

WCES-2010

Prospective elementary teachers' conceptions of volume

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Received October 16, 2009; revised December 25, 2009; accepted January 8, 2010

Abstract

The current study sheds light on how and to what extent prospective elementary mathematics teachers have access to concept definitions of volume and how compatible are their concept definitions. It provides information about twelve prospective elementary teachers' (two from US and 10 from Turkey) concept images of volume and the extent to which those images coincide with concept definitions of volume. The study highlights the importance of coordination of concept images (filled-up or occupied space) and concept definitions (multiplication of three dimensional measures for prisms) as well as coordination in between different but related concept definitions of volume. The article also illustrates the effects of lack of coordination. © 2010 Elsevier Ltd. Open access under [CC BY-NC-ND license](http://creativecommons.org/licenses/by-nc-nd/3.0/).

Keywords: Teacher knowledge; teacher competency; mathematical ideas; understanding; mathematical concepts.

1. Introduction

Measurement is a crucial area in both mathematics and science. As some leading organizations in science and mathematics education pointed out in different documents (e.g., National Council of Teachers of Mathematics [NCTM], 2000) students should be able to analyze characteristics and properties of measurement systems and understand measurement concepts (e.g., volume, area, length). Even though these goals are targeted in school mathematics, the emphasis is mostly on the formulae before giving students enough opportunities to understand concepts (Banchoff, 1990). The early emphasis on formulae creates difficulties for learners in understanding measurement concepts (Battista and Clements, 1996). Zevenbergen (2005) points to the reliance on the use of formulae to solve the problems rather than on the conceptual meanings among prospective elementary teachers. Studies like these highlight the fact that teachers are not well equipped to teach the concept of volume by having their students primarily focus on the conceptual meanings. In order to give the priority to the concepts in school mathematics, educators first need to know enough about what is involved in operating with these concepts as well as how students usually think about these concepts. The current study aimed at serving the education community in these regards for the volume concept. The overall purpose of the study is to get an insight into prospective elementary teachers' understanding of volume concept. Such pursuit is believed to help in teaching and curriculum development.

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2. Conceptual Analysis of Measurement and Relevant Literature

Pursuing the aforesaid goal requires identification and clarification of certain meanings and definitions based on the literature. Bright (1976) describes measuring as a process of comparison of an attribute of an object with a selected unit for the purpose of quantifying that attribute. This is one of the fundamental definitions referred to in articles focusing on measurement. There are several hidden components being highlighted in this definition. These hidden components as well as other important meanings (as mostly related to volume concept) are detailed in the below argument.

1. Measuring is considered as a process in Bright's (1976) description. This process is dependent on iteration of units, counting of units and size of units (Curry and Outhred, 2005). For example, in order to find the volume of a given rectangular right prism (hereafter, abbreviated as RRP) with the dimensions 4cm (width), 6cm (length), 3cm (height), one first needs to decide on a certain size unit (e.g., 1cm \times 1cm \times 1cm). Then, he needs to use a type of iteration to count the number of units that can fit into the given RRP. Note that such counting through iteration is done in three dimensions (Curry and Outhred, 2005) and requires a spatial structuring (Outhred and Mitchelmore, 2000) as partially illustrated in Figure 1. This spatial structuring is not easily understood by students (Battista et al., 1998) and requires partitioning regions into arrays (as in Figure 1) as well as coordination with iteration (Reynolds and Wheatley, 1996).

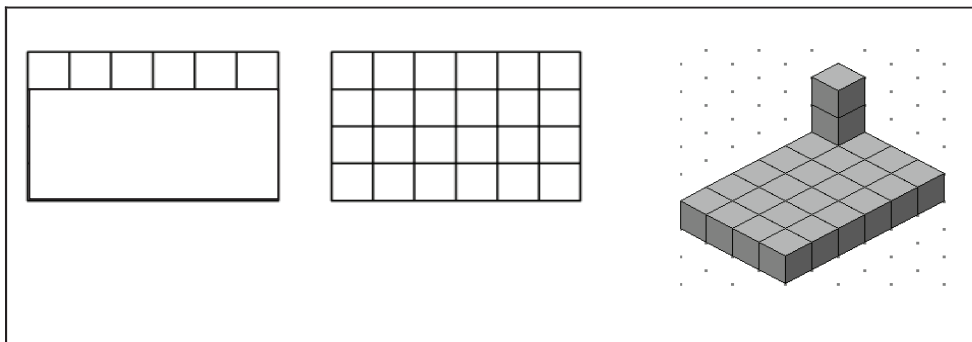


Figure 1. Spatial structuring of the RRP with the dimensions, 6cm(length) \times 4cm(width) \times 3cm(height).

2. Measuring process consists of an activity of comparison. Considering the RRP in the given example, one needs to compare the volume of the chosen unit (e.g., 1cm³) to the space covered by the RRP, which is basically the core of iteration. Since measuring is about attributes (volume, area, length, etc.) of figures and a unit to quantify that attribute through comparison, the unit and the figure under consideration should have matching attributes (Nitabach and Lehrer, 1996) so that they are comparable. In the previous example, one can measure the volume of the given RRP using a cubic unit, not a stick.

3. In measuring volume, a coordination of three dimensions is required. Further, for a given RRP, one needs to understand that dimensions and the number of units that can be laid along a dimension are related (Outhred and Mitchelmore, 2000). For example, if the length of a given RRP is 6cm as in the previous example, one can put mostly 6cm-long units along the length of the RRP. Additionally, covering the space occupied by an RRP (see Figure 1) using a unit cube is a one dimensional and additive act, whereas finding the volume of that RRP through the formula (length \times width \times height) is three dimensional and multiplicative (Outhred and Mitchelmore, 2000). Such distinction may confuse students as recommended by researchers.

There are some recent efforts that also highlight the importance of unit and the aforesaid process of comparison. Reynolds and Wheatley (1996) pointed to the importance of unitizing (an activity of constructing units) (Steffe & Cobb, 1988) and coordination of units as fundamental components of learners' activity in conceptualizing area, which can also be applied to volume. For example, to find the volume of an RRP, one needs to form a rectangular layer of unit cubes (as in middle part of Figure 1) and iterate that layer as a new unit (unit of units) a number of times (corresponding to height, as in rightmost part of Figure 1) to find the volume of an RRP. Hence, one needs to think about layers as iterable units and the volume of an RRP as a composite unit as a result of unitizing.†

Even though understanding of unit is very important in conceptualizing volume, students have a lack of understanding of units used in one-, two-, and three-dimensional measurement situations (Raghavan et al., 1998) and do not fully understand the importance of unit in measuring (Curry and Outhred, 2005). Kamii and Kysh (2006) also stress that a unit is not considered as a unit (or having a space covering feature) in area situations by 4-8 grade students. Students do not have a good grasp of principles behind the formulas and use inappropriate formulas for given situations either (Raghavan et al., 1998). For example, most students used the formula ‘base times height’ to find the volume of nonregular shapes.

The presented literature suggest that learners are weak in making sense of measurement concepts and need to understand certain constructs in measurement (e.g., unit, iteration, dimension, spatial structuring, coordination) in order to be able to analyze characteristics and properties of measurement systems and understand measurement concepts.

3. Method

This study is a combination of two efforts, one made in U.S. as a pilot study and one made in Turkey as the main study. The researcher initially explored the common difficulties of prospective elementary teachers’ and their reasoning about the volume concept through a number of piloting efforts conducted in U.S. In the last of those piloting efforts, two senior prospective elementary teachers participated in the study. These teachers were from a U.S. university and were taking mathematics methods courses. The researcher investigated these two teachers’ understanding of measurement ideas (especially volume) using up-to-two hour one-on-one semi-structured interviews through videotaping. Based on a detailed analysis of this last piloting effort and available literature, the researcher then developed a series of research questions that may be of interest to the mathematics education field in the area of volume measurement. The research questions pursued in this study were: (1) What concept images do prospective elementary teachers hold of ‘volume’? In what ways are these concept images related to (their) concept definition of volume? (2) How and in what ways do prospective elementary teachers’ personal concept definitions of Cavalieri’s Principle (the main idea for base area \times height) guide their concept images of ‘volume’? (3) What is teachers’ concept image/definition of ‘dimension’? (4) To what extent prospective elementary teachers could coordinate their concept images/definitions of volume, Cavalieri’s Principle and ‘dimension’? (5) What is the nature of the constructing and coordinating required for conceptualizing volume concept?

To pursue these research questions, the researcher then conducted the main study consisting of one-hour-long one-on-one interviews with 10 more prospective elementary teachers from a metropolitan-city university in Turkey. In order to capture participants’ work clearly and for retrospective analyses purposes, all the interviews were both video- and audio-taped. The participants were informed about the interview process and they were asked to think aloud during the interviews.

The researcher investigated the aforesaid research questions based on qualitative analyses of the interviews using Tall and Vinner’s (1981) framework. Data analysis process for the main study still continues and the author will only provide an example to illustrate an answer to the first of these research questions. The rest of the results and implications will be shared with the audience during the session.

3.1. Data Analysis Procedure and Conceptual Framework

The interview data has been analyzed using Tall and Vinner’s (1981) framework about concept images and concept definitions. Tall and Vinner (1981) considers all mental attributes (conscious or unconscious), mental pictures, properties or processes associated with a concept as the *concept image*. Concept image is “built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures” (Tall and Vinner, p.2). In this framework, a portion of the concept image that is activated at a particular time is labelled as *evoked concept image*. On the other hand, these scholars consider “form of words used to specify that concept [...] [or] a personal reconstruction by the student of a definition” (ibid, p.2) as *concept definition*. This framework suggests that concept definitions are to some degree related to the concept. Individuals learn concept definitions in either a rote fashion or meaningfully. In their given description, Tall and Vinner (1981) made a distinction between two types of concept definitions. A concept definition that can be revised by individuals from time to time is called *personal concept definition*. On the other hand, a concept definition that is accepted by the mathematics community at large is

called *formal concept definition*. At times, an individual may experience a conflict between part of a concept image/definition and another part of the concept image/definition. Such conflict is called a *potential conflict factor*. What follows is an example illustrating such conflict among an individual's concept definitions and images.

4. Results

One formal concept definition for the volume of a figure is the amount of space contained within the boundaries of the figure. Piaget called this type of volume as interior volume and it is widely accepted by mathematicians and educators. In the interviews of both the pilot and the main study, all the participants without any exception answered the questions of "What is volume? How would you describe it?" by referring to this concept definition of volume at some point in the interviews. The purpose of asking such a question was to determine participants' personal concept definitions and related concept images. One of the participants' dilemmas about this concept definition is highlighted below as an example.

Dan (an American teacher) provided two personal concept definitions for volume. One was stated as "length \times width \times height," whereas the other one was "volume is the amount of space something takes up." Interestingly these two concept definitions, even though sound and reasonable, did not allow him to find the volume of a given cylinder. Dan's reasoning with cylinder was particularly interesting in terms of seeing (lack of) coordination among different concept definitions and concept images of volume. When asked to explain what he thought about volume of the given cylinder, he referred to one of his concept definitions, "length \times width \times height," and then went on to search for the number of units that can fit along each dimension. His concept image of cylinder was that since cylinder is three dimensional, it has length, width and height. Therefore, he went on to finding these three measures to find the volume of cylinder. He first associated length with circumference of the one of the bases of the cylinder, "I've got to find the circumference." By assigning approximate values to diameter (as 10cm) and π (as 3), he calculated circumference as 30cm and mentioned "So, there we've got the length [writing L next to 30] ... circumference is length." He then continued: "I'm imagining this [pointing to the ready-made regular cylinder] flattened out." The flattened out version of the cylinder is a rectangle and length of that rectangle corresponds to the circumference of the upper base of the cylinder for Dan. He then continued his venture by looking for height and width as follows.

D: Let's just say ... [measuring height of the cylinder using the ruler]. [...] So, let's just make it twenty [finding height as 23cm but taking an approximate value of 20cm and writing "h" next to it], is that all right? [...] Okay, so I have the length, the height. How come I can't see the width right now! [paused for 10 seconds] Be the same [writing 30 (same as height) and then labeling it as "w"]?

Dan seemed to have the concept image that a three-dimensional figure suggests having the measures of length, width, and height. For Dan, these measures for cylinder were the circumference of the base corresponding to the length and height of the cylinder corresponding to the height as well as the width. He used these concept images to make sense of his concept definition of "length \times width \times height." He multiplied those three measures (30 \times 20 \times 30) and mistakenly found the result as 6000 cubic centimeters. Without any interruption the researcher then asked if it is possible to place 6000 plastic cubes (laid out on the table) into the cylinder. Such a question puzzled him and caused him to shift his thinking to his second concept definition (amount of space inside figures) and to reconsider his strategy since "6000 is quite a big number." Using the closed version of the cylinder he was moving his hand around the circumference of the bases from inside (tracing the lateral faces from inside) the cylinder and mentioning that cube blocks should be placed vertically inside the cylinder around the lateral faces from inside (see Figure 2). Dan then continued by stating that "the diameter shows how far they [referring to the block of cubes that are aligned with height] can go across like this [moving his hand in the air along the diameter of the upper base of the closed cylinder]" (see Figure 2). He then continued as follows.

D: So you have twenty deep going up and down, Okay? (See Figure 2) [...] So, and then you can go all the way around [pointing to the inside of the cylinder in a circular motion by moving the cube block in circular motion]. If this [referring to the cut-open cylinder] was closed, you would go all the way around with these [blocks]. And then the diameter would show you how far you can go like this [moving the block of cubes along with a made-up diameter in air]. [...]

R: Okay, so diameter tells, so if diameter is ten, does it mean that you are going to go ten- Like this, this way [moving his hand along with a made-up diameter in air].

D: Ten, you should go ten from any edge. Any edge [pointing to two points on the circular border of the upper base as if those two points are the end points of the line segment representing the diameter of the base of the cylinder], you should be able to go 10 across.



Figure 2. Dan's description of how 6000 cubes would fit in the given cylinder.

Dan's case is an example illustrating the lack of coordination among his concept definitions and his conflicting concept image about the dimensions of the cylinder. This was partly because of the fact that he was not able to coordinate units of measure (Reynolds and Wheatley, 1996). Further results will be provided in the presentation.

5. Concluding Remarks

Preliminary results of the current study suggest that there is a lack of coordination among participants' concept definitions, and between their concept images and their concept definitions of volume. It would be unreasonable to expect future teachers to design learning environments promoting such coordination for their future prospective students if they do not have a conceptualization of such coordination themselves. .

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