Fuzzy Rule-Based Models of Predator/Prey Interactions

A Preliminary Study

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1. Introduction

Behavioural patterns associated with natural predator/prey interactions often comprise complex sequences of distinct behaviour. Interpretation of this *global* behaviour and identification of the determining factors forms the basis of much experimental effort (see, for example, Young & Taylor [1988]). While the mechanisms leading to the transition from one type of behaviour to another are difficult to assess, the nature of the latter stages of pursuit/evasion are dependent, primarily, on the *local* behavioural characteristics of both predator and prey. These characteristics are often associated with particular objectives or goals and the outcomes of interactions are influenced, to a significant degree, by the individual sensing and performance capabilities of the predator and prey.

Attempts to model such local interactions have focused on simple time-evolving deterministic representations of the predator/prey kinematics and detection processes (Weihs & Webb [1984]). Although models of this kind provide some insight to the factors which influence the outcome of predator/prey interactions, such models are unrepresentative of natural interactions in which the principal processes are event driven and may be probabilistic or known only imprecisely. Furthermore, purely deterministic models are not easily reconciled with experimental observations in which wide variability of data is present.

Efforts to model analogous pursuit/evasion problems in other fields have met with similar difficulties (Baron *et al.* [1970]). To circumvent these deficiencies, a more robust *stochastic* modelling approach based on state-increment dynamic programming is adopted in which the system description is absorbed into a discrete sequence of controllable transition probabilities representing the event-dependent progression from one system state to another under the action of a control (*cf.* Eaton & Zadeh [1962]). More recently, this approach has been extended to accommodate *fuzzy* system descriptions in which the system dynamic and state/control environment are defined imprecisely (Baldwin & Pilsworth [1982], Yoshida [1994]). In conjunction with robust procedures for the construction of fuzzy system descriptions (Yager & Filev [1994]), this approach offers the potential for the introduction of more representative models of natural predator/prey interactions while maintaining simple heuristic descriptions of the constituent processes. Additionally, the fuzzy system description provides a convenient basis for the identification from input/output data of rules governing internal processes

The aim of the present work is to outline a fuzzy modelling methodology applicable to local descriptions of a class of predator/prey interaction problems and to indicate how such a methodology might be used to test hypotheses on the internal control actions and sensing mechanisms adopted by predator and/or prey. The work concludes with a detailed example of fuzzy modelling procedures applied to a simplified pursuit/evasion problem.

2. Template-Based Fuzzy Modelling of Dynamical Systems

2.1 Fuzzy Decomposition of the State/Control Space

In the template-based approach to fuzzy system modelling, the model structure is pre-assigned on the basis of expert knowledge. The state space, $X = X_1 \times \ldots \times X_n$, and control space, $U = U_1 \times \ldots \times U_m$, are assumed to be partitioned into a finite number of fuzzy regions such that

$$X = \bigcup_i X^{(i)} \quad , \quad U = \bigcup_i U^{(i)}$$

where $X^{(i)}$, $U^{(i)}$ represent appropriate Cartesian products of templates in X and U. Each of the template fuzzy subsets, $X_1^{(i)}, \ldots, X_n^{(i)}$, $U_1^{(i)}, \ldots, U_m^{(i)}$, is associated with the *linguistic* label, $Lx_1^{(i)}, \ldots, Lx_n^{(i)}$, $Lu_1^{(i)}, \ldots, Lu_m^{(i)}$, respectively.

In general, each of the template fuzzy subsets is associated with a *membership function* , $\chi_{x_j^{(i)}}(x_j), \chi_{u_k^{(i)}}(u_k) \in [0,1]$, indicating the degree to which any state or control component is a member of the respective set.

2.2 Fuzzy System Model

The partitioning of the state and control space forms the template used to construct the rule-base representing the fuzzy system model. In general, the rule-base may be expressed as a collection of rules of the form

IF <u>current state</u> is $\mathbf{Lx}^{(i)}$ AND <u>control</u> is $\mathbf{Lu}^{(i)}$
THEN <u>transition state</u> is $L\bar{\mathbf{x}}^{(i)}$

where $\mathbf{Lx}^{(i)} = Lx_1^{(i)} \times \ldots \times Lx_n^{(i)}$ and $\mathbf{Lu}^{(i)} = Lu_1^{(i)} \times \ldots \times Lu_m^{(i)}$.

Each rule defines a fuzzy relation, $R^{(i)}$, on $X \times U \times X$ and the aggregation of individual fuzzy relations, $R = \bigcup_i R^{(i)}$, defines the *fuzzy system model.*

The membership function of the aggregate fuzzy relation is denoted by $\chi_R(u, x; \bar{x})$.

If $\mathbf{x} \subset X$ denotes the fuzzy current state with membership function $\chi_{\mathbf{x}}(x)$, then under the action of the fuzzy control $\mathbf{u} \subset U$ with membership function $\chi_{\mathbf{u}}(u)$, the fuzzy transition state $\bar{\mathbf{x}} \subset X$ is characterized by the membership function $\chi_{\bar{\mathbf{x}}}$ defined by the max-min inference rule (Yager & Filev [1994])

$$\chi_{\bar{\mathbf{x}}}(\bar{x}) = \vee_{(x,u)\in X\times U}\left(\chi_{(\mathbf{x},\mathbf{u})}(x,u) \wedge \chi_R(u, x; \bar{x})\right)$$

where

$$\chi_{(\mathbf{x},\mathbf{u})}(x,u) = \chi_{\mathbf{x}}(x) \wedge \chi_{\mathbf{u}}(u)$$

and $\vee$, $\wedge$ denote max, min operators, respectively.

A computationally efficient algorithm for the determination of the transition state membership function is defined by

$$\chi_{\bar{x}_j}(\bar{x}_j) = \vee_i \tau_i \wedge \chi_{L\bar{x}_j^{(i)}}(\bar{x}_j)$$

where

$$\tau_i = \left[\vee_{x_1}\left(\chi_{Lx_1^{(i)}}(x_1) \wedge \chi_{x_1}(x_1)\right)\right] \wedge \ldots \wedge \left[\vee_{x_n}\left(\chi_{Lx_n^{(i)}}(x_n) \wedge \chi_{x_n}(x_n)\right)\right]$$
$$\wedge \left[\vee_{u_1}\left(\chi_{Lu_1^{(i)}}(u_1) \wedge \chi_{u_1}(u_1)\right)\right] \wedge \ldots \wedge \left[\vee_{u_m}\left(\chi_{Lu_m^{(i)}}(u_m) \wedge \chi_{u_m}(u_m)\right)\right]$$

2.3 Fuzzy Decision Processes

A multi-stage decision process is considered in which the fuzzy control sequence $\underline{\mathbf{u}} = (\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{T-1})$ leads, via the transition mapping, to the fuzzy final state $\mathbf{x}_T$.

Fuzzy constraints, defined by the membership function sequence $\{\chi_0, \chi_1, \ldots, \chi_{T-1}\}$, are assumed on the controls u_i, $0 \le i \le T-1$. Also, a fuzzy goal constraint, defined by the membership function $\hat{\chi}_T$, is imposed on the final state x_T.

A measure of the degree to which these constraints are satisfied is provided by the *truth function*

$$T(\underline{\mathbf{u}}\mathbf{R}\mathbf{x}_T) = \chi_{\underline{\mathbf{u}}} \circ \mathbf{R} \circ \chi_{\mathbf{x}_T}$$

where $\circ$ denotes the max-min composition and the relational matrix $\mathbf{R}$ is defined as

$$\mathbf{R}(u_0, u_1, \ldots, u_{T-1}, x_T) = \chi_0(u_0) \wedge \chi_1(u_1) \wedge \ldots \wedge \chi_{T-1}(u_{T-1}) \wedge \hat{\chi}_T(x_T)$$

It is easily verified (Baldwin & Pilsworth [1982]) that

$$T(\underline{\mathbf{u}}\mathbf{R}\mathbf{x}_T) = (\chi_0 \circ \chi_{\mathbf{u}_0}) \wedge (\chi_1 \circ \chi_{\mathbf{u}_1}) \wedge \ldots \wedge (\chi_{T-1} \circ \chi_{\mathbf{u}_{T-1}}) \wedge (\hat{\chi}_T \circ \chi_{\mathbf{x}_T})$$

The optimal fuzzy decision sequence is defined as the control sequence which maximizes the truth function and is obtained via dynamic programming.

That is, with $S_k(\mathbf{x}_k)$ defined as

$$S_k(\mathbf{x}_k) = \max_{\mathbf{u}_k \ldots \mathbf{u}_{T-1}} T(\mathbf{u}_k \ldots \mathbf{u}_{T-1}\mathbf{R}\mathbf{x}_T)$$

the optimal decision sequence is determined from

$$S_k(\mathbf{x}_k) = \max_{\mathbf{u}_k}\left[(\chi_k \circ \chi_{\mathbf{u}_k}) \wedge S_{k+1}(\mathbf{x}_{k+1})\right]$$

$$k = T-1, T-2, \ldots, 1, 0$$

subject to the terminal condition

$$S_T(\mathbf{x}_T) = (\hat{\chi}_T \circ \chi_{\mathbf{x}_T})$$

Here, $\mathbf{x}_{k+1}$ is the fuzzy state resulting from the fuzzy control action $\mathbf{u}_k$ when the system is in state $\mathbf{x}_k$. Evaluation of $S_{k+1}(\mathbf{x}_{k+1})$ is accomplished by means of a *fuzzy interpolation* procedure (Baldwin & Pilsworth [1982]).

3. Fuzzy Models of Predator/Prey Interactions

The *local* characteristics of predator/prey interactions are assumed to be defined via a finite-dimensional *fuzzy* state-increment model of the kinematics and detection and control processes. Such a model consists of a set of rules defined on fuzzy subsets of the state/control space which describe the fuzzy transition state of the system for given fuzzy values of the current state and control action.

In the context of natural predator/prey interactions, the decomposition of the state space into a finite number of fuzzy regions corresponds to fuzzy event partitions derived from *a priori* knowledge of the detection envelope and detection resolution of predator and/or prey. The fuzzy event partitions are presumed to initiate a change of control action. The decomposition of the control space into fuzzy partitions is accomplished via vague descriptors of the possible predator/prey control actions.

The system state, $x(t) \in X$, is assumed to be a composite state defining the state of the predator and prey, and the control, $u(t) \in U$, is a composite control comprising elective predator/prey manoeuvres. The predator/prey state variables and control actions are assumed to be resolved only to within a fuzzy subset of the state/control space. Prescribed pursuit/evasion rules are incorporated, implicitly, in the definition of the model while elective manoeuvres are selected to optimize some pre-defined objective over a fixed (finite) horizon.

To illustrate the rule-based modelling methodology in the context of predator/prey interactions, a simple prey evasion/avoidance problem is described in which a *generic* decomposition of state/control space is employed.

3.1 Kinematic State/Control Variables

The principal kinematic variables in a predator centred co-ordinate system are illustrated in Fig. 1. Motion is assumed restricted to the plane Oxy. Here, r is the relative prey range, ξ is the azimuth of the prey relative to the predator, θ is the orientation of the prey relative to the predator, and ψ is the prey heading relative to the prey orientation. The (variable) speeds of predator and prey are denoted V_a and V_e, respectively.

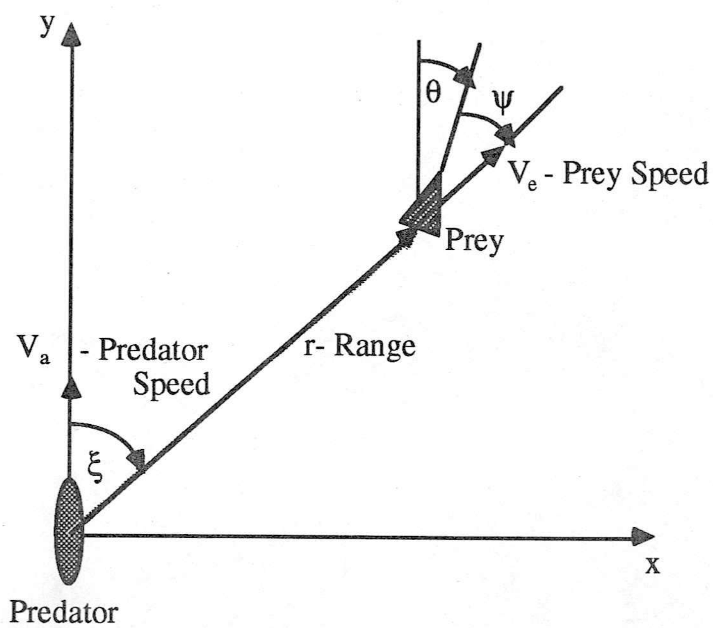

Figure 1

In prey-centred co-ordinates $(r, \xi, \theta) \rightarrow (r', \xi', \theta')$ (see Fig. 2). An appropriate set of independent state variables is defined by (r', ξ', ξ, V_r) where V_r is the speed of the prey relative to the predator. The inclusion of ξ as a state variable allows for a convenient description of the detection envelope of the predator. The set of control variables comprises the prey acceleration, $\dot{V}_e$, and prey heading, ψ, while the prescribed pursuit rule is defined in terms of the predator speed, V_a, and instantaneous turn rate, $\dot{\theta}'$.

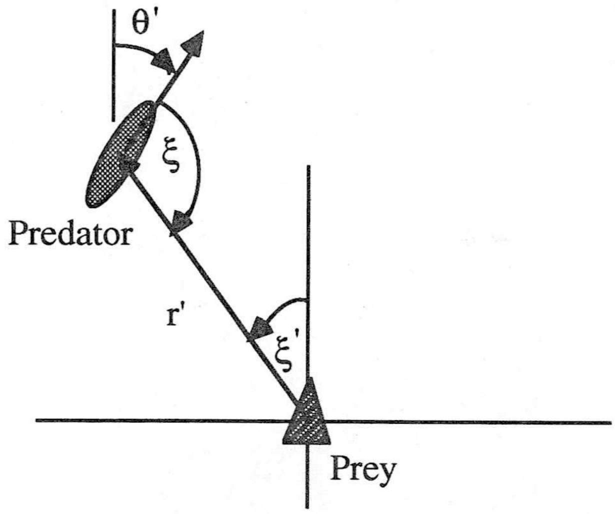

Figure 2

3.2 Decomposition of State/Control Space

The fuzzy model structure is assigned on the basis of 'expert' knowledge of the sensing, kinematic and other behavioural characteristics of the predator and prey. These include predator and prey detection envelopes, detection resolutions, relative speed ranges of predator and prey, manoeuvre ranges, and prescribed pursuit/evasion rules.

To facilitate application of the proposed modelling methodology to a broad class of predator/prey interactions in which only limited details of the detection mechanisms and kinematic characteristics of predator and prey are available, the structure of the state/control space is based on a generic decomposition of the principal spatial and kinematic variables. This decomposition implicitly defines the event partitions which initiate a (change of) control action. In general, the control actions of predator and prey are affected asynchronously.

Details of the decomposition and parameterization of state/control space appropriate to the restricted prey evasion/avoidance problem considered here are summarized in Table 1 and Table 2.

Table 1

State Space			
Variable	**Subset**	**Linguistic Label**	**Membership Function**
r'	$r'_{\min} \le r' \le r'_1$	Near Field (NF)	$\chi_{r'(NF)}$
	$r'_2 \le r' \le r'_{\max}$	Far Field (FF)	$\chi_{r'(FF)}$
ξ'	$-\xi'_1 \le \xi' \le \xi'_1$	Centre Field (CF)	$\chi_{\xi'(CF)}$
	$-\pi \le \xi' \le 0$	Left Field (LF)	$\chi_{\xi'(LF)}$
	$0 \le \xi' \le \pi$	Right Field (RF)	$\chi_{\xi'(RF)}$
ξ	$-\xi_1 \le \xi \le \xi_1$	Line of Sight (LoS)	$\chi_{\xi(LoS)}$
	$-\pi \le \xi \le -\xi_2$ $\xi_2 \le \xi \le \pi$	No Line of Sight (NL)	$\chi_{\xi(NL)}$
V_r	$0 \le V_r \le 1$	Slow (S)	$\chi_{V_r(S)}$
	$1-(V_{r_1}-1) \le V_r \le V_{r_1}$	Relative(R)	$\chi_{V_r(R)}$
	$1 \le V_r \le V_{r_{\max}}$	Fast (F)	$\chi_{V_r(F)}$

Table 2

Control Space			
Variable	**Subset**	**Linguistic Label**	**Membership Function**
$\dot{V}_e$	$0 \le \dot{V}_e \le \dot{V}_{e_1}$	Low (Lo)	$\chi_{\dot{V}_e(Lo)}$
	$0 \le \dot{V}_e \le \dot{V}_{e_{\max}}$	High (Hi)	$\chi_{\dot{V}_e(Hi)}$
ψ	$-\pi/4 \le \psi \le \pi/4$	Forward (Q0)	$\chi_{\psi(Q0)}$
	$0 \le \psi \le \pi/2$	Quadrant 1 (Q1)	$\chi_{\psi(Q1)}$
	$\pi/2 \le \psi \le \pi$	Quadrant 2 (Q2)	$\chi_{\psi(Q2)}$
	$\pi \le \psi \le 3\pi/2$	Quadrant 3 (Q3)	$\chi_{\psi(Q3)}$
	$3\pi/2 \le \psi \le 2\pi$	Quadrant 4 (Q4)	$\chi_{\psi(Q4)}$

3.3 Rule-Base Tableau

The rule-base is constructed in accordance with known predator/prey kinematic behaviour and pre-defined predator pursuit rules. Where explicit (deterministic) forms of the kinematic relations are available, construction of the rule-base is achieved via direct simulation*.

Table 3 illustrates the format of the matrix of rules associated with the decomposition of state/control space defined in §3.2.

Table 3

$Lx^{(i)}$ / $Lu^{(i)}$	Lo×Q0	Hi×Q0	Lo×Q1	...	Lo×Q4	Hi×Q4
NF×CF×LoS×S						
NF×LF×LoS×S						
NF×RF×LoS×S						
FF×CF×LoS×S						
FF×LF×LoS×S						
FF×RF×NL×S						
NF×CF×NL×S						
NF×LF×NL×S						
NF×RF×NL×S						
FF×CF×NL×S						
FF×LF×NL×S						
FF×RF×NL×S						
.						
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.						
NF×CF×NL×F						
NF×LF×NL×F						
NF×RF×NL×F						
FF×CF×NL×F						
FF×LF×NL×F						
FF×RF×NL×F						
	$L\bar{x}^{(i)}$					

* A simple procedure for the generation of the rule-base derives from fuzzification of the 'crisp' output of the deterministic kinematic equations to 'crisp' inputs corresponding to de-fuzzified values of the fuzzy input sets.

3.4 Fuzzy Control Strategy

Fuzzy control policies* $((\dot{V}_{e0}, \psi_0), (\dot{V}_{e1}, \psi_1), \ldots, (\dot{V}_{eT-1}, \psi_{T-1}))$for $T = 1, 2, 3, \ldots$ stage decision sequences are determined for prescribed fuzzy initial states $(r_0', \xi_0', \xi_0, V_{r0})$ and pre-defined constraints via fuzzy dynamic programming, as described in §2.3. The prey evasion/ avoidance strategy is expressed as a collection of rules on fuzzy sets corresponding to appropriate fuzzy initial states.

Goal constraints for evasion/avoidance are defined in terms of the capture/detection envelope of the predator as summarized in Table 4. Additionally, control constraints can be imposed at each decision stage (*e.g.* to account for energy depletion of the prey).

Table 4

Goal Constraints			
Variable	**Subset**	**Linguistic Label**	**Membership Function**
r_T'	$r'_{\text{capture}} \leq r_T' \leq r'_{\text{detection}}$	Evasion (Ev)	$\hat{\chi}_{r_T'(Ev)}$
ξ_T'	$-\pi \leq \xi_T' \leq \xi_1'$ $\xi_1' \leq \xi_T' \leq \pi$	Avoidance (Av)	$\hat{\chi}_{\xi_T'(Av)}$

The T stage manoeuvre strategy evaluated by dynamic programming is optimal in the sense that the truth function associated with the terminal (goal) constraints and control constraints is maximized. However, the resulting optimal manoeuvre strategy relates to a particular decomposition and parameterization of state/control space and to a particular rule-base. Consequently, the optimal manoeuvre strategy depends implicitly on the predator/prey detection mechanisms, kinematic characteristics and prescribed predator pursuit rules assigned to the model.

* At each stage, the control action comprises either a single manoeuvre or a presribed sequence of manoeuvres.

4. Conclusions and Recommendations for Further Research

Fuzzy system modelling techniques offer scope for more representative models of natural predator-prey interactions. In contrast to *classical* predator-prey models in which pre-defined goals are assumed unequivocally for predator and prey, and in which both protagonists are assumed to perceive environmental change in a precise and continuous manner, the fuzzy system model accommodates event-driven (or threshold) internal processes and imprecise sensory information. The fuzzy system description also provides a convenient basis for the identification and assessment of particular detection and control mechanisms via appropriate state/control decompositions and rule-bases.

A natural extension of the proposed rule-based modelling methodology is the characterization of each rule by its credibility; that is, the *possibility* that the output satisfies the rule consequent if the input satisfies the rule antecedent. The fuzzy decision process is then formulated in the context of a controllable Markov process (*cf.* Yoshida [1994]). This approach supports elimination of certain low credibility rules from the rule-base. The high order of the rule-base in the present application is a major deficiency of the modelling procedure. Moreover, the system attributes employed in the construction of the rule-base are essentially kinematic in origin. Formal methods for the reduction of a rule-base to a *minimal* rule-base are the focus of much current research effort in fuzzy modelling and it is expected that some of the techniques reported in the literature are applicable to the present problem, in which case an alternative set of system attributes may be appropriate.

Rule induction algorithms - based on statistical procedures in which relationships between system attributes and outcomes are formalized by a set of rules - provide an alternative route to the identification of a minimal rule-base. The application of rule induction techniques requires the availability of appropriate experimental or simulated data sets from which the underlying behavioural rules may be identified. Additionally, experimental or simulated data may be used to identify the structure and parameterization of the decomposition of state/control space thereby alleviating the need to prescribe the decomposition in advance.

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