - 1\right) cov\left(\beta_{t-1} | \mathcal{D}_{t-1}\right)$ where $\mathcal{D}_{t-1}$ denotes all the available data at time $t-1$ and scalar $\Lambda \in (0, 1]$ is a 'forgetting factor' discounting older observations. The covariance matrix Σ_t evolves according to a Wishart matrix discount process (Prado and West (2010)) of the form:

$$\Sigma_t \sim iW\left(S_t, n_t\right), \tag{2.5}$$

$$n_t = \delta n_{t-1} + 1, \tag{2.6}$$

$$S_t = \delta S_{t-1} + f\left(v_t' v_t\right), \tag{2.7}$$

where n_t and S_t are the degrees of freedom and scale matrix, respectively, of the inverse Wishart distribution, δ is a 'decay factor' discounting older observations, and $f\left(v_t' v_t\right)$ is

[8] Bauer and Rudebusch (2017) test the knife-edge restrictions and point out these restrictions, though statistically rejected, have only small effects on cross-sectional fit and estimated term premia. Therefore, we follow Joslin, Priebsch and Singleton (2014) and Coroneo, Giannone and Modugno (2015) for the tractability of our proposed model.

a specific function of the squared residuals of our model and explained in the Appendix A.1. The pair of forgetting/decay factors (Λ, δ) can be interpreted as a Bayesian prior on the amount of time-variation expected in the drifting coefficients β_t and the volatilities Σ_t, respectively. Following recommendations in Koop and Korobilis (2013) and Dangl and Halling (2012), we set the forgetting factor $\Lambda = 0.99$ and the decay factor $\delta = 0.95$.

To sum up, we have specified a VAR with drifting coefficients and stochastic volatility which allows for modeling structural instability and regime changes in the joint dynamics of the NS factors and the macroeconomic and financial factors. The Nelson-Siegel restrictions allow for straightforward estimation and interpretation, and shift our focus on the time-series dynamics of the estimated factors. Given ample empirical evidence about the importance of structural change and volatility in forecasting macroeconomic and financial variables,[9] our approach is more pragmatic: by specifying a flexible VAR structure with time-varying parameters and stochastic volatility, as well as relevant macro-finance information that can switch over time, we expect to obtain better forecasts of the underlying yield curve.

2.2 Model Uncertainty and the Role of Macro-Finance Factors

This paper argues that the possible set of risk factors relevant for characterizing the yield curve can change over time. We focus on Eq. (2.3) and work with three possible information sets: small, medium, and large. The small-size (NS) model only contains the three yield factors extracted from the Nelson-Siegel model and zero macro variable, therefore $q = 0$ in Eq. (2.3). The middle-size (NS + macro) model includes, in addition to the Nelson-Siegel factors, Federal Fund Rate, CPI inflation and Industrial Production, so $q = 3$. The large (NS + macro-finance) model includes $q = 15$ macroeconomic and financial variables.

Having three candidate models $i = 1, 2, 3$, in our model space, we use the recursive nature of the Kalman filter to choose among different models at each point in time. That is, for each t we obtain the probability/weight for each model i

$$\pi_t^{(i)} = f\left(L_{t-1} = i | D_{t-1}\right)$$

under the regularity conditions $\sum_{i=1}^{K} \pi_t^i = 1$ and $\pi_t^i \in [0, 1]$, and where L_{t-1} is the model selected at time $t - 1$. We estimate these model weights in a recursive manner, in the spirit of the Kalman filtering approach. We follow Koop and Korobilis (2013) and define

[9] See for example, Dangl and Halling (2012), Koop and Korobilis (2012), Van Dijk et al. (2014) and Byrne, Korobilis and Ribeiro (2018).

the updating step

$$\pi_{t|t}^{(i)} \propto \pi_{t|t-1}^{(i)} p^{(i)} \left(z_t | D_{t-1} \right). \tag{2.8}$$

where the quantity $p^{(i)} \left(z_t | D_{t-1} \right)$ is the time t predictive likelihood of model i, using information up to time $t-1$. This quantity is readily available from the Kalman filter and it provides an out-of-sample measure of fit for each model which allows us to construct model probabilities. In this paper we focus on the predictive likelihoods of the three Nelson-Siegel factors when implementing DMA. The time t prior $\pi_{t|t-1}^{(i)}$ is given by

$$\pi_{t|t-1}^{(i)} = \frac{\left(\pi_{t-1|t-1}^{(i)} \right)^{\alpha}}{\sum_{i=1}^{K} \left[\left(\pi_{t-1|t-1}^{(i)} \right)^{\alpha} \right]} \tag{2.9}$$

where $0 < \alpha \leq 1$ is a decay factor which allows discounting exponentially past forecasting performance, see Koop and Korobilis (2013) for more information. When $\alpha \to 0$ we have the case that at each point in time we update our beliefs with a prior of equal weights for each model. When $\alpha = 1$ the predictive likelihood of each observation has the same weight which is basically equivalent to recursively implementing static Bayesian Model Averaging. For all other values between $(0, 1)$ Dynamic Model Averaging occurs. In this paper a sufficiently small value is used for α such that the time t prior is flat, and we subsequently show this can capture changing economic conditions and increase the predictive performance.

3 Empirics

3.1 Data

The smoothed yields provided from the US Federal Reserve by Gürkaynak, Sack and Wright (2007) are used in the term-structure model specified in the previous section. We also include 3- and 6-month Treasury Bills (Secondary Market Rate). The empirical analysis focuses on yields with maturities of 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108 and 120 months. The key macroeconomic and financial variables that enter our Dynamic Model Averaging model are obtained from St. Louis Federal Reserve Economic Data (FRED). These include inflation, real activity indicators, monetary variables, as well as the stock market, exchange rate, house prices and other financial market indicators; the details can be found in the Data Appendix. The full sample is from November 1971 to November 2013 and we use end of the month yield data. We present the yields' descriptive statistics in Table 1. As expected, the mean of yields increase with maturity, consistent with the existence of a risk premium for long maturities. Yields have high autocorrelation which declines with lag length and increases with maturity. The short end of the yield curve is more volatile than the long end.

Different numbers of macro-finance variables are selected for the three VAR sizes entering the dynamic model averaging framework. As mentioned above, the small-size VAR (NS) does not include any macro or financial variables, but only the Nelson-Siegel factors. The middle-size VAR (i.e. NS + macro) includes Federal Fund Rate, inflation and Industrial Production, which are also used in related literature such as Ang and Piazzesi (2003) and Diebold, Rudebusch and Aruoba (2006). The large VAR (i.e. NS + macro-finance) includes all 15 macro and financial variables, which should comprehensively include the information the market players are able to acquire.

Table 1: Descriptive Statistics of Bond Yields

	Mean	Std. Dev.	Minimum	Maximum	$\hat{\rho}(1)$	$\hat{\rho}(12)$	$\hat{\rho}(30)$
3	5.154	3.341	0.010	16.300	0.987	0.815	0.533
6	5.284	3.320	0.040	15.520	0.988	0.827	0.557
12	5.675	3.440	0.123	16.110	0.987	0.842	0.599
24	5.910	3.355	0.188	15.782	0.988	0.858	0.648
36	6.102	3.259	0.306	15.575	0.989	0.868	0.677
48	6.266	3.161	0.454	15.350	0.990	0.873	0.695
60	6.411	3.067	0.627	15.178	0.990	0.876	0.707
72	6.539	2.980	0.815	15.061	0.990	0.877	0.714
84	6.653	2.902	1.007	14.987	0.990	0.878	0.718
96	6.754	2.833	1.197	14.940	0.990	0.878	0.721
108	6.843	2.772	1.380	14.911	0.990	0.878	0.722
120	6.920	2.720	1.552	14.892	0.990	0.877	0.723
Level	7.437	2.379	2.631	14.347	0.989	0.866	0.700
Slope	-2.277	1.940	-5.824	4.522	0.954	0.492	-0.114
Curvature	-1.424	3.222	-8.948	5.282	0.903	0.634	0.369

Notes: This table presents descriptive statistics for monthly yields at 3- to 120-month maturity, and for the yield curve Level, Slope and Curvature factors extracted from the Nelson-Siegel model. The sample period is 1971:11–2013:11. We use following abbreviations. **Std. Dev.**: Standard Deviation; $\hat{\rho}(\mathbf{k})$: Sample Autocorrelation for Lag k.

3.2 Evidence on Parameter Instability

In this section we seek to motivate the use of time-varying parameter methods. There is a vast selection of different tests of parameter instability and structural breaks in the literature from both a frequentist and a Bayesian perspective; see for example, Andrews and Ploberger (1994), Hanson (2002) and Rossi (2005). McCulloch (2007) suggests a likelihood-based approach to test parameter instability in a TVP model. The limiting distribution of the proposed test statistic may not be standard and, consequently, its critical values need to be bootstrapped. In the spirit of McCulloch (2007), we construct a likelihood-based test on the small VAR system of the factor dynamics, using the 1983-2013 sample. We bootstrap 5000 samples to recover the test statistics following Feng and McCulloch (1996). Based on our test, the null hypothesis that the coefficients of the VAR are constant over time is rejected at 1% significance level, which means employing the TVP-VAR model is appropriate.

Nevertheless, such tests of parameter instability are in-sample tests and fail to provide evidence concerning structural instability and predictability out-of-sample. Instead, we follow a different strategy and we note that the constant parameter version of the Nelson-Siegel model can be obtained as a special case of our proposed time-varying specification.[10] Since our ultimate purpose is to obtain optimal forecasts of the yield curve, "testing" for parameter instability can conveniently boil down to a comparison of pseudo out-of-sample predictive power between the TVP-VAR and a constant parameter VAR. We employ the test proposed by Diebold and Mariano (1995) and evaluate the predictive power of competing models across four forecast horizons ($h = 1, 3, 6, 12$ months) and for all of our maturities. The p-values of the tests are reported in Table 2, which correspond to the test of the null hypothesis that the competing TVP-VAR model has equal expected square prediction error relative to the benchmark forecasting model constant parameter VAR (i.e. Diebold and Li (2006)), against the alternative hypothesis that the competing TVP-VAR forecasting model has a lower expected square prediction error than the benchmark forecasting model. Table 2 indicates the TVP-VAR consistently outperforms the constant parameter VAR. The test statistic rejects the null for most of the maturities, and especially at longer forecast horizons, so the time-varying parameter model should be preferred as it can provide more robust estimates.

To highlight the importance of the TVP feature, Figure 1 sets out the time-varying persistence of factor dynamics in the small-size VAR. This can be examined by considering the behavior of the eigenvalues. We can detect significant changes in all eigenvalues, which reflects indispensable changes in the persistence of pricing factors over time. The first eigenvalue seem relatively stable, but the mild variation in the eigenvalue would translate into sufficiently large changes in long-term expectations. Another observation is the clear rising trend for the third eigenvalue, which implies the third pricing factor is becoming

[10] In particular, as Koop and Korobilis (2013) show, by setting the forgetting and decay factors $\Lambda = \delta = 1$, our model is equivalent to the recursive estimation of a model with constant coefficients and constant variance.

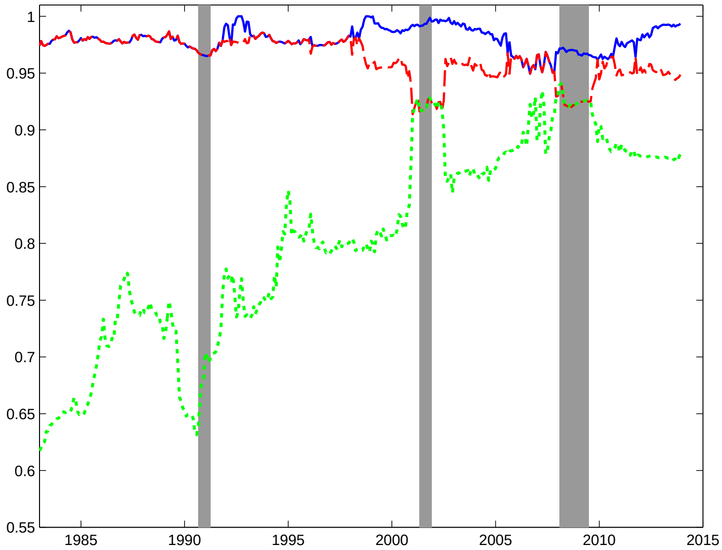
Table 2: Parameter Instability Test

P-Values: TVP-VAR vs. VAR												
Maturity	3	6	12	24	36	48	60	72	84	96	108	120
$h = 1$	0.02	0.00	0.54	0.14	0.02	0.00	0.00	0.00	0.01	0.08	0.33	0.68
$h = 3$	0.03	0.01	0.13	0.04	0.01	0.01	0.00	0.01	0.02	0.05	0.13	0.28
$h = 6$	0.00	0.00	0.04	0.02	0.01	0.01	0.01	0.01	0.02	0.04	0.08	0.16
$h = 12$	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.03

Notes: 1. This table reports the statistical significance for the relative forecasting performance, based on the Diebold and Mariano (1995) test. We conduct 1, 3, 9 and 12 months ahead forecasts for bond yields at maturities ranging from 3 months to 120 months. The predictive period is between 1983:11 and 2013:11.
2. Statistical significance for the relative MSFE statistics is based on the p-value for the Diebold and Mariano (1995) statistic; the statistic corresponds to the test of the null hypothesis that the competing TVP-VAR model has equal expected square prediction error relative to the benchmark forecasting model constant parameter VAR (that is, a model identical to the Diebold and Li (2006) model), against the alternative hypothesis that the competing TVP-VAR model has a lower expected square prediction error than the benchmark forecasting model.

more persistent. Moreover, we find that the second and third eigenvalues have important changes in near recession periods, which is connected to the shifting dynamics of Slope and Curvature factors. This is evidence of sudden structural changes. As macro-finance information is considered important during recessions as suggested by Bernanke, Gertler and Gilchrist (1996), it is uncertain whether the small-size VAR can still produce plausible forecasts when faced with structural instability.

Figure 1: Time-Varying Persistence of Factor Dynamics



Notes: The graph shows the largest three eigenvalues of the factor time-series dynamics in the small-size TVP model. The shaded areas are recession periods according to the NBER Recession Indicators.

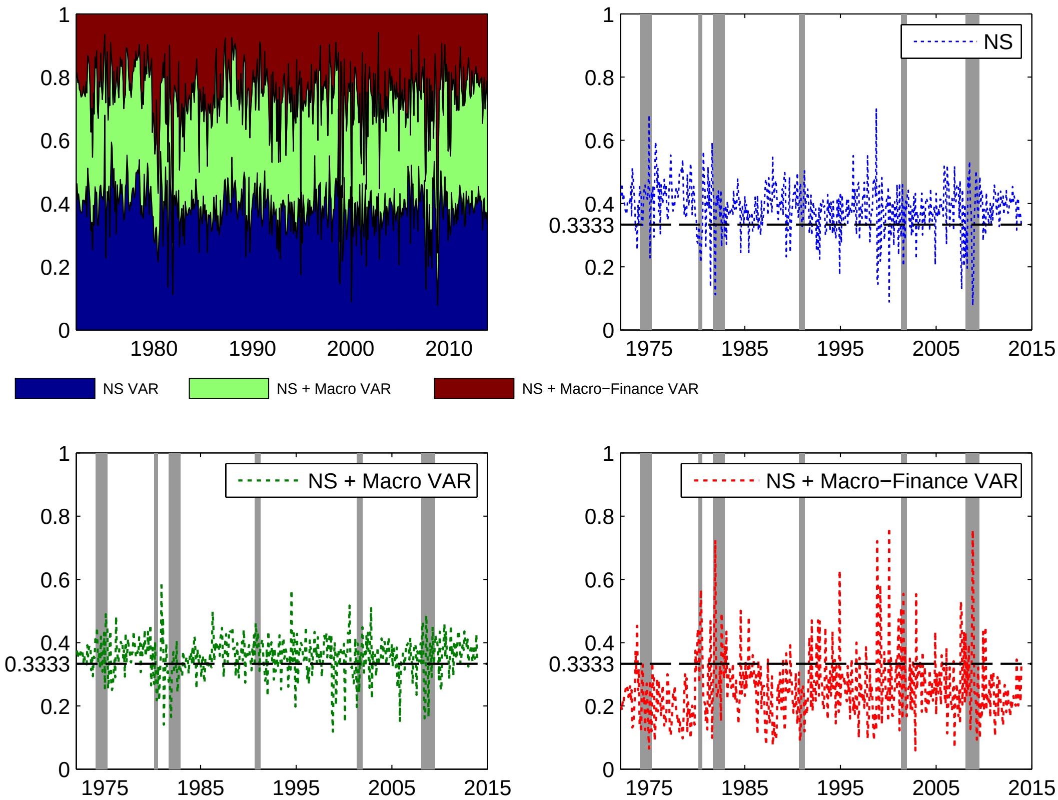
3.3 Model Dynamics

Graphical evidence of the usefulness of our model averaging approach is provided by the Figure 2. The upper two panels set out the relative importance of the small, medium and large VAR models used in DMA. In general, there is substantial time variation in the weights, and the empirical observations are of economic importance.

It is worth reiterating the importance of the large macro-finance VAR, as Altavilla, Giacomini and Ragusa (2014) indicate that the original version of the dynamic NS model without macro information has weaker predictive power in recent years. We show the large-size VAR significantly boosts the forecast performance because of its superior performance during the recession periods. Moreover, model averaging expands the model set when compared with a single-model setup or model selection, and potentially mitigates the misspecification problem. Intuitively, the consideration of models with richer information sets allows to effectively 'hedge' the risk of using a single model as Elliott and Timmermann (2008) suggest.

Since the changes in model weights are very sensitive to new information, DMA reacts to sudden, rather than smooth, changes in coefficients. Without model averaging or selection, a time-varying parameter model with a specific information set may have volatile forecasting performance, as the true dynamics may not be captured well during certain periods. The DMA approach encompasses moderate to sudden changes in the economic environment and accordingly is promising in producing consistently superior forecasting

Figure 2: Model Weights for TVP, TVP-M and TVP-L Models



Notes:

1. This figure sets out the time-varying probabilities of our three models in our Dynamic Model Averaging (DMA) approach. The probabilities for DMA are updated from a Kalman filter based on the predictive accuracy, see Eq. (A.5); the probabilities/weights of the VAR models sum up to 1.

2. The upper left panel shows the probability weights of all models. The upper right and the lower panels display the weights of the TVP(NS VAR), the TVP-M (NS + Macro VAR) and the TVP-L (NS + Macro-Finance VAR), respectively. The shaded areas are the recession periods based on NBER Recession Indicators.

performance, a point we discuss in detail in the following sections.

3.4 Forecasting Performance

At each time, we generate forecasts using the proposed DMA model with the data only up to that period. As described in the methodology section, we obtain the full posterior distribution of state variables and parameters, and the predictive density of yields, using only data up to time t. Specifically, the out-of-sample forecasts of yields are generated in the following two-step procedure.

The first stage is using the Kalman filter and DMA to generate predictions of the three Nelson-Siegel yield factors with macro variables. That is, at each time, β_t is estimated using the Kalman filter and Σ_t is estimated according to the forgetting factor method. For each of the three candidate models (the small, meddle or large), we use Eq. (2.3) with the predicted $\hat{\beta}^{(i)}_{t+1} = \beta^{(i)}_t$ to forecast pricing factors. Also at each time a model averaging method is recursively implemented to generate a weighted average of the forecasts by three candidate models, as decried in Section 2.2. The second stage is forecasting the yields with the predicted NS factors and the fixed NS loadings. This step is straightforward as bond yields are just linear combinations of three NS factors. The macro variables are not directly used to predict the yields in the second step, because of the knife-edge cross-sectional restrictions.

We use a training sample from 1971:11 to 1983:10, which gives a long forecast evaluation period from 1983:11 to the 2013:11. To thoroughly investigate the predictive power of our proposed model, we produce out-of-sample monthly forecasts for 30 years, and the predictive horizons range from one month to twelve months. To better evaluate the predictive performance of DMA, we have the following seven variants of dynamic Nelson-Siegel models: recursive estimation of factor dynamics using a standard VAR following Diebold and Li (2006) (DL), 10-year rolling-window VAR estimations (DL-R10), recursive VAR estimation with three macro variables (DL-M), recursive estimations of standard VAR with macro-finance principal components following Stock and Watson (2002) (DL-SW), time-varying parameter VAR estimations of factor dynamics without macro information (TVP), time-varying parameter VAR estimations of factor dynamics with three macro variables (TVP-M), and Dynamic Model Selection (DMS).

DL is the two-step forecasting model proposed by Diebold and Li (2006), which recursively estimates the factor dynamics using a standard VAR. In other words, DL estimates the VAR model of factors recursively with historical data, extending through all the following periods. We have four variations of the DL model: 10-year rolling-window estimations (DL-R10); recursive estimations with three macro variables of Fed Fund Rate, Inflation and Industrial Production (DL-M); and recursive estimations with three principal components of our whole macro-finance dataset (DL-SW). In the DL-SW model, three macro principal components are drawn using the method proposed by Stock and Watson (2002) to augment DL. Lastly, we include two extensions of DL using a time-varying parameter VAR without macro information and a time-varying parameter VAR with three macro variables to characterize the factor dynamics, denoted TVP and TVP-M, respectively; the latter is essentially the model estimated in Bianchi, Mumtaz and Surico (2009) using MCMC methods. We report the performance of all models relative to the random walk (RW) forecast, which allows us to evaluate whether the term structure models successfully capture the high persistence in bond yields.

The point forecast precision of all term structure models is assessed in Table 3 which displays the one and three period ahead Mean Squared Forecast Error (MSFE) of the competing specifications relative to the MSFE of the random walk. Lower values of the relative MSFEs indicate better performance in general, while values lower than one indicate better performance relative to the random walk in particular. The core empirical results are very encouraging for the proposed method. As can be seen in this table, the DMA model consistently outperforms all the benchmark models. In many cases the relative MSFEs are also below one, meaning that the model fairs well compared to the random walk. Nevertheless, in some cases the relative MSFEs are higher than one, or they are not substantially lower than one in order to be considered significant.[11] Table 4 shows relative MSFEs for horizons of six and twelve months ahead and it reveals that the DMA is also preferred at relatively long forecast horizons. Moving from point to density forecasts, that is forecasts that take into account the uncertainty in the predictive distribution, we find that also DMA performs well. In Tables 3 and 4 we find that DMA has the highest value of the log predictive score; see Geweke and Amisano (2010) for a definition of this metric. Given that DMA is outperforming all other specifications in all instances on average using this measure, we simply denote this in the two tables by using the symbol †. Figure 3 goes one step further and instead of quoting the average predictive log-scores, it plots the cumulative sum of predictive log-scores during the whole evaluation period, for selected maturities and for the DMA/DMS and Diebold-Li approaches. The predictive density of the DMA specification is more accurate compared to the predictive density of the Diebold-Li (DL) across all maturities, especially for short rates.

Among all models, the results indicate DMA is the only one comparable in forecasting

[11] Significance of relative MSFEs can also be measured using the Diebold and Mariano (1995) statistic we used in Table 2 to measure the predictive differences between constant and time-varying parameter specifications.

Table 3: One and three month ahead relative MSFEs of competing term structure models

	DMA	**DMS**	**TVP**	**TVP-M**	**DL**	**DL-R10**	**DL-M**	**DL-SW**
Maturity	One month ahead relative MSFE ($h = 1$)							
3	**0.706**†	0.781	0.747	0.710	0.848	1.085	0.885	1.417
6	**0.818**†	0.927	0.894	0.908	1.068	1.313	1.130	1.668
12	0.971†	1.031	0.983	1.011	0.930	**0.897**	0.979	1.547
24	1.000 †	1.075	1.044	1.060	1.064	1.105	1.103	1.461
36	**0.977**†	1.039	1.032	1.026	1.123	1.223	1.144	1.237
48	**0.965**†	1.008	1.016	1.002	1.130	1.266	1.143	1.099
60	**0.965**†	0.996	1.011	0.997	1.116	1.273	1.129	1.051
72	**0.971**†	0.998	1.015	1.006	1.096	1.259	1.114	1.055
84	**0.982**†	1.008	1.026	1.024	1.074	1.226	1.098	1.090
96	**0.996**†	1.023	1.040	1.046	1.052	1.173	1.083	1.139
108	1.009†	1.038	1.055	1.068	1.031	1.108	1.068	1.183
120	1.020†	1.050	1.065	1.084	1.015	1.043	1.053	1.214
Mean	**0.964**†	1.009	1.008	1.010	1.053	1.162	1.083	1.237
Maturity	Three month ahead relative MSFE ($h = 3$)							
3	**0.765**†	0.873	0.864	0.845	1.105	1.514	1.070	1.795
6	**0.863**†	0.976	0.976	0.997	1.305	1.646	1.283	1.907
12	**0.931**†	1.003	0.997	1.019	1.131	1.231	1.119	1.727
24	**0.988**†	1.046	1.062	1.068	1.255	1.390	1.249	1.537
36	1.002†	1.044	1.073	1.060	1.295	1.482	1.292	1.358
48	1.006†	1.037	1.069	1.049	1.294	1.528	1.293	1.246
60	1.006†	1.032	1.063	1.043	1.269	1.539	1.272	1.196
72	1.005†	1.030	1.057	1.041	1.233	1.525	1.239	1.189
84	1.002†	1.029	1.053	1.044	1.190	1.488	1.201	1.207
96	**0.999**†	1.031	1.050	1.049	1.146	1.431	1.160	1.238
108	**0.996**†	1.033	1.049	1.055	1.102	1.360	1.120	1.272
120	**0.994**†	1.035	1.048	1.061	1.062	1.283	1.083	1.302
Mean	**0.969**†	1.018	1.035	1.032	1.205	1.449	1.205	1.405

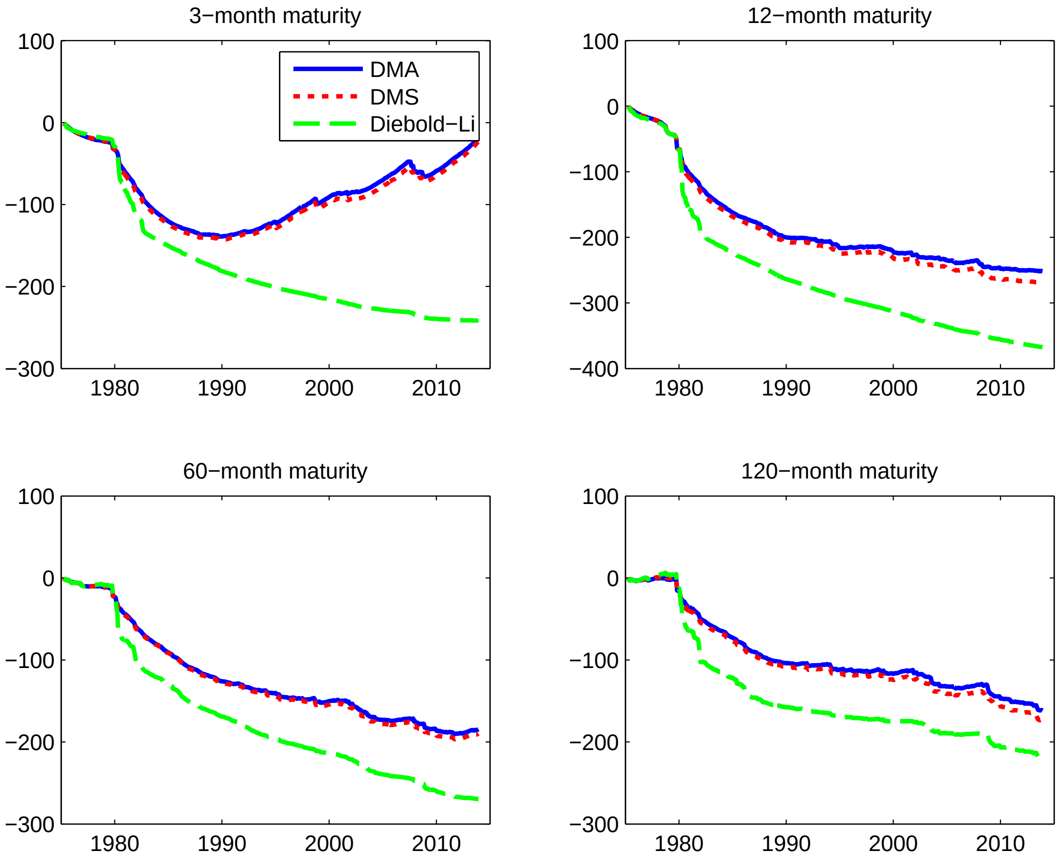
Notes: 1. This table presents one and three month ahead forecast statistics of bond yields with maturities ranging from three months to 120 months. The evaluation period is 1983:11 to 2013:11.
2. We report the ratio of each model's Mean Squared Forecast Errors (MSFE) relative to the MSFE of the random walk, and the preferred values are in bold. The symbol † indicates the model with the highest value of average predictive log-scores, a measure that takes into account the whole predictive density; see Geweke and Amisano (2010) for details.
3. In this table, we use following abbreviations: **MSFE**: Mean Squared Forecasting Error; **Mean**: Averaged MSFE across all sample maturities. In our proposed Nelson-Siegel (**NS**) framework, **DMA** (Dynamic Model Averaging) averages all the models with probabilities in each step, while **DMS** (Dynamic Model Selection) chooses the best model with the highest probability at any point in time. **TVP**: a time-varying parameter model without macro information; **TVP-M**: a time-varying parameter model with three macro variables: fund rate, inflation and industrial production, similar to Bianchi Mumtaz and Surico (2009) but estimated with a fast algorithm without the need of MCMC; **DL**: Diebold and Li (2006) model, i.e. constant coefficient Vector Autoregressive model with recursive (expanding) estimations; **DL-R10**: Diebold and Li (2006) estimates based 10-year rolling windows; **DL-M**: factor dynamics in Diebold and Li (2006) are augmented with three macro variables: fund rate, inflation and industrial production, using recursive estimations; **DL-SW**: factor dynamics in Diebold and Li (2006) are augmented with three principal components (see Stock and Watson (2002)) of our macro/finance data, using recursive estimations.

Table 4: Six and twelve month ahead relative MSFEs of competing term structure models

	DMA	**DMS**	**TVP**	**TVP-M**	**DL**	**DL-R10**	**DL-M**	**DL-SW**
Maturity	SIX MONTH AHEAD RELATIVE MSFE ($h = 6$)							
3	**0.871**†	0.976	0.974	1.012	1.332	1.703	1.405	1.908
6	**0.947**†	1.051	1.053	1.120	1.446	1.796	1.514	1.999
12	**0.969**†	1.072	1.057	1.080	1.304	1.501	1.322	1.825
24	1.025†	1.109	1.106	1.105	1.393	1.623	1.407	1.707
36	1.038†	1.107	1.110	1.090	1.416	1.685	1.427	1.574
48	1.038†	1.097	1.101	1.073	1.403	1.709	1.414	1.481
60	1.032†	1.085	1.088	1.060	1.368	1.702	1.381	1.432
72	1.021†	1.073	1.076	1.051	1.322	1.673	1.336	1.417
84	1.009†	1.063	1.064	1.046	1.270	1.627	1.286	1.422
96	**0.997**†	1.055	1.056	1.044	1.218	1.568	1.236	1.438
108	**0.987**†	1.048	1.049	1.043	1.167	1.502	1.187	1.458
120	**0.978**†	1.043	1.045	1.044	1.122	1.433	1.142	1.477
Mean	**0.994**†	1.067	1.067	1.066	1.323	1.632	1.348	1.607
Maturity	TWELVE MONTH AHEAD RELATIVE MSFE ($h = 12$)							
3	**0.980**†	1.073	1.021	1.240	1.349	1.605	1.517	1.677
6	1.034†	1.128	1.079	1.292	1.419	1.703	1.579	1.784
12	1.025†	1.139	1.082	1.210	1.353	1.592	1.458	1.661
24	1.075†	1.191	1.139	1.208	1.474	1.757	1.573	1.664
36	1.091†	1.202	1.152	1.188	1.528	1.848	1.623	1.625
48	1.087†	1.193	1.145	1.163	1.532	1.885	1.625	1.591
60	1.070†	1.175	1.127	1.140	1.505	1.884	1.596	1.577
72	1.049†	1.154	1.106	1.121	1.459	1.858	1.549	1.581
84	1.025†	1.133	1.086	1.105	1.405	1.816	1.494	1.599
96	1.003†	1.115	1.068	1.092	1.349	1.766	1.437	1.623
108	**0.983**†	1.100	1.054	1.083	1.294	1.711	1.381	1.649
120	**0.966**†	1.089	1.043	1.077	1.243	1.655	1.329	1.673
Mean	1.035†	1.143	1.093	1.174	1.415	1.748	1.524	1.648

Notes: 1. This table presents six and twelve month ahead forecast statistics of bond yields with maturities ranging from three months to 120 months. The evaluation period is 1983:11 to 2013:11.
2. We report the ratio of each model's Mean Squared Forecast Errors (MSFE) relative to the MSFE of the random walk, and the preferred values are in bold. The symbol † indicates the model with the highest value of average predictive log-scores, a measure that takes into account the whole predictive density; see Geweke and Amisano (2010) for details.
3. In this table, we use following abbreviations: **MSFE**: Mean Squared Forecasting Error; **Mean**: Averaged MSFE across all sample maturities. In our proposed Nelson-Siegel (**NS**) framework, **DMA** (Dynamic Model Averaging) averages all the models with probabilities in each step, while **DMS** (Dynamic Model Selection) chooses the best model with the highest probability at any point in time. **TVP**: a time-varying parameter model without macro information; **TVP-M**: a time-varying parameter model with three macro variables: fund rate, inflation and industrial production, similar to Bianchi Mumtaz and Surico (2009) but estimated with a fast algorithm without the need of MCMC; **DL**: Diebold and Li (2006) model, i.e. constant coefficient Vector Autoregressive model with recursive (expanding) estimations; **DL-R10**: Diebold and Li (2006) estimates based 10-year rolling windows; **DL-M**: factor dynamics in Diebold and Li (2006) are augmented with three macro variables: fund rate, inflation and industrial production, using recursive estimations; **DL-SW**: factor dynamics in Diebold and Li (2006) are augmented with three principal components (see Stock and Watson (2002)) of our macro/finance data, using recursive estimations.

Figure 3: Cumulative Sum of Predictive Log-Likelihood of 3-, 12-, 60- or 120-Month Maturities



Notes: These are one month ahead cumulative sums of predictive log-likelihood for predicted yields from early 1975 to late 2013. From top left clockwise we have maturities of 3, 12, 120 and 60 months. The models are DMA (solid), DMS (dotted) and Diebold-Li (dashed). A higher log-likelihood implies improved density predictability.

performance to, or better than, the RW. In fact, DMA not only successfully captures the persistence in bond yields, but also reveals robust short rate expectations and risk premium estimates because of its superior performance in short rate forecasts. It is worth noting that the rolling-window forecasts perform much less favorably. In addition, the predictive power of DL-SW is not satisfactory. The macro principal components alone cannot provide useful information in terms of yield forecasting, since the method fails to exclude irrelevant information in a time-varying manner. This is an example of an unstable forecasting relationship, where the same information in macro-finance variables may not be useful in forecasting at all periods and forecast horizons. Hence, this result indicates the relative advantages of DMA as a plausible adaptive method for forecasting using only relevant information at each time period.

In the Nelson-Siegel setup long-term yields are almost exclusively driven by the Level factor which is very persistent and has relatively lower volatility, so long-rate forecasts at longer horizons should be quite stable for capable term structure models. For long yields, the forecast performance of a term structure model should be very close to the random walk if the model successfully captures the high persistence as suggested by Duffee (2011*a*). In contrast, if short yields are anchored by policy rates, this implies short-horizon forecasts of short yields are accurate as long as monetary policy is predictable in the short run. However, without further information, forecasts of short yields at longer forecast horizons deteriorate substantially, given that the monetary policy target or market expectations may shift in the long run. In comparing our results to the existing literature, Diebold and Li (2006) shows the DL beats the RW for forecast horizons up to 12 months before 2000. But Diebold and Rudebusch (2013) and Altavilla, Giacomini and Ragusa (2014) imply NS can no longer beat a RW, which is in line with the increased persistence as we have shown previously. Our extended NS model consistently improves upon DL across all horizons and maturities, which is confirmed by relative MSFEs, predictive log-scores, and the Diebold-Mariano test. Moreover, at shorter horizons, and to some extent at longer horizons, our proposed method improves upon the RW.

Fluctuation Test In addition to the relative MSFE and the cumulative log predictive likelihoods, we formally test for forecasting ability over time in the presence of instabilities by implementing the Giacomini and Rossi (2010) one-sided Fluctuation test (Ft-test). Under the null hypothesis, the Ft-test gauges whether the local relative forecasting performance (based on Diebold and Mariano (1995) test) of the competing model and the benchmark model (say DL) is equal at each point in time. The alternative is that the competing model forecasts better than the DL. Hence, when the Ft-test statistic is above its critical value at the 10% level of significance, the competing model forecasts significantly better than the DL at that point in time. Otherwise, if the Ft-test is below its critical value, the evidence suggests the absence of forecasting ability of the model. To compute the test, we follow the recommendations in Giacomini and Rossi (2010) and set the size of moving local window to a third of the in-sample observations.[12]

The Ft-tests for the $h = 3, 6$ and 12 month forecast horizons are reported in Figures 4, 5 and 6. At the 3-month and 6-month horizons, both DMA and TVP(TVP-M) display significant forecasting ability except the very long maturity, as the Ft-test is usually above its critical value. At the twelve month forecast horizon, DMA and TVP(TVP-M) perform even better, as the Ft-test is consistently above the critical value for all maturities. Overall, DMA performs better than TVP or TVP-M as the test statistics are consistently higher, especially for shorter maturities. This is a further piece of evidence regarding the importance of systematically taking into account time-variation in parameters of term structure models, as well as model uncertainty.

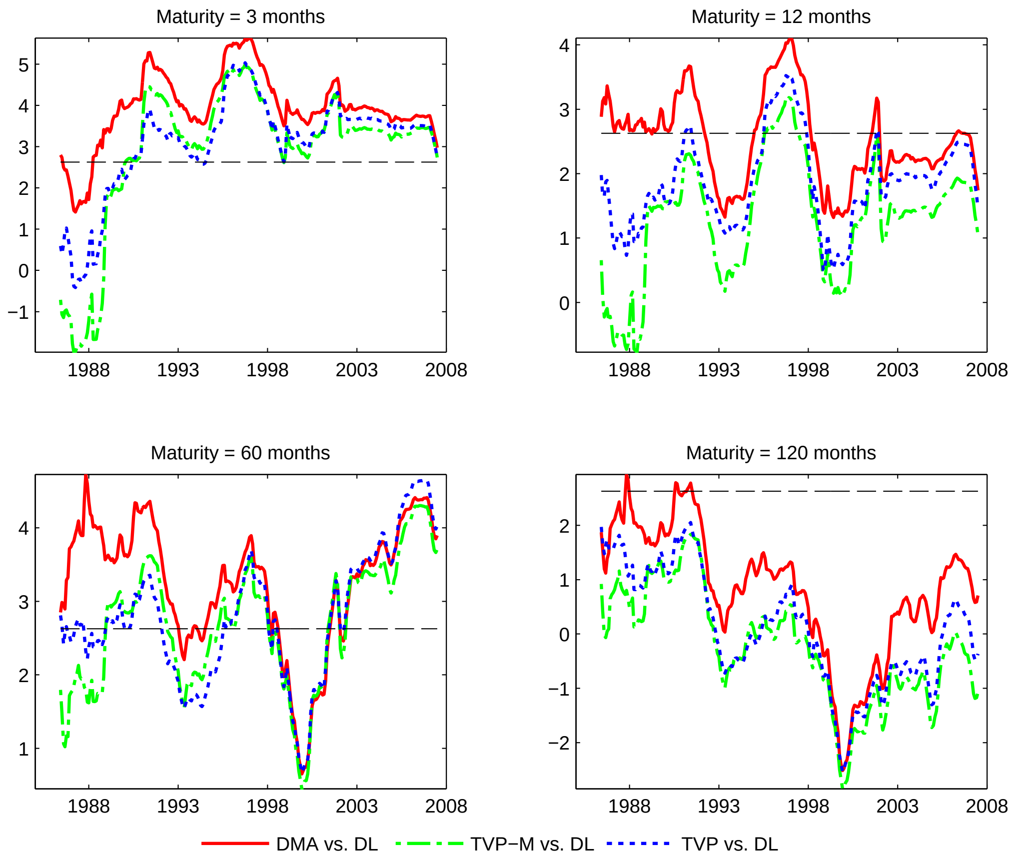
3.5 Predictive Gains and Sources of Instability

Since the pricing dynamics of most competing methods are fully characterized by the NS restrictions, we can conclude that the predictive gains in our preferred specification are purely from its time-series dynamics. Specifically these gains seem to be stemming from the fact that parameter and model uncertainty are fully taken into account. In this subsection our desire is to further highlight and decompose the various sources of these predictive gains. In the previous results we have found that time-varying parameter models improve over constant parameter versions, whether these include macro-finance information or not. Further improvements can be achieved by allowing the dimension of the model to switch over time. In order to pin down the exact sources of predictive gains we first conduct a statistical test to evaluate the out-of-sample forecasting performance. In Table 5 we show results of the Diebold and Mariano (1995) test, in order to evaluate the forecasting performance of DMA relative to DL and TVP-M. The Diebold and Mariano (1995) statistic is also used by Diebold and Li (2006) and Altavilla, Giacomini and Ragusa (2014). The relative MSFE is shown in Table 5 for forecasting horizons 1, 3, 6 and 12 months. These results indicate that the DMA clearly outperforms the DL and TVP-M, not only since MSFE are consistently lower but also the differences are statistically significant.

However, to what extent the constituent components of our DMA model contribute

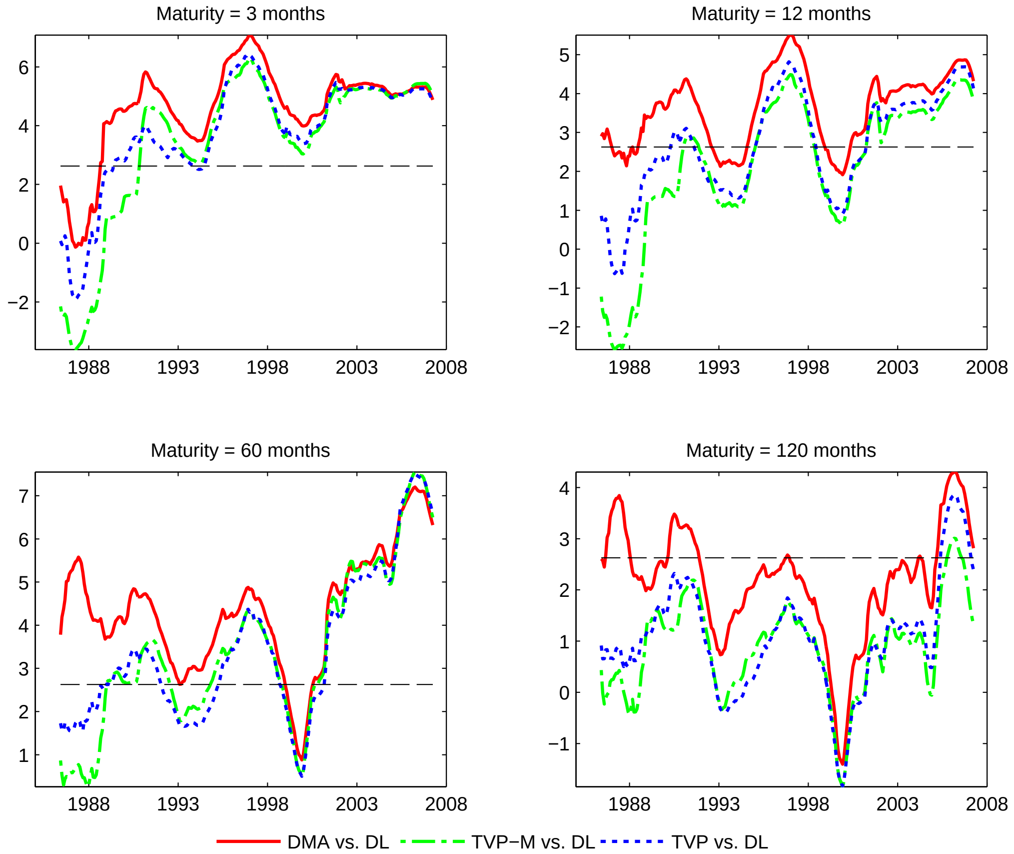
[12] According to Giacomini and Rossi (2010) Monte Carlo evidence, the Fluctuation test has good properties when implemented using a local moving window size that is a small, such as a third of the in-sample estimation window.

Figure 4: Fluctuation Test at 3-Month Forecasting Horizon



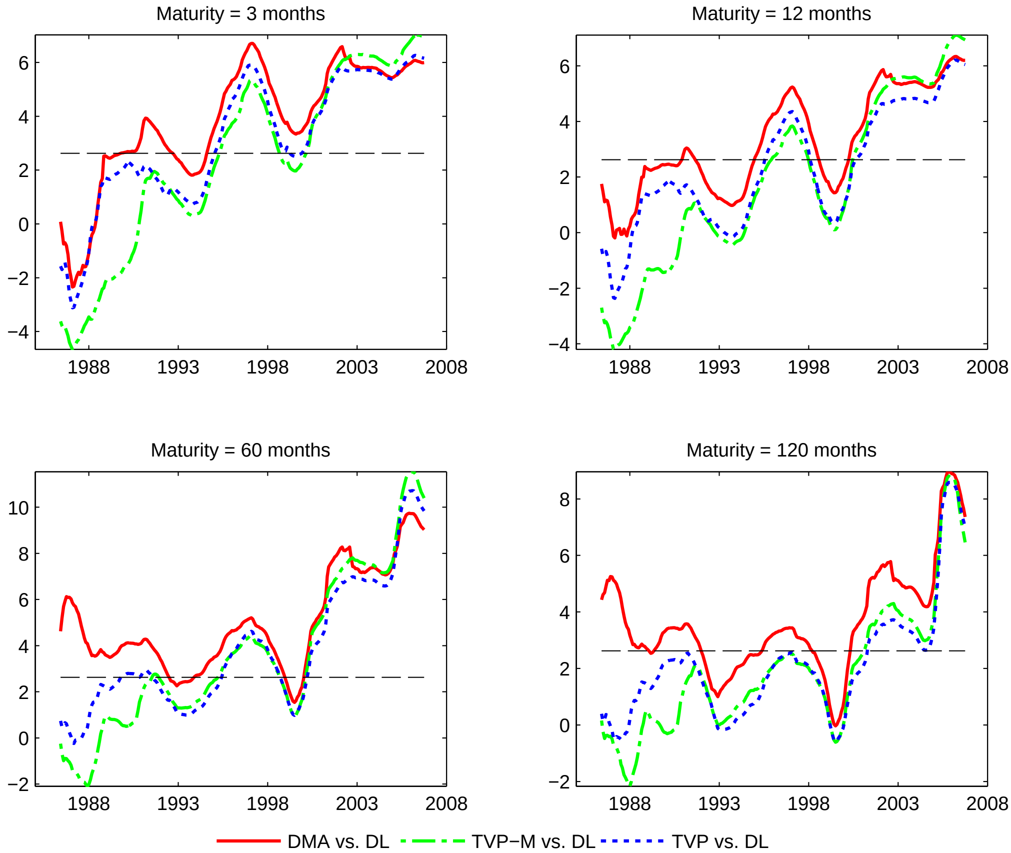
Notes: The Figure shows the Giacomini and Rossi's (2010) one-sided Fluctuation test (Ft-test) based on DMW-test for three competing models: DMA, TVP-M and TVP. The benchmark model is the Diebold and Li (2006) (DL). It also displays the one-sided Ft-test critical value at 10% level of significance. When the Ft-test statistic is above its critical value, we reject the null of equal local relative forecasting performance between the competing model and the DL, concluding that the method under consideration forecasts significantly better than the DL at that point in time. When the Ft-test is below its critical value, the evidence is consistent with the absence of forecasting ability of the method under consideration. The forecasting horizon is h = 3 months.

Figure 5: Fluctuation Test at 6-Month Forecasting Horizon



Notes: The Figure shows the Giacomini and Rossi's (2010) one-sided Fluctuation test (Ft-test) based on DMW-test for three competing models: DMA, TVP-M and TVP. The benchmark model is the Diebold and Li (2006) (DL). It also displays the one-sided Ft-test critical value at 10% level of significance. When the Ft-test statistic is above its critical value, we reject the null of equal local relative forecasting performance between the competing model and the DL, concluding that the method under consideration forecasts significantly better than the DL at that point in time. When the Ft-test is below its critical value, the evidence is consistent with the absence of forecasting ability of the method under consideration. The forecasting horizon is h = 6 months.

Figure 6: Fluctuation Test at 12-Month Forecasting Horizon



Notes: The Figure shows the Giacomini and Rossi's (2010) one-sided Fluctuation test (Ft-test) based on DMW-test for three competing models: DMA, TVP-M and TVP. The benchmark model is the Diebold and Li (2006) (DL). It also displays the one-sided Ft-test critical value at 10% level of significance. When the Ft-test statistic is above its critical value, we reject the null of equal local relative forecasting performance between the competing model and the DL, concluding that the method under consideration forecasts significantly better than the DL at that point in time. When the Ft-test is below its critical value, the evidence is consistent with the absence of forecasting ability of the method under consideration. The forecasting horizon is h = 12 months.

Table 5: MSFE from DMA Relative to Other Models

	DMA vs. DL				DMA vs. TVP-M			
Maturity	$h = 1$	$h = 3$	$h = 6$	$h = 12$	$h = 1$	$h = 3$	$h = 6$	$h = 12$
3	0.833***	0.693***	0.653***	0.843***	0.995	0.906*	0.860*	0.790**
6	0.766***	0.661***	0.655***	0.846***	0.901**	0.865**	0.845**	0.800**
12	1.045	0.824**	0.743***	0.866***	0.961**	0.914**	0.897*	0.847**
24	0.939**	0.788***	0.735***	0.849***	0.943***	0.925**	0.927*	0.890*
36	0.870***	0.774***	0.733***	0.845***	0.952***	0.945**	0.952	0.918
48	0.854***	0.777***	0.740***	0.842***	0.963**	0.959*	0.967	0.934
60	0.864***	0.793***	0.754***	0.844***	0.967**	0.965*	0.973	0.939
72	0.886***	0.815***	0.773***	0.846***	0.965**	0.965*	0.971	0.936
84	0.914***	0.842***	0.794***	0.849***	0.959**	0.960*	0.965	0.928
96	0.947**	0.872**	0.819**	0.851***	0.951**	0.953**	0.955	0.918
108	0.978*	0.904**	0.845**	0.854***	0.945***	0.944**	0.946	0.907
120	1.004	0.936	0.872*	0.860***	0.941***	0.937***	0.937	0.897

Notes: 1. This table reports MSFE-based statistics of DMA forecasts of bond yields at maturities ranging from 3 months to 120 months, relative to the forecasts of Diebold and Li (2006) (DL) or TVP-M (similar to Bianchi Mumtaz and Surico (2009)). The predictive period is between 1983:11 and 2013:11.
2. Statistical significance for the relative MSFE statistics is based on the p-value for the Diebold and Mariano (1995) statistic; the statistic corresponds to the test of the null hypothesis that the competing DMA model has equal expected square prediction error relative to the benchmark forecasting model (DL or TVP-M) against the alternative hypothesis that the competing forecasting model has a lower expected square prediction error than the benchmark forecasting model. *, ** and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

to the predictive gains is still unclear. To provide a straightforward answer to this question, we perform a simple but informative decomposition to pin down various sources of instability that might affect the out-of-sample forecasting performance of DMA. In particular, this decomposition quantifies exactly the relative contribution of time-varying parameters or model uncertainty to the prediction variance of bond yields, highlighting sources that potentially hinder the bond yield forecasts.

We use the law of total variance to decompose the variance of a random variable into its constituent parts. Following Dangl and Halling (2012), we begin with the decomposition of pricing factor forecasts with respect to different choices of forecasting model L:

$$Var(F^{NS}) = E_L\Big(Var(F^{NS}|L)\Big) + Var_L\Big(E(F^{NS}|L)\Big). \tag{3.1}$$

After some algebra and using the expressions detailed in previous sections, we have

$$\begin{aligned} Var(F^{NS}_{t+1}) \quad = \quad & \underbrace{\sum_i (\Sigma_t|L_i, D_t)P(L_i|D_t)}_{\text{Observational variance}} \\ & \underbrace{+\sum_i (X_t\Phi_{t|t-1}X'_t|L_i, D_t)P(L_i|\ D_t)}_{\text{Parameter uncertainty}} \\ & \underbrace{+\sum_i (\hat{F}^{NS}_{t+1,i} - \hat{F}^{NS}_{t+1})^2 P(L_i|D_t)}_{\text{Model uncertainty}}, \end{aligned}$$

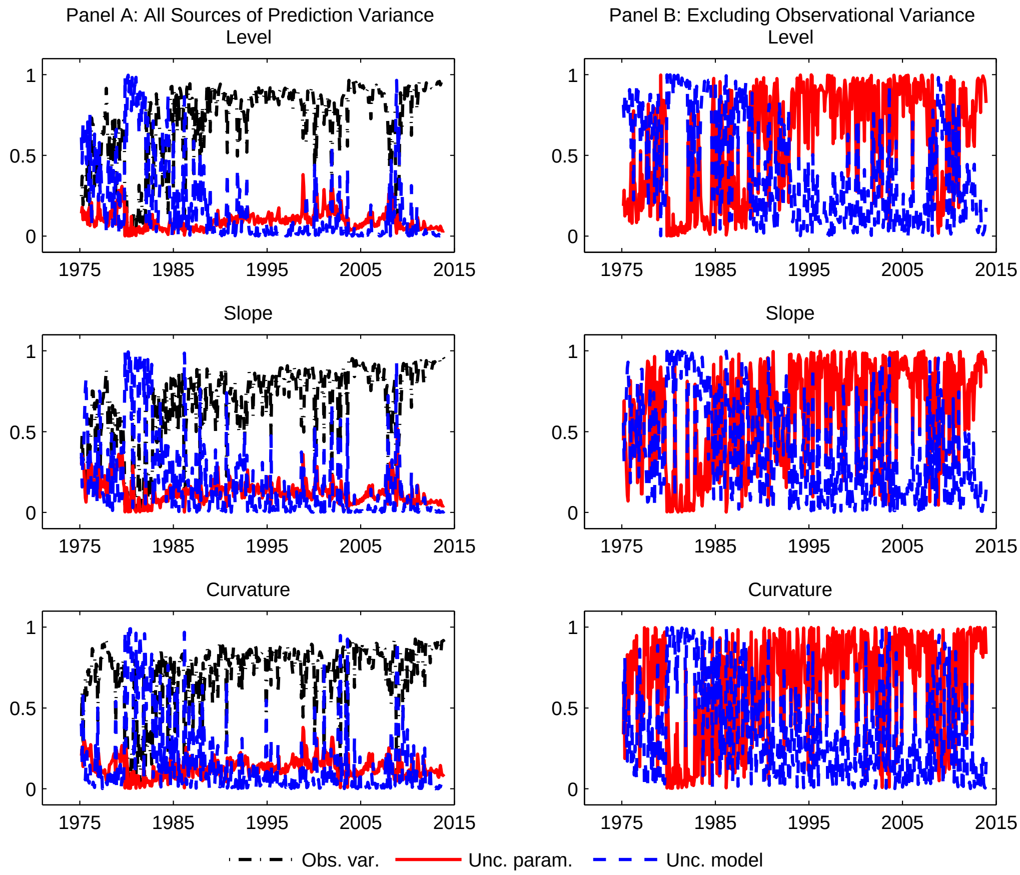
where Σ_t denotes the variance of the disturbance term in the observation equation, $\Phi_{t|t-1}$ denotes the unconditional variance of the time-t prior of the coefficient vector β_t, $\hat{F}^{NS}_{t+1,i}$ is the forecast conditional on L_i and $\hat{F}^{NS}_{t+1}$ is the weighted average over all candidate models.

The individual terms of Equation (3.2) state the sources of prediction uncertainty and have intuitive interpretations. The first term measures the expected observational variance, calculated over different choices of forecast model L. This term in fact captures the random fluctuations or risks in the pricing factors, relative to the predictable drift component. The second term is the expected variance from errors in the estimation of the coefficient vector, which can be interpreted as the source of estimation or parameter uncertainty. The third term captures model uncertainty, which can also be considered as the time variability of predictors used to generate forecasts.

Figure 7 displays the variance decomposition for three pricing factors, where Panel A shows that the predominant source of uncertainty is observational variance except during the 1980s. On average observational variance accounts for more than 70% of predictive variance for all three pricing factors. This is consistent with the findings of Dangl and

Halling (2012), as the asset prices frequently fluctuate randomly over their expected values. These fluctuations serve as the source of risk premia, and dominate the drift components in the term structure model. Therefore the fluctuations in fact contaminate the predictive power of term structure models, especially during the periods when pricing factors are highly persistent.[13]

Figure 7: Sources of Prediction Variance



Notes: This figure displays the decomposition of the prediction variance with respect to different sources. In Panel A, the prediction variance is split into observational variance (Obs. var.), variance caused by errors in the estimation of parameters (Unv. param.), variance caused by the model uncertainty (Unc. model). The illustration shows the relative weights of these components. Panel B masks out observational variance and shows relative weights of the remaining variance.

In Panel B of Figure 7, by excluding the observational variance we can focus upon the relative weights of the remaining sources of prediction uncertainty. The parameter uncertainty turns out to be a main source of prediction uncertainty after 1990, on average above 60%, which implies parameter instability is another crucial reason causing interest

[13]This does not at all mean term structure models are not useful. For instance, term structure models can reveal informative dynamics of market prices of risks and have reliable term premia of long-term bonds, which can not be offered by the random walk model.

rate unpredictability during that time. Therefore, a successful forecasting model should at least consider the feature of time-varying parameters. The model uncertainty is also important especially during certain periods. For example, model uncertainty rises steeply during the 1980s, accounting for more than 90% of total variance. This observation suggests without considering model uncertainty the predictive power of term structure models may be significantly compromised. The contribution of each source is time-varying but comparable for the three pricing factors. It highlights that *the consideration of both parameter uncertainty and model uncertainty regarding different choices of indicators* is a better way to produce more reliable interest rate forecasts.

3.6 Model-Implied Term Premia

In this section we set out a visual comparison of our term premium estimates.[14] We plot the DMA time-varying risk premia from 1985 for a medium-term bond (maturity 36 months) and a long-term bond (maturity 120 months) in Figure 8. For comparison, we also plot the model-implied term premia estimated from no-arbitrage term structure models proposed by Kim and Wright (2005), Wright (2011) and Bauer, Rudebusch and Wu (2014), all of which use full-sample data.[15] Note that we use monthly data when applying the methods of Wright (2011) and Bauer, Rudebusch and Wu (2014), and the physical VAR dynamics are all augmented with three macro variables as in our medium-size model in this paper. As a result, the term premium measures from these two methods are similar, which helps resolve a discrepancy indicated in Bauer, Rudebusch and Wu (2014).

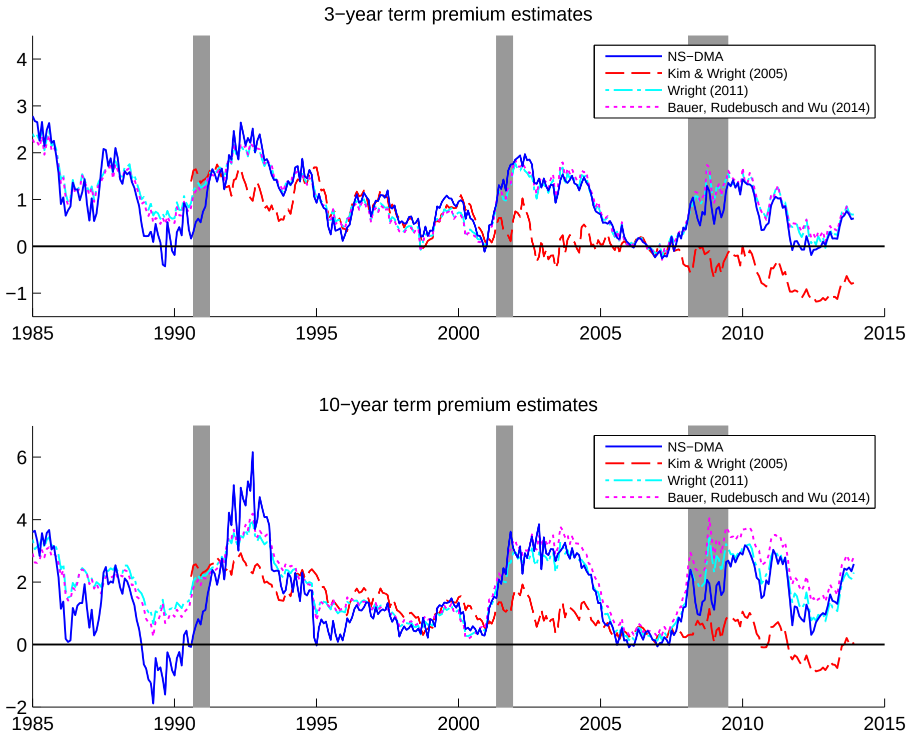
It is worth emphasizing that DMA captures plausible term premia using conditional information only. As it is shown in the upper panel of Figure 8, the 36-month term premium estimates of DMA are highly consistent with the full-sample estimates of Wright (2011) and Bauer, Rudebusch and Wu (2014). In general all term premia estimates display countercyclical behavior, as they rise in and around US recessions, except from the estimates of Kim and Wright (2005). The difference between the estimates of Kim and Wright (2005) (KW) and other models is due to the estimated expectation of future short rate. As indicated in Christensen and Rudebusch (2012), in the KW measure the factor dynamics tend to display distinctively different persistence from other measures

[14] A more thorough discussion about term premia and risk-neutral rates, as well as the underlying drivers, can be found in Appendix C.5.

[15] The comparison between the DMA term premia and recursively estimated term premia from dynamic Nelson-Siegel is shown in Appendix C.5.1. The DMA approach seems to be more robust than the constant-parameter dynamic Nelson-Siegel model, as the dynamic Nelson-Siegel model proposed by Diebold and Li (2006) tends to overestimate the future short rates and hence underestimate the term premia.

because of the augmentation of survey data. According to the observations here, the expected future short rates from the survey tend to be very stable, so the KW term premia has a relatively lower variance and may display an acyclical pattern.

Figure 8: Time-Varying Term Premia of 36-and 120-Month Bonds



Notes:
1. The top panel is the 36-month term premia and the bottom is the 120-month term premia. The EH consistent 36- and 120-month bond yields are estimated using Eq. (C.1); we then calculate the term premia using Eq. (C.2).
2. In addition to DMA, we use the whole sample to separately estimate two types of term premia employing the methods proposed by Wright (2011) and Bauer, Rudebusch and Wu (2014). The Kim and Wright (2005) term premia can be obtained from the Federal Reserve Board website.
3. Shaded areas are recession periods based on the NBER Recession Indicators. The unit is percentage.

Among all measures considered, the DMA term premia seem to be more sensitive to changes in the economic environment, which can be seen more clearly from the lower panel of Figure 8 of the long-term term premia. The reason is that expectations of the future short rates move flexibly in DMA and, hence, the 10-year term premia presents a more significant countercyclical pattern. For example, the short rate was continuously decreasing from 1990 to 1993 so the expectation of future short rates was also decreasing. Long rates were relatively stable in contrast, which leads to the increasing risk premia that peaked in 1993.

4 Conclusion

The Nelson-Siegel approach of yield curve modeling has been extended by Diebold and Li (2006), Diebold, Rudebusch and Aruoba (2006) and Bianchi, Mumtaz and Surico (2009). We further extend this literature by proposing a Dynamic Model Averaging (DMA) approach with the consideration of a large set of macro-finance factors, in order to better characterize the nonlinear dynamics of yield factors and further improve yield forecasts. We explore the time-varying predictive power of term structure models and unfold the time variation of sources that significantly drive the predictive variance of the yield curve. The DMA method significantly improves the predictive accuracy for bond yields, short rates in particular, and successfully identifies plausible dynamics of term premia in real time.

Specifying the interactions between the yield factors and macro-finance information using time varying parameters, stochastic volatility, and switching information set, causes additional econometric challenges in terms of tractability of estimation. Such challenges are addressed here by bringing in a fast and simple estimation technique. The proposed yield curve specification is robust to various sources of structural instabilities, and it is highly consistent with the theoretical and empirical findings in the previous yield curve literature. Future research could employ a one-step estimation approach to provide forecasts with higher accuracy, in which case a trade-off should be made between predictive accuracy and estimation efficiency. Finally, disentangling the real part of the term structure from inflation expectations is meaningful and desirable, but it is beyond the scope of this paper and can also be considered in further work.

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Data Appendix

Table 6: List of Yields and Macro-Finance Variables

Series ID	**Description**
TB	3- and 6-month Treasury Bills (Secondary Market Rate) [1]
ZCY	Smoothed Zero-coupon Yield from Gürkaynak, Sack and Wright (2007) [1]
IND	Industrial Production Index [5]
CPI	Consumer Price Index for All Urban Consumers: All Items Less Food & Energy [5]
FED	Effective Federal Funds Rate, End of Month [1]
SP	S&P 500 Stock Price Index, End of Month [5]
TCU	Capacity Utilization: Total Industry [1]
M1	M1 Money Stock [5]
TCC	Total Consumer Credit Owned and Securitized, Outstanding (End of Month) [5]
LL	Loans and Leases in Bank Credit, All Commercial Banks [5]
DOE	DOE Imported Crude Oil Refinery Acquisition Cost [5]
MSP	Median Sales Price for New Houses Sold in the United States [5]
TWX	Trade Weighted U.S. Dollar Index: Major Currencies [1]
ED	Eurodollar Spread: 3m Eurodollar Deposit Rate - 3m Treasury Bill Rate [1]
WIL	Wilshire 5000 Total Market Index [5]
DYS	Default Yield Spread: Moodys BAA-AAA [1]
NFCI	National Financial Conditions Index [1]

Notes:
1. In square brackets [·] we have a code for data transformations used in this data set: [1] means original series is used; [5] means log first-order difference is used to detrend and ensure stationarity. The series are seasonally adjusted when appropriate.
2. Data are obtained from St. Louis Federal Reserve Economic Data [http://research.stlouisfed.org/], spanning from Nov. 1971 to Nov. 2013. The smoothed zero-coupon yield is available on the Federal Reserve Board website [http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html/].
3. National Financial Conditions Index, provided by the Chicago Fed, is available on the website [http://www.chicagofed.org/webpages/publications/nfci/].
4. The small-size VAR model includes no macro variables. The medium-size VAR model includes only three macro variables: IND, CPI and FED. The large-size VAR model uses all the macro and financial variables in this data list.

Online Appendix for "Forecasting the Term Structure of Government Bond Yields in Unstable Environments"

Joseph P. Byrne, Shuo Cao and Dimitris Korobilis

Sunday 1st October, 2017

Appendix A Econometric Methods

A.1 Bayesian Kalman Filter with Forgetting Factor

We conduct the Kalman filter estimation for the state space model with Eq. (2.3) and Eq. (2.4):

$$z_t = X_t \beta_t + v_t,$$

$$\beta_{t+1} = \beta_t + \mu_t,$$

where z_t is an $n \times 1$ vector of variables, $X_t = I_n \otimes \left[z'_{t-1}, ..., z'_{t-p}\right]'$, β_t are VAR coefficients, $v_t \sim N\left(0, \Sigma_t\right)$ with Σ_t an $n \times n$ covariance matrix, and $\mu_t \sim N\left(0, Q_t\right)$.

Given that all the data from time 1 to t denoted as D_t, the Bayesian solution to updating about the coefficients β_t takes the form

$$\begin{aligned} p\left(\beta_t | D_t\right) &\propto \mathbf{L}\left(\beta_t; z_t\right) p\left(\beta_t | D_{t-1}\right), \\ p\left(\beta_t | D_{t-1}\right) &= \int_{\wp} p\left(\beta_t | D_{t-1}, \beta_{t-1}\right) p\left(\beta_{t-1} | D_{t-1}\right) d\beta_{t-1}, \end{aligned}$$

where $\wp$ is the support of β_{t-1}. The solution to this problem can be defined using a Bayesian generalization of the typical Kalman filter recursions. Given an initial condition $\beta_0 \sim N\left(m_0, \Phi_0\right)$ we can define (cf. West and Harrison (1997))[16]:

[16] For a parameter θ we use the notation $\theta_{t|s}$ to denote the value of parameter θ_t given data up to time s (i.e. $D_{1:s}$) for $s > t$ or $s < t$. For the special case where $s = t$, I use the notation $\theta_{t|t} = \theta_t$

1. Posterior at time $t-1$

$$\beta_{t-1}|D_{t-1} \sim N\left(m_{t-1}, \Phi_{t-1}\right),$$

2. Prior at time t

$$\beta_t|D_{t-1} \sim N\left(m_{t|t-1}, \Phi_{t|t-1}\right),$$

where $m_{t|t-1} = m_{t-1}$ and $\Phi_{t|t-1} = \Phi_{t-1} + Q_t$.

3. Posterior at time t

$$\beta_t|D_t \sim N\left(m_t, \Phi_t\right), \tag{A.1}$$

where $m_t = m_{t|t-1} + \Phi_{t|t-1} X_t' (V_t^{-1})' \widetilde{v}_t$ and $\Phi_t = \Phi_{t|t-1} - \Phi_{t|t-1} X_t' (V_t^{-1})' X_t \Phi_{t|t-1}'$, with $\widetilde{v}_t = z_t - X_t m_{t|t-1}$ the prediction error and $V_t = X_t \Phi_{t|t-1} X_t' + \Sigma_t$ its covariance matrix.

Following the discussion above, we need to find estimates for Σ_t and Q_t in the formulas above. We define the time t prior for Σ_t to be

$$\Sigma_t|D_{t-1} \sim iW\left(S_{t-1}, \delta n_{t-1}\right), \tag{A.2}$$

while the posterior takes the form

$$\Sigma_t|D_t \sim iW\left(S_t, n_t\right),$$

where $n_t = \delta n_{t-1} + 1$ and $S_t = \delta S_{t-1} + n_t^{-1}\left(S_{t-1}^{0.5} V_{t-1}^{-0.5} \widetilde{v}_{t|t-1} \widetilde{v}_{t|t-1}' V_{t-1}^{-0.5} S_{t-1}^{0.5}\right)$. In this formulation, v_t is replaced with the one-step ahead prediction error $\widetilde{v}_{t|t-1} = z_t - m_{t|t-1} X_t$. The estimate for Σ_t is approximately equivalent numerically to the Exponentially Weighted Moving Average (EWMA) filter $\hat{\Sigma}_t = \delta \hat{\Sigma}_{t-1} + (1-\delta)\, v_t v_t'$. The parameter δ is the decay factor, where for $0 < \delta < 1$. In fact, Koop and Korobilis (2013) apply such a scheme directly to the covariance matrix Σ_t, which results in a point estimate. In this case by applying variance discounting methods to the scale matrix S_t, we are able to approximate the full posterior distribution of Σ_t.

Regarding Q_t, we use the forgetting factor approach in Koop and Korobilis (2013); see also West and Harrison (1997) for a similar discounting approach. In this case Q_t is set to be proportionate to the filtered covariance $\Phi_{t-1} = cov\left(\beta_{t-1}|D_{t-1}\right)$ and takes the following form

$$Q_t = \left(\Lambda^{-1} - 1\right) \Phi_{t-1}, \tag{A.3}$$

for a given forgetting factor Λ.

The brief interpretation of forgetting factors is that they control how much 'recent past' information will be used. With the exponential decay for the forgetting factors, if it takes a value of 0.99, the information 24 periods ago (two years for monthly data) receives around 80% as much weight as the information of last period. If forgetting factor takes 0.95, then forecast performance 24 periods ago receives only about 30% as much weight. The similar implication holds for the decay factor.

A.2 Probabilities for Dynamic Selection and Averaging

To obtain the desire probabilities for dynamic selection or averaging, we need updating at each point in time. In papers such as Raftery, Kárnỳ and Ettler (2010) or Koop and Korobilis (2012) the models are TVP regressions with different sets of explanatory variables. The analogous result of the model prediction equation, when doing DMA or DPS, is

$$p(\beta_{t-1}|D_{t-1}) = \sum_{i=1}^{K} p(\beta_{t-1}^{(i)}|L_{t-1} = i, D_{t-1})\mathbf{Pr}(L_{t-1} = i|D_{t-1}), \tag{A.4}$$

where $L_{t-1} = i$ means the i_{th} model[17] is selected and $p(\beta_{t-1}^{(i)}|L_{t-1} = i, D_{t-1})$ is given by the Kalman filter (Eq. A.1). To simplify notation, let $\pi_{t|s}^{(i)} = \mathbf{Pr}(L_t = i|D_s)$.

The model updating equation is

$$\pi_{t|t}^{(i)} = \frac{\pi_{t|t-1}^{(i)} p^{(i)}(z_t|D_{t-1})}{\sum_{l=1}^{K} \pi_{t|t-1}^{(l)} p^{(l)}(z_t|D_{t-1})}, \tag{A.5}$$

where $p^{(i)}(z_t|D_{t-1})$ is the predictive likelihood. Raftery, Kárnỳ and Ettler (2010) used an empirically sensible simplification that

$$\pi_{t|t-1}^{(i)} = \frac{\left(\pi_{t-1|t-1}^{(i)}\right)^{\alpha}}{\sum_{l=1}^{K} \left(\pi_{t-1|t-1}^{(l)}\right)^{\alpha}}, \tag{A.6}$$

where $0 < \alpha \leq 1$. A forgetting factor is also employed here, of which the meaning is

[17] For example, the i_{th} model in Dynamic Model Selection/Averaging, or the i_{th} candidate γ value in Dynamic Prior Selection.

discussed in the last section. The huge advantage of using the forgetting factor α is that it does not require an MCMC algorithm to draw transitions between models or a simulation algorithm over model space.

When proceeding with Dynamic Model Selection, the model with the highest probability is the best model we would like to select. Alternatively, we can conduct Dynamic Model Averaging, which average the predictions of all models with respective probabilities.

In our Bayesian empirical analysis of the factor dynamics, we can begin by selecting the prior parameter γ with Dynamic Prior Selection (DPS), then the best prior will be selected for each of the three VAR models. Next we update the model weights with Dynamic Model Averaging (DMA), and finally we update on the parameters using a Bayesian Kalman filter.

As the degree of the shrinkage potentially affects the forecasting results, we allow for a wide grid for the reasonable candidate values of γ: $[10^{-10}, 10^{-6}, 0.001, 0.005, 0.01, 0.05, 0.1]$. The best prior γ can be selected dynamically according to the forecasting accuracy each value in the grid generates. That is, following Koop and Korobilis (2013) we select γ for each of the three models $M^{(i)} = 1, 2, 3$ and for each time period. In the Dynamic Prior Selection step, we find that the best prior γ value in Eq. (A.7) is stable, i.e. fixed at 0.1, for all three VAR models, given the associated forgetting factor fixed. The associated forgetting factor controls the persistence of probabilities, and the results do not change substantially as long as it is sufficiently large: the best γ values is relatively stable for all three sizes of models when the forgetting factor is larger than 0.90. The evidence concludes that a relatively flexible and consistent prior can generate accurate yield forecasts. For simplicity and tractability, we fix the value at $\gamma = 0.1$ such that the coefficients tend to be more flexible. In fact, we find that holding γ constant at 0.1 does not affect our results, as the uncertainty of posterior predictive densities seems trivial with a sufficiently large sample as in our paper.

A.3 Prior Specification

We define a Minnesota prior for our VAR, which provides shrinkage that could prevent overfitting of our larger models. This prior is of the form $\beta_0 \sim N\left(\mathbf{0}, V^{MIN}\right)$ where V^{MIN} is a diagonal matrix with element V_i^{MIN} given by

$$V_i^{MIN} = \begin{cases} \gamma/r^2, & \text{for coefficients on lag } r \text{ where } r = 1, ..., p \\ \underline{\alpha}, & \text{for the intercept} \end{cases}, \tag{A.7}$$

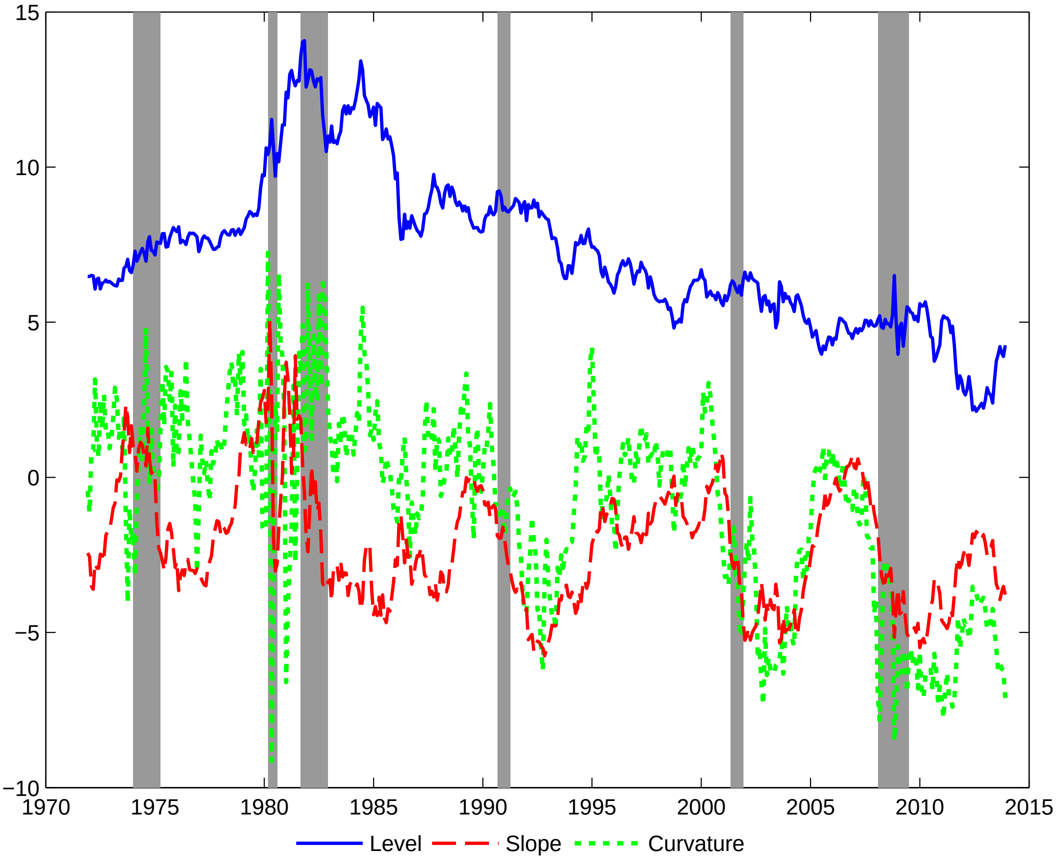
where p is the lag length determined by the Bayesian information criterion (BIC) using a training sample, and $\underline{\alpha} = 1$. The prior covariance matrix controls the degree of shrinkage on the VAR coefficients. To be more specific, the larger the prior parameter γ is, the more flexible the estimated coefficients are and, hence, the lower the intensity of shrinkage towards zero.[18]

[18]However, we test the robustness of prior parameters by implementing a dynamic model selection procedure with a grid of reasonable values, and the results are available upon request.

Appendix B Interpretation of Factor Dynamics

We illustrate the factor dynamics in this section and try to shed light on the economic implications of the latent factors. The extracted NS factors are shown in Figure 9. The Level factor has a downward trend since the early 1980s. The Level factor also has greater persistence compared with the other more volatile factors. The downward trend in the Level factor is consistent with the descriptive statistics in Table 1 and the results of Koopman, Mallee and Van der Wel (2010). The latter suggest a strong link between the Level factor and (expected) inflation, as they share high persistence. Evans and Marshall (2007) also indicate that there is a link between the level of yields and inflation with structural VAR evidence. In particular, the Level factor fell significantly after the financial crisis, which may indicate that the market had low inflation expectations. The Level factor rises in 2013, potentially associated with rising inflation and the impact of the Fed's Quantitative Easing (QE) pattern.

Figure 9: Nelson-Siegel Factor Dynamics



Notes: The graph shows the Nelson-Siegel Level, Slope and Curvature factors, which are drawn from Eq. (2.1). The shaded areas are recession periods according to the NBER Recession Indicators.

The Slope factor tends to fall sharply within recession periods, as indicated in Figure 9 by the shaded areas. Therefore, this factor could be closely related to real activity. The Slope factor is often considered as a proxy for the term spread, see Diebold, Rudebusch and Aruoba (2006). It can also be considered as a proxy for the stance of monetary policy, as the short end is influenced by policy rates.

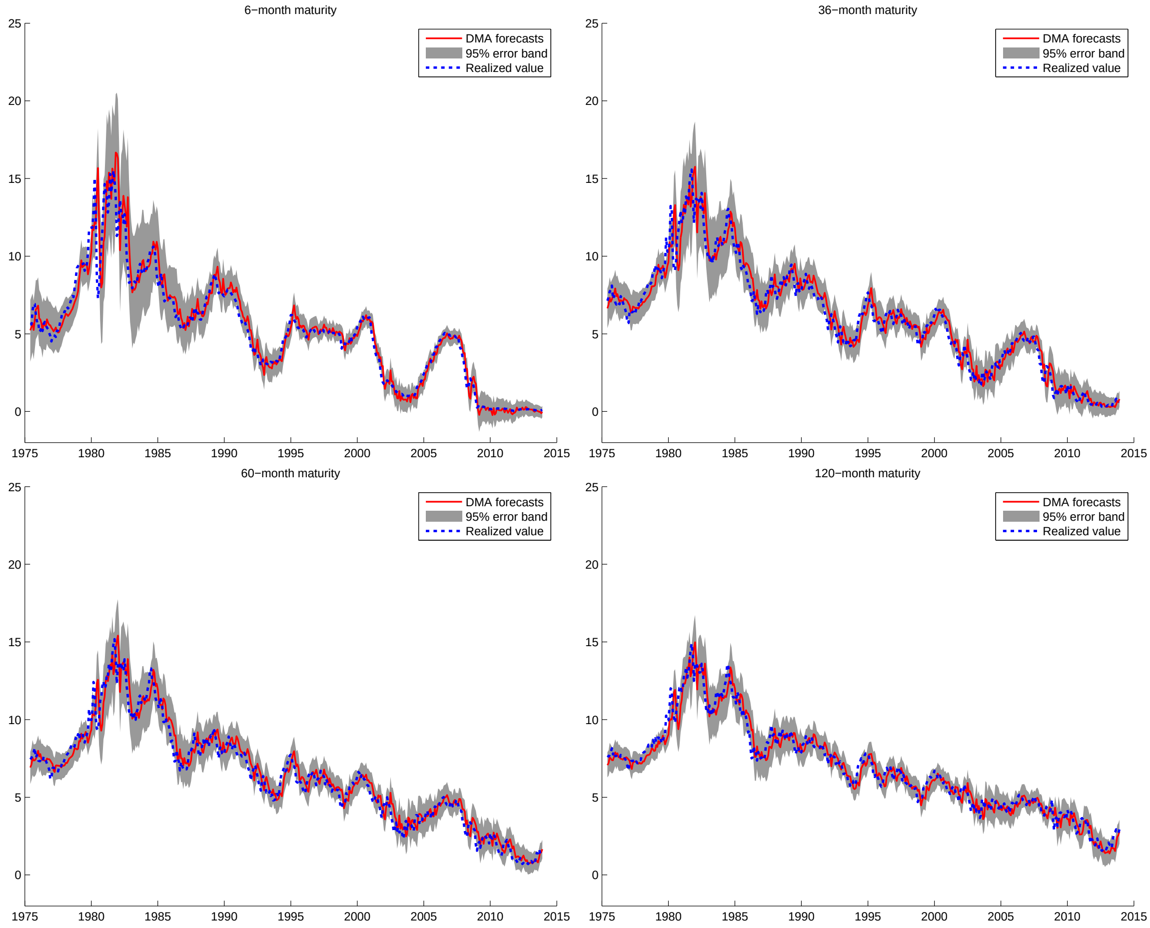
Lastly, the Curvature factor is harder to interpret and Diebold and Rudebusch (2013) indicate that this factor is less important than the other factors. On one hand, Litterman, Scheinkman and Weiss (1991) link the Curvature factor to the volatility of the level factor, via the argument of yield curve convexity, which can also be seen in Neftci (2004).[19] On the other hand, medium rates can be linked to expect short rates in the future, and therefore should be linked to current and expected future policies, which may potentially contain useful macro information missing in the first two factors.

[19] Generally, higher convexity means higher price-volatility or risk, and vice versa.

Appendix C Additional Results

C.1 Forecasting Results

Figure 10: DMA Forecasts of Yields



Notes: These are 3 months ahead forecasts (95% error band) for yields against realized values with maturities 6, 36, 60 and 120 months, from early 1975 to late 2013. The forecasts are two-step forecasting using DMA, which can be summarized by Eq. (2.1), (2.3) and (2.4).

Table 7: Relative MAFE Performance of Term Structure Models

FH	h=1								h=3							
Maturity	**DMA**	**DMS**	**TVP**	**TVPM**	**DL**	**DLR10**	**DLM**	**DLSW**	**DMA**	**DMS**	**TVP**	**TVPM**	**DL**	**DLR10**	**DLM**	**DLSW**
3	0.851†	0.890	0.866	0.858	0.968	1.050	0.979	1.275	0.884†	0.948	0.939	0.926	1.077	1.301	1.092	1.527
6	0.976†	1.036	1.025	1.007	1.101	1.218	1.120	1.462	0.957†	1.019	1.021	1.028	1.189	1.391	1.206	1.621
12	1.023†	1.047	1.021	1.039	0.984	0.963	1.013	1.324	0.987†	1.031	1.017	1.041	1.059	1.128	1.067	1.408
24	1.028†	1.053	1.041	1.047	1.047	1.059	1.054	1.272	1.006†	1.034	1.039	1.053	1.122	1.201	1.127	1.289
36	0.990†	1.010	1.011	1.015	1.062	1.142	1.073	1.131	1.013†	1.033	1.047	1.044	1.145	1.242	1.151	1.194
48	0.973†	0.987	1.000	0.992	1.065	1.164	1.072	1.049	1.012†	1.020	1.041	1.031	1.145	1.255	1.149	1.128
60	0.976†	0.986	1.000	0.988	1.061	1.167	1.066	1.023	1.011†	1.013	1.035	1.028	1.130	1.254	1.134	1.096
72	0.984†	0.995	1.006	1.000	1.051	1.160	1.059	1.030	1.008†	1.011	1.031	1.028	1.110	1.243	1.113	1.090
84	0.993†	1.006	1.014	1.013	1.041	1.140	1.047	1.050	1.003†	1.011	1.027	1.028	1.082	1.218	1.091	1.094
96	0.997†	1.014	1.020	1.025	1.030	1.110	1.038	1.071	0.998†	1.011	1.023	1.028	1.059	1.186	1.068	1.108
108	1.004†	1.023	1.027	1.037	1.015	1.069	1.029	1.091	0.994†	1.014	1.021	1.031	1.039	1.150	1.048	1.126
120	1.010†	1.028	1.034	1.044	1.004	1.028	1.019	1.110	0.988†	1.013	1.017	1.030	1.018	1.109	1.029	1.139
Mean	0.988†	1.008	1.008	1.009	1.036	1.104	1.047	1.143	0.991†	1.015	1.023	1.027	1.098	1.220	1.106	1.224

Notes: 1. This table shows 1-month and 3-month ahead forecasts of bond yields with maturities ranging from 3 months to 120 months. The predictive duration is from 1983:11 to 2013:11.

2. The MAFE-based statistics relative to the RW are reported. The dagger (†) indicates, in terms of the sum of predictive log-likelihood, the model has the preferred value among all models at certain maturities (or in total), see Geweke and Amisano (2010).

3. In this table, we use following abbreviations. **MAFE**: Mean Absolute Forecasting Error; **Mean**: Averaged MSFE across all sample maturities. **DMA** (Dynamic Model Averaging) averages all the models with probabilities in each step, while **DMS** (Dynamic Model Selection) chooses the best model with the highest probability at any point in time. **TVP-M**: a time-varying parameter model with three macro variables: fund rate, inflation and industrial production, similar to Bianchi Mumtaz and Surico (2009) but estimated with a fast algorithm without the need of MCMC; **DL**: Diebold and Li (2006) model, i.e. constant coefficient Vector Autoregressive model with recursive (expanding) estimations; **DL-R10**: Diebold and Li (2006) estimates based 10-year rolling windows; **TVP**: a time-varying parameter model without macro information; **DL-M**: factor dynamics in Diebold and Li (2006) are augmented with three macro variables: fund rate, inflation and industrial production, using recursive estimations; **DL-SW**: factor dynamics in Diebold and Li (2006) are augmented with three principal components (see Stock and Watson (2002)) of our macro-finance data, using recursive estimations; **RW**: Random Walk.

Table 8: Relative MAFE Performance of Term Structure Models (Continued)

FH	h=6								h=12							
Maturity	**DMA**	**DMS**	**TVP**	**TVPM**	**DL**	**DLR10**	**DLM**	**DLSW**	**DMA**	**DMS**	**TVP**	**TVPM**	**DL**	**DLR10**	**DLM**	**DLSW**
3	$0.951^{\dagger}$	1.009	1.020	1.003	1.206	1.360	1.249	1.522	$1.028^{\dagger}$	1.074	1.078	1.127	1.220	1.321	1.296	1.374
6	$0.999^{\dagger}$	1.055	1.068	1.061	1.250	1.411	1.290	1.581	$1.054^{\dagger}$	1.098	1.101	1.141	1.245	1.365	1.313	1.438
12	$0.995^{\dagger}$	1.052	1.044	1.047	1.131	1.236	1.148	1.445	$1.031^{\dagger}$	1.085	1.073	1.082	1.190	1.308	1.229	1.389
24	$0.999^{\dagger}$	1.040	1.036	1.033	1.165	1.263	1.176	1.348	$1.055^{\dagger}$	1.095	1.093	1.068	1.242	1.392	1.274	1.363
36	$1.017^{\dagger}$	1.040	1.046	1.027	1.187	1.297	1.197	1.290	$1.063^{\dagger}$	1.092	1.097	1.054	1.258	1.433	1.287	1.313
48	$1.022^{\dagger}$	1.036	1.046	1.024	1.187	1.305	1.194	1.243	$1.061^{\dagger}$	1.086	1.089	1.046	1.259	1.448	1.287	1.291
60	$1.017^{\dagger}$	1.029	1.040	1.021	1.174	1.296	1.177	1.212	$1.054^{\dagger}$	1.079	1.080	1.039	1.250	1.448	1.276	1.287
72	$1.008^{\dagger}$	1.022	1.033	1.019	1.153	1.277	1.155	1.199	$1.048^{\dagger}$	1.073	1.070	1.034	1.239	1.442	1.266	1.295
84	$0.999^{\dagger}$	1.015	1.025	1.016	1.127	1.250	1.128	1.196	$1.041^{\dagger}$	1.069	1.062	1.029	1.227	1.431	1.254	1.313
96	$0.993^{\dagger}$	1.011	1.020	1.014	1.102	1.227	1.104	1.205	$1.032^{\dagger}$	1.066	1.054	1.025	1.212	1.415	1.239	1.336
108	$0.985^{\dagger}$	1.008	1.015	1.012	1.079	1.205	1.080	1.215	$1.019^{\dagger}$	1.061	1.045	1.020	1.193	1.393	1.219	1.357
120	$0.979^{\dagger}$	1.007	1.012	1.012	1.061	1.183	1.061	1.228	$1.007^{\dagger}$	1.056	1.038	1.016	1.171	1.369	1.198	1.378
Mean	$0.998^{\dagger}$	1.028	1.034	1.025	1.153	1.276	1.164	1.306	$1.042^{\dagger}$	1.079	1.075	1.060	1.227	1.395	1.264	1.346

Notes: 1. This table shows six-month and twelve-month ahead forecasts of bond yields with maturities ranging from 3 months to 120 months. The predictive duration is from 1983 to the end of 2013.
2. The MAFE-based statistics relative to the RW are reported. The dagger (†) indicates, in terms of the sum of predictive log-likelihood, the model has the preferred value among all models at certain maturities (or in total), see Geweke and Amisano (2010).
3. In this table, we use following abbreviations. **MAFE**: Mean Absolute Forecasting Error; **Mean**: Averaged MAFE across all sample maturities. **DMA** (Dynamic Model Averaging) averages all the models with probabilities in each step, while **DMS** (Dynamic Model Selection) chooses the best model with the highest probability at any point in time. **TVP-M**: a time-varying parameter model with three macro variables: fund rate, inflation and industrial production, similar to Bianchi Mumtaz and Surico (2009) but estimated with a fast algorithm without the need of MCMC; **DL**: Diebold and Li (2006) model, i.e. constant coefficient Vector Autoregressive model with recursive (expanding) estimations; **DL-R10**: Diebold and Li (2006) estimates based 10-year rolling windows; **TVP**: a time-varying parameter model without macro information; **DL-M**: factor dynamics in Diebold and Li (2006) are augmented with three macro variables: fund rate, inflation and industrial production, using recursive estimations; **DL-SW**: factor dynamics in Diebold and Li (2006) are augmented with three principal components (see Stock and Watson (2002)) of our macro-finance data, using recursive estimations; **RW**: Random Walk.

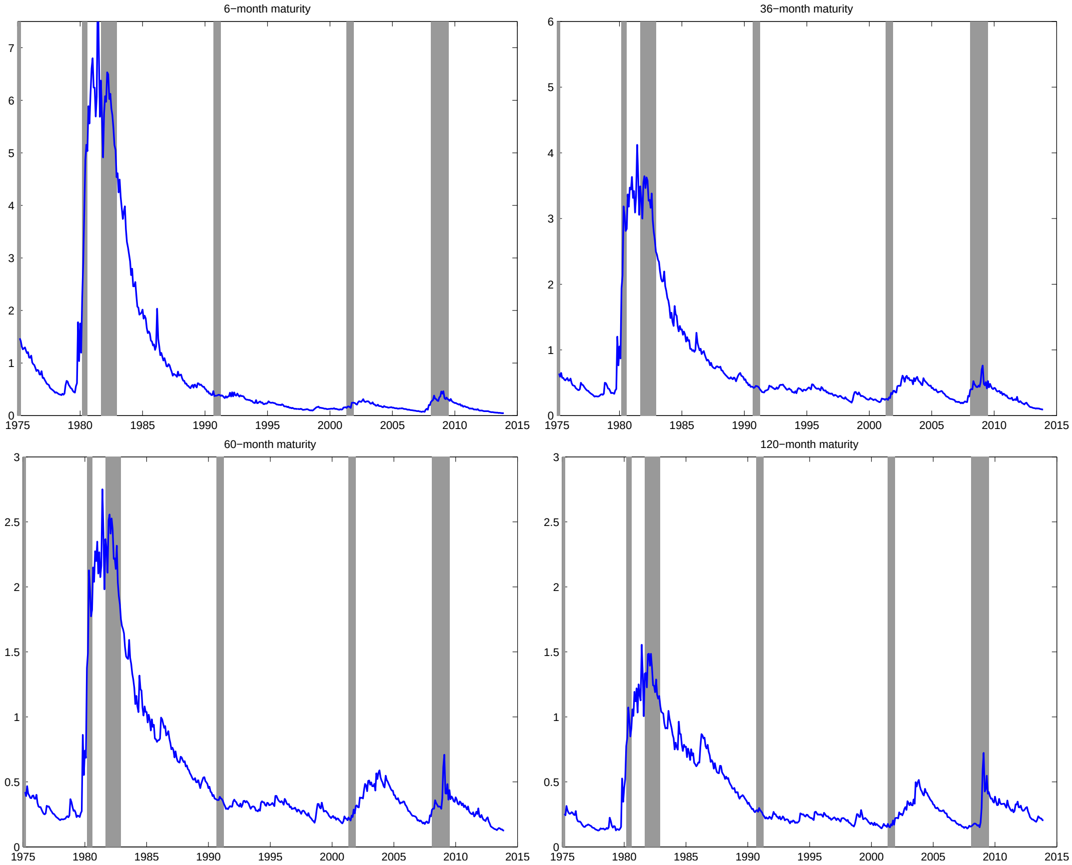
C.2 Time-Varying Volatility

It has been indicated by Bianchi, Mumtaz and Surico (2009) that homoskedasticity is a frequent and potentially inappropriate assumption in much of the macro-finance literature. Cieslak and Povala (2016) show that stochastic volatility can have a non-trivial influence on the conditional distribution of interest rates. Piazzesi (2010) indicates that fat tails in the distribution of bond factors can be modeled by specifying an appropriate time-varying volatility. The DMA model allows for heteroskedastic variances and this assumption is crucial for its good density forecast performance; this evidence is consistent with Hautsch and Yang (2012).

The DMA not only provides more sensible results in terms of density forecasts, but also captures the desirable evolutionary dynamics of the economic structure. Figure 11 shows the time-varying second moments of 3 month ahead forecasts from the DMA model. The figure displays distinct time variation in the evolution of volatility. The stable decline of volatility before the financial crisis matches the conclusions of Bianchi, Mumtaz and Surico (2009), who refer to this empirical result as the 'Great Moderation' of the term structure. We observe that yields with longer maturities have lower volatilities. This feature is counter-intuitive. Theoretically, long yields are mainly driven by three components: the expected future (real) short yields; inflation expectations; and the term premia. Inflation expectations may change abruptly and frequently during a short period of time, so do the expected future short yields. At the same time, term premia can also be quite volatile. Therefore, summing up the movements of these three components, the variance of long yields should be larger than the short yields; nevertheless, the empirical result implies the opposite. As indicated in Duffee (2011*b*), the reason causing this result is that the factor driving up the expected future short yields or inflation expectations may drive down the term premia, thus, offsetting the variation in these components.

From the perspective of time dimension, the volatilities of yields (especially shorter-term) are high in the 1980s, while the bond yield level is also relatively high. The high volatilities are due to large forecast variances of forecast models as well as a high degree of forecast dispersion in forecasts. It is clear that the volatilities are declining during the Great Moderation, and therefore the variances of bond forecasts are rather small between 1990 and 2007, except during the 2004-05 episode of 'Greenspan's Conundrum'. In around 2009, the volatilities surge to a high level since the 1990's, although the short yields stay at a relatively low level (restricted by the zero lower bound) among all periods. Even after the financial crisis, ambiguity in yield forecasts still exists as the volatilities remain at a relatively high level.

Figure 11: Time-Varying Second Moments



Notes: These are time-varying second moments of 3-month ahead forecasts for bonds at maturities 6, 36, 60 and 120 months, from early 1975 to late 2013. The variance of NS factors is estimated from Eq. (A.2), and then the variances of yield forecasts generated by each candidate model in the DMA, can be easily calculated as linear combinations of factor variances.

C.3 Discussion: Do We Need Strict Arbitrage-Free Restrictions?

As we have discussed in Section 2, we impose NS restrictions on the pricing dynamics and leave the physical dynamics unconstrained. Feunou et al. (2014) show that the NS model is the continuous time limit of their near arbitrage-free class with a unit root in the pricing dynamics. Joslin, Singleton and Zhu (2011) show that no-arbitrage Nelson-Siegel restrictions cannot improve out-of-sample forecasts in the context of canonical Gaussian affine term structure models. By allowing for parameter and model uncertainty in the physical dynamics, we are able to acquire significant predictive gains.

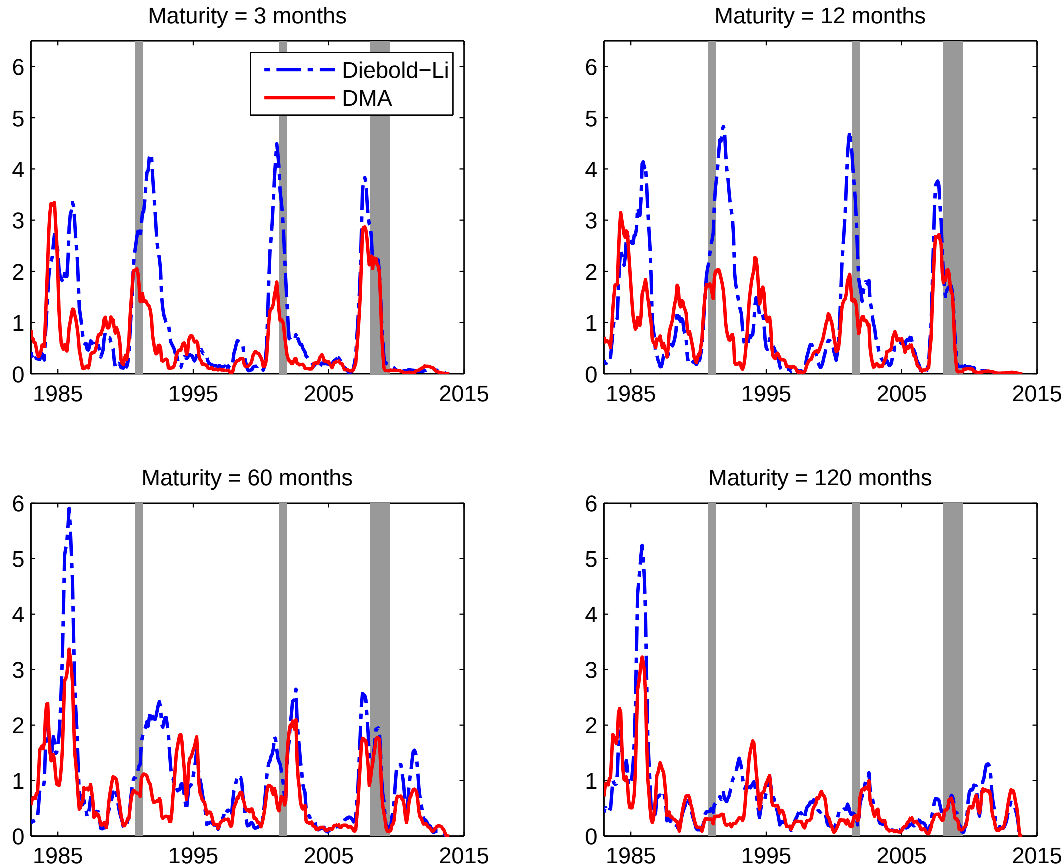
Our DMA approach does not explicitly impose 'hard' arbitrage-free restrictions. From a theoretical perspective, Filipović (1999) and Björk and Christensen (1999) show that the Nelson-Siegel family does not impose the restrictions necessary to eliminate opportunities for riskless arbitrage. From a practical perspective, our implementation allows all bond yields to be priced with errors, which naturally breaks their original assumptions of the Nelson-Siegel family in their papers. Therefore, the potential loss of not imposing arbitrage-free restrictions may be mitigated. The reason is that our focus here is not on the dynamic structure of market price of risks. Duffee (2014) indicates that the no-arbitrage restrictions are unimportant, if a model aims to pin down physical dynamics but not equivalent-martingale dynamics that specify the pricing of risk. In order to capture expectations of investors, we aim to improve forecasts of the interest rate term structure. Joslin, Singleton and Zhu (2011) show that no-arbitrage cross-sectional restrictions are irrelevant for out-of-sample forecasts if the factor dynamics are unrestricted. In practice, the arbitrage-free restrictions are not important in terms of forecasting in models assuming bond yields are priced with errors, see for example, Coroneo, Nyholm and Vidova-Koleva (2011) and Carriero and Giacomini (2011).

To ensure the robustness of our DMA approach, we extend the three-factor arbitrage-free Nelson-Siegel model proposed by Christensen, Diebold and Rudebusch (2011) and evaluate the forecast performance of the arbitrage-free version of DMA. The key difference between arbitrage-free DMA and DMA is a 'yield-adjustment term', which only depends on the maturity and factor volatility. See Christensen, Diebold and Rudebusch (2011) and Diebold and Rudebusch (2013) for more details. The forecast performances of two models are very close, implying that the DMA is almost arbitrage-free, which is consistent with theoretical evidence in Feunou et al. (2014) and Krippner (2015) that the NS models are near arbitrage-free. Hence, following Duffee (2014), we choose not to impose arbitrage-free restrictions to avoid potential misspecification.

C.4 Time-Varying Predictability

Figure 12 shows six-month ahead Squared Forecasting Errors of DL and DMA across the whole out-of-sample forecast period. It is evident that the DMA significantly and consistently outperforms the DL across all maturities. We detect a pattern that the predictive power of term structure models, DL in particular, tends to be procyclical. The forecast errors are in general higher during periods when economic conditions deteriorate, especially for short-term rates. Economic theories suggest that central banks can influence short rates to achieve policy goals, so the deteriorated predictability of yields implies unexpected or abrupt changes in the behavior of policy makers. For long-term yields, the predictability seems more acyclical, as the movements in long yields are affected not only by short rate expectations but also by the expected risk compensation.

Figure 12: Squared Forecasting Errors for Yields of 3-, 12-, 60- and 120-Month Maturities



Notes: These are 6 months ahead Squared Forecasting Errors for predicted yields from 1983 to late 2013. We calculate 9-month moving averages for clarity and plot the statistics for maturities of 3, 12, 60 and 120 months. The models are DMA (solid) and Diebold-Li (dashed and dotted).

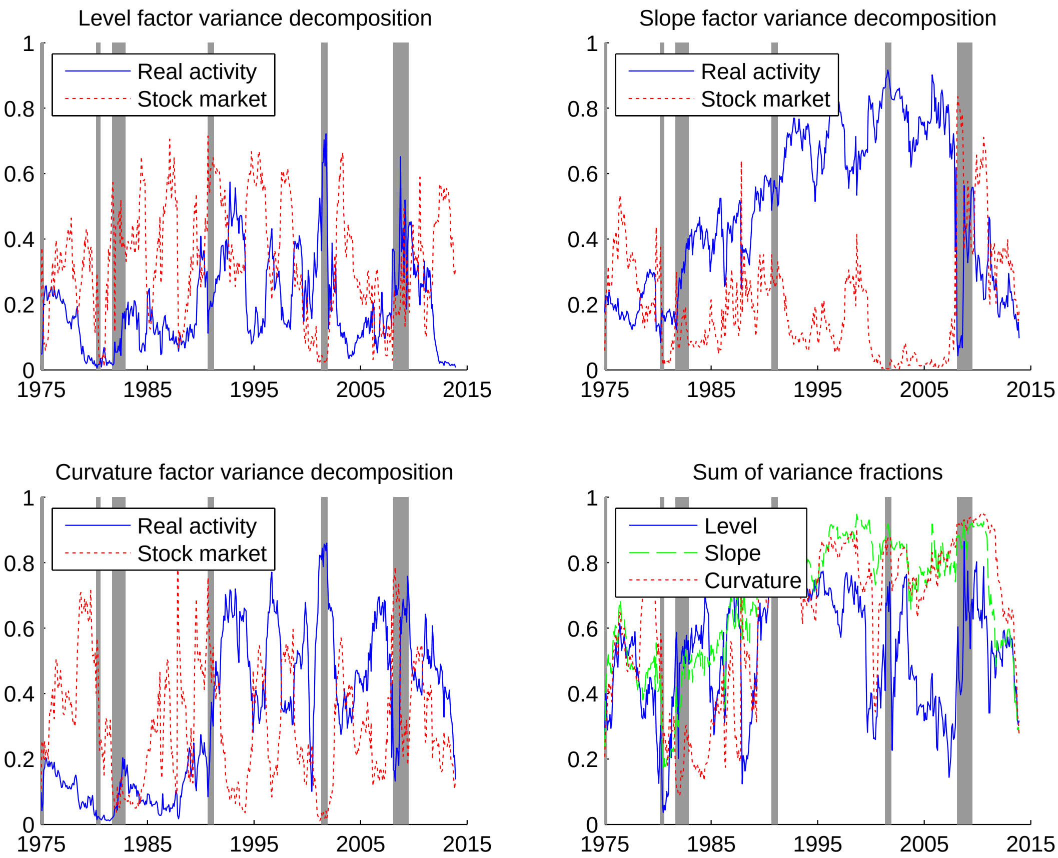
As we have discussed earlier, the DL fails to account for a larger information set and parameter instability, which reduces its forecasting performance. Additionally, our approach allows for model uncertainty, and the large macro-finance VAR significantly contributes to the superior performance of DMA during recession periods. It is of importance to include the large-size VAR, as the increase in the weight assigned to this model significantly reduces forecast errors of DMA when compared with the DL benchmark.[20] Moreover, the DMA has better performance than TVP or TVP-M models especially for short rates as shown in Table 3. As we have discussed, DMA allows the model to capture the sudden changes, which in this case are potentially related to the Fed's policy targets.

We are very interested in why the large-size model has distinctive performance during contraction periods. The question is: What are the underlying economic sources that contribute to the pricing factor movements? Following Koop, Pesaran and Potter (1996) and Diebold and Yilmaz (2014), we conduct the *generalized forecast error variance decomposition* to evaluate the contributions of shocks to respective macro-finance variables.[21] Among 15 variables, our results in Figure 13 suggest that the most important variables driving large-size VAR's predictive power are indicators of real activity and the stock market. In particular, real activity and stock markets contribute to more than 80% of the 60-month forecast error variance of bond factors during the recent three recessions. There is substantial time variation in the role of these variables, and the contributions of two groups tend to be negatively correlated. Specifically, the economic content of Slope and Curvature factors can be largely explained by real activity since the Great Moderation, but the stock market condition is still indispensable. This observation is in line with Kurmann and Otrok (2013) and Bansal, Connolly and Stivers (2014), but contrasts with the evidence from the UK economy provided by Bianchi, Mumtaz and Surico (2009). In the Nelson-Siegel framework, pricing factors are closely related to short rate expectations and term premia, which we will discuss in detail in the following.

[20] The regression results are not shown for the sake of brevity but are available upon request.

[21] We encourage readers to consult the original papers for motivation and background. The generalized variance decomposition is invariant to the ordering of the variables in the VAR, but sums of forecast error variance contributions are not necessarily unity. Here we calculate the normalized weights which add up to unity following Diebold and Yilmaz (2014).

Figure 13: Variance Decomposition of Bond Pricing Factors



Notes:

1. This figure sets out the generalized forecast error variance decomposition of pricing factors using the large-size VAR model. The upper panels and the bottom left panel show the average contributions of our target variables to the forecast error variance of the respective bond factors over time. At each point in time, the fractions are calculated based on the 60-month forecast error variance. *Real activity* corresponds to the information of Industrial Production Index and Total Industry Capacity Utilization, and *Stock market* corresponds to the information of S&P 500 Stock Price Index and Wilshire 5000 Total Market Index.
2. The lower right panel displays for each pricing factor the sum of the variance fractions of the two groups of target variables shown in the previous panels. The shaded areas are the recession periods based on NBER Recession Indicators.

C.5 Expectation Hypothesis and Term Premium

Within our empirical framework we shall set out the formal modeling of the term premia, which has been used to explain the failure of the Expectations Hypothesis and provides important information for the conduct of monetary policy, see Gürkaynak and Wright (2012). The Expectations Hypothesis (EH) consistent bond yield $y_t(\tau)^{EH}$ is given by:[22]

$$y_t(\tau)^{EH} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} E_t y_{t+i}(1), \tag{C.1}$$

where $y_t(\tau)$ is the yield at time t for a bond of τ-period maturity. That is to say, the EH consistent long yield is equal to the average of expected short yields $E_t y_{t+i}(1)$. The time-varying term premium is therefore,

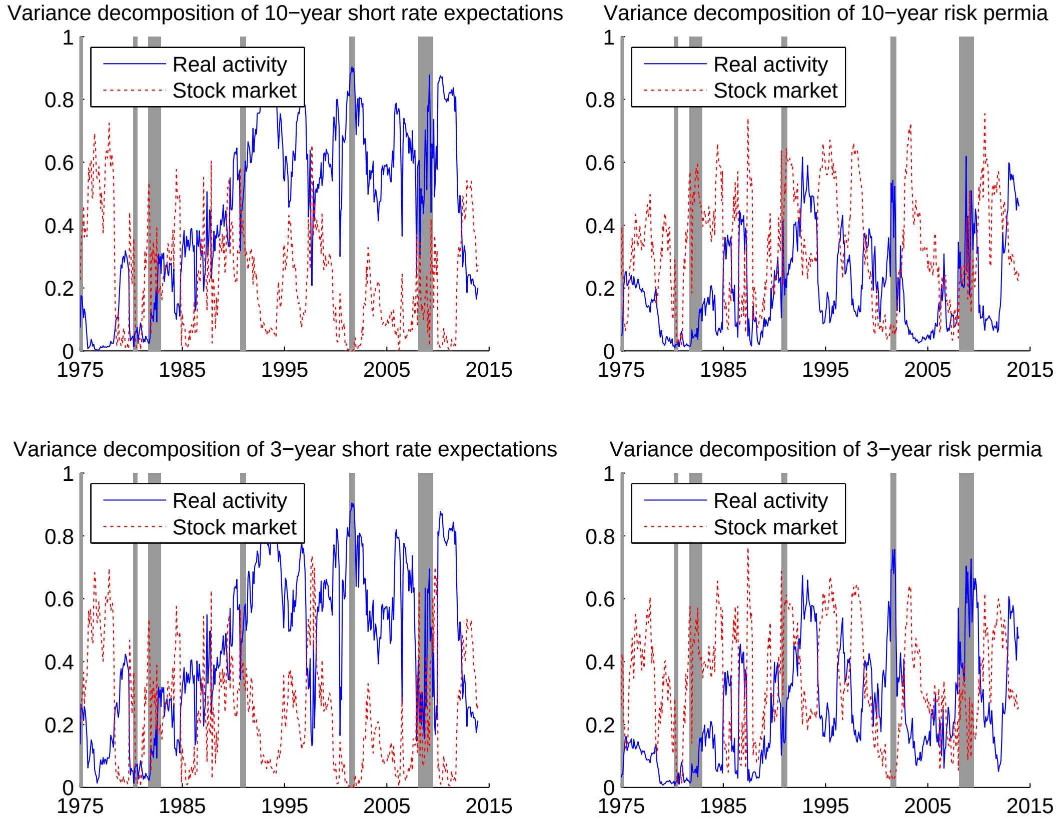
$$TP_t(\tau) = y_t(\tau) - y_t(\tau)^{EH}. \tag{C.2}$$

In the large VAR system, both the short rate expectations and the term premia are linear functions of pricing factors and macro and finance variables, see Diebold, Rudebusch and Aruoba (2006). By the linearity of expectation, we can directly employ the generalized variance decomposition for these quantities.

The patterns in variance decompositions displayed in Figure 14 have intuitive appeal, revealing the relative importance of macro-finance variables in driving short rate expectations and risk permia. Standard theory such as the Taylor rule suggests that policy rates should react at least partially to real activity, and our evidence shows short rate expectations are indeed mainly driven by real activity indicators. In contrast, we find that there is strong time variation regarding the main source of risk compensation required by investors, and the underlying sources differ sharply for different horizons. In particular, short-term risk permia is largely explained by real activity shocks during recessions, while long-term risk premia is much less sensitive to real activity during the same periods and more related to the stock market condition in normal times. This observation is interesting but not surprising: As suggested by finance theories, investors' risk attitude influences the demand for bonds and stocks, and Bansal, Connolly and Stivers (2014) show there is a strong link between these two types of assets.

[22] The expectation here is under the physical measure.

Figure 14: Variance Decomposition of Short Rate Expectations and Term Premia

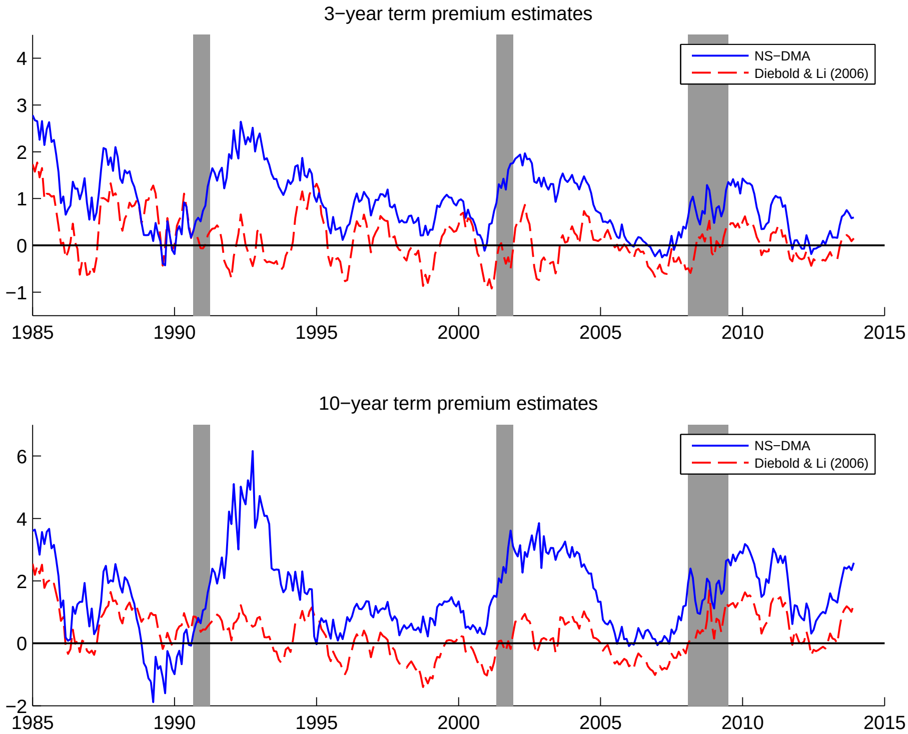


Notes:

This figure sets out the generalized forecast error variance decomposition of short rate expectations and risk premia using the large-size VAR model. The left panels show at each point in time, the average contributions of our target variables to the forecast error variance of 10-year and 3-year short rate expectations, respectively. The right panels show at each point in time, the average contributions of our target variables to the forecast error variance of 10-year and 3-year risk premia, respectively. The time-varying fractions are calculated based on the 60-month forecast error variance. *Real activity* corresponds to the information of Industrial Production Index and Total Industry Capacity Utilization, and *Stock market* corresponds to the information of S&P 500 Stock Price Index and Wilshire 5000 Total Market Index. The shaded areas the recession periods based on NBER Recession Indicators.

C.5.1 Term Premia of Diebold-Li and DMA

Figure 15: Time-Varying Term Premia of 36-and 120-Month Bonds



Notes:
1. The top panel is the 36-month term premia and the bottom is the 120-month term premia. The EH consistent 36- and 120-month bond yields are estimated using Eq. (C.1); we then calculate the term premia using Eq. (C.2).
2. In addition to DMA, we plot the recursively estimated term premia employing the methods proposed by Diebold and Li (2006).
3. Shaded areas are recession periods based on the NBER Recession Indicators. The unit is percentage.