Multi-objective integrated robust H∞ control for attitude tracking of a flexible spacecraft

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Abstract: This paper investigates the multi-objective attitude tracking problem of a flexible spacecraft in the presence of disturbances, parameter uncertainties and imprecise collocation of sensors and actuators. An integrated robust H∞ controller, including an output feedback component and a feedforward component, is proposed, and its gains are calculated by solving Linear Matrix Inequalities. The output feedback component stabilizes the integrated control system while the feedforward component can drive the attitude motion to track the desired angles. The system robustness against disturbances, parameter uncertainties and imprecise collocation is addressed by the H∞ approach and convex optimization. Numerical simulations are finally provided to assess the performance of the proposed controller.

Keywords: Attitude control, Flexible spacecraft, Robust control, Output feedback, Multi-objective

1. Introduction

Attitude control is a key requirement for most space missions, and many different approaches to addressing this problem are proposed [1-5]. The problem of spacecraft attitude control can be generally classified in two separate

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issues: one dealing with tracking and the other with stabilization. In the attitude stabilization problem, a controller that drives the system states to the equilibrium points from an initial configuration is designed [6-9], while a controller, that drives the system outputs to track the reference angles, is developed for attitude tracking problem [10-16]. It is generally more challenging to tackle the attitude tracking problem than the stabilization problem, since the attitude tracking problem is not only to track the reference angles but also guarantee the system stabilization in the meantime. Recent years have however witnessed plentiful research in the attitude tracking control of flexible spacecraft, as this is a key requirement of future missions that rely on paradigms of formation flying and stereoscopic mapping and spacecraft on-orbit servicing [18-20]. The primary challenges in the control system design for the flexible spacecraft, are as follows: (i) large system orders; (ii) weakly damped oscillating behavior of the flexible appendages; (iii) parameter uncertainties caused by identification inaccuracy or appendages motion; (iv) environmental disturbances; [21]. Besides, it may not be possible in practice to place sensors and actuators exactly at the same locations of a large spacecraft. All these issues make it extremely difficult to achieve ideal control performance for spacecraft attitude tracking. Previous studies have proposed various approaches to solve this problem. The static and dynamic controllers in the presence and absence of measurement are proposed for attitude tracking of a spacecraft with flexible appendages [11]. To achieve globally asymptotic stability of the attitude tracking errors, simple Lyapunov-based controllers are developed in the presence of parameter uncertainties and external disturbances [12]. The finite-time control technique for flexible spacecraft attitude tracking, which demonstrated that the attitude control system has better convergence and robustness, is proposed in [13-14]. A sliding-mode control (SMC) algorithm is derived and applied to quaternion-based spacecraft attitude tracking maneuvers [15]. A simple variable structure controller, including a PD term and a switching function, is designed for tracking maneuvers in the presence of disturbances and model uncertainties [16]. Hu has studied many novel control approaches for attitude tracking of a flexible spacecraft subject to different constraints [17, 23-25]. A novel fault tolerant attitude tracking control scheme, to perform attitude tracking in presence of actuator effectiveness fault, is investigated in [17]. Another fault-tolerant control scheme considering thruster redundancy is proposed for attitude tracking of a flexible spacecraft, and a $H\infty$ performance index is introduced to describe the disturbance attenuation performance of the closed-loop system [23]. Based on the sliding mode control, adaptive control and backstepping technique, the attitude tracking controllers are developed in the presence of parameter variation, disturbances and input saturation and singularity [24, 25]. Quasi-continuous second and third order sliding mode controllers, to perform quaternion-based spacecraft attitude tracking maneuvers, are investigated in [26]. An adaptive control system, to perform rotational maneuver tracking and vibration suppression of a flexible spacecraft, is designed in [27]. The adaptive sliding mode control with hybrid sliding surface is used to minimize the effects of uncertainties, disturbances and the difficulties arising from measurement [28].

Although the above mentioned control algorithms have shown adequate performance in flexible spacecraft attitude tracking maneuvers, they cannot be currently used in practice. Alternatively, a controller with a simple structure and low orders is more desirable for real case applications [29]. The simple state or output feedback controller based on H∞ control design methodology has been used in space missions [30-31], since it has a PD-like structure and clear physical meaning. Besides, the problem of spacecraft parameter uncertainties has been widely addressed, and a commonly-used methodology is to assume that the uncertain parameters, typically the moments of inertia, always consist of the nominal component and the uncertain one [4, 6, 12-16, 18, 25, 27]. Sometimes, spacecraft mass and modal parameters are uncertain and occasionally time-varying due to on-orbit maneuvers such as solar array and antenna re-orientation [32]. Spacecraft sensors and actuators are not placed at the same locations exactly [33]. On this occasion, the above assumption is invalid, and the uncertain parameters should be then dealt with as a whole throughout the controller design process. Hence, it is obvious that the attitude tracking control is a multi-objective problem, and we must synthetically consider the requirements of the system stability and performance, controller simpleness, and meanwhile disturbances, parameter uncertainties and imprecise collocation of sensors and actuators.

To advance the research in the attitude tracking of a flexible spacecraft, we make use of the robust H∞ control design technique. The integrated robust controller, including an output feedback component and a feedforward component, is proposed to meet the multi-objective design requirements in the presence of disturbances, uncertain or time-varying parameters and imprecise collocation. The output feedback component can stabilize the integrated control system and the feedforward component can drive the attitude angles to track the desired angles, in which the existence conditions for admissible controllers are formulated in the form of linear matrix inequalities (LMIs). The stability and robustness of the integrated control system are discussed, and the controller design problem is cast into a convex optimization problem subject to LMIs constraints. The effectiveness of the proposed controllers is finally demonstrated through numerical simulations. It should be noted that the proposed approach is used to avoid excessive complexity of the controller, as well as to reduce the dependency of control efficiency from accurate knowledge of the system parameters.

2. Problem Definition

2.1 Attitude Dynamics of the Flexible Spacecraft

The spacecraft is described as a rigid body with three flexible appendages, and the attitude dynamics is governed by the following differential equations [14, 31]

$$\begin{gathered} J\ddot{\vartheta}+\sum_{\mathrm{i}=1}^{3}\varGamma_{\mathrm{i}}\ddot{\eta}_{\mathrm{i}}=T_{\mathrm{u}}+T_{\mathrm{d}} \\ \ddot{\eta}_{\mathrm{i}}+2\xi_{\mathrm{i}}\omega_{\mathrm{i}}\dot{\eta}_{\mathrm{i}}+\omega_{\mathrm{i}}^{2}\eta_{\mathrm{i}}+\varGamma_{\mathrm{i}}^{\mathrm{T}}\ddot{\vartheta}=0 \end{gathered} \tag{1}$$

where J denotes the inertia matrix, $\vartheta=[\vartheta_1,\vartheta_2,\vartheta_3]^{\mathrm{T}}$ is the spacecraft attitude angle vector, $\varGamma_{\mathrm{i}}$ and η_{i} are the coupling coefficient matrix and modal coordinate of the i-th appendage. ξ_{i} and ω_{i} are the modal damping ratio and the modal frequency matrix, T_{u} denotes the control torques and T_{d} represents the bounded disturbance torques. The system outputs are attitude angles ϑ and their angular velocities $\dot{\vartheta}$, which are measured by different sensors in practice. Then the measurement output is given by

$$y=[y_1^{\mathrm{T}},y_2^{\mathrm{T}}]^{\mathrm{T}}=[\vartheta^{\mathrm{T}},\dot{\vartheta}^{\mathrm{T}}]^{\mathrm{T}} \tag{2}$$

For the controller design, the attitude dynamics are rewritten as following Lagrange-like dynamics

$$\begin{gathered} M\ddot{\psi}+D\dot{\psi}+K\psi=LT_{\mathrm{u}}+LT_{\mathrm{d}} \\ y_1=L^{\mathrm{T}}\psi,\ y_2=L^{\mathrm{T}}\dot{\psi} \end{gathered} \tag{3}$$

where $\psi=[\vartheta^{\mathrm{T}},\eta_1^{\mathrm{T}},\eta_2^{\mathrm{T}},\eta_3^{\mathrm{T}}]^{\mathrm{T}}$, $M=\begin{bmatrix} J & \varGamma_1 & \varGamma_2 & \varGamma_3 \\ \varGamma_1^{\mathrm{T}} & E & 0 & 0 \\ \varGamma_2^{\mathrm{T}} & 0 & E & 0 \\ \varGamma_3^{\mathrm{T}} & 0 & 0 & E \end{bmatrix}$, $D=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2\xi_1\omega_1 & 0 & 0 \\ 0 & 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 0 & 2\xi_3\omega_3 \end{bmatrix}$,

$K=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \omega_1^2 & 0 & 0 \\ 0 & 0 & \omega_2^2 & 0 \\ 0 & 0 & 0 & \omega_3^2 \end{bmatrix}$, $L=[E,0,0,0]^{\mathrm{T}}$ and E denotes the identity matrix.

2.2 Definitions and Lemmas

Defination 1 (H∞ performance): For such a continuous system,

$$\begin{aligned} \dot{x}_1 &= Q_1 x_1 + Q_2 w \\ x_2 &= Q_0 x_1 \end{aligned} \tag{4}$$

Define the transfer function matrix ϕ from w to x_2

$$\phi(\mathrm{s}) = Q_0 (sE - Q_1)^{-1} Q_2 \tag{5}$$

The H∞ norm of ϕ is given by

$$\|\phi\| = \sup_{\omega} \delta_{\max} (\phi(j\omega)) \tag{6}$$

where Q_0, Q_1 and Q_2 are system gains, $\delta_{\max}$ denotes the maximum singular value, $\sup$ represents the supremum, $\|\square\|$ is the H∞ norm, and ω denotes the system frequency. The H∞ performance is governed by the following inequality:

$$\|\phi\| < \kappa \tag{7}$$

where κ is a positive constant.

According to Eq. (6), the physical meaning of H∞ norm represents a kind of generalized energy caused by a disturbance w. Generally, with lower values of H∞ norm, the system has better performances in suppressing disturbances.

Lemma 1: The system (3) has two features: firstly

$$M > 0 \, , D \geq 0 \, , K \geq 0 \tag{8}$$

holds from the modal identity [34], and secondly the system (3) is stabilizable and detectable since the rank conditions satisfy [33]

$$\mathrm{rank}[D, L] = \mathrm{rank}[K, L] = n \tag{9}$$

Lemma 2: (Schur Complement Lemma) For a symmetric matrix $\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ {\Omega_{12}}^{\mathrm{T}} & \Omega_{22} \end{bmatrix}$, the following three inequalities hold [35]

$$\begin{aligned}&\varOmega < 0\\&\varOmega_{11} < 0,\varOmega_{22} - {\varOmega_{12}}^{\mathrm{T}}{\varOmega_{11}}^{-1}\varOmega_{12} < 0\\&\varOmega_{22} < 0,\varOmega_{11} - {\varOmega_{12}}^{\mathrm{T}}{\varOmega_{22}}^{-1}\varOmega_{12} < 0\end{aligned}\tag{10}$$

Lemma 3: Consider a continuous-time transfer function $\varGamma(s)$ of the form $\varGamma(s) = D_1 + H_1(sE - A_1)^{-1}B_1$. The following statements are equivalent [35]

(i) $\left\| D_1 + H_1(sE - A_1)^{-1}B_1 \right\|_\infty < \gamma$, and A_1 is stable in the continuous-time sense;

(ii) there exists a symmetric positive definite solution X to the following LMI

$$\begin{bmatrix} {A_1}^{\mathrm{T}}X+XA_1 & XB_1 & {H_1}^{\mathrm{T}} \\ X^{\mathrm{T}}B_1 & -\gamma E & {D_1}^{\mathrm{T}} \\ H_1 & D_1 & -\gamma E \end{bmatrix} < 0 \tag{11}$$

(iii) $${A_1}^{\mathrm{T}}X+XA_1 + \gamma^{-1}{H_1}^{\mathrm{T}}H_1 + \gamma(XB_1 + \gamma^{-1}{H_1}^{\mathrm{T}}D_1)(\gamma^2 E - {D_1}^{\mathrm{T}}D_1)^{-1}({B_1}^{\mathrm{T}}X + \gamma^{-1}{D_1}^{\mathrm{T}}H_1) < 0 \tag{12}$$

Lemma 4: Let $\bar{w}_2$ and w_2 be the unique positive definite solution of the following Lyapunov equation

$$\bar{w}_2 A_2 + {A_2}^{\mathrm{T}}\bar{w}_2 = -2\bar{q} \tag{13}$$

$$w_2 A_2 + {A_2}^{\mathrm{T}}w_2 = -2q \tag{14}$$

where

$$\bar{q} = kq \tag{15}$$

and k is a positive scalar. Then, it yields

$$\frac{\min\lambda(\bar{q})}{\max\lambda(\bar{w}_2)} = \frac{\min\lambda(q)}{\max\lambda(w_2)} = a \tag{16}$$

where λ denotes the eigenvalue. Besides, the bound value a is maximum when q is the identity matrix in Eq.(14).

Proof: Using Eq.(15), Eq.(14) can be rewritten as

$$\left(\frac{1}{k}\bar{w}_2\right)A_2 + {A_2}^{\mathrm{T}}\left(\frac{1}{k}\bar{w}_2\right) = -2q \tag{17}$$

Due to the uniqueness of solutions of above Lyapunov equations Eqs.(13), (14) &(17), it yields $\frac{1}{k}\bar{w}_2 = w_2$.

Therefore, it can be concluded that $max\,\lambda(\bar{w}_2) = max\,\lambda(kw_2) = k \cdot max\,\lambda(w_2)$. Then it yields

$$\frac{\min\lambda(\bar{q})}{\max\lambda(\bar{w}_2)} = \frac{\min\lambda(kq)}{k\cdot\max\lambda(w_2)} = \frac{\min\lambda(q)}{\max\lambda(w_2)} \tag{18}$$

Let $k = \frac{1}{\min\lambda(q)}$, then

$$\min\lambda(\bar{q}) = 1 \tag{19}$$

The solution of above Lyapunov equation (13) is

$$\bar{w}_2 = 2\int_0^\infty \mathrm{e}^{A_2^{\mathrm{T}}t}\bar{q}\mathrm{e}^{A_2 t}\mathrm{d}t \tag{20}$$

Then Eq.(16) is rewritten as

$$a = \frac{\min\lambda(\bar{q})}{\max\lambda(\bar{w}_2)} = \frac{1}{\max\lambda(\bar{w}_2)} \tag{21}$$

Consider a second solution of the Lyapunov equation $\tilde{w}_2 A_2 + A_2^{\mathrm{T}}\tilde{w}_2 = -2E_1$, and the solution is given by

$$\tilde{w}_2 = 2\int_0^\infty \mathrm{e}^{A_2^{\mathrm{T}}t}E_1\mathrm{e}^{A_2 t}\mathrm{d}t \tag{22}$$

Then we can obtain another bound value $a' = \frac{\min\lambda(E_1)}{\max\lambda(\tilde{w}_2)} = \frac{1}{\max\lambda(\tilde{w}_2)}$. From Eqs.(20) & (22), we see $\bar{w}_2 - \tilde{w}_2 = 2\int_0^\infty \mathrm{e}^{A_2^{\mathrm{T}}t}(\bar{q} - E_1)\mathrm{e}^{A_2 t}\mathrm{d}t$. Since $\lambda(\bar{q}) \geq 1$, $\mathrm{e}^{A_2^{\mathrm{T}}t}$ and $\mathrm{e}^{A_2 t}$ are nonsingular, it then yields the conclusion that

$$\bar{w}_2 - \tilde{w}_2 = 2\int_0^\infty \mathrm{e}^{A_2^{\mathrm{T}}t}(\bar{q} - E_1)\mathrm{e}^{A_2 t}\mathrm{d}t \geq 0 \tag{23}$$

Namely $max\lambda(\bar{w}_2) \geq max\lambda(\tilde{w}_2)$. Therefore, the result $a \leq a'$ holds for all positive definite $\bar{q}$.

3. Controller Design

3.1 The Control Objective

During on-orbit operation, the inertia matrix and modal parameters of a flexible spacecraft are uncertain and could even be time-varying. Vibration of the flexible appendages may arise from external disturbances or attitude maneuvers. The objective of controller design is:

(i) A controller is developed to guarantee system stability, while making system output track the reference attitude signals.

(ii) The stability of the attitude control system should be guaranteed at all times, even if there exist unknown changes and errors in model parameters.

(iii) The proposed controller should be robust to external disturbances and imprecise collocation of sensors and actuators.

3.2 The Output Feedback Controller

To achieve attitude tracking, an integrated H∞ controller, including an output feedback component and a feedforward component, is proposed in the presence of disturbances and uncertain parameters. The output feedback component is used to stabilize the attitude control system of the flexible spacecraft, and is then given by

$$T_{\mathrm{u}} = -K_0 y \tag{24}$$

where $K_0 = \begin{bmatrix} K_{01} & K_{02} \end{bmatrix}$, and the gains K_{01} and K_{02} are positive definite. Substituting Eq. (24) into Eq. (3) yields

$$M\ddot{\psi} + D^{\times}\dot{\psi} + K^{\times}\psi = LT_{\mathrm{d}} \tag{25}$$

where $D^{\times} = D + LK_{02}L^{\mathrm{T}}$, $K^{\times} = K + LK_{01}L^{\mathrm{T}}$. According to Lemma 1, the following inequalities hold

$$D^{\times} > 0 \, , K^{\times} > 0 \tag{26}$$

The theorem 1 is therefore given below for controller design.

Theorem 1: The closed-loop system (25) is asymptotically stable if $M > 0$, $D^{\times} > 0$, $K^{\times} > 0$ when the disturbance torque $T_{\mathrm{d}} = 0$.

Proof: Let $z = [\psi, \dot{\psi}]^{\mathrm{T}}$, then the system (25) is rewritten as

$$\begin{gathered} \dot{z} = C_1 z + C_2 T_{\mathrm{d}} \\ \sigma = C_3 z \end{gathered} \tag{27}$$

where $C_1 = \begin{bmatrix} 0 & E \\ -M^{-1}K^{\times} & -M^{-1}D^{\times} \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix}$, $C_3 = \begin{bmatrix} L^{\mathrm{T}} & \beta L^{\mathrm{T}} \end{bmatrix}$, σ represents the output and β is a weighting coefficient. Once $T_{\mathrm{d}} = 0$, the asymptotic stability of system (25) is equivalent to the matrix C_1 being stable. C_1 is stable if and only if there exists a positive definite symmetric matrix Z such that

$$ZC_1 + C_1^{\mathrm{T}} Z < 0 \tag{28}$$

The Lyapunov inequality obviously has a solution Z_0 such as

$$Z_0 = \begin{bmatrix} K^{\times} & \varepsilon M \\ \varepsilon M & M \end{bmatrix} \tag{29}$$

Since $M > 0$, $K^{\times} > 0$, the following inequalities hold for any vector $\varXi \neq 0$

$$\begin{gathered} \varXi^{\mathrm{T}} K^{\times} \varXi = \nu_1 > 0 \\ \varXi^{\mathrm{T}} M \varXi = \nu_2 > 0 \end{gathered} \tag{30}$$

Then there exists $0 < \varepsilon < \varepsilon_1 = \sqrt{\dfrac{\nu_1}{\nu_2}}$ such that the following inequality holds

$$\varXi^{\mathrm{T}} K^{\times} \varXi - \varepsilon^2 \varXi^{\mathrm{T}} M \varXi > 0 \tag{31}$$

namely

$$\begin{gathered} K^{\times} - \varepsilon^2 M > 0 \\ K^{\times} - (\varepsilon M^{\mathrm{T}}) M^{-1} (\varepsilon M) > 0 \end{gathered} \tag{32}$$

According to lemma 2, we have

$$Z_0 > 0 \tag{33}$$

Let

$$Z_0 C_1 + C_1^{\mathrm{T}} Z_0 = -Q \tag{34}$$

where $Q = \begin{bmatrix} 2\varepsilon K^{\times} & \varepsilon D^{\times} \\ \varepsilon D^{\times} & 2D^{\times} - 2\varepsilon M \end{bmatrix}$. According to inequalities (8) and (26), there is $\varXi \neq 0$ such that

$$\begin{aligned} &\varXi^{\mathrm{T}} D^{\times} \varXi = \nu_3 > 0 \\ &\varXi^{\mathrm{T}} \left(2\mathrm{M} + \frac{D^{\times}(K^{\times})^{-1} D^{\times}}{2} \right) \varXi = \nu_4 > 0 \end{aligned} \tag{35}$$

Similarly, there exists $0 < \varepsilon < \varepsilon_2 = \dfrac{\nu_3}{\nu_4}$ such that

$$\varXi^{\mathrm{T}} \left(2D^{\times} - \varepsilon \left(2M + \frac{D^{\times}(K^{\times})^{-1} D^{\times}}{2} \right) \right) \varXi > 0 \tag{36}$$

Namely

$$2D^{\times} - \varepsilon \left(2M + \frac{D^{\times}(K^{\times})^{-1} D^{\times}}{2} \right) > 0 \tag{37}$$

Therefore we have $Q > 0$. Let $0 < \varepsilon < \min\{\varepsilon_1, \varepsilon_2\}$ and there exists $Z_0 > 0$, then the following inequality holds

$$Z_0 C_1 + C_1^{\mathrm{T}} Z_0 = -Q < 0 \tag{38}$$

Based on the above proofs, theorem 1 is therefore proved, and the closed-loop system (25) is asymptotically stable when $T_{\mathrm{d}} = 0$.

Remark 1: As can be seen in theorem 1, the stability of the closed-loop system (25) is only related to the positive definite property of M , $D^{\times}$ and $K^{\times}$, regardless of the number of modes retained in the reduced-order model and the uncertainties in the parameter values. This consequence is attributable to the fact that, since the positive definite K_{01} and K_{02} are chosen, the control-induced stiffness and damping matrixes $D^{\times}$, $K^{\times}$ are always positive definite regardless of whether full-order or reduced-order models are used. Thus, by using such an output feedback controller, the potential instability problems due to spillover can be completely avoided, and the parameters do not have to be known accurately to guarantee stability. This stability analysis leads us to the conclusion that if the system is controllable, then the closed-loop system is at least asymptotically stable as long as the gain matrices are chosen as positive definite matrices. In practice, these optimistic stability arguments must be then adjusted, of course, by consideration of system robustness to disturbances, uncertain or time-varying parameters and imprecise collocation of sensors and actuators.

3.3 Stability Robustness Analysis

The robustness of the attitude control system to disturbances is highly desirable in practice. For this purpose, the transfer function from T_{d} to σ is given below according to Eq.(27)

$$T(s) = C_3 (sE - C_1)^{-1} C_2 \tag{39}$$

By the definition 1, the H∞ norm of T is given by

$$\|T\|_{\infty} = \sup_{\omega} \delta_{\max} (T(j\omega)) \tag{40}$$

From Eq.(40), the H∞ norm denotes the maximum of singular values of the system frequency response, which represents the influence of the disturbances on the attitude system. In principle, a smaller value of H∞ norm also represents a smaller influence, namely, the closed-loop system has better robustness. Let δ denote the upper bound of the maximum singular value, then Eq.(40) is rewritten as

$$\|T\|_{\infty} = \left\| C_3 (sE - C_1)^{-1} C_2 \right\|_{\infty} < \delta \tag{41}$$

Theorem 2, to firstly discuss the robustness to disturbances, is then given below.

Theorem 2: There always exists a controller (24) such that the closed-loop system has a H∞ norm less than $\delta > 0$.

Proof: From lemma 3, the inequality (41) is satisfied only if there is a symmetric matrix $Z^{\times} > 0$ such that

$$\begin{bmatrix} C_1^{\mathrm{T}} Z^{\times} + Z^{\times} C_1 & Z^{\times} C_2 & C_3^{\mathrm{T}} \\ Z^{\times \mathrm{T}} C_2 & -\delta E & 0 \\ C_3 & 0 & -\delta E \end{bmatrix} < 0 \tag{42}$$

According to lemmas 2&3, the inequality (42) is equivalent to the following two inequalities

$$C_1^{\mathrm{T}} Z^{\times} + Z^{\times} C_1 < 0 \tag{43}$$

$$C_1^{\mathrm{T}} Z^{\times} + Z^{\times} C_1 + \delta^{-1} \begin{bmatrix} Z^{\times} C_2 & C_3^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} C_2^{\mathrm{T}} Z^{\times} \\ C_3 \end{bmatrix} < 0 \tag{44}$$

As stated in theorem 1, the inequality (43) has a solution $Z_0^{\times} > 0$. From Eq.(34), inequality (44) is rewritten as

$$\delta^{-1} \begin{bmatrix} \left(\varepsilon^2 + 1\right) LL^{\mathrm{T}} & \left(\varepsilon^2 + \beta\right) LL^{\mathrm{T}} \\ \left(\varepsilon^2 + \beta\right) LL^{\mathrm{T}} & \left(\beta^2 + 1\right) LL^{\mathrm{T}} \end{bmatrix} - Q < 0 \tag{45}$$

The inequality (45) holds for a sufficiently large δ. By solving the linear matrix inequality (45), the controller K_0 is therefore obtained. Indeed, the lower bound of δ can be calculated as

$$\delta_{\mathrm{low}} = max \left\{ \lambda_{max} \left(Q^{-1/2} N N^{\mathrm{T}} Q^{-1/2} \right) \right\} \tag{46}$$

by the eigenvalue analysis of the inequality (45), where $N = \begin{bmatrix} \varepsilon L & L \\ L & 0 \end{bmatrix}$.

In a real mission, the spacecraft mass and modal parameters, such as inertia matrix, modal damping ratio and modal frequency, could be uncertain and time-varying due to the rotations of flexible appendages or inaccurate on-orbit identification. Due to the presence of unknown changes, the accurate attitude dynamics cannot be obtained. To deal with this issue, a commonly-used approach mentioned above is to assume that the spacecraft parameters consist of a nominal value plus an unknown term. The unknown term typically has an upper bound, which is then used during controller design. To reduce the dependence on priori knowledge of upper bound value, an alternative solution is to consider the spacecraft parameters as single unknown or time-varying variable. Then attitude dynamics Eq. (3) is actually a multi input multi output time-varying system. The convex optimization approach is addressed below for controller design [36, 37]. $M(\Delta)$, $D(\Delta)$ and $K(\Delta)$ represent the uncertain spacecraft parameters, and are given by:

$$\begin{gathered} M(\Delta) = \sum_{\mathrm{i}=1}^{\tau} \lambda_{\mathrm{i}} M_{\mathrm{i}} \, , D(\Delta) = \sum_{\mathrm{i}=1}^{\tau} \lambda_{\mathrm{i}} D_{\mathrm{i}} \, , K(\Delta) = \sum_{\mathrm{i}=1}^{\tau} \lambda_{\mathrm{i}} K_{\mathrm{i}} \\ \lambda_{\mathrm{i}} \geq 0 \, , \sum_{\mathrm{i}=1}^{\tau} \lambda_{\mathrm{i}} = 1 \end{gathered} \tag{47}$$

where M_{i}, D_{i} and K_{i} are constant matrixes. Then the inequality (45) can be rewritten as

$$\sum_{\mathrm{i}=1}^{\tau} \lambda_{\mathrm{i}} \left(\delta^{-1} \begin{bmatrix} \left(\varepsilon^2+1\right) LL^{\mathrm{T}} & \left(\varepsilon^2+\beta\right) LL^{\mathrm{T}} \\ \left(\varepsilon^2+\beta\right) LL^{\mathrm{T}} & \left(\beta^2+1\right) LL^{\mathrm{T}} \end{bmatrix} - Q_{\mathrm{i}} \right) < 0 \tag{48}$$

where $Q_{\mathrm{i}} = \begin{bmatrix} 2\varepsilon\left(K_{\mathrm{i}} + LK_{01}L^{\mathrm{T}}\right) & \varepsilon\left(D_{\mathrm{i}} + LK_{02}L^{\mathrm{T}}\right) \\ \varepsilon\left(D_{\mathrm{i}} + LK_{02}L^{\mathrm{T}}\right) & 2\varepsilon\left(D_{\mathrm{i}} + LK_{02}L^{\mathrm{T}}\right) - 2\varepsilon M_{\mathrm{i}} \end{bmatrix}$. Then the design condition (45) becomes

$$\delta^{-1} \begin{bmatrix} \left(\varepsilon^2+1\right) LL^{\mathrm{T}} & \left(\varepsilon^2+\beta\right) LL^{\mathrm{T}} \\ \left(\varepsilon^2+\beta\right) LL^{\mathrm{T}} & \left(\beta^2+1\right) LL^{\mathrm{T}} \end{bmatrix} - Q_{\mathrm{i}} < 0 \tag{49}$$

We can choose different operating modes of a flexible spacecraft from launch to on-orbit to determine the value of τ . Then the uncertain spacecraft parameters $M(\Delta)$, $D(\Delta)$ and $K(\Delta)$ are obtained by the weighted average of M_{i}, D_{i} and K_{i} at i-th operating mode.

For a general configuration, the actuators are placed in the main body of a spacecraft, and sensors are also installed in the main body. This is known as actuator/sensor collocation. We have established in the previous section that, with ideal conditions (i.e., exact actuator/sensor collocation, perfect actuators and sensors), then the vibration of flexible appendages can be regards as a kind of disturbance for spacecraft, and the stability of proposed closed-loop system is likely to be guaranteed. However, it may not be possible in practice to place sensors and actuators exactly at the same locations, which could result in the interaction between flexible mode and closed-loop system. We therefore investigate the robustness of the proposed output feedback controllers to imprecise collocation of sensors and actuators. Suppose the measurement output of Eq.(3) is rewritten as

$$y_1 = (L+\varDelta_1)^{\mathrm{T}} \psi,\ y_2 = (L+\varDelta_2)^{\mathrm{T}} \dot{\psi} \tag{50}$$

where Δ_1 and Δ_2 represent bounded noncollocation perturbation matrices. According to theorem 1, system (25) is asymptotically stable and there are $Q > 0$ and $Z_0 > 0$ that satisfy Eq.(38). To discuss the robustness and solve the control gains, theorem 3 is then given below.

Theorem 3: The closed-loop system with imprecisely collocated sensors and actuators is asymptotically stable if

$$\left\|K_{01}\right\|_{\mathrm{s}}\left\|\Delta_1\right\|_{\mathrm{s}}+\left\|K_{02}\right\|_{\mathrm{s}}\left\|\Delta_2\right\|_{\mathrm{s}}<\frac{\lambda_{\mathrm{m}}(Q)}{2\left\|L\right\|_{\mathrm{s}}\lambda_{\mathrm{M}}(Z_0)}\leq\frac{1}{2\left\|L\right\|_{\mathrm{s}}\lambda_{\mathrm{M}}(\tilde{Z}_0)} \tag{51}$$

where $\left\|\square\right\|_{\mathrm{s}}$ represents the spectral norm (maximum singular value) of a matrix, $\tilde{Z}_0$ is the solution of Eq.(38) when Q is an identity matrix, λ_{M} and λ_{m} denote the largest and the smallest eigenvalues.

Proof: The closed-loop system considering imprecise collocation of sensors and actuators is given by

$$\dot{z}=C_1 z-\begin{bmatrix}0 & 0\\ M^{-1}LK_{01}\varDelta_1^{\mathrm{T}} & M^{-1}LK_{02}\varDelta_2^{\mathrm{T}}\end{bmatrix}z=(C_1+H)z \tag{52}$$

where $T_{\mathrm{d}}=0$. The candidate Lyapunov function is defined as

$$U(z)=z^{\mathrm{T}}Z_0 z \tag{53}$$

Computing the derivative of U yields

$$\begin{aligned}\dot{U}(z)&=\dot{z}^{\mathrm{T}}Z_0 z+z^{\mathrm{T}}Z_0\dot{z}\\ &=z^{\mathrm{T}}(C_1^{\mathrm{T}}Z_0+Z_0C_1)z+2z^{\mathrm{T}}Z_0Hz\end{aligned} \tag{54}$$

For $\dot{U}$ to be negative definite, the sufficient condition is

$$2\left|z^{\mathrm{T}}Z_0Hz\right|<z^{\mathrm{T}}Qz \tag{55}$$

Since

$$\begin{gathered}\left|z^{\mathrm{T}}Z_0Hz\right|\leq\left\|Z_0\right\|_{\mathrm{s}}\left\|H\right\|_{\mathrm{s}}\left\|z\right\|^2\\ \lambda_{\mathrm{m}}(Q)\left\|z\right\|^2\leq z^{\mathrm{T}}Qz\end{gathered} \tag{56}$$

Then $\dot{U}$ is negative definite if

$$\left\|H\right\|_{\mathrm{s}}\leq\frac{\lambda_{\mathrm{m}}(Q)}{2\lambda_M(Z_0)} \tag{57}$$

From the definition of H in Eq.(52), it yields

$$\|H\|_{\mathrm{s}} \le \left\|M^{-1}LK_{01}\Delta_1^{\mathrm{T}}\right\|_{\mathrm{s}} + \left\|M^{-1}LK_{02}\Delta_2^{\mathrm{T}}\right\|_{\mathrm{s}} \le \left[\|K_{01}\|_{\mathrm{s}}\|\Delta_1\|_{\mathrm{s}} + \|K_{02}\|_{\mathrm{s}}\|\Delta_2\|_{\mathrm{s}}\right]\left\|M^{-1}L\right\|_{\mathrm{s}} \tag{58}$$

Using the upper bound on $\|H\|_{\mathrm{s}}$ in Eq.(57) gives

$$\|K_{01}\|_{\mathrm{s}}\|\Delta_1\|_{\mathrm{s}} + \|K_{02}\|_{\mathrm{s}}\|\Delta_2\|_{\mathrm{s}} \le \frac{\lambda_{\mathrm{m}}(Q)}{2\left\|M^{-1}L\right\|_{\mathrm{s}}\lambda_M(Z_0)} < \frac{\lambda_{\mathrm{m}}(Q)}{2\|L\|_{\mathrm{s}}\lambda_M(Z_0)} \tag{59}$$

According to lemma 4, the following inequality holds

$$\|K_{01}\|_{\mathrm{s}}\|\Delta_1\|_{\mathrm{s}} + \|K_{02}\|_{\mathrm{s}}\|\Delta_2\|_{\mathrm{s}} < \frac{\lambda_{\mathrm{m}}(Q)}{2\|L\|_{\mathrm{s}}\lambda_M(Z_0)} \le \frac{1}{2\|L\|_{\mathrm{s}}\lambda_{\mathrm{M}}(\tilde{Z}_0)} \tag{60}$$

The theorem 3 is then proven, and the closed-loop system is asymptotically stable in the presence of imprecise collocation of sensors and actuators. The output feedback controller K_0 can be therefore obtained by solving inequalities (49) & (60).

3.4 The Integrated Controller

Eq.(24) presents an output feedback controller, which is used to stabilize the attitude control system. To track a desired angle, a feedforward controller is proposed on the basis of the above closed-loop system (25), and an integrated H∞ controller is therefore achieved. The feedforward controller T_{v} is given by

$$T_{\mathrm{v}} = \left[K_{\mathrm{v1}}^{\mathrm{T}}\ K_{\mathrm{v2}}^{\mathrm{T}}\right]^{\mathrm{T}} r_{\mathrm{d}} \tag{61}$$

where K_{v1}, K_{v2} are positive definite gains, and r_{d} denotes the desired angle that is to be tracked. Substituting Eq.(61) into the closed-loop system (25) yields

$$M\ddot{\psi} + D^{\times}\dot{\psi} + K^{\times}\psi = LT_{\mathrm{d}} + LK_0K_{\mathrm{v1}}r_{\mathrm{d}} + LK_{\mathrm{v2}}r_{\mathrm{d}} \tag{62}$$

Then the theorem 3 is therefore proposed below for controller design.

Theorem 3: The integrated control system (62) is asymptotically stable if the following inequality is satisfied when the disturbance torque $T_{\mathrm{d}} = 0$.

$$K^{\times} - LK_0K_{\mathrm{v1}} - LK_{\mathrm{v2}} > 0 \tag{63}$$

Proof: As mentioned in Theorem 1, the asymptotic stability of Eq.(62) is equivalent to the following matrix being stable.

$$C_4 = \begin{bmatrix} 0 & E \\ -M^{-1}K^{\circ} & -M^{-1}D^{\circ} \end{bmatrix} \tag{64}$$

where $K^{\circ} = K^{\times} - LK_0K_{v1} - LK_{v2}$ and $D^{\circ} = D + LK_{02}L^{T}$. C_4 is stable if and only if there exists a positive definite symmetric matrix Z such that

$$ZC_4 + C_4^{T}Z < 0 \tag{65}$$

Since $M > 0$ and $K^{\circ} > 0$, the solution of the above inequality (65) can be then given by

$$Z_1 = \begin{bmatrix} K^{\circ} & \varepsilon^{\circ}M \\ \varepsilon^{\circ}M & M \end{bmatrix} \tag{66}$$

The proof can be readily accomplished by following the lines in the proof of theorem 1, and thus we have

$$\begin{aligned} & Z_1C_4 + C_4^{T}Z_1 \\ & = \begin{bmatrix} K^{\circ} & \varepsilon^{\circ}M \\ \varepsilon^{\circ}M & M \end{bmatrix} \begin{bmatrix} 0 & E \\ -M^{-1}K^{\circ} & -M^{-1}D^{\circ} \end{bmatrix} + \begin{bmatrix} 0 & -M^{-1}K^{\circ} \\ E & -M^{-1}D^{\circ} \end{bmatrix} \begin{bmatrix} K^{\circ} & \varepsilon^{\circ}M \\ \varepsilon^{\circ}M & M \end{bmatrix} \\ & = -Q \end{aligned} \tag{67}$$

According to Lemma 2, we have $Q > 0$. Theorem 3 is proved, and the integrated system is asymptotically stable. The feedforward control gains K_{v1} and K_{v2} can be therefore obtained by solving the above inequalities (49) & (63). The integrated robust H∞ controller, including an output feedback component and feedforward component, is therefore implemented to stabilize the attitude control system and while tracking the desired angle.

Remark 2: The proposed convex optimization approach could avoid the dependence on priori knowledge of upper bound value of uncertain parameters in practice. Besides, the time-varying spacecraft parameters $M(\Delta)$, $D(\Delta)$ and $K(\Delta)$ can be obtained more accurately using the weighted average of M_i, D_i and K_i by selecting more operating modes. The computational work required to solve the control gains will however increase since more operating modes will result in more inequalities. A proper τ is therefore the key to balance the computational work in terms of the parameter uncertainties.

Remark 3: The proposed integrated controller is a PD-like controller, and has clear physical significance. That means the collocated attitude angle and angular velocity feedback can guarantee the asymptotic stability of the attitude control system. A PD plus feedforward controller (PD+) is then addressed to compare with the proposed integrated controller. The PD feedback component stabilizes the closed-loop system and the feedforward component provides the attitude tracking maneuvers.

4. Numerical Simulation

In this section, three simulation cases are presented to demonstrate the performance of the proposed integrated controller. The spacecraft has two solar panels, and three operating modes of a flexible spacecraft are chosen.

Case 1, the spacecraft parameters at the beginning of life are given as $J_{\mathrm{b}} = \mathrm{diag}([3040,1800,3950])\ kg \cdot m^2$, $\Gamma_{\mathrm{b1}} = \begin{bmatrix} 0 & -0.002 & -31.714 \\ 32.485 & 0.3867 & 0 \end{bmatrix} kg^{1/2}m$, $\Gamma_{\mathrm{b2}} = \begin{bmatrix} 0 & 0.002 & -31.73 \\ 32.5 & 0.389 & 0 \end{bmatrix} kg^{1/2}m$, $\omega_{\mathrm{b1}} = \left[0.02,\ 0.061\right] rad/s$, $\omega_{\mathrm{b2}} = \left[0.021, 0.059\right] rad/s$, $\xi_{\mathrm{b1}} = \xi_{\mathrm{b2}} = 0.005$.

Case 2, the spacecraft parameters at the middle of life are given as $J_{\mathrm{m}} = \mathrm{diag}([2840,1700,3630])\ kg \cdot m^2$, $\Gamma_{\mathrm{m1}} = \Gamma_{\mathrm{b1}}$, $\Gamma_{\mathrm{m2}} = \Gamma_{\mathrm{b2}}$, $\omega_{\mathrm{m1}} = \left[0.019, 0.059\right] rad/s$, $\omega_{\mathrm{m2}} = \left[0.02, 0.059\right] rad/s$, $\xi_{\mathrm{m1}} = \xi_{\mathrm{m2}} = 0.005$.

Case 3, the spacecraft parameters at the end of life are given as $J_{\mathrm{e}} = \mathrm{diag}([2735,1650,3535])\ kg \cdot m^2$, $\Gamma_{\mathrm{e1}} = \Gamma_{\mathrm{b1}}$, $\Gamma_{\mathrm{e2}} = \Gamma_{\mathrm{b2}}$, $\omega_{\mathrm{e1}} = \left[0.019, 0.055\right] rad/s$, $\omega_{\mathrm{e2}} = \left[0.02, 0.059\right] rad/s$, $\xi_{\mathrm{e1}} = \xi_{\mathrm{e2}} = 0.005$.

The controller gains are $K_{01} = \mathrm{diag}([190.2,190.2,190.2])$, $K_{02} = \mathrm{diag}([20.52, 20.05, 20.28])$, $K_{\mathrm{v2}} = \mathrm{diag}([0.0052, -236.27, -236.27])$ and $K_{\mathrm{v1}} = \begin{bmatrix} 0.987 & 0 & 0 & 0.1057 & 0 & 0 \\ 0 & 2.2241 & 0.0002 & 0 & 0.2321 & 0 \\ 0 & -0.0001 & 2.2325 & 0 & -0.0009 & 0.2395 \end{bmatrix}^{\mathrm{T}}$. The gains of the PD+ controller are $K_{\mathrm{p}} = \mathrm{diag}([200, 200, 200])$ and $K_{\mathrm{D}} = \mathrm{diag}([15.2,15.7,15.4])$. The initial attitude angle is $\vartheta_0 = [11.5, -8.6, 6.9]^{\mathrm{T}}$ *deg*. The desired attitude angle and angular velocity are $r_{\mathrm{d}} = \begin{bmatrix} 0.01 sin 0.01t \\ -0.01 sin 0.01t \\ 0.015 sin 0.015t \end{bmatrix}$ *deg* and $\dot{r}_{\mathrm{d}} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ *deg/s* respectively. The numerical results are given below.

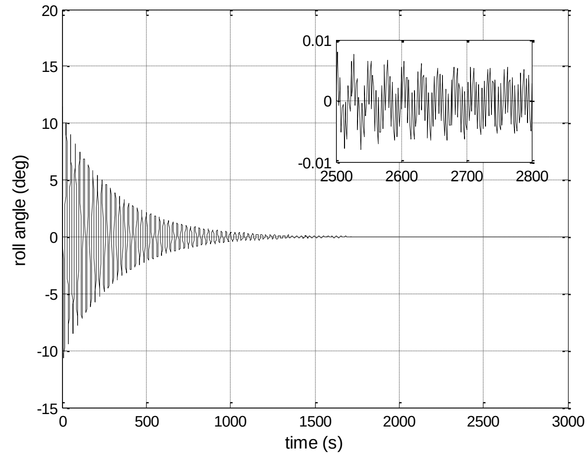

Fig. 1 Tracking error of roll angle

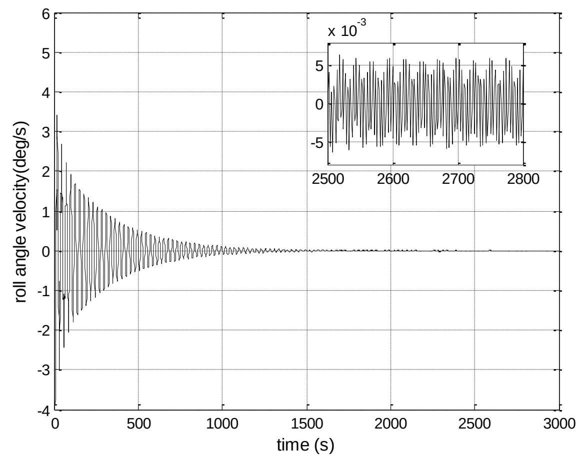

Fig. 2 Tracking error of roll angular velocity

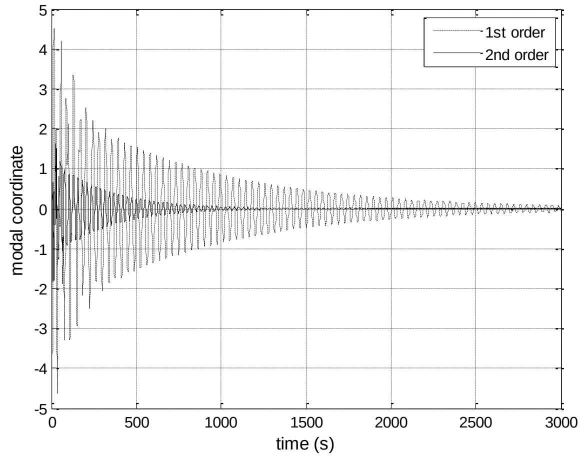

Fig. 3 Modal coordinate

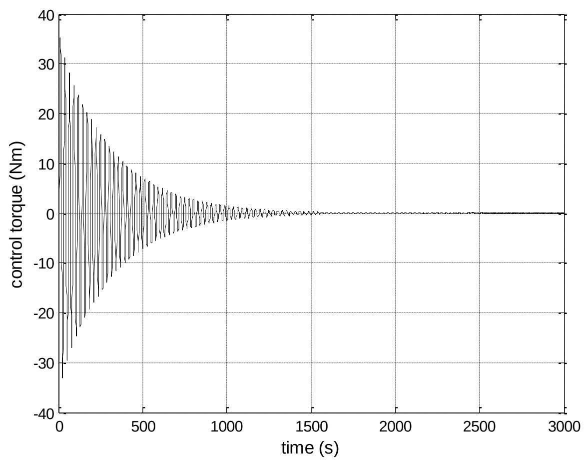

Fig. 4 Control torque

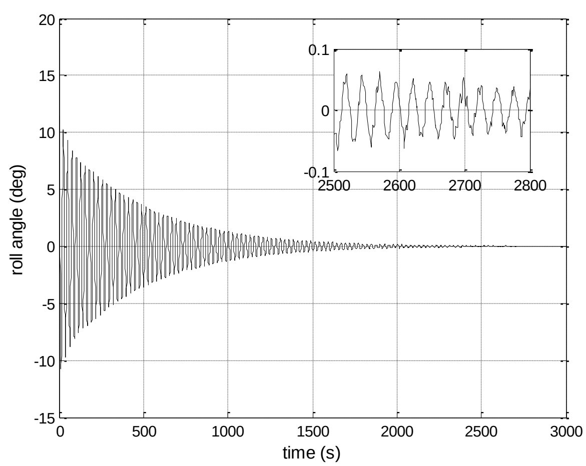

Fig. 5 Tracking error of roll angle — PD+

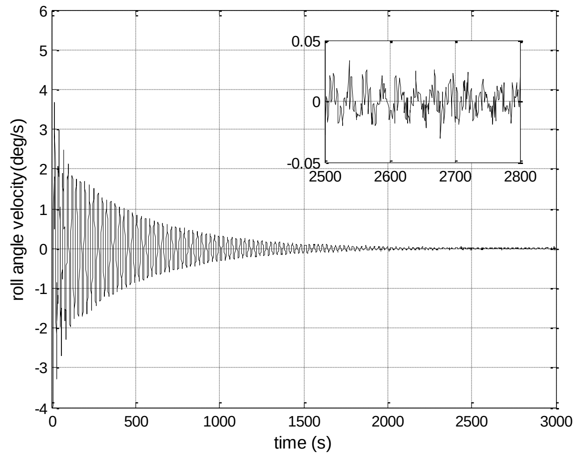

Fig. 6 Tracking error of roll angular velocity — PD+

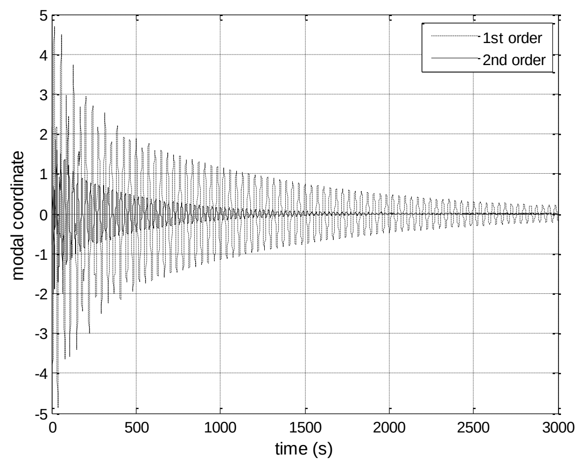

Fig. 7 Modal coordinate — PD+

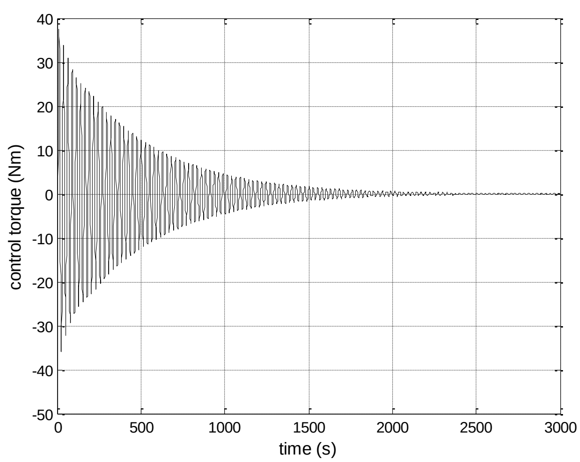

Fig. 8 Control torque — PD+

Table 1 Comparison of control performance

	Tracking error (attitude accuracy)	Tracking error (attitude stability)	Convergence time	Control torque
The proposed controller	$\pm 0.005°$	$\pm 0.005° / \mathrm{s}$	2700 s	< 35 N.m
PD+ controller	$\pm 0.05°$	$\pm 0.05° / \mathrm{s}$	2700 s	< 38 N.m

Figs. 1-4 show the roll-axis results of the attitude tracking maneuver performed by the integrated controller, using the attitude dynamic model with two modal coordinates of solar panels. As can be seen in Figs 1-2, the tracking errors of the roll angle and angular velocity converge to $\pm 0.005°$ and $\pm 0.005° / \mathrm{s}$ in approximately 2700 seconds. The modal coordinate and control torque are presented in Figs. 3-4 respectively. The broken line denotes the first order modal coordinate and the solid line represents the second one. It can be clearly seen that the vibration of the solar panels is effectively suppressed. The results demonstrate that the proposed integrated robust H∞ controller can drive the attitude motion to follow the desired angle in the presence of disturbances and parameter uncertainties. A value of $\delta = 0.007$ is calculated by solving LMIs, which represents the H∞ system performance. The roll angle can converge to a smaller steady-state error if we choose different values of the parameters ε and β by trial and error. This will however increase the control torque, which could lead to actuator saturation.

The simulation results of roll-axis under a PD+ controller are shown in Figs. 5-8. The roll angle is observed to oscillate between $-0.05°$ and $+0.05°$ in approximately 2700 seconds. The response of the roll angle and angular velocity is seen to follow the desired values, and the control torque in Fig. 8 has amplitude similar trend as that in Fig. 4. The comparison of control performance is summarized in Table 1. As can be seen, it is apparent that the proposed integrated robust H∞ controller is effective in increasing the steady-state accuracy and transient performance compared with the PD+ controller. This is because the optimization of the control performance using LMIs has functioned more effectively. Only the roll-axis results are presented since the performance along the other two axes shows analogous behavior and is thus omitted in this paper.

5. Conclusion

The multi-objective problem of the attitude tracking control of a flexible spacecraft in the presence of external disturbances, parameter uncertainties and imprecise collocation of sensors and actuators is addressed in this paper. An integrated robust $H\infty$ controller, including an output feedback component and a feedforward component, is proposed to perform attitude tracking maneuver. The control gains are calculated by solving LMIs. The robustness of closed-loop system against disturbances and imprecise collocation is discussed by the $H\infty$ approach, and the convex decomposition is developed to optimize the controller subject to model uncertainties. Numerical simulations are finally presented, and the results show that the proposed integrated controller has better attitude tracking accuracy and transient performance than a PD+ controller. Besides, the proposed controller has a simple structure, low orders and clear physical significance, which therefore avoids excessive complexity and could provide a possible solution for real engineering applications.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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