Generation of Entropy during Forced Convection of Heat in Nanofluid Stagnation-Point Flows over a Cylinder Embedded in Porous Media

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Abstract

Thermodynamics and heat transfer processes during impingement of a nanofluid flow upon a cylinder with constant surface temperature and embedded in porous media are investigated. The surface of the cylinder can feature uniform or non-uniform transpiration and is hotter than the incoming nanofluid flow. Appropriate similarity parameters are employed to reduce the three-dimensional governing equations of nanofluid motion and heat transfer in porous media to simpler equations solvable through using a finite difference scheme. The numerical solutions of these equations reveal the flow velocity and temperature fields as well as the Nusselt number and induced shear stress. These are then used to calculate the rate of entropy generation within the system by viscous and heat transfer irreversibilities. It is demonstrated that changes in the concentration of nanoparticles result in the variation of thermal and hydrodynamic boundary layers and hence can modify the Nusselt number and entropy generation considerably. However, the shear stress on the surface of the cylinder is observed to be less affected by the variations in the concentration of nanoparticles. Further, the Reynolds number of the impinging nanofluid flow and the functional form of transpiration are shown to have significant effects upon the Nusselt number and entropy generation. In particular, it is

argued that the influences of Reynolds number on the boundary layer thickness can majorly modify the level of irreversibility and the value of Bejan number.

Keywords: Nanofluid; Stagnation-point flow; Porous media; Entropy generation; Similarity solution; Non-uniform transpiration

Nomenclature

A_1, A_2, A_3	Constants	T_w	wall temperature
a	cylinder radius	u, w	velocity components along (r,z)-axis
Be	Bejan number	$U_0(\varphi)$	transpiration
Br	Brinkman number $Br = \dfrac{\mu_f (\bar{k}.a)^2}{k_f (T_w - T_\infty)}$	z	axial coordinate
c_p	specific heat at constant pressure	**Greek symbols**	
$f(\eta, \varphi, \tau)$	function related to u-component of velocity	α	Effective thermal diffusivity of the porous medium
h	heat transfer coefficient	η	similarity variable, $\eta = \left(\dfrac{r}{a}\right)^2$
k	thermal conductivity	$\theta(\eta, \varphi, \tau)$	non-dimensional temperature
$\bar{k}$	freestream strain rate	λ	Permeability parameter, $\lambda = \dfrac{a^2}{4k_1}$
k_1	permeability of the porous medium	ε	porosity
N_G	entropy generation number $N_G = \dfrac{\dot{S}'''_{gen}}{\dot{S}'''_0}$	Λ	dimensionless temperature difference $\Lambda = \dfrac{(T_w - T_\infty)}{T_w}$
Nu	Nusselt number	μ	dynamic viscosity
p	fluid pressure	υ	kinematic viscosity
P	non-dimensional fluid pressure	ρ	fluid density
P_0	The initial fluid pressure	σ	shear stress
$\Pr$	Prandtl number	ϕ	nanoparticle volume fraction
q_w	heat flow at the wall	φ	angular coordinate
r	radial coordinate	**Subscripts**	
Re	Freestream Reynolds number $\mathrm{Re} = \dfrac{\bar{k}a^2}{2\upsilon_f}$	w	condition on the surface of the cylinder
$S(\varphi)$	transpiration rate function $S(\varphi) = \dfrac{U_0(\varphi)}{\bar{k}a}$	∞	far field
$\dot{S}'''_0$	characteristic entropy generation rate	nf	nanofluid
$\dot{S}'''_{gen}$	rate of entropy generation	f	base fluid
T	temperature	s	nano-solid-particles
T_∞	freestream temperature		

1. Introduction

Forced convection of nanofluids through porous media remains as a priority area for the research communities of transport in porous media and nanofluids [1], [2]. Natural convection of nanofluids in porous media has already received a considerable attention, see for example

[3],[4]. Yet, the equivalent problem under forced convection is comparatively much less explored. In particular, entropy generation process by forced convection of nanofluids in porous media has been identified as an underdeveloped area demanding more research efforts [2].

The general problem of forced convection of nanofluids through porous media has been so far visited by a limited number of researchers[5]. The existing studies can be broadly categorised into two groups of nanofluid flow in porous conduits [6, 7]and in rotating porous discs [8, 9]. Only the latter can involve boundary layer flows and hence, is further discussed here. In a numerical investigation, Bachok et al. [10]examined flow and heat transfer of nanofluids in a configuration including rotating porous disks. These authors employed two different models of the effective thermal conductivity and analysed the fluid dynamics and thermal behaviours of the system [10]. Hatami et al. [11] investigated the nanofluid flow between two counter-rotating disks with porous faces. They considered water based nanofluids with a number of different metal and metal oxide as nanoparticles [11]. Their study included an extensive analysis of the influences of nanoparticle size and type on the heat transfer characteristics of the system [11]. The problem of convective heat transfer by nanofluid flows between rotating porous disks was also investigated by Hosseini et al. [12]. They used homotopy perturbation method and demonstrated that increasing the concentration of nanoparticles magnifies the Nusselt number. This finding was later confirmed by other authors for other configurations [12]. According to Hooseini et al. [12],there is a monotonic and nearly linear relationship between the volumetric fraction of nanoparticles and the growth in Nusselt number. A three dimensional analysis was conducted by Saidi and Tamim [8]to predict the heat and mass transfer behaviour of a system including two rotating porous disks. The Brownian motion of nanoparticles were included in this study and the magnetohydrodynamic effects were investigated [8]. Amongst other findings, it was reported that increasing the permeability of the porous disks boosts the heat and mass transfer coefficients on the surfaces of the disks [8]. Considering a moving permeable surface, Khazayinejad et al. [13]solved the equations that dominate the transport of momentum in a nanofluid boundary layer. They developed a similarity solution for the problem and included the influences of nanofluid suction and injection in their solution [13].

A specific type of boundary layer flows in porous media includes stagnation-points. This class of flow finds wide applications in cooling technologies and therefore, has been subjected to several investigations, mostly focused on stagnation flows of ordinary fluids over flat porous inserts. Here, a short summary of the literature in this area is put forward. A pioneering work on the hydrodynamics of stagnation-point, isothermal flow on a flat porous insert was conducted by Wu et al. [14]. They assumed a Darcy-Brinkman flow and developed an asymptotic solution for the hydrodynamic field in a horizontal porous plate under an impinging jet [14]. In a numerical investigation Jeng and Tzeng [15]examined the transport of heat when a slot jet impinges on the surface of a metallic foam heat sink. These authors reported that the location of the maximum Nusselt number varies with the jet Reynolds number [15]. These authors later set an experiment on the same problem [16]and showed that by increasing the jet Reynolds number the Nusselt number increases, yet the flow pressure drop is also intensified [16]. Wong and Saeid [17, 18] did a heat transfer optimisation on the problem of jet flow blowing on the surface of a horizontal porous insert heated from below.

Harris et al. [19] developed a similarity solution for boundary layer formed near the stagnation point on a vertical porous plate. A numerical work on mixed convection in jet impingement on a flat porous plate showed that increasing the jet width and the Reynolds number results in magnifying the average Nusselt umber [20]. It also showed that decreasing the distance between the jet and the heated section increases the Nusselt number [20]. Kokubun and Fichini [21]presented an analytical solution for the stagnation point flow in an infinitely long, horizontal porous insert subject to different thermal boundary conditions. This work showed that a dimensionless parameter, including information on the transport properties of the fluid and solid, dominates the heat transfer process. In an experimental and numerical study, Feng et al. [22] examined the problem of tube flow impingement on a heated porous insert. They analysed metal foam and finned metal foam and showed that through increasing the thickness of the metal foam heat transfer coefficient decreases. However, this was not the case for the metal finned foam [22]. Recently, Buonomo et al. [23] tackled the problem of interactions between a downward vertical, laminar jet and a confined, horizontal porous layer in an axisymmetric configuration. They showed that Peclet number determines the opposing or supporting arrangements of natural and forced convection [23].

All cited literatures, so far, have been exclusively concerned with flows over flat porous inserts. A survey of literature shows that the problem of stagnation point flow over curved surfaces in porous media has been rarely investigated. An exception to this is the most recent work of the authors, in which they developed a semi-similar solution for the stagnation-flow on the surface of cylinder embedded in a homogenous porous medium [5]. This involved the hydrodynamics and heat convection for an ordinary fluid without consideration of entropy generation [5]. Another highly unexplored area includes nanofluid stagnation-point flow in porous media. The shortage of research in this area extends to both flat and curved configurations. An early investigation of flow over a cylinder embedded in porous media was reported by Abu-Hijleh [24]. A laminar flow of ordinary fluid through the porous media and over the cylinder was examined in this work and the rate of entropy generation was calculated numerically [24]. It was shown that increasing the thickness of the porous layer covering an isothermal cylinder reduces the total entropy generation [24]. Entropy generation in magneto-hydrodynamic (MHD) flow of nanofluids in porous media has been analysed in a few recent works. Rashidi and Freidoonmehr [25]considered the MHD and nanofluid equivalent of the classical configuration of Heimenz [26]when the solid plate was replaced by a flat porous insert. Their work was exclusively concerned with generation of entropy and made the conclusion that the effects of increasing the values of Hartmann, Brinkman and magnetic interaction numbers and, reducing Prandtl and Reynolds numbers aresimilar and lead to an increase in the entropy generation. This study was then extended to the configurations including rotating porous disks with ordinary fluids [25]and nanofluid [28]. In addition to these studies there exists a series of investigations on entropy generation by nanofluid flow over permeable surfaces [28, 29, 30]. Although mathematical equations similar to those of porous media are used in these works, the physical differences between them and stagnation flow inside porous media are rather significant. Hence, these investigations are not further discussed here.

The preceding review of literature reveals a number of points, which can be summarised as follows.

- The general problem of forced convection of nanofluids in porous media and the particular problem of entropy generation by such flows have been highlighted as largely unexplored fields.
- Stagnation point flows in curved porous media have so far received very little attention. Yet, there has been several studies on impingement of external flows in flat porous plates.
- Few existing studies on boundary layer nanofluid flows in porous media are concerned with flat porous inserts or preamble surfaces. Thus, there is currently no study of nanofluid stagnation-point flow in non-flat porous inserts. Most importantly, entropy generation in such configurations has not been evaluated in the past.

In practice, many curved objects are covered with porous layers [5] and nanofluids are increasingly used as the cooling agents in such configurations [31]. However, there is currently no systematic evaluation of the heat transfer and second law performance of such systems. The present work, therefore, aims to fill this gap through a theoretical study of a cylindrical object embedded in porous media and subject to non-axisymmetric, nanofluid stagnation-point flow. This study advances the recent work of the authors [5] to nanofluid flows and also adds entropy generation analysis to that.

2. Theoretical and numerical methods

2.1. Problem configuration, assumptions and governing equations

Figure 1 shows schematically the problem under investigation. This includes a cylinder with radius a centred at $r = 0$covered with a porous medium. The surface of the cylinder can include uniform or non-uniform transpiration with prescribed circumferential distributions, while the temperature of the external surface of the cylinder is maintained constant. An external axisymmetric radial stagnation-point flow of strain rate of $\bar{k}$ impinges on the cylinder. Because of the non-uniformity of transpiration, the flow configuration around the cylinder can be asymmetric. The following assumptions are made through this work.

- The flow is steady, incompressible and laminar.
- The nanofluid is assumed to be Newtonian and single phase.
- The cylinder is assumed to be infinitely long.
- The porous medium is homogenous, isotropic and under local thermal equilibrium.
- The radiation heat transfer is ignored.
- The viscous dissipation of kinetic energy of the flow and the gravitational effects are ignored.
- Physical properties such as porosity, specific heat, density and thermal conductivity are assumed to be constant and hence, the thermal dispersion effects are negligible.
- A moderate range of pore-scale Reynolds number is considered in the porous medium and therefore, non-linear effects in momentum transfer are negligibly small.

A three dimensional Darcy-Brinkman model of transport of momentum along with the one-equation model of transport of thermal energy in cylindrical coordinate are used in this work [32, 33, 34]. The governing equations and boundary conditions, in the cylindrical coordinate system shown in Fig. 1, can be summarised as follows.

The continuity of mass reads,

$$\frac{\partial (ru)}{\partial r}+\frac{\partial v}{\partial \varphi}+r\frac{\partial w}{\partial z}=0 \tag{1}$$

The transport of momentum in the radial direction is

$$\frac{\rho_{nf}}{\varepsilon^2}\left(u\frac{\partial u}{\partial r}+\frac{v}{r}\frac{\partial u}{\partial \varphi}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{\mu_{nf}}{\varepsilon}\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}+\frac{\partial^2 u}{\partial z^2}\right]-\frac{\mu_{nf}}{k_1}u\,, \tag{2}$$

and that in the angular direction is given by

$$\frac{\rho_{nf}}{\varepsilon^2}\left(u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \varphi}+\frac{uv}{r}+w\frac{\partial v}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \varphi}+\frac{\mu_{nf}}{\varepsilon}\left[\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{1}{r^2}\frac{\partial^2 v}{\partial \varphi^2}+\frac{2}{r^2}\frac{\partial u}{\partial \varphi}+\frac{\partial^2 v}{\partial z^2}\right]-\frac{\mu_{nf}}{k_1}v\,. \tag{3}$$

The transport of momentum in axial direction takes the form of

$$\frac{\rho_{nf}}{\varepsilon^2}\left(u\frac{\partial w}{\partial r}+\frac{v}{r}\frac{\partial w}{\partial \varphi}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{\mu_{nf}}{\varepsilon}\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}+\frac{\partial^2 w}{\partial z^2}\right]-\frac{\mu_{nf}}{k_1}w\,. \tag{4}$$

The transport of thermal energy is expressed by

$$u\frac{\partial T}{\partial r}+\frac{v}{r}\frac{\partial T}{\partial \varphi}+w\frac{\partial T}{\partial z}=\frac{k_{nf}}{(\rho C_p)_{nf}}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}+\frac{\partial^2 T}{\partial z^2}\right]. \tag{5}$$

In Eqs. (1-5) p , ρ_{nf} , μ_{nf} , T , $(\rho C_p)_{nf}$, k_{nf} and k_1 are the pressure, density, kinematic viscosity of the nanofluid, temperature, the heat capacitance of the nanofluid, thermal conductivity of the nanofluid and permeability of the porous medium, respectively. These properties are evaluated inside the boundary layer and in the vicinity of the flow impingement point. The nanofluid parameters are defined by [6, 7, 35],

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \quad , \quad (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \quad , \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + 2\phi(k_f - k_s)} \tag{6}$$

where ϕ denote the nanoparticle volume fraction. In Eq. (6) the subscripts “f” and “s”, refer to fluid and solid fraction properties, respectively. The thermo-physical properties of the base fluid (water) and different nanoparticles are given in Table 1.

The velocity boundary conditions are as follows.

$$r = a : w = 0, v = 0, u = -U_0(\varphi), \tag{7}$$

$$r \to \infty : w = 2\bar{k}z, \lim_{r\to\infty} rv = 0, u = -\bar{k}\left(r - \frac{a^2}{r}\right). \tag{8}$$

Further, the two boundary conditions with respect to φ (angular coordinate) are given by

$$u(r,0) = u(r,2\pi), v(r,0) = v(r,2\pi), \tag{9}$$

$$\frac{\partial u(r,0)}{\partial \varphi} = \frac{\partial u(r,2\pi)}{\partial \varphi}, \quad \frac{\partial v(r,0)}{\partial \varphi} = \frac{\partial v(r,2\pi)}{\partial \varphi}.$$

Equation (7) represents no-slip conditions on the external surface of the cylinder. Further, Eq. (8) indicates that the viscous flow solution approaches, in a manner analogous to the Hiemenz flow, the potential flow solution as $r \to \infty$ [32, 33, 36]. This can be verified by starting from the continuity equation in the followings. $-\frac{1}{r}\frac{\partial}{\partial r}(ru) - \frac{\partial v}{\partial \varphi} = \frac{\partial w}{\partial z} =$ Constant $= 2\bar{k}z$ and integrating in r and z directions with boundary conditions, $w = 0$ when $z = 0$ and $u = -U_0(\varphi)$ when $r = a$.

The boundary condition for the transport of thermal energy is given by

$$r=a: T = T_w = \text{Constant}, \tag{10}$$

$$r \to \infty: T \to T_\infty$$

and the two boundary conditions with respect to angular coordinate, φ are

$$T(r,0) = T(r,2\pi), \tag{11}$$

$$\frac{\partial T(r,0)}{\partial \varphi} = \frac{\partial T(r,2\pi)}{\partial \varphi},$$

in which T_w is the cylinder surface temperature and T_∞ is the free-stream temperature.

2.2 Self-similar solutions

A reduction of the governing Eqs. (1-5) is obtained through applying the following similarity transformations.

$$u = -\frac{\bar{k}a}{\sqrt{\eta}} f(\eta,\varphi), \quad v = \frac{\bar{k}a}{\sqrt{\eta}} G(\eta,\varphi), w = \left[2\bar{k}f'(\eta,\varphi) - \frac{\bar{k}}{\eta}\frac{\partial G}{\partial \varphi}\right] z \quad , p = \rho_f \bar{k}^2 a^2 P, \tag{12}$$

Where $\eta = \left(\frac{r}{a}\right)^2$ is the dimensionless radial variable. Transformations (12) satisfy Eq. (1) automatically and their substitution into Eqs. (2), (3) and (4) leads to the following system of coupled differential equations.

$$\begin{aligned}&\varepsilon.\left(\eta f''' + f'' - \frac{1}{8\eta^2}\frac{\partial^3 G}{\partial \varphi^3} - \frac{1}{2}\frac{\partial G''}{\partial \varphi} + \frac{1}{2\eta}\frac{\partial G'}{\partial \varphi} - \frac{1}{2\eta^2}\frac{\partial G}{\partial \varphi} + \frac{1}{4\eta}\frac{\partial^2 f'}{\partial \varphi^2}\right)\\ &+\mathrm{Re}\,A_1.(1-\phi)^{2.5}\left[1 + ff'' - (f')^2 - \frac{f}{2\eta}\frac{\partial G'}{\partial \varphi} + \frac{f}{2\eta^2}\frac{\partial G}{\partial \varphi} - \frac{G}{2\eta}\frac{\partial f'}{\partial \varphi} + \frac{G}{4\eta^2}\frac{\partial^2 G}{\partial \varphi^2} + \frac{f'}{\eta}\frac{\partial G}{\partial \varphi} - \frac{1}{4\eta^2}\left(\frac{\partial G}{\partial \varphi}\right)^2\right] + \varepsilon^2.\lambda\left[1 - f'\right] = 0\end{aligned}, \tag{13}$$

$$\begin{aligned}P - P_0 = &\frac{1}{\varepsilon^2}\frac{1}{2\bar{k}^2}\int_1^\eta \frac{1}{\eta^2}\left[G^2 + G\frac{\partial f}{\partial \varphi}\right]d\eta - \frac{1}{\varepsilon A_1.(1-\phi)^{2.5}}\left[\left(\frac{f'}{\mathrm{Re}} - \frac{1}{4\,\mathrm{Re}}\int_1^\eta \frac{1}{\eta^2}\frac{\partial^2 f}{\partial \varphi^2}d\eta + \frac{1}{2\,\mathrm{Re}}\int_1^\eta \frac{1}{\eta^2}\frac{\partial G}{\partial \varphi}d\eta\right) + \frac{\lambda}{\mathrm{Re}}\int_1^\eta \frac{f}{\eta}d\eta\right]\\ &-2\left[\frac{1}{\varepsilon^2} + \frac{1}{A_1.(1-\phi)^{2.5}}\frac{\lambda}{\mathrm{Re}}\right]\left(\frac{z}{a}\right)^2 - \frac{1}{2\varepsilon^2}\left(\frac{f^2}{\eta}\right)\end{aligned} \tag{14}$$

In which $\mathrm{Re} = \dfrac{\bar{k}a^2}{2\upsilon_f}$ is the free stream Reynolds number, $\lambda = \dfrac{a^2}{4k_1}$ is called permeability parameter and prime indicates differentiation with respect to η. Considering Eqs. (6), (7), and (8), the boundary conditions for Eqs. (13) and (14) reduce to:

$$\eta = 1 : f\,'(1,\varphi) = 0\,, f\,(1,\varphi) = S\,(\varphi) \tag{15}$$

$$\eta \to \infty : f\,'(\infty,\varphi) = 1 \tag{16}$$

$$f\,(\eta,0) = f\,(\eta,2\pi)\,, \frac{\partial f\,(\eta,0)}{\partial\varphi} = \frac{\partial f\,(\eta,2\pi)}{\partial\varphi}\,, \tag{17}$$

in which, $S\,(\varphi) = \dfrac{U_0(\varphi)}{\bar{k}a}$ is the transpiration rate function. Note that Equations (13) and (14) are the complete form of Equations (9) and (11) in Ref. [37]. Substitution of Eq. (12) into Eqs. (3) and (4) results in a differential equation in terms of $G\,(\eta,\varphi)$ as well as an expression for the pressure. This reads

$$\varepsilon.\left(\eta G'' + \frac{1}{4\eta}\frac{\partial^2 G}{\partial\varphi^2} - \frac{1}{2\eta}\frac{\partial f}{\partial\varphi}\right) + \mathrm{Re}.A_1.(1-\phi)^{2.5}\left[f\,.G' - \frac{G}{2\eta}\frac{\partial G}{\partial\varphi}\right] - \varepsilon^2.\lambda.G = 0\,, \tag{18}$$

Considering conditions (7)-(9), the boundary and initial conditions for Eq. (18) can be written as

$$\eta = 1 : G\,(1,\varphi) = 0\,,\ \frac{\partial G\,(1,\varphi)}{\partial\varphi} = 0\,, \tag{19a}$$

$$\eta \to \infty : G\,(\infty,\varphi) = 0 \tag{19b}$$

$$G\,(\eta,0) = G\,(\eta,2\pi)\,,\ \frac{\partial G\,(\eta,0)}{\partial\varphi} = \frac{\partial G\,(\eta,2\pi)}{\partial\varphi}\,. \tag{20}$$

To transform the energy Eq. (5) into a dimensionless form, the following transformation is introduced,

$$\theta(\eta,\varphi)=\frac{T\ (\eta,\varphi)-T_\infty}{T_W\ -T_\infty} \tag{21}$$

Substitution of Eqs. (12) and (20) into Eq. (5)and ignoring the small dissipation terms yields

$$\eta\theta''+\theta'+\frac{1}{4\eta}\frac{\partial^2\theta}{\partial\varphi^2}+\mathrm{Re}\,.\,\mathrm{Pr}\,.\frac{A_2}{A_3}.\left(f\ \theta'-\frac{G}{2\eta}\frac{\partial\theta}{\partial\varphi}\right)=0\,, \tag{22}$$

while the boundary conditions reduce to

$$\eta=1:\ \theta(1,\varphi)=1 \tag{23a}$$

$$\eta\to\infty:\ \theta(\infty,\varphi)=0 \tag{23b}$$

$$\theta(r,0)=\theta(r,2\pi)\,,\quad \frac{\partial\theta(r,0)}{\partial\varphi}=\frac{\partial\theta(r,2\pi)}{\partial\varphi}\,. \tag{24a,b}$$

In Eqs. (18) and (22), A_1, A_2 and A_3 are constants with the following forms:

$$A_1=(1-\phi)+\frac{\rho_s}{\rho_f}\phi\,,A_2=(1-\phi)+\frac{(\rho C_p)_s}{(\rho C_p)_f}\phi\,,A_3=\frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2\phi(k_f-k_s)}\,. \tag{25}$$

It is recalled here that Eq. (22) is the complete form of Equations (14) in Ref. [37]. Equations (12), (18) and (22), together with the boundary conditions (15-17), (19-20), (23) and (24), were solved numerically using an implicit, iterative tri-diagonal finite-difference method similar to that discussed in Refs. [38, 39]. It should be noted that, although not shown here, a three dimensional solution of the problem clearly shows that the numerical values of G (η,φ) are negligibly small [5]. Thus, in the rest of the analysis it is assumed that $G(\eta,\varphi)=0$.

$$\varepsilon.\left(\eta f'''+f''+\frac{1}{4\eta}\frac{\partial^2 f'}{\partial\varphi^2}\right)+\mathrm{Re}A_1.(1-\phi)^{2.5}\left[1+ff''-(f')^2\right]+\varepsilon^2.\lambda[1-f']=0\,, \tag{26}$$

$$P - P_0 = -\frac{1}{2\varepsilon^2}\left(\frac{f^2}{\eta}\right) - \frac{1}{\varepsilon . A_1 . (1-\phi)^{2.5}}\left[\left(\frac{f'}{\mathrm{Re}} - \frac{1}{4\,\mathrm{Re}}\int_1^{\eta}\frac{1}{\eta^2}\frac{\partial^2 f}{\partial\varphi^2}d\eta\right) + \frac{\lambda}{\mathrm{Re}}\int_1^{\eta}\frac{f}{\eta}d\eta\right], \tag{27}$$
$$-2\left[\frac{1}{\varepsilon^2} + \frac{1}{A_1 . (1-\phi)^{2.5}}\frac{\lambda}{\mathrm{Re}}\right]\left(\frac{z}{a}\right)^2$$

$$\eta\theta'' + \theta' + \frac{1}{4\eta}\frac{\partial^2\theta}{\partial\varphi^2} + \mathrm{Re}.\mathrm{Pr}.\frac{A_2}{A_3}.\left(f\,\theta'\right) = 0 \cdot \tag{28}$$

2.3 Shear stress and Nusselt number

The shear-stress induced by the nanofluid flow on the external surface of the cylinder is given by [5, 34]

$$\sigma = \mu_{nf}\left[\frac{\partial w}{\partial r}\right]_{r=a}, \tag{29}$$

Where μ_{nf} is the nanofluid viscosity. Employing Eq. (12), a semi-similar solution for the shear stress on the surface of the cylinder can be developed. This reads:

$$\sigma = \mu_{nf}\,\frac{2}{a}\left[2\bar{k}zf''(1,\varphi)\right] \Rightarrow \frac{\sigma a}{4\mu_f\,\bar{k}z} = (1-\phi)^{-2.5} f''(1,\varphi) \cdot \tag{30}$$

For the current problem with iso-thermal boundaries, the local heat transfer coefficient and rate of heat transfer are defined as

$$h = \frac{q_w}{T_w - T_\infty} = \frac{-k_{nf}\left(\frac{\partial T}{\partial r}\right)_{r=a}}{T_w - T_\infty} = -\frac{2k_{nf}}{a}\frac{\partial\theta(1,\varphi)}{\partial\eta}, \tag{31}$$

and

$$q_w = -\frac{2k_{nf}}{a}\frac{\partial\theta(1,\varphi)}{\partial\eta}(T_w - T_\infty) \cdot \tag{32}$$

Hence,Nusselt number can be written as

$$Nu = \frac{ha}{2k_f} = -\frac{k_{nf}}{k_f}\theta'(1,\varphi) = -A_3.\theta'(1,\varphi). \tag{33}$$

2.4 Entropy generation

Considering the assumption stated in section 2.1, the volumetric rate of local entropy generation in the problem is given by[40, 41]:

$$\begin{aligned}\dot{S}'''_{gen} &= \frac{k_{nf}}{T_W^2}\left[\left(\frac{\partial T}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial T}{\partial \varphi}\right)^2\right] + \frac{2\mu_{nf}}{T_W}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] \\ &+ \frac{\mu_{nf}}{T_W}\left[\left(\frac{1}{r}\frac{\partial w}{\partial \varphi}\right)^2 + \left(\frac{\partial w}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u}{\partial \varphi}\right)^2\right] + \frac{\mu_{nf}}{k_1 T_W}\left[u^2 + w^2\right]\end{aligned} \tag{34}$$

Using the similarly variables given in Eqs. (12) and (34), the local entropy generation becomes:

$$\begin{aligned}\dot{S}'''_{gen} &= \frac{4k_{nf}\left(T_W - T_\infty\right)^2}{a^2 T_W^2}\left[\eta\theta'^2 + \frac{1}{4\eta^2}\left(\frac{\partial \theta}{\partial \varphi}\right)^2\right] \\ &+ \frac{4\bar{k}^2\mu_{nf}}{T_W}\left[\eta f''^2 + 4f'^2 + \frac{f^2}{\eta^2} - \frac{2ff'}{\eta} + \frac{1}{\eta}\left(\frac{\partial f'}{\partial \varphi}\right)^2 + \frac{1}{4\eta^2}\left(\frac{\partial f}{\partial \varphi}\right)^2\right] + \frac{\bar{k}^2\mu_{nf}a^2}{k_1 T_W}\left[\left(\frac{f}{\eta}\right)^2 + 4f'^2\right]\end{aligned} \tag{35}$$

in which $N_G = \frac{\dot{S}'''_{gen}}{S'''_0}$ and $S'''_0 = \frac{4k_f\left(T_W - T_\infty\right)}{a^2 T_W}$ is the characteristic entropy generation rate.

The dimensionless form of volumetric rate of local entropy generation (N_G) can be presented as follows:

$$\begin{aligned}N_G &= \Lambda.A_3\left[\eta\theta'^2 + \frac{1}{4\eta^2}\left(\frac{\partial \theta}{\partial \varphi}\right)^2\right] \\ &+ Br.\left(1-\phi\right)^{-2.5}\left\{\left[\eta f''^2 + 4f'^2 + \frac{f^2}{\eta^2} - \frac{2ff'}{\eta} + \frac{1}{\eta}\left(\frac{\partial f'}{\partial \varphi}\right)^2 + \frac{1}{4\eta^2}\left(\frac{\partial f}{\partial \varphi}\right)^2\right] + \lambda\left[\left(\frac{f}{\eta}\right)^2 + 4f'^2\right]\right\}\end{aligned} \tag{36}$$

Where $\Lambda = \frac{\left(T_W - T_\infty\right)}{T_W}$ is the dimensionless temperature difference, and $Br = \frac{\mu_f\left(\bar{k}.a\right)^2}{k_f\left(T_W - T_\infty\right)}$ is the Brinkman number. The Bejan number, defined as the ratio of entropy generation due to heat transfer to the total entropy generation, is used to facilitate understanding the mechanisms of entropy generation. Bejan number for thecurrent problem can be expressed as

$$Be = \frac{\Lambda . A_3 \left[\eta \theta'^2 + \frac{1}{4\eta^2} \left(\frac{\partial \theta}{\partial \varphi} \right)^2 \right]}{Br.(1-\phi)^{-2.5} \left\{ \left[\eta f''^2 + 4f'^2 + \frac{f^2}{\eta^2} - \frac{2ff'}{\eta} + \frac{1}{\eta} \left(\frac{\partial f'}{\partial \varphi} \right)^2 + \frac{1}{4\eta^2} \left(\frac{\partial f}{\partial \varphi} \right)^2 \right] + \lambda \left[\left(\frac{f}{\eta} \right)^2 + 4f'^2 \right] \right\}} \tag{37}$$

2.5Grid independency and validation

To establish grid independency of the developed numerical solution, Fig. 2 plots $f'(\eta,\varphi)$ as a function of $\eta$with varying mesh sizes of 51×18, 102×36, 204×72, 408×144 and 816×288. It is clear from Fig. 2a that there are no considerable changes of $f'(\eta,\varphi)$ for (η,φ) mesh sizes of (204×72), $(408 \times 144)$and (816×288). Hence, a (408×144) grid in $\eta - \varphi$ directionswas used for the computational domain reported in this work. A non-uniform grid was applied in η-direction to capture the sharp gradients around the external surface of the cylinder, and a uniform mesh was implemented in φ direction. The computational domain extends over $\varphi_{max} = 360°$ and $\eta_{max} = 15$. In this expression,η_{max}corresponds to $\eta \to \infty$, which for all investigated cases, is located outside the momentum and thermal boundary layers. Figure 2b shows the computational mesh utilised in the current study. A convergence criterion was employed in the numerical simulations. This was such that when the difference between the two consecutive iterations became less than10^{-7}, the solution was assumed to have converged and hence the iterative process was terminated. On the basis of the implemented numerical scheme, the numerical error is of $O(\Delta\eta)^2$[37, 38]. The solutions developed in sections 2.2 and 2.3 were validated by comparing the Nusselt number calculated by Eq. (33) with those from the literature for flows over cylinders with no transpiration and large permeability (no porous layer). Table 2 shows the outcomes of this comparison. The close agreement between the two sets of Nusselt number ensures the validity of the numerical simulations. Further, at $\phi = 0$ the temperature and Nusselt numbers reported in this work reduce to those in Ref.[5]for an ordinary fluid flow on a cylinder embedded in porous media.

3. Results and discussion

Table 3 summarises the default values of parameters that the results presented in this section are based on. Any changes to these default values have been explicitly stated in the figures and tables. Further, three types of transpiration functions including $S=0$, $S=\text{const.}$, $S=\cos(\varphi)$ (see Fig. 1) have been used in this section.

3.1Temperature field, Nusselt number and friction coefficient

Numerical solution of the equations of section 2.2 results in calculating the non-dimensional velocity and temperature fields. For conciseness reasons, however, only temperature results are presented here. Figure 3 shows the radial and angular variations of the dimensionless temperature for different values of volumetric concentration of nanoparticles. This figure also includes temperature fields corresponding to uniform transpiration in the form of suction and injection as well as a non-uniform transpiration. It is clear from Fig. 3a that by increasing η, the dimensionless temperature drops rapidly. The radial region over which the dimensionless temperature has a finite value, denotes thickness of the thermal boundary layer developed around the cylinder. Figure 3a shows that this thickness varies considerably with the type of transpiration function and also the concentration of nanoparticles. Addition of nanoparticles boosts the thermal conductivity of the nanofluid and consequently makes the thermal boundary layer thicker. Figure 3b shows that under uniform transpiration the angular distribution of the temperature is nearly uniform and invariant. However, introduction of non-uniform transpiration, in the form of $s = \cos(\varphi)$,causes significant angular variations in which temperature rises between $0°$ and $180°$. Interestingly, increasing the concentration of nanoparticles tends to reduce the maximum temperature, while it sharpens the angular temperature gradient near $\varphi = 0°$.

Figure 4 indicates that Reynolds number of the impinging flow and the type of transpiration function can strongly influence the temperature field. As expected, increasing the Reynolds number results in reducing the thickness of the thermal boundary layer (see Fig. 4a). Further, it is clear from Fig. 4a that for the same Reynolds number and under uniform suction the radial temperature distribution is quite similar to that under non-uniform transpiration. Blowing, however, leads to major restructuring of the thermal boundary layer in which the temperature is generally higher in comparison with other cases and the boundary layer is thicker. Figure 4b shows that for the given value of η in Table 3 and under Re=0.1 and Re=1 the thermal boundary layer exists for all value ofφ (i.e. around the entire cylinder). However, by increasing the Reynolds number to Re=10, the thermal boundary layer becomes detectable only after $\varphi = 60°$, reflecting the pronounced effects of Reynolds number on the boundary layer thickness.

Figures 5-8 depict the effects of various parameters on the Nusselt number and dimensionless shear stress. Figure 5 shows the influences of the functional form of transpiration upon the angular distribution of Nusselt number for a given concentration of nanoparticles. It is evident from this figure that regardless of the type of transpiration, at $\varphi = 0$ the numerical value of Nusselt number is quite large. This is a commonly observed behaviour in impinging flows upon curved objects [5, 33, 34] and is related to the development of the boundary layers at $\varphi = 0°$, which involves small thickness and therefore, magnifies the convection coefficient. Increasing the value of φ leads to a sharp decline of the Nusselt number and for uniform transpiration, the Nusselt number takes a constant value for $\varphi \gtrsim 20°$. However, the angular variation of Nusselt number is more involved under non-uniform transpiration, as such for $S(\varphi) = \cos(\varphi)$,the Nusselt number decreases monotonically as φ decreases,featuring extremum points for $S(\varphi) = \sin(\varphi)$ and $S(\varphi) = -\sin(\varphi)$. The observed trend for the dimensionless shear stress, shown in Fig. 5b, is considerably simpler and includes a monotonic increase with respect to φ for all investigated transpiration functions. Nonetheless, Fig. 5b indicates that the numerical value of dimensionless shear stress is altered by variations in the transpiration function.

It is inferred from Fig. 6a that the permeability of the porous medium has marginal effects upon the Nusselt number, while as already discussed the type of transpiration function is by far more influential. According to this figure, increasing the permeability parameter from 10 to 1000 results in a small increase in the Nusselt number. However, Fig. 6b shows that such increase in the permeability parameter substantially increases the induced shear stress on the surface of the cylinder. It is concluded from this figure that for most values of $\varphi,$ the numerical value of the dimensionless shear stress for $\lambda = 100$ is more than twice of those for $\lambda = 10$. However, by increasing the permeability parameter to 1000 a smaller increase in the value of dimensionless shear stress is observed.

Variations in the concentration of nanoparticles affects the thermal conductivity and viscosity of the nanofluid and hence can alter the values of Nusselt number and shear stress. Figure 7a shows that increasing the volumetric concentration of nanoparticles has an angle-dependent effect on the Nusselt number. For $\varphi < 90°$, increasing the concentration of nanoparticles reduces the Nusselt number. This peculiar behaviour is quite noticeable for $10° < \varphi < 80°$, and can be attributed to the growth of the boundary layer thickness in this region. However, for $\varphi > 90°$ the nanofluids with higher concentration of nanoparticles feature higher Nusselt number. This is consistent with the previously reported Nusselt number trend for forced convection of nanofluids through porous media [6, 7]. The observed angular dependency of Nusselt number under different concentrations of nanoparticles is expected to be related to the scaling of the convection coefficient with thermal conductivity and thickness of thermal boundary layer (i.e. $h \sim k_f / \delta_t$[42, 43]). Addition of nanoparticles directly affects the thermal conductivity and as discussed earlier can also considerably increase the thickness of the boundary layer. Interaction of these with the complex hydrodynamics of the current problem [5] renders the behaviour of Nusselt number depicted in Fig. 7a. Similar to those discussed in earlier figures, the angular variation of the dimensionless shear stress with concentration of nanoparticles is quite straight forward. Increasing the concentration of nanoparticles results in a relatively small augmentation of the shear stress as a result of the boosted viscosity of the nanofluid.

Figure 8 depicts the effects of Reynolds number of the impinging flow on the Nusselt number and induced shear stress. It is clear from Fig. 8a that Reynolds number can strongly influence the rate of heat transfer. For all investigated Reynolds numbers in this figure, as the value of φ increases the Nusselt number decreases. This decrease is to be expected and is related to the growth of the boundary layer thickness at higher values of φ. Yet, rate of the decline appears to be very strongly dependent upon Reynolds number. According to Fig. 8a, for Re=0.1 there is a gradual drop of Nusselt number around most of the cylinder ($20° < \varphi < 180°$). This decline becomes considerably stronger for Re=1 and then very sharp at Re=10. These drops correspond to the growth of the thermal boundary layer for the same Reynolds numbersas depicted in Fig. 4. Strong dependency upon the Reynolds number can be also seen in the dimensionless shear stress shown in Fig. 8b. As expected, increasing Reynolds number results in the magnification of the induced shear stress. A common feature in Figs. 8a and 8b, is the non-uniformity of the Reynolds number effects. Increasing the Reynolds number from 0.1 to 1 (a factor of ten) results in a relatively small change in the Nusselt number and shear stress modifications. Nonetheless, increasing the Reynolds number again for ten time (Re=10) leads to a very significant increase in the Nusselt number and shear stress.

Further quantitative information about the effects of Reynolds number and permeability parameter upon the dimensionless shear stress can be found in Table 4. This table clearly reflects the significant strengthening effects of Reynolds number and permeability parameter on the induced shear stress over the surface of the cylinder. Also, Table 4 shows the influences of the type of nanoparticles on the average Nusselt number for a given value of volumetric concentration of nanoparticles. An interesting qualitative agreement is observed between the

Nusselt numbers reported in Table 4 and those extracted from literature for a non-porous configuration and presented in Table 2 [35]. In both of these tables, the Nusselt numbers corresponding to Cu and Al_2O_3 are quite close to each other, while those of Al_2O_3 are very slightly larger. However, the Nusselt numbers associated with TiO_2 are noticeably larger than all other Nusselt numbers.

3.2 Thermodynamic irreversibilities

Figure 9 shows the radial and angular distribution of the entropy generation number for three different values of Brinkman number. Examination of the radial distribution of entropy generation, shown in Fig. 9a, reveals an interesting behaviour of entropy generation process. In case of low Brinkman number there is a drop of entropy generation close to the surface of the cylinder, while, for higher values of Brinkman number the entropy generation number grows radially and reaches an almost asymptotic value. This is due to the dominance of viscous entropy generation at higher Brinkman numbers and will be discussed later in the context of Bejan number. Angular distribution of entropy generation number (Fig. 9b) includes a sharp decrease of entropy in the vicinity of $\varphi = 0°$ followed by mild fluctuations for non-uniform transpirations and constant values for uniform transpirations. Similar to that discussed for Fig. 9a, high Brinkman number results in larger values of entropy generation number.

Figure 10 shows Bejan number distributions under the conditions investigated in Fig. 9. This figures shows that the radial (Fig. 9a) distributions of Bejan number include a sharp decrease at small values of η and almost zero Bejan number for larger radii. This reflects the fact that contribution of the thermal irreversibility is limited to the thermal boundary layer. Figure 10a shows that the largest value of Bejan number correspond to the lowest value of Brinkman number. This, once again, represents the dominance of thermal irreversibility in the boundary layer for low strain rate flows. Angular distributions of Bejan number, shown in Fig. 10b, all include a value close to zero at $\varphi = 0$ followed by an increase at small values of φ. This is due to the fact that at $\varphi = 0$ the stagnant flow is in thermal equilibrium with the surface of the cylinder and hence the thermal irreversibility is zero. Such equilibrium does not exists at other angles and thus the Bejan number grows for non-zero angles.

The effects of increasing the concentration of nanoparticles upon entropy generation have been depicted in Figs. 11 and 12. It is clear from Fig. 11 that entropy generation is intensified by increasing the concentration of nanoparticles. This figure further shows that transpiration can majorly affect the thermodynamic irreversibilities. For instance, Figure 11a shows that for large radii the entropy generation number for uniform transpiration and for$\phi = 0.05$ becomes similar to the non-uniform transpiration with the base fluid. Radial distributions of Bejan number, shown in Fig.12a, shows that this parameter is almost unresponsive to changes in nanoparticles concentration. It is inferred from this observation that as the concentration of nanoparticles increases, the contribution of thermal irreversibility with the total entropy generation remains nearly constant. Thus, the main cause of increase in the entropy generation number at higher values ϕ (as shown Fig. 11) is the boosted viscosity of the nanofluid, which reinforces flow irreversibility. The negligible contribution of concentration alterations with Bejan number is also reflected in Fig. 12b, which has been plotted for a small range of Bejan number.

Figures 13 and 14 depict the influences of Reynolds number on the entropy generation number and Bejan number. As expected, higher Reynolds number of the impinging flows render higher irreversibility. Figure 13 shows that for high Reynolds numbers the entropy generation drops sharply very close to the surface of the cylinder, reaches a minimum value and then grows fast before reaching a plateau. Decreasing the Reynolds number reduces the intensity of the sharp drop at near surface and completely eliminates that for the smallest investigated Reynolds number. It further pushes the location of plateau development towards higher radii, which is due to the thickening of the hydrodynamic boundary layer at lower values of Reynolds number. In Fig. 13a, the extremum behaviour of entropy generation at high Reynolds number can be attributed to two important factors. First, as shown previously, thermal boundary layer is limited to small radii around the surface of the cylinder and larger radial distances from the cylinder fall outside the thermal boundary layer. Thus, the thermal irreversibility does not exist at these radii (see Fig. 14a) and the total entropy generation drops significantly. Second, the same series of events occur for the hydrodynamic boundary layer and majorly reduces the flow irreversibility by taking a small radial distance from the surface of the cylinder. However, since hydrodynamic boundary layer is thicker than the thermal boundary layer (Pr$>$ 1) the flow irreversibility continues to be generated at higher values of η. This generates the minimum point and the following increase in the total entropy plot of Re=10. At lower values of Reynolds number, the boundary layers are thicker and therefore the drop of entropy generation close to the surface is much small.

An interesting behaviour is observed in Fig. 14b in which the angular distribution of Bejan number for the high Reynolds number flow is nonzero only for $80° \lesssim \varphi \lesssim 120°$. Yet, for the lower Reynolds numbers the Bejan number is nonzero for all values of φ. This is due to the fact that, as shown in Fig. 4, for $80° \lesssim \varphi$ the thermal boundary layer does not reach the radial location for which Fig. 14b has been generated (see Table 3). For $80° \lesssim \varphi \lesssim 120°$ a thermal boundary layer exists and hence thermal irreversibilities are nonzero. Beyond $\varphi \sim 120°$, a thermal equilibrium has been reached and hence there is no thermal entropy generation.

Figures 9-14 all demonstrated that the functional form of transpiration rate can considerably affect the entropy generation and Bejan number. To further investigate these effects, Fig. 15 shows the radial and angular distribution of entropy generation number for different non-uniform transpiration functions. It is clear from this figure that in general, irreversibilities associated with blowing are smaller than those of suction. It was shown in section 3.1 that boundary layers are thicker and hotter for blowing cases. Thus, the velocity and temperature gradients are not as sharp as the suction boundary layers and the value of entropy generation becomes smaller. This finding, once again, highlights the significance of boundary layer thickness as a key parameter determining the intensity of thermodynamic irreversibility in the current problem.

4. Conclusions

Semi-similar solutions were developed for the velocity and temperature fields induced by the impingement of nanofluid flows on the surface of a cylinder embedded in a porous medium. The cylinder was assumed to have a higher temperature compared to the incoming nanofluid flow and can include uniform and non-uniform mass transpiration. The temperature and flow solutions were used to find the Nusselt number and shear stress on the surface of the cylinder. Further, entropy generation in the flow through viscous and thermal mechanisms were calculated. An extensive parametric study was conducted and the most important outcomes can be summarised as follows.

- Increasing the concentration of nanoparticles results in thickening the thermal boundary layer around the cylinder. However, thermal boundary layer was observed to become significantly thinner by increasing the Reynolds number.
- It was shown that the functional form of transpiration rate can majorly change the radial and angular distributions of nanofluid flow temperature and hence can modify the Nusselt number and entropy generation distributions significantly.
- Non-uniform transpirations were found to be highly influential upon the value of the local Nusselt number. However, their effects on the dimensionless shear stress is considerably smaller.

- Nusselt number was found to be mostly indifferent to the type of nanoparticles. However, the use of TiO_2 nanoparticles was shown to produce a noticeably higher Nusselt number.
- Increasing the concentration of nanoparticles intensifies the irreversibilities noticeably. However, its effect on Bejan number is significantly smaller, indicating that the observed increase in entropy generation is majorly due to the flow irreversibilities.
- Reynolds number and the resultant changes in the thickness of the thermal and hydrodynamic boundary layers were shown to impart strong effects upon the total entropy generation and Bejan number.

In addition to offering physical insight, the results of this study can be used for the validation of future computational and theoretical works.

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References

[1] K.Vafai(Ed.), “Handbook of porous media,” CRC Press, 2015.

[2] O. Mahian et al., “A review of entropy generation in nanofluid flow,” Int.J. Heat Mass Transf., vol. 65, pp. 514-532, 2013.

[3] R. A. Mahdi, H. A. Mohammed, K. M. Munisamy, and N. H. Saeid, “Review of convection heat transfer and fluid flow in porous media with nanofluid,” Renew. Sust. Energ. Rev., vol.41, pp. 715-734, 2015.

[4] A. Kasaeian et al., “Nanofluid flow and heat transfer in porous media: a review of the latest developments,” Int.J. Heat Mass Transf., vol. 107, pp. 778-791, 2017.

[5] R. Alizadeh, A. B. Rahimi, N. Karimi, and A. Alizadeh, “On the hydrodynamics and heat convection of an impinging external flow upon a cylinder with transpiration and embedded in a porous medium,” Transport Porous Med., vol. 120, no. 3, pp. 579-604, 2017.

[6] C. Dickson, M. Torabi, N. Karimi, “First and second law analysis of nanofluid convection through a porous channel-The effects of partial filling and internal heat sources,” Appl. Therm. Eng., vol, 103, pp. 459-480, 2016.

[7] M. Torabi, C. Dickson, N. Karimi, “Theoretical investigation of entropy generation and heat transfer by forced convection of copper-water nanofluid in a porous channel- Local thermal non-equilibrium and partial filling effects,” Powder Technol., vol. 301, pp. 234-254, 2016.

[8] M. H. Saidi, H. Tamim, “Heat transfer and pressure drop characteristics of nanofluid inunsteady squeezing flow between rotating porous disks considering the effects of thermophoresis and Brownian motion,” Adv. Powder Technol., vol. 27, no. 2, pp. 564-574, 2016.

[9] M. M. Rashidi, S. Mahmud, N. Freidoonimehr, B. Rostami, “Analysis of entropy generation in an MHD flow over a rotating porous disk with variable physical properties,” Int.J. Exergy., vol. 16, no. 4, pp. 481-503, 2015.

[10] N. Bachok, A. Ishak, I. Pop, “Flow and heat transfer over a rotating porous disk in a nanofluid,” Phys. B., vol. 406, no. 9, pp. 1767-1772, 2011.

[11] M. Hatami, M. Sheikholeslami, D. D. Ganji, “Laminar flow and heat transfer of nanofluid between contracting and rotating disks by least square method,” Powder Technol., vol. 253, pp. 769-779, 2014.

[12] M. Hosseini, E. Mohammadian, M. Shirvani, S. N. Mirzababaei, and F. S. Aski, “Thermal analysis of rotating system with porous plate using nanofluid,” Powder Technol., vol. 254, pp. 563-571, 2014.

[13] M. Khazayinejad, M. Hatami, D. Jing, M. Khaki, and G. Domairry, “Boundary layer flow analysis of a nanofluid past a porous moving semi-infinite flat plate by optimal collocation method,” Powder Technol., vol. 301, pp. 34-43, 2016.

[14] Q. Wu, S. Weinbaum, Y. Andreopoulos, “Stagnation-point flows in a porous medium,” Chem. Eng. Sci., vol. 60, no.1, pp. 123-134, 2005.

[15] T. M. Jeng, S. C. Tzeng, “Numerical study of confined slot jet impinging on porous metallic foam heat sink,” Int.J. Heat Mass Transf., vol. 48, no. 23-24, pp. 4685-4694, 2005.

[16] T. M. Jeng, S. C. Tzeng, “Experimental study of forced convection in metallic porous block subject to a confined slot jet,” Int.J. Therm. Sci., vol. 46, no. 12, pp. 1242-1250, 2007.

[17] K. C. Wong, N. H. Saeid, “Numerical study of mixed convection on jet impingement cooling in a horizontal porous layer-using Brinkman-extended Darcy model,” Int.J. Therm. Sci., vol. 48, no. 1, pp. 96-104, 2009.

[18] Wong, K. C., Saeid, N. H. “Numerical study of mixed convection on jet impingement cooling in a horizontal porous layer under local thermal non-equilibrium conditions,” Int.J. Therm. Sci., vol. 48, no. 5, pp. 860-870, 2009.

[19] S. D. Harris, D. B. Ingham, I. Pop, “Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip,” Transport Porous Med., vol. 77, no. 2, pp. 267-285, 2009.

[20] A. Sivasamy, V. Selladurai, P. R. Kanna, "Mixed convection on jet impingement cooling of a constant heat flux horizontal porous layer," Int.J. Therm. Sci., vol. 49, no. 7, pp. 1238-1246, 2010.

[21] M. A. Kokubun, F. F. Fachini, "An analytical approach for a Hiemenz flow in a porous medium with heat exchange," Int.J. Heat Mass Transf., vol. 54, no. 15-16, pp. 3613-3621, 2011.

[22] S. S. Feng, J. J. Kuang, T. Wen, T. J. Lu, K. Ichimiya, "An experimental and numerical study of finned metal foam heat sinks under impinging air jet cooling. Int.J. Heat Mass Transf., vol. 77, pp. 1063-1074, 2014.

[23] B. Buonomo, G. Lauriat, O. Manca, S. Nardini, "Numerical investigation on laminar slot-jet impinging in a confined porous medium in local thermal non-equilibrium," Int.J. Heat Mass Transf., vol. 98, pp. 484-492, 2016.

[24] B. A. K. Abu-Hijleh, "Entropy generation due to cross-flow heat transfer from a cylinder covered with an orthotropic porous layer," Heat mass transfer., vol. 39, pp. 27-40, 2002.

[25] M. M. Rashidi, N. Freidoonimehr, "Analysis of entropy generation in MHD stagnation-point flow in porous media with heat transfer," Int. J. Comput. Meth. Eng. Sci. Mech., vol. 15, no. 4, pp. 345-355, 2014.

[26] K. Hiemenz, "Die Grenzschicht an einem in den gleichformigenFlussigkeitsstromeingetauchtengeradenKreiszlynder," Dinglers Polytech. J., vol. 326, pp. 321-324, 1911.

[27] M. M. Rashidi, S. Abelman, N. F. Mehr, "Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid," Int.J. Heat Mass Transf., vol. 62, pp. 515-525, 2013.

[28] J. Qing, M. M. Bhatti, M. A. Abbas, M. M. Rashidi, and M. E. S. Ali, "Entropy generation on MHD Cassonnanofluid flow over a porous stretching/shrinking surface," Entropy., vol.18, no. 4, pp. 123, 2016,

[29] N. Freidoonimehr, A. B. Rahimi, "Exact-solution of entropy generation for MHD nanofluid flow induced by a stretching/shrinking sheet with transpiration: Dual solution," Adv. Powder Technol., vol. 28, no. 2, pp. 671-685, 2017.

[30] M. M. Bhatti, M. M. Rashidi, "Numerical simulation of entropy generation on MHD nanofluid towards a stagnation point flow over a stretching surface," Int. J. Appl. Comput. Math., vol. 3, no. 3, pp. 2275-2289, 2017.

[31] , A. Kasaeian et al., "Nanofluid flow and heat transfer in porous media: a review of the latest developments," Int.J. Heat Mass Transf., vol. 107, pp. 778-791, 2017.

[32] R. Alizadeh, A.B. Rahimi, M. Najafi, "Unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid on a moving cylinder with time-dependent axial velocity," J. Braz. Soc. Mech. Sci. & Eng., vol. 38, no. 1, pp. 85-98, 2016.

[33] R. Alizadeh, A.B. Rahimi, R. Arjmandzadeh, M. Najafi, and A. Alizadeh, "Unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid with variable viscosity on a cylinder in constant heat flux," Alexandria Eng. J., vol. 55, no. 2, pp. 1271-1283, 2016.

[34] R. Alizadeh, A.B. Rahimi, M. Najafi, “Magnetohydrodynamic unaxisymmetric stagnation-point flow and heat transfer of a viscous fluid on a stationary cylinder,” Alexandria Eng. J., vol. 126, no. 6, pp. 997-1005, 2004.

[35] H. R. Ashorynejad, M. Sheikholeslami, I. Pop, and D. D. Ganji, “Nanofluid flow and heat transfer due to a stretching cylinder in the presence of magnetic field,” Heat mass transfer., vol. 49, no. 3, pp. 427-436, 2013.

[36] G. M. Cunning, A. M. J. Davis, P. D. Weidman, “Radial Stagnation Flow on a Rotating Cylinder with Uniform Transpiration,” J. Eng. Math., vol. 33, no. 2, pp. 113-128, 1998.

[37] R. Saleh, A.B. Rahimi, “Axisymmetric Stagnation-Point Flow and Heat Transfer of a Viscous Fluid on a Moving Cylinder with Time- Dependent Axial Velocity and Uniform Transpiration,” J. Fluid Eng., vol. 126, no. 6, pp. 997-1005, 2004.

[38] J. W. Thomas, “Numerical partial differential equations: finite difference methods,” Springer Science & Business Media, vol. 22, 2013.

[39] P. Ganesan, G. Palani, “Finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux,” Int.J. Heat Mass Transf., vol. 47, no. 19-20, pp. 4449-4457, 2004.

[40] M. Torabi, N. Karimi, G. P. Peterson, S. Yee, “Challenges and progress on modeling of entropy generation in porous media: A review,” Int.J. Heat Mass Transf., vol. 114, pp. 31-46, 2017.

[41] M. Torabi, K. Zhang, N. Karimi, G. P. Peterson, “Entropy generation in thermal systems with solid structures-a concise review,” Int.J. Heat Mass Transf., vol. 97, pp. 917-931, 2016.

[42] T.L. Bergman, P. Frank, “Incropera. Fundamentals of Heat and Mass Transfer,” John Wiley & Sons, 2011.

[43] W.M Deen, “Analysis of Transport Phenomena,” Oxford University Press, 1998.