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Performance Evaluation for Direction of Arrival Estimation Using 4-Element Linear Array

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***Abstract*—This paper presents the performance evaluation of Direction of Arrival (DOA) estimation techniques such as Root-MUSIC, Root-WSF and Beamspace-ESPRIT (BS-ESPRIT) algorithms under common Wi-Fi conditions such as operating in the IEEE radio frequency spectrum of 5 GHz band, the presence of noise and correlated, closely sourced signals. In addition, we will consider the computational time required for these algorithms to complete the estimation process. Results have shown that the 3 techniques provide a mean of 98.8% accuracy in detecting closely-sourced signals. Although Root-WSF has the longest average computational time of 36.7ms as opposed to BS-ESPRIT and Root-MUSIC at 29.4ms and 22.5ms respectively, Root-WSF technique performed best overall especially in detecting coherent signals with 99.78% accuracy as compared to BS-ESPRIT at 95.27% and Root-MUSIC at 64.77%.**

***Index Terms*—Antenna Array, Direction-Of-Arrival, Wi-Fi.**

I. INTRODUCTION

A. Overview & Motivation

Over the recent years, the direction of arrival (DOA) estimation has been considered a crucial field in signal processing with applications such as mobile communication, marine communication, space communication & localization, radar surveillance, vehicle auto-navigation & avoidance and medical diagnosis [4] [5] [17] [19]. Coupled with an antenna array and beamforming technology to form a smart antenna, these systems have gained a positive reputation due to its ability to have adaptive radiation pattern, high directivity gain as compared to a conventional omnidirectional antenna and multipath signal attenuation [1] [2] [15].

With Wi-Fi systems in the IEEE 802.11 protocol enabling smart cities and infrastructures, there is a need for the development of simplified smart antennas to cater to the exponential growth of wireless usage by improving the wireless transmission efficiency as discussed in [6] and [16]. Ruckus Wireless – a company that develops wireless Access Points (AP) has developed the “Beamflex+” adaptive antenna technology which enables antennas to adapt to client device orientation in addition to client device location. This is done by adapting the individual antenna's radiation patterns together with physical data rates which minimize interference and increase system capacity [25].

Some of the wireless infrastructure examples are wireless hotspot in crowded areas such as in shopping malls and in public transportation networks such as trains and buses. To that end, a robust and accurate DOA technique must be employed to cater to the high usage of a wireless network to reduce wireless traffic congestion and improve efficiency without sacrificing the data bandwidth as much as possible.

B. Existing Work & Progress for DOA Techniques

There has been an increase in demand for research for a DOA-enabled smart antenna system – typically in the transportation market. One example would be in [18] where a measurement campaign was carried out in high-speed train (HST) environment where the downlink (Access Point to Client) signals was deployed along an HST railway. The DOA were estimated based on the Space-Alternating Generalized Expectation-maximization (SAGE) principle by the construction of a virtual antenna array constructed based on the knowledge of the train speed by exploiting the characteristics of the Doppler frequency trajectory and least-square method. [18] only carried out the test in the third generation (3G) Universal Mobile Telecommunications System (UMTS) frequency band of 2-3 GHz Results demonstrated reasonable estimation using the SAGE principle. However, this technique is deemed complex as it requires parallel computation of geometrical parameters of the train, formation of a virtual antenna array and DOA algorithm application.

Another example would be in [20] which employs a 12-element Electronically Steerable Parasitic Array Radiator (ESPAR) antenna designed for DOA estimation as part of a railway control system. The DOA techniques employed was the cross-correlation and MUSIC algorithm. It was discovered in [20] that the technique employed in combination with ESPAR is susceptible to residual in-band interference which results in a broad and a less precise DOA estimation.

C. Proposed Solution

In this paper, we will focus on determining a suitable subspace-based DOA estimation technique which is applicable for the Wi-Fi frequency band with real-world scenarios which are the Root-MUSIC, Root-WSF and BS-ESPRIT techniques. It was shown in works of literature that subspace methods can evaluate DOA at a faster and accurate pace [17] [22]. Evaluation is based on high operating frequency performance corresponding to 5 GHz Wi-Fi band, closely-sourced signals, computational time and accuracy in a coherent condition.

It has been shown through simulation in this paper that the Root-WSF technique is the most applicable in terms of performance. However, further improvements to the Root-WSF technique needs to be implemented due to the higher computational time in comparison with the Beamspace-ESPRIT and Root-MUSIC technique.

D. Organisation Of This Paper

This paper is organized into 5 sections. Section II provides an overall view of the system design and the DOA algorithms. Section III provides an explanation of the evaluation criterion and why the criteria are important for Wi-Fi application. Section IV and V presents the simulation results and conclusion for future works respectively.

II. SMART ANTENNA DESIGN & DOA TECHNIQUES

A. System Design

In this section, a simplified 4-element Uniformed Linear Array (ULA) is defined for the smart antenna system. We assume that only azimuth information is required for the tracking of signal source with no elevation changes. As in real-world application in the implementation of a smart antenna system for a smart transportation market, there aren't much elevation changes. One example would be that the height between a mounted AP to the track always remains the same. Any slight deviation changes to the elevation can be countered by a wider main-lobe beamwidth.

It has been shown in [24] that the ULA geometry provides superior performance as compared to other arrays like Uniformed Rectangular Array (URA) and Uniformed Circular Array (UCA) especially in the smart infrastructure application such as the HST.

With reference from figure 1, the array elements are equally spaced by a distance d, and a plane wave arrives at the array from a direction of θ off the array broadside.

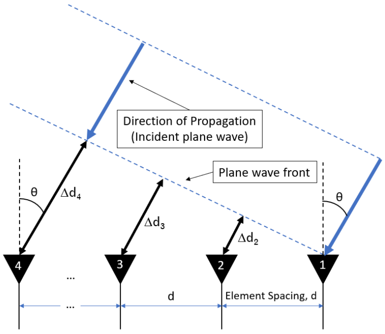

Figure 1: 4-Element ULA Model

As in [21], the total signals received by the antenna array elements is expressed as:

$$\boldsymbol{X}(t) = S(t) + \boldsymbol{N}(t) \tag{1}$$

Or in its compact vector form for a 4-element ULA as:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = S(t) \begin{bmatrix} 1 \\ e^{-j\emptyset} \\ e^{-j2\emptyset} \\ e^{-j3\emptyset} \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ n_4(t) \end{bmatrix} \tag{2}$$

Alternatively, the array output can be represented as:

$$\boldsymbol{x}(t) = \sum_{n=1}^{N} \boldsymbol{a}(\emptyset_k) s_k(t) + \boldsymbol{n}(t) \tag{3}$$

Where **x**(t) is the vector of signals received by the array antenna, N is the number of signal source, $s_k(t)$ is the signal emitted by the k^th source as received from the reference sensor, $\boldsymbol{a}(\emptyset_k)$ is the steering vector towards the direction of $\emptyset_k$ and **n**(t) is the noise vector.

B. DOA Estimation Algorithms

The primary objective of the DOA algorithms is to collect the incoming signal source data received from the antenna array and estimate the direction of the signal source relative to the array's location.

1) Root-MUSIC Algorithm

The Root-MUSIC technique is a modification to the MUSIC algorithm proposed by Schmidt [3] and is based on polynomial rooting from the noise subspace which provides higher angular resolution. However, it is only applicable to a uniformed linear array [14]. The expression of the Root-MUSIC technique as shown in [17] can be presented as:

$$P_{rootmusic}(\theta) = \frac{1}{|a(\theta)^H \boldsymbol{C} a(\theta)|} \tag{4}$$

Where $a(\theta)$ is the steering vector and **C** is a Hermitian matrix given as:

$$\boldsymbol{C} = \boldsymbol{U_n U_n^H} \tag{5}$$

Where $\boldsymbol{U_n}$ is the noise subspace. The poles of the MUSIC pseudospectrum is the corresponding roots that lie closest to the unit circle. With reference from [27], DOA is calculated by:

$$\theta_i = \sin^{-1}\left(\frac{Im(\log z_i)}{kd}\right) \tag{6}$$

Where, $z_i = e^{j\frac{2\pi}{\lambda}d\sin\theta}$ and $k = \frac{2\pi}{\lambda}$

2) Root-WSF Algorithm

Another technique that can be employed for DOA estimation would be the Root-weighted subspace fitting (Root-WSF) algorithm. With the capability to detect coherent signals such as in a multipath environment, this is one of the most sought out technique to employ – especially in a Wi-Fi environment where the multipath environment is common. However, it is important to note that this technique is iterative to obtain its accuracy and differentiate coherent signal source and therefore it is demanding in terms of computational complexity leading to a longer computational time. Details of the Root-WSF algorithm can be found in [7][8]. This technique has been chosen for this paper as past works of literature have proven that it provides good angular resolution and multipath attenuation. As analyzed by [13], it has been shown that the root-WSF technique provides good resolution performance.

3) Beamspace-ESPRIT Algorithm

The beamspace-ESPRIT algorithm is a method of reducing computational complexity which derives from the ESPRIT algorithm. This approach solves a subspace DOA estimation problem with reduced dimensions in beamspace.

The main drawback of this technique is that it always requires a priori knowledge of the sector where the signals are located to position the center of the output beam fan. More details regarding the Beamspace-ESPRIT algorithm can be found in [9] [10]. This technique was chosen for the analysis as it has been shown in past works of literature such as [26] presenting significant computational savings and performance.

III. EVALUATION CRITERION

In an actual wireless communication scenario, propagated signals are susceptible to changes due to the medium environment, signal correlation, multipath attenuation as well as noise. The evaluation criterion is modeled like [13] and [23]. For the performance evaluation of DOA algorithms, we set the following criteria represented as follows:

A. Low Frequency & High-Frequency Performance

Typically, in a 5 GHz Wi-Fi scenario, there is a list of channels that can be selected in accordance with the 802.11ac wave 2 protocol. The different channel will have different performance levels. For example, one channel may be noisier and/or congested than another. The frequency of a signal directly influences the DOA performance. At low frequency, it was proven that we can achieve good coherence and well-correlated signals while on high frequency, we can have bad coherence and uncorrelated signals.

B. Closely-Sourced Signal Performance

In real-world wireless communication environment, sources can be close to each other. For example, a user may be connected via Wi-Fi with their phone and laptop at the same time. Therefore, this evaluation criterion is necessary to determine the DOA algorithm with high precision and accuracy. For this part, the variance of each sample is taken which is expressed as:

$$V = \frac{1}{N-1}\sum_{i=1}^{N}|A_i - \mu|^2 \tag{7}$$

Where V is the variance value, N is the number of observation sample, A is the estimated DOA data and μ is the mean of A which is expressed as:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} A_i \tag{8}$$

C. Computational time

Computational time taken is crucial especially in real-world wireless communication scenario especially in a scenario where fast-tracking of the signal source is required. Theoretically, we want faster computational time without sacrificing angular resolution accuracy. However, in practice, this may not be the case as most of the time, DOA techniques are either accurate or fast – hardly both at the same time. This criterion will allow us to observe which technique is the fastest and correlate the results to accuracy to select the best one for real-world application.

D. Coherent Signal Source Performance

A coherent signal can be defined as a signal that shares similar properties with another signal source in terms of phase and magnitude. It can also be defined as a signal where there is a strong statistical correlation between samples.

An incoherent signal represents a signal where any sample correlation has been removed – or in other words, every sample taken is different than one another.

If several sources are coherent as in a multipath environment, the spatial covariance matrix becomes rank deficient and thus, subspace-based DOA estimation technique that relies on the covariance matrix will fail to localize signal source accurately. One of the common situations where coherent signals would occur in a Wi-Fi environment would be multipath attenuation [11] which must be addressed for accurate DOA estimation.

IV. SIMULATION RESULTS

To evaluate the performance of the DOA algorithms, we assume the following standard model which is based on [12]:

- Number of elements: 4
- Element spacing: 27.25mm
- Number of samples: 1024
- Noise Power: -20dBW/0.01W

For the number of signal source, it is assumed that this value is known a priori (number of clients does not change per detection). As for the array element spacing, it is assumed to be fixed as in real-world application and can perform across multiple Wi-Fi channels. In this case, element-spacing half wavelength of 5500 MHz operating frequency. Noise is assumed to be an Additive White Gaussian Noise (AWGN) at 0.01W/-20dBW power considering propagation and cable loss. All simulation ran for 500 times and tabulated as mean values across all iteration unless otherwise stated. The simulation was performed using Matlab R2018b.

A. Low frequency vs high frequency in 5 GHz Wi-Fi band

This section presents an evaluation of DOA algorithms with varying operating frequencies. For the simulation, we assume to use the 5 GHz Wi-Fi band regulated in Singapore which ranges from 5160 MHz to 5845 MHz. The results of the simulation can be found in table 1. The 3 DOA tested are -45°, 10° and 45°.

	Root-MUSIC			**Root-WSF**			**Beamspace-ESPRIT**		
	-45	10	45	-45	10	45	-45	10	45
5160 MHz	-45.01	10.01	45.141	-45.01	10.01	45.138	-45.1	9.98	45.115
5500 MHz	-45.01	10.01	45.115	-45.001	10.01	45.112	-45.06	9.997	45.103
5845 MHz	-45.001	10.007	45.09	-45.0004	10.0068	45.0882	-45.036	9.991	45.0942

Table 1: Varying Operating Frequency Performance

Based on the simulation results in the table, all 3 techniques can estimate the true direction of arrival. The Root-MUSIC and Root-WSF algorithms perform similarly at the lowest operating frequency. Both algorithms perform much better than the Beamspace-ESPRIT technique in terms of its resolution accuracy.

However, at higher operating frequency, we can see that the results deviate slightly between Root-MUSIC and Root-WSF. At high frequency, the root-WSF technique performs much better than Root-MUSIC and BS-ESPRIT.

B. Closely-Sourced Signals

Consider a scenario where the signal sources are closely spaced within one another. For this section, we assume the same standard model parameters. The operating frequency is fixed at 5500 MHz. In addition, we also assume that the array is receiving 2 signal sources at the same time. The 2 sources are spaced and simulated at 8° and 4° apart.

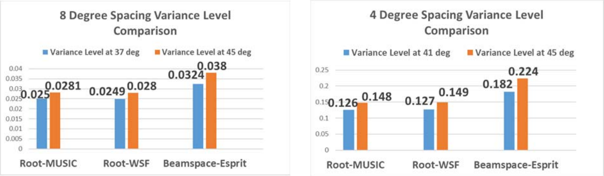

Figure 2 (top left) & 3 (top right): Variance level of 8 degrees spacing and 4 degrees spacing respectively

	Root-MUSIC		**Root-WSF**		**Beamspace-Esprit**	
True DOA	37	45	37	45	37	45
Spaced 8 degrees	37.349	44.934	37.342	44.942	37.279	44.879
Variance	0.025	0.0281	0.0249	0.028	0.0324	0.038

Table 2: 8 Degree Separation Performance

	Root-MUSIC		**Root-WSF**		**Beamspace-Esprit**	
True DOA	41	45	41	45	41	45
Spaced 4 degrees	41.912	45.016	41.875	45.051	41.772	44.872
Variance	0.126	0.148	0.127	0.149	0.182	0.224

Table 3: 4 Degree Separation

Based on the simulation results, we observe that all techniques can estimate sources that are closely spaced apart. Based on the variance results, we can see that the Root-WSF technique performs slightly better at 8 degrees spacing while the Root-MUSIC technique performs better at 4-degree spacing at a small margin. The BS-ESPRIT technique performs worst overall in both cases as compared to Root-MUSIC and Root-WSF.

C. Computational time performance

In this section, the computational time performance is being evaluated across the 3 DOA techniques. For this scenario, we assume the same standard model with an operating frequency of 5500 MHz. In addition, the array is only receiving 1 signal source.

	Root-MUSIC	**Root-WSF**	**Beamspace-Esprit**
Computational Time	22.5ms	36.7ms	29.4ms

Table 4: Computation Time Performance

Based on the simulation results tabulated in the table above, we can see that Root-MUSIC is the fastest in terms of computational time, followed by the BS-ESPRIT and the Root-WSF technique. This is in line with the theory that accuracy comes at the expense of computational time. This is since the Root-WSF technique is iterative in obtaining the DOA and therefore the most computational complex. However, in almost all cases, the Root-WSF technique provides the best overall performance in terms of accuracy.

D. Coherent Signal Source Performance

If several sources are coherent such as in terms of a multipath scenario, the spatial covariance matrix will become rank deficient and will cause subspace-based DOA estimation technique to fail. To ensure robustness, DOA techniques must be able to perform in a coherent environment.

In this coherent signal source scenario, we assume that there are 2 signal sources with a true DOA of 20° and 45°. We employ the same standard model as before with an operating frequency of 5500 MHz. To simulate a multipath environment, we assume that 1 of the 2 signals are the multipath reflection of the first source with a magnitude of ¼ of that first source.

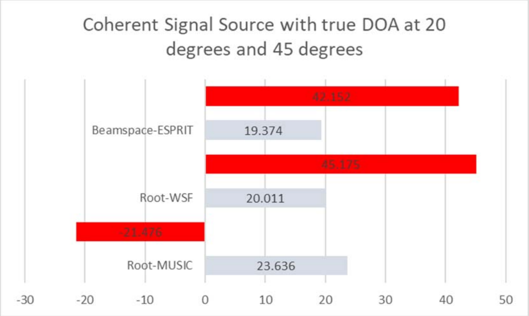

Figure 4: DOA Detection in Coherent Signal Environment

	Root-MUSIC		**Root-WSF**		**Beamspace-ESPRIT**	
	20	45	20	45	20	45
Coherent Signal Source	23.636	-21.476	20.011	45.175	19.374	42.152

Table 5: Coherent Signal Source Performance

With reference from figure 4 and table 5, we can see that the Root-WSF algorithm performs the best as compared to the Root-MUSIC and BS-ESPRIT technique with a mean accuracy of 99.78% as compared to 64.77% and 95.27% respectively. We can observe in the context of correlated signals, Root-MUSIC will fail. This is because, for the Root-MUSIC and all the derivatives of the MUSIC algorithm, it can only estimate if sources are uncorrelated which is true in accordance with literature studies.

V. CONCLUSION & FUTURE WORK

The main objective of this paper is to find the best DOA algorithm that performs well under Wi-Fi condition using a simplified ULA array. This will lead to optimal performance with lower manufacturing costs in fabricating the hardware required for the smart antenna to be used in a Wi-Fi environment.

According to the performance analysis and evaluation carried out in this paper, we can conclude that the best overall performing DOA estimation algorithm would be the Root-WSF algorithm with respect to the performance criterion. It has the capability to detect signals accurately in a multipath environment, optimal performance across the 5 GHz Wi-Fi frequency band as well as performing well in a closely-source signal environment which is crucial in a Wi-Fi application.

Although the computational time is longer as compared to the Root-MUSIC and BS-ESPRIT algorithm, the benefit outweighs the disadvantage of having a slightly longer computational time. This can be resolved by adaptively changing the iteration with just enough cycles to obtain the most accurate DOA estimation.

As part of future work, we will explore the hardware implementation of the DOA algorithm in Wi-Fi condition. This will provide more realistic results as compared to the simulation when actual noise is a significant factor. In addition, we will also explore the possibility of improving the Root-WSF algorithm by automatically controlling the iterative step which is causing longer computational time.

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REFERENCES

[1] Allen, B., & Ghavami, M. (2005). *Adaptive array systems: Fundamentals and Applications*, *2nd Ed.* Chichester, England: Wiley.

[2] Monzingo, R. A., Haupt, R. L., & Miller, T. W. (2011). *Introduction to adaptive arrays.* Raleigh, NC: SciTech Pub.

[3] R. Schmidt, "Multiple emitter location and signal parameter estimation," in *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276-280, Mar 1986.

[4] C. Uysal and T. Filik, "Contactless respiration rate estimation using MUSIC algorithm," *2017 10th International Conference on Electrical and Electronics Engineering (ELECO)*, Bursa, 2017, pp. 606-610.

[5] Â. M. C. R. Borzino, J. A. Apolinário, and M. L. R. de Campos, "Robust DOA estimation of heavily noisy gunshot signals," *2015 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, South Brisbane, QLD, 2015, pp. 449-453.

[6] M. Yanovsky, A. Gorbenko and V. Kharchenko, "Adaptive WiFi systems: Principles of design and application support," *The International Conference on Digital Technologies 2013*, Zilina, 2013, pp. 203-206.

[7] M. Jansson, A. L. Swindlehurst and B. Ottersten, "Weighted subspace fitting for general array error models," in *IEEE Transactions on Signal Processing*, vol. 46, no. 9, pp. 2484-2498

[8] M. Viberg, B. Ottenten and T. Kailath, "Direction-of-arrival estimation and detection using weighted subspace fitting," *Twenty-Third Asilomar Conference on Signals, Systems and Computers, 1989.*, Pacific Grove, California, USA, 1989, pp. 604-608.

[9] Guanghan Xu, S. D. Silverstein, R. H. Roy and T. Kailath, "Beamspace ESPRIT," in *IEEE Transactions on Signal Processing*, vol. 42, no. 2, pp. 349-356, Feb 1994.

[10] T. N. Ferreira, S. L. Netto and P. S. R. Diniz, "Beamspace covariance-based DOA estimation," *2008 IEEE 9th Workshop on Signal Processing Advances in Wireless Communications*, Recife, 2008, pp. 136-140.

[11] L. Cikovskis and I. Slaidins, "Smart antennas for multi-path routing in ad-hoc wireless networks," *2017 Advances in Wireless and Optical Communications (RTUWO)*, Riga, 2017, pp. 268-271.

[12] M. C. Tan, Minghui Li, Qammer H. Abbasi1, Muhammad Imran, "A Smart and Low-cost Enhanced Antenna System for Industrial Wireless Broadband Communication" 12th European Conference on Antenna and Propagation (EUCAP), 2018.

[13] B. Boustani, A. Baghdad, A. Sahel, A. Ballouk and A. Badr, "Performance analysis of direction of arrival estimation under hard condition," *2018 4th International Conference on Optimization and Applications (ICOA)*, Mohammedia, 2018, pp. 1-5.

[14] B. D. Rao and K. V. S. Hari, "Performance Analysis Of Root-MUSIC/sup */," *Twenty-Second Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, 1988, pp. 578-582.

[15] M. F. Ünlerşen and E. Yaldiz, "Direction of Arrival Estimation by Using Artificial Neural Networks," *2016 European Modelling Symposium (EMS)*, Pisa, 2016, pp. 242-245.

[16] M. Seufert, C. Moldovan, V. Burger and T. HoBfeld, "Applicability and limitations of a simple WiFi hotspot model for cities," *2017 13th International Conference on Network and Service Management (CNSM)*, Tokyo, 2017, pp. 1-7.

[17] R. K, H. F and M. M A, "A comprehensive analysis and performance evaluation of different direction of arrival estimation algorithms," *2012 IEEE Symposium on Computers & Informatics (ISCI)*, Penang, 2012, pp. 256-259.

[18] Xuesong Cai, Xuefeng Yin and A. P. Yuste, "Direction-of-arrival estimation using single antenna in high-speed-train environments," *2016 10th European Conference on Antennas and Propagation (EuCAP)*, Davos, 2016, pp. 1-4.

[19] D. M. Vijayan and S. K. Menon, "Direction of arrival estimation in smart antenna for marine communication," *2016 International Conference on Communication and Signal Processing (ICCSP)*, Melmaruvathur, 2016, pp. 1535-1540.

[20] J. Webber, S. Tsukamoto, S. Ano, N. Kukutsu and T. Kumagai, "Direction of arrival estimation apparatus for communication based train control system using ESPAR antenna," *2015 Third International Conference on Digital Information, Networking, and Wireless Communications (DINWC)*, Moscow, 2015, pp. 49-54.

[21] F. Gross, *Smart Antennas for Wireless Communications with MATLAB. New York: McGraw-Hill, 2005, pp 193-196.*

[22] P. Gupta, K. Aditya and A. Datta, "Comparison of conventional and subspace based algorithms to estimate Direction of Arrival (DOA)," *2016 International Conference on Communication and Signal Processing (ICCSP)*, Melmaruvathur, 2016, pp. 0251-0255.

[23] M. P. Priyadarshini and R. Vinutha, "Comparative performance analysis of MUSIC and ESPRIT on ULA," *2012 International Conference on Radar, Communication and Computing (ICRCC)*, Tiruvannamalai, 2012, pp. 120-124.

[24] B. Chen, Z. Zhong, B. Ai and X. Chen, "Comparison of Antenna Arrays for MIMO System in High Speed Mobile Scenarios," *2011 IEEE 73rd Vehicular Technology Conference (VTC Spring)*, Yokohama, 2011, pp. 1-5.

[25] Anon, (2015). *BeamFlex, 11ac Wave 2, and MIMO: The art of RF engineering*. [online] Available at: https://ruckus-www.s3.amazonaws.com/pdf/wp/wp-art-of-rf-engineering.pdf [Accessed 23 Oct. 2018].

[26] Guanghan Xu, S. D. Silverstein, R. H. Roy and T. Kailath, "Beamspace ESPRIT," in *IEEE Transactions on Signal Processing*, vol. 42, no. 2, pp. 349-356, Feb. 1994.

[27] R. K, H. F and M. M A, "A comprehensive analysis and performance evaluation of different direction of arrival estimation algorithms," *2012 IEEE Symposium on Computers & Informatics (ISCI)*, Penang, 2012, pp. 256-259.