Parametric Resonance Analyses for Spar Platform in Irregular Waves

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ABSTRACT

The parametric instability of a spar platform in irregular waves is analyzed. Parametric resonance is a phenomenon that may occur when a mechanical system parameter varies over time. When it occurs, a spar platform will have excessive pitch motion and may capsize. Therefore, avoiding parametric resonance is an important design requirement. The traditional methodology includes only a prediction of the Mathieu stability with harmonic excitation in regular waves. However, real sea conditions are irregular, and it has been observed that parametric resonance also occurs in non-harmonic excitations. Thus, it is imperative to predict the parametric resonance of a spar platform in irregular waves. A Hill equation is derived in this work, which can be used to analyze the parametric resonance under multi-frequency excitations. The derived Hill equation for predicting the instability of a spar can include non-harmonic excitation and random phases. The stability charts for multi-frequency excitation in irregular waves are given and compared with that for single frequency excitation in regular waves. Simulations of the pitch dynamic responses are carried out to check the stability. Three-dimensional stability charts with various damping coefficients for irregular waves are also investigated. The results show that the stability property in irregular waves has notable differences compared to that in case of regular waves. In addition, the stability chart obtained using the Hill equation is an effective method to predict the parametric instability of spar platforms. Moreover, some suggestions for designing spar platforms to avoid parametric resonance are presented, such as increasing the damping coefficient, using an appropriate RAO and increasing the metacentric height.

Key words: *Spar platform; instability; parametric resonance; Mathieu equation; irregular waves; Dynamic responses*

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1. Introduction

Understanding parametric resonance is very important for the safety of floating structures. Parametric resonance may occur when a mechanical system parameter varies over time. For platforms and ships, this parameter is usually the metacentric height. Parametric resonance is known to occur during the roll motion of ships under certain conditions (Fossen and Nijmeijer 2011). Parametric roll is a dangerous phenomenon for ships, and has been considered to be the reason for some accidents (Galeazzi, Blanke et al. 2013). If a vessel experiences parametric rolling, accidents may occur, which can lead to high economic losses (Ginsberg 1998, France, Levadou et al. 2003). Because accidents can occur as a result of parametric resonance in irregular waves, parametric resonance should be regarded as a real danger in not only regular waves but also irregular waves. For many years, researchers have paid more attention to parametric rolling for ships in regular seas (Paulling 1961, Dunwoody 1989, Dunwoody 1989, Spyrou 2000, Francescutto 2001, Bulian 2006, Neves and Rodríguez 2007, Chang， B.C 2008, Spyrou, Tigkas et al. 2008). Similar to ship parametric rolling, spar platform parametric resonance in regular waves has also been observed in experimental tests (Haslum and Faltinsen 1999, Hong, Lee et al. 2005, Neves, Sphaier et al. 2008). Several researchers have suggested parametric instability analyses for offshore structures in irregular waves using wave spectrums (Witz 1995, Spyrou 2000, Zhang, Zou et al. 2002, Radhakrishnan, Datla et al. 2007). Pettersen (Pettersen and Machado-Damhaug 2007) found that the parametric instability of a spar platform was triggered by irregular waves in experimental tests. Most studies of parametric instability have been based on harmonic excitation and characterized by regular waves. The parametric instability prediction methodology for a spar platform in irregular waves is still at the level of a recommendation and requires further investigation. The accurate experimental investigation of the parametric resonance of offshore structures is not easy; moreover, it is very time-consuming (Bulian, Francescutto et al. 2008). Stability analyses should be extended to "realistic" seaways (Spyrou and Thompson 2000). Therefore, the development of a methodology for predicting parametric resonance in irregular waves is urgently required for spar platforms.

Several studies have been carried out to predict spar parametric resonance in regular waves on the basis of the Mathieu equation (Haslum and Faltinsen 1999, Zhang, Zou et al. 2002, Rho, Choi et al. 2005, Radhakrishnan, Datla et al. 2007, Zhao, Tang et al. 2010). A spar platform in real sea conditions may experience non-harmonic forces. However, the Mathieu equation was derived using harmonic excitations in regular waves. Thus, predicting the parametric instability of a spar platform in irregular waves is a challenging task. The Mathieu equation was used by several researchers to discuss the instability properties of ships and offshore structures (Koo, Kim et al. 2004, Rho, Choi et al. 2005, Radhakrishnan, Datla et al. 2007, Chang，B.C 2008, Spyrou, Tigkas et al. 2008). Haslum and Faltinsen (1999) (Haslum and Faltinsen 1999) studied the instability property of spar platforms using a simplified method. Mathieu’s instability phenomenon was found using both numerical simulations and model tests. Tao and Cai (Tao and Cai 2004) presented heave motion suppression of a spar platform with a heave plate. They calculated the heave damping forces by directly solving the Navier-Stokes equation. Zhang et al. (Zhang, Zou et al. 2002) drew spar instability regions on a Mathieu chart with damping changes. He pointed out that the probability of occurrence of regular waves with period above 20s under the real sea conditions is very small. It is more important to develop stability diagrams in irregular waves with wave spectrums. Radhakrishnan et al. (Radhakrishnan, Datla et al. 2007) presented an experimental analysis of the instability of a tethered buoy in regular waves. He recommended further investigation of the instability that occurs in irregular waves. Parametric resonance has been found in the experiments for spar platforms in irregular waves (Pettersen and Machado-Damhaug 2007). However, very few investigations have predicted the instability by developing a stability diagram on the basis of the damping effects for irregular waves with a specific spectrum (Wang and Zou 2006).

In this paper, the parametric resonance of a spar in irregular waves is discussed using a Hill equation. It is derived from the motion equation for a spar platform based on the linear wave theory and response amplitude operator (RAO). The corresponding stability charts for a spar platform in irregular waves are developed by analyzing the equation. A comparison between Mathieu’s equation for a single frequency and Hill’s equation for multiple frequencies is also carried out. This work also provides design guidelines to avoid the occurrence of parametric instability in a spar platform.

2. Theory and Method

2.1 Irregular wave theory

Irregular waves can be represented as the sum of harmonic wave components with random phases according to a wave spectrum. The Joint North Sea Wave Analysis Project (JONSWAP) spectrum is one of the most commonly used wave spectrum, its formula is as follows:

$$S(\omega)=\frac{5{H_s}^2{\omega_p}^2}{16\omega^5}\cdot(1-0.287\lg\gamma)\cdot\exp\left(-\frac{5{\omega_p}^4}{4\omega^4}\right)\cdot\gamma^{\exp\left[-\frac{(\omega-\omega_p)}{2\sigma^2{\omega_p}^2}\right]} \tag{1}$$

where H_S is the significant wave height, ω is the wave frequency, ω_p is the peak wave frequency, γ is the peak enhancement factor, and σ is given as

$$\sigma = \begin{cases} 0.07 & \text{if } \omega \le \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases} \tag{2}$$

The wave spectrum shows the distribution of the energy by frequency. Assume that the wave frequencies are in the $\omega_0 \sim \omega_n$ range. Some frequencies ω_1, $\omega_2, \ldots,$ ω_{n-1} can then be chosen using equal division of the frequency range or energy.

The wave height at one point can be expressed as follows:

$$\zeta(t) = \sum_{i=1}^{n} \sqrt{2S(\bar{\omega}_i) \cdot \Delta\omega_i} \cdot \cos(\hat{\omega}_i t + \varepsilon_i) = \sum_{i=1}^{n} \zeta_i \cos(\hat{\omega}_i t + \varepsilon_i) \tag{3}$$

Where $\Delta\omega_i = \omega_i - \omega_{i-1}$, $\bar{\omega}_i = (\omega_{i-1} + \omega_i)/2$, $\hat{\omega}_i$ is random with the intervals (ω_{i-1}, ω_i), ε_i is random phase for each element.

2.2 Dynamic responses of spar platform in irregular waves

Obtaining the heave motion of a platform on the basis of a numerical model is a complex and time-consuming process. Therefore, the response amplitude operator (RAO), which is easier to apply, is more often used when performing motion analyses of offshore floating structures.

The main purpose of RAO is to show the relationship between the structure's motion and the incoming unit wave. In this study, the pitch motion is assumed to be within a quite small range before parametric resonance. Thus, the nonlinear effects of pitch motion on heave motion can be ignored before parametric pitch occurs, and it can be assumed that the response of the heave motion of a spar platform to an incoming wave is a linear system. And when parametric pitch occurs, though the nonlinear effects cannot be ignored, it is already determined to be unstable. So, it is proper to apply the RAO of heave to simulate the heave motion in prediction of parametric pitch.

Both the amplitude of the wave and that of the spar motion can be represented as the superposition of sine/cosine waves with the same frequencies. For each component of the superposition of the spar motion, the amplitude and phase can be obtained by the corresponding component of the superposition of incoming wave with the same frequency and RAO.

The RAO curve for the $H(T)$ value of the studied spar platform is shown in Fig. 1, with a JONSWAP spectrum.

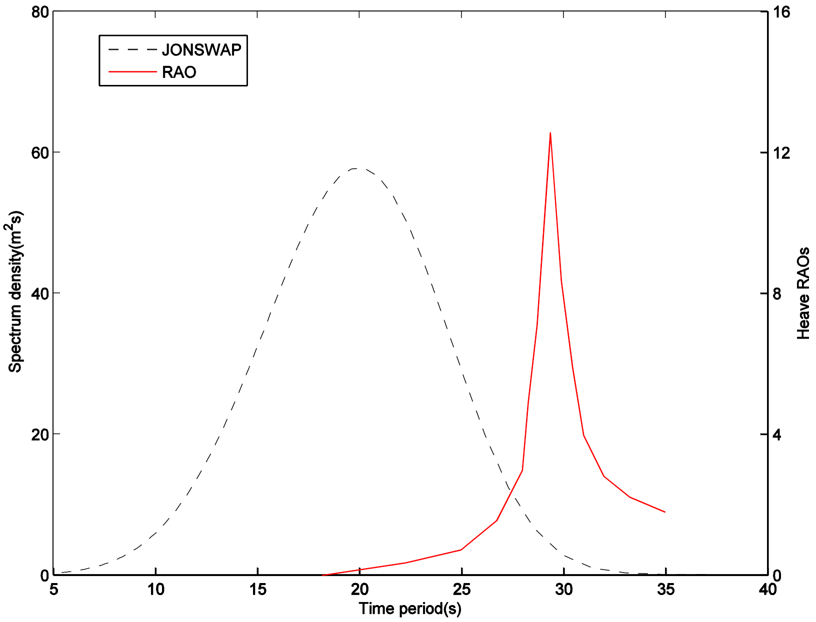


Fig.1. Heave response of spar and JONSWAP spectrum

For a single wave frequency, the heave motion can be given by

$$\eta(t)=\eta_0\cos(\omega t+\varepsilon)=H(2\pi/\omega)\zeta_0\cos(\omega t+\varepsilon) \tag{4}$$

According to the linear wave theory, assuming that the response of the spar platform is a linear system, the heave motion of the spar under random waves is

$$\eta(t)=\sum_{i=1}^{n}H(\hat{\omega}_i)\zeta_i\cos(\hat{\omega}_i t+\varepsilon_i) \tag{5}$$

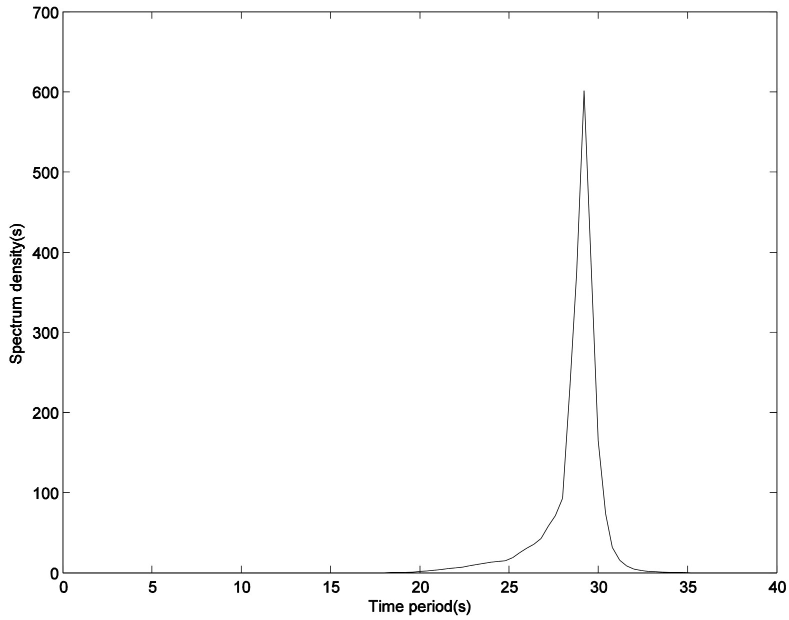


Fig.2. Heave spectrum of spar under JONSWAP spectrum

The heave motion amplitudes can also be obtained from the spectrum of the spar heave motion by applying RAO to the wave spectrum, as shown below:

$$S_\eta(\omega)=\left|H(\omega)\right|^2 S_\zeta(\omega)$$

(6)

The heave spectrum in Fig. 2 shows the spectrum density of the heave motion energy with the time period. Comparing with Fig. 1, it can be observed that the peak frequency of heave motion is the peak of RAO, namely the natural frequency of heave, but not that of wave spectrum. So in condition of irregular, the frequency is focused on the relation between natural frequencies of heave and pitch motions.

The discretization of the spectrum is conducted by choosing a set of time periods and calculating the amplitude for each time period, as in the case of the wave spectrum. The heave motion is then given by their superposition, which can be rewritten to be the same of Eq.5.

$$\eta(t)=\sum_{i=1}^{n}\sqrt{2S_\eta(\bar{\omega}_i)\cdot\Delta\omega_i}\cdot\cos(\hat{\omega}_i t+\varepsilon_i)=\sum_{i=1}^{n}H(\bar{\omega}_i)\zeta_i\cos(\hat{\omega}_i t+\varepsilon_i)$$

(7)

2.3 Instability prediction using Mathieu and Hill equations

Most of the previous studies on spar platform parametric resonance analysis have assumed harmonic behavior for the time-varying restoring arm of metacentric height. The parametric resonance has been modeled as a Mathieu type equation in this assumption. However, a Mathieu equation has several limitations. First, the accuracy of the instability prediction is low. Second, a Mathieu equation is only valid for regular waves with a constant phase. The regular wave assumption is physically unrealistic, except perhaps under certain swell conditions (Witz 1995). A stability prediction using a Mathieu equation is limited to a single harmonic excitation frequency. In this study, the Hill equation derived for spar instability prediction can include non-harmonic excitation and random phases.

2.3.1 *Mathieu equation*

One classic method for studying parametric stability is the use of a Mathieu equation. In this method, the heave-pitch motion equation for a spar platform is simplified as

$$(I+M)\ddot{\phi}+C\dot{\phi}+\Delta\left(GM-\frac{1}{2}\eta\cos\omega t\right)\phi=F\cos(\omega t+\gamma)$$

(8)

where, I is the moment of inertia of the pitch, M is the added mass coefficient, C is the coefficient of linear damping, Δ is the displacement, GM is the initial value of metacentric height,

$\eta cos\omega t$ is the single-frequency heave motion, and $Fcos(\omega t+\gamma)$ is the force of the external excitation.

One method that is used to determine whether the result is stable is to see whether the solutions of a variation equation tend to be zero. The variation equation is

$$(I+M)\delta\ddot{\phi}+C\delta\dot{\phi}+\Delta\left(GM-\frac{1}{2}\eta\cos\omega t\right)\delta\phi=0 \tag{9}$$

By using the following variable substitution

$\lambda=\frac{\pi}{2}+\omega t,\ x=\delta\phi$

The equation can be rewritten as

$$(I+M)\omega^2\ddot{x}+C\omega\dot{x}+\Delta\left(GM+\frac{1}{2}\eta\cos\lambda\right)x=0 \tag{10}$$

This equation can then be written as a damped Mathieu equation as follows:

$$\ddot{x}+c\dot{x}+(a+q\cos\lambda)x=0 \tag{11}$$

The substitutions are as follows

$\dot{x}=\frac{dx}{d\lambda},\ a=\frac{\Delta GM}{(I+M)\omega^2}=\left(\frac{\omega_5}{\omega}\right)^2,\ q=\frac{\Delta\cdot\eta}{2(I+M)\omega^2},\ c=\frac{C}{(I+M)\omega}$

where ω_5 is the natural pitch frequency.

2.3.2 *Hill's equation*

In condition of irregular waves, the heave motion cannot be simply described by a cosine wave. The equation of motion can then be written as

$$(I+M)\ddot{\phi}+C\dot{\phi}+\Delta\left(GM-\frac{1}{2}\eta(t)\right)\phi=F(t) \tag{12}$$

Similar to the above, the variation equation is

$$(I+M)\delta\ddot{\phi}+C\delta\dot{\phi}+\Delta\left(GM-\frac{1}{2}\eta(t)\right)\delta\phi=0 \tag{13}$$

In this study, the analyses mainly focus on the effects of the size of these parameters, thus the values of these parameters are assumed to be constant, ignoring the effects of different wave frequencies and some other aspects.

The heave motion, which is the linear superposition of a group of sine and cosine waves in irregular waves, can be written in form of Fourier expansion, as shown below:

$$\eta(t)=\sum_{k=1,2,...} A_k \sin\omega_k t + B_k \cos\omega_k t = \sum_{k=2,4,...} A_{\frac{k}{2}} \sin\omega_{\frac{k}{2}} t + B_{\frac{k}{2}} \cos\omega_{\frac{k}{2}} t \tag{14}$$

where ω_k=2$k\omega_0$, ω_0 is half the chosen base frequency, $A_k=\eta_k \sin\varepsilon_k$ and $B_k=\eta_k \cos\varepsilon_k$.

The normal form of Hill's equation is

$$\ddot{x}+2c\dot{x}+\left(a+2q\varphi(\tau)\right)x=0 \tag{15}$$

To deal with the heave motion of the platform, $\varphi(\tau)$ is taken as

$$\varphi(\tau)=\sum\nolimits_{k=1,3,...} a_k \sin(k+1)\tau + \sum\nolimits_{k=2,4,...} a_k \cos k\tau = -\eta(t) \tag{16}$$

where $\tau=\omega_0 t$, $a_k=-A_{(k+1)/2}\,(k=1,3,...)$, $a_k=-B_{k/2}\,(k=2,4,...)$.

The variation equation can then be rewritten as Hill equation by making the following substitutions:

$$x=\delta\phi,\ \dot{x}=\frac{dx}{d\tau},\ a=\frac{\Delta GM}{(I+M)\omega_0^{\ 2}}=\left(\frac{\omega_5}{\omega_0}\right)^2,\ q=\frac{\Delta}{4(I+M)\omega_0^{\ 2}},\ c=\frac{C}{(I+M)\omega_0}$$

Using Bubnov-Galerkin approach, the stability of the solution of Hill equation can be determined (Pedersen 1980). Though, with the determinant order larger, the obtained boundary could be more accurate, it will cost much more time in calculation. Thus, in order to get a accurate boundary within acceptable time, the determinant order is recommended to be 2n+1, where n is the order of the expansion of $\varphi(\tau)$. In this study, n is chosen to be 15 to give a more efficient analysis (Yang and Xu 2015) .

According to the method above, to study the parametric instability by using this equation, only the natural pitch frequency and metacentric height are needed. This is very convenient for avoiding parametric instability in a preliminary design. The Hill equation can capture non-harmonic excitations under real sea conditions. The effects of damping and random phases are also considered in the stability prediction.

3. Stability Charts in Regular Waves and Irregular Waves

The Mathieu equation is very general and does not capture the non-harmonic excitation property. If the excitation is not a single frequency harmonic, the system cannot be represented by a

Mathieu equation. In such a case we can represent the time-varying coefficients as a Fourier expansion. The derived equation is called a Hill equation.

3.1 Stability chart in regular sea

The stability chart shows the stability region for the corresponding values of a and q according to the solution of the Mathieu equation. By changing damping c, instability regions with different damping coefficients can be obtained.

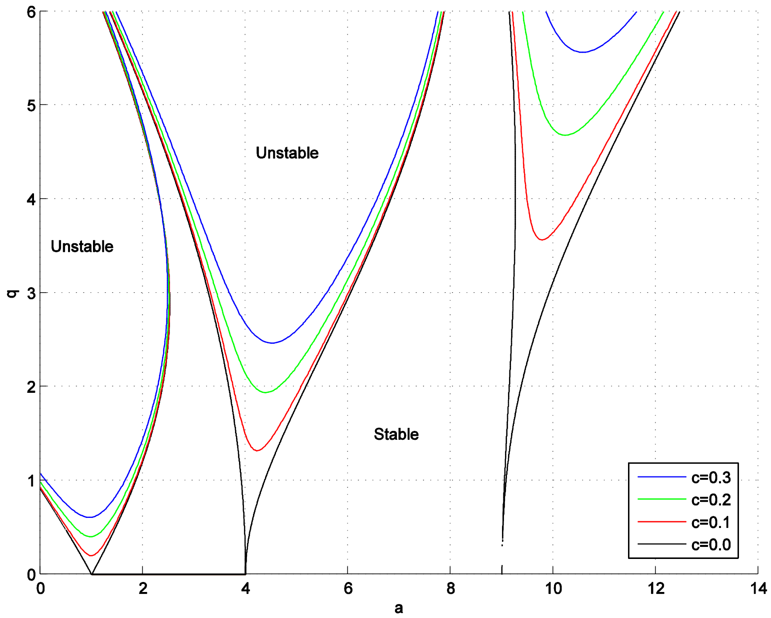


Fig.3. Stability chart for single frequency with varying damping

Most of the previous studies have used stability charts that do not indicate the effect of damping. Damping dramatically affects the boundaries between the stable and unstable regions. As shown in Fig. 3, the instability region is suppressed when damping increases. A two dimensional chart is the most common method for predicting the parametric resonance of a spar platform. A major drawback of the method is that the chart does not depend on the spar characteristic. A three-dimensional chart that depends on the spar parameters would be a more practical approach for an engineering project. By applying the chart, the occurrence of parametric resonance can be predicted more conveniently in the preliminary design stage. To visualize the effect of damping on the instability regions directly, a three-dimensional stability region figure in regular waves is given in Fig. 4, in which the added axis corresponds to the varying damping coefficient.

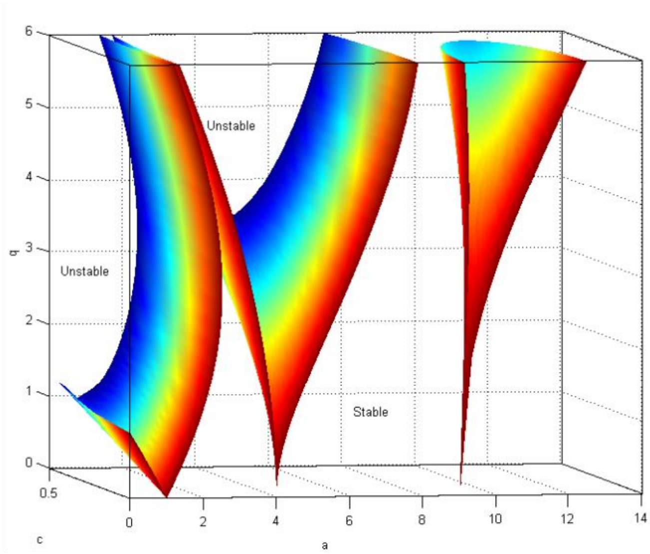


Fig.4. Three-dimensional stability chart for single frequency

It can be seen from the above figures that increasing the damping can reduce the region of parametric instability, with a higher order resulting in a greater damping effect. The results of these charts agree with the results of previous studies.

3.2 Stability prediction for spar platform in irregular sea

The heave motion is simulated according to the wave spectrum and RAO. With the Fourier expansion, the heave-pitch coupled motion equation can be rewritten as Hill equation, and the stability chart can be given. Boundary lines in Fig. 5 with c=0 shows the stability chart for a spar platform with a damping coefficient of zero. Besides, in this section the initial phases are chosen to be zero for convenience.

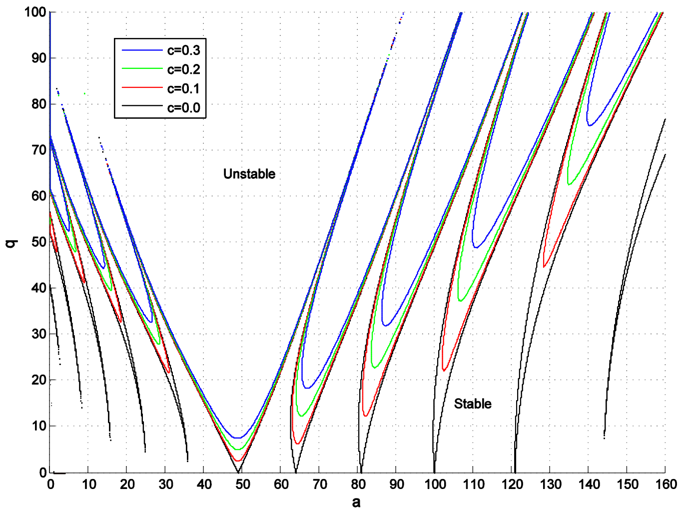


Fig.5. Stability chart for multiple frequencies with varying damping

In Fig. 5, it can be observed that there are more instability regions, with most being much thinner. Compared to a single frequency condition, the stability regions are distributed more widely.

Thus, in irregular waves, the occurrence of parametric resonance is more random.

3.2.1 *Effects of damping changes*

As in the case of a single wave, increasing the damping coefficient can also decrease the instability region in the stability chart. Fig. 5 shows the different instability regions with various damping coefficients.

The three-dimensional stability region for multiple frequencies is shown in Fig. 6, from which the effect of the damping coefficient can be directly observed. In this figure, the two outside axes show the values of variable a and q, where the value of a shows the relationship between the natural frequency of the pitch and that of the heave, and value of q shows the effect of the amplitude and the initial value of the metacentric height. The third axis represents the value of the damping coefficient. The first three instability regions are omitted because they are too small to show in the figure. Unlike the single frequency condition, where the first-order region is the least effected by damping, with the multi-frequency condition, the effects of the damping on the regions show a slope shape. The region which corresponds to the first region in condition of single frequency is least affected by the damping,. It is supposed that the property of the instability regions under the multi-frequency condition is affected the most by the wave element with the largest amplitude among the set of harmonic waves. These stability charts can act as a guide in the preliminary design of a spar.

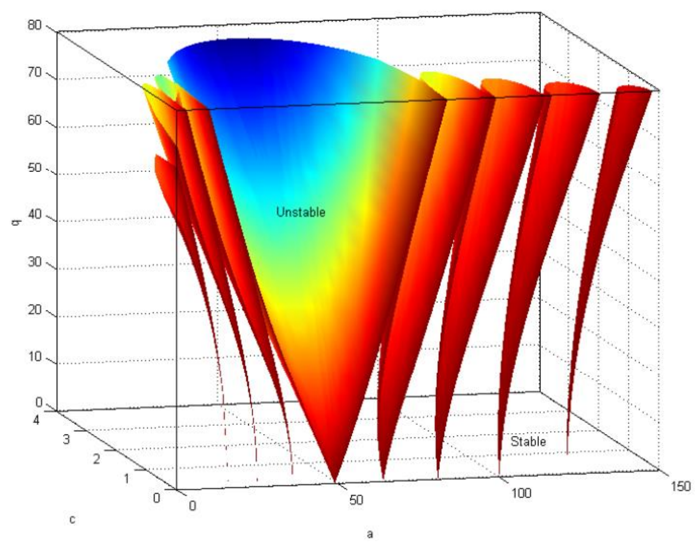


Fig.6. Three-dimensional stability chart for multiple frequencies

3.2.2 *Checking for instability using stability charts*

To verify the effectiveness of the charts, some typical design points are chosen to check the stability. The parametric values of the design points are applied in the equation of motion, and the time histories of the pitch are given. The results of the numerical simulations are compared with the positions of the points in the charts to see whether the charts predict the stability correctly.

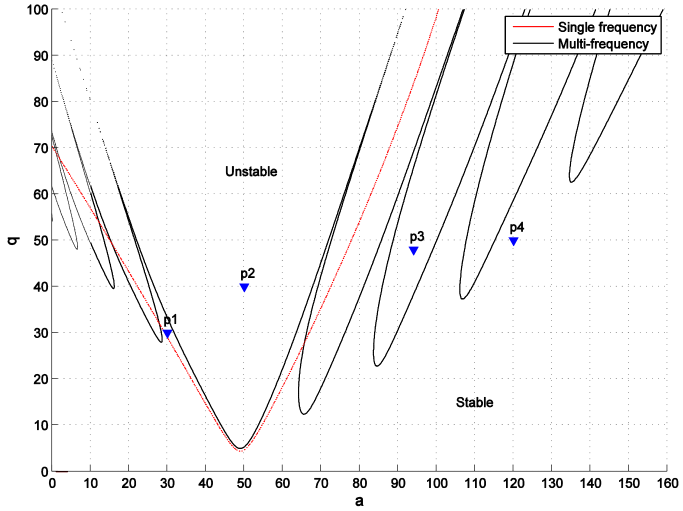
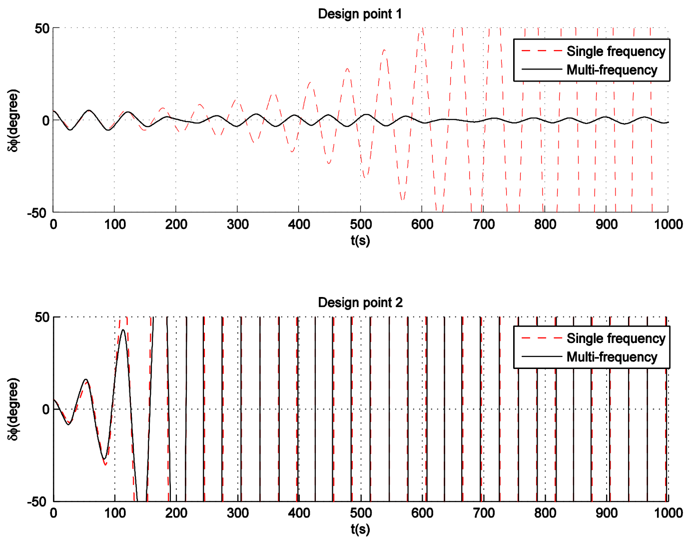


Fig.7. Chosen design points in stability chart with damping of 0.2

The results of the simulations are listed in Fig. 8 and Table 1. The results show that the stability charts can effectively distinguish the stability and instability regions.

Table 1 Simulations results for design points

Design points	c = 0.2	
	Single frequency	Multiple frequencies
p1	stable	stable
p2	unstable	stable
p3	unstable	unstable
p4	stable	unstable



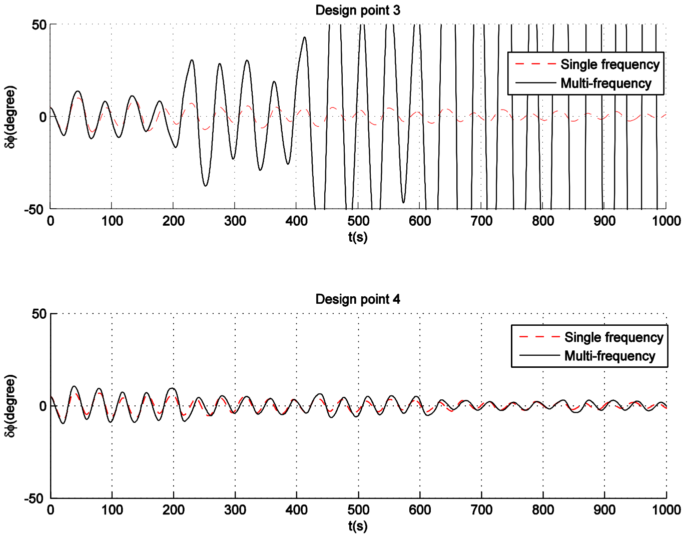


Fig.8. Numerical checks for chosen design points

3.2.3 *Effects of metacentric height and heave amplitude*

The design points for a spar platform in a stability chart can be regarded as a set of points along a line that passes through the origin point with a slope of 4η/GM, which can be called the design line. When the design points fall in an instability region, the design is thought to be a failure. Therefore, if the design line has less intersection with an instability region, it is easier to avoid parametric pitch in the design.

In previous studies, it has been suggested that a smaller GM amplitude and higher initial GM value could reduce the possibility of instability. In Fig. 9, the effect of smaller GM amplitude and higher initial GM value is that the slope of design line becomes smaller. The damping of the pitch is also considered in this figure, which is shown as a decrease in the area of the instability regions.

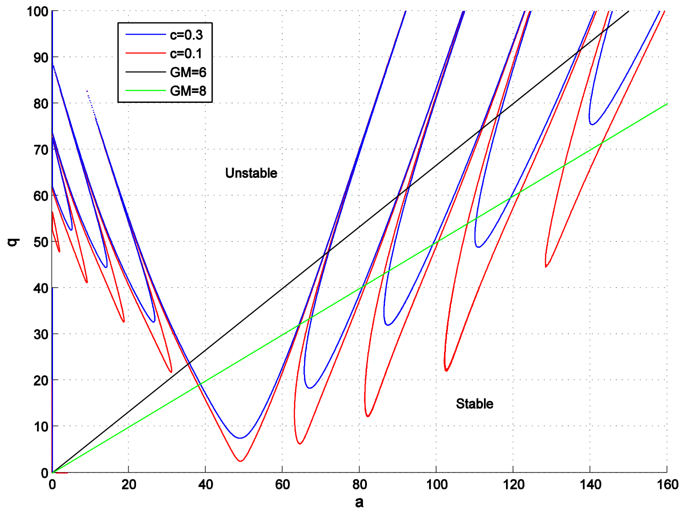


Fig.9. Stability chart for different damping and design lines with different GM values

The effects of these parameters can be easily observed in this figure. With a higher initial GM value, smaller amplitude for the heave motion, and larger damping coefficient, the instability is more likely to be avoided. These results agree with those of previous studies. However, for different regions, increasing the damping of the system and the metacentric height has different levels of effect. For the largest region, a higher GM can significantly decrease the area of instability, while for other thinner regions, increasing the damping works better.

3.2.4 *Effects of random phases*

In regular waves, the initial phase of wave has little effects on the responses of spar platforms. And in the motion equation, the phase can be eliminated through variable substitution. However, in condition of irregular waves, when simulating it with multi-frequency waves, the effects of random phases cannot be dealt in this way. Thus, it is important to analyze how the random phases affect the property of parametric instability of a spar platform.

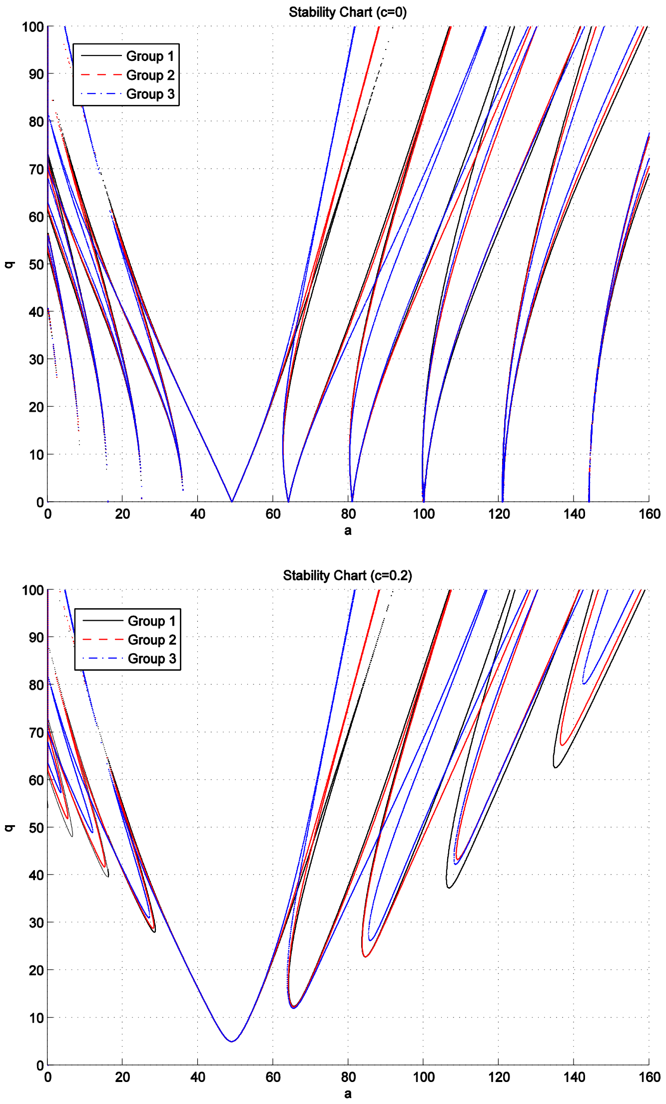


Fig.10. Stability chart for different initial phases

Fig. 10 shows the instability regions with three groups of initial phases. For the first group, all initial phases are chosen to be zero. The initial phases in other two groups are chosen randomly based on average distribution in range of (-π, π].

It can be observed that, though the regions are different under different initial phases, the differences are quite small, especially the main region. Also, for each region, with smaller value of q, the effect becomes smaller. Besides, for most regions, the ranges are wider when initial phases are chosen to be zero.

Thus, it is acceptable to ignore the effects of random initial phases when analyzing the property of parametric pitch of spar platform. However, for more accurate prediction in numerical simulation, it should be considered. The stability charts are more recommended to be applied for qualitative analyses.

4. Numerical Cases

4.1 Numerical model of spar platform

The main parameters of the classical spar platform being investigated are listed in Table 2, where the parameters are the initial values when no heave or pitch motion occurs.

Table 2 Main parameters of studied spar platform

Parameters	Symbol	Value
Diameter	D	37.5 m
Draft	T_d	202.50 m
Displacement	Δ	229246 t
Heave natural period	T_{n3}	29.4 s
Pitch natural period	T_{n5}	99 s
Metacentric height	GM	4.00 m

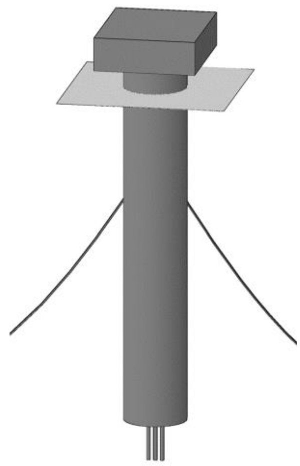


Fig.11. Hull shape of classical spar

The JONSWAP wave spectrum with peak wave frequency ω_p=0.314rad/sec and peak enhancement factor λ=1.05 is used for simulating the irregular waves.

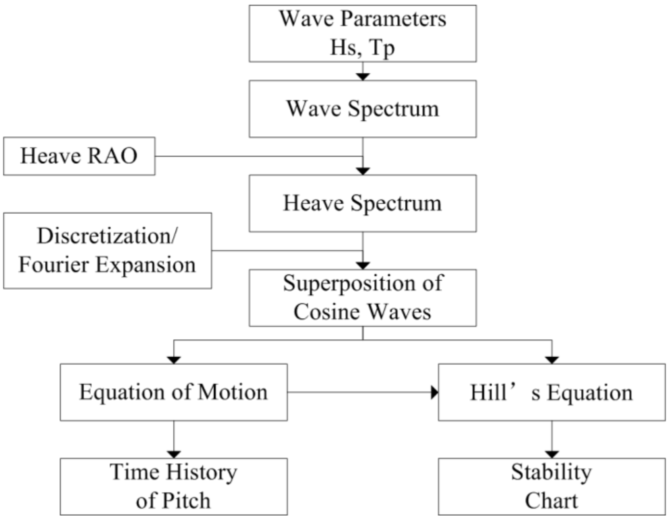


Fig.12. Flow diagram of parametric resonance analysis

The steps for analyzing the parametric resonance of a spar platform are shown in Fig. 12. These can be divided into three main steps. First, the wave spectrum for real sea conditions is given with the chosen wave parameters. In this work, the JONSWAP spectrum is used. By applying the heave RAO of a spar platform, the spectrum of the heave motion is obtained. Second, the heave motion is simulated using a Fourier expansion or obtained directly from the spectrum. Then, the heave motion can be discretisized into the superposition of a set of cosine waves, the periods of which are T_0, $T_0/2$, $T_0/3$,… and the amplitude of each is calculated. Third, the values of the amplitudes are substituted into the equation of motion and the Hill equation derived from it. The time history of the pitch motion can then be simulated by solving the motion of equation with a numerical solution. The stability chart can be given by analyzing Hill’s equation as shown in the previous section.

4.2 Numerical results

A prediction of the parametric resonance of a classical spar platform is carried out. The heave motion is simulated for both a single frequency and multiple frequencies. A length of time of 600 s is selected.

The simulations of the pitch motion are carried out on the basis of the equation of motion. The amplitudes of the heave motion are imported into the equation and the equation is solved using the Runge-Kutta algorithm. The results under two conditions are shown in Fig. 13 and Fig. 14. The length of the simulation time is also set to 600 s. The pitch motion is simulated for different values of damping coefficient c and metacentric height GM.

Several conclusions can be drawn from Fig. 13 and Fig. 14. First, with increasing damping, the pitch amplitudes decreases with time, and a larger damping coefficient produces, the faster decrease in the pitch motion. Second, a higher metacentric height makes the pitch amplitude smoother, and even though for some parts the amplitudes may be larger for a higher GM, the maximum amplitude of the higher GM is smaller always. Third, even when the heave motion is regular, the pitch motion may become irregular.

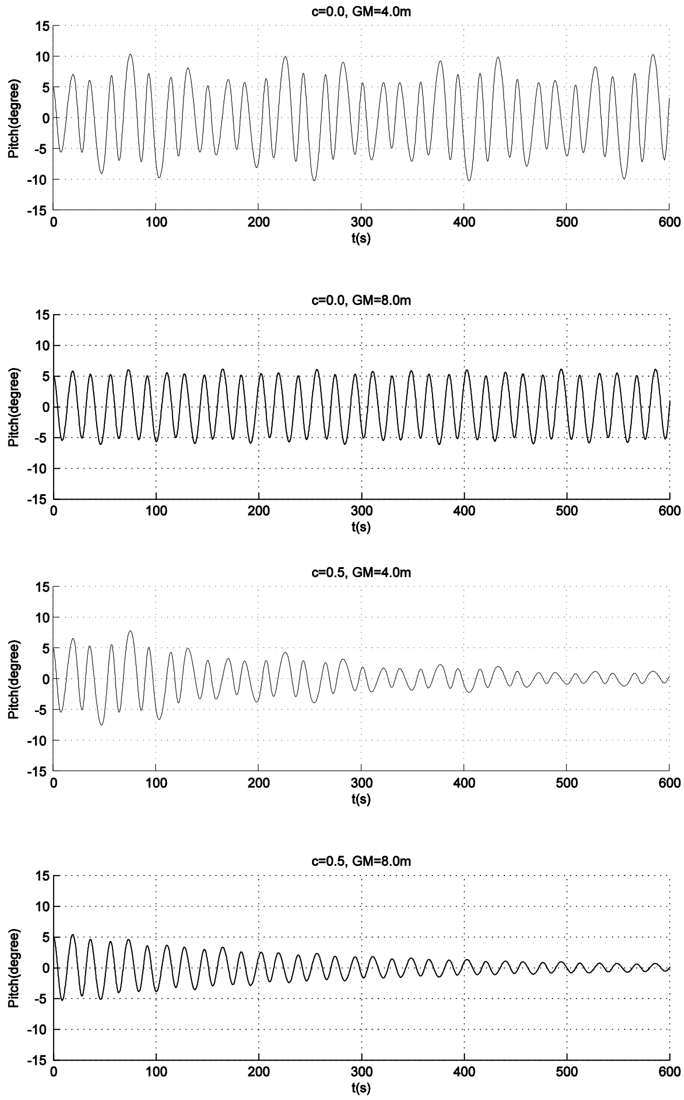


Fig.13. Time history of pitch motion with single frequency heave

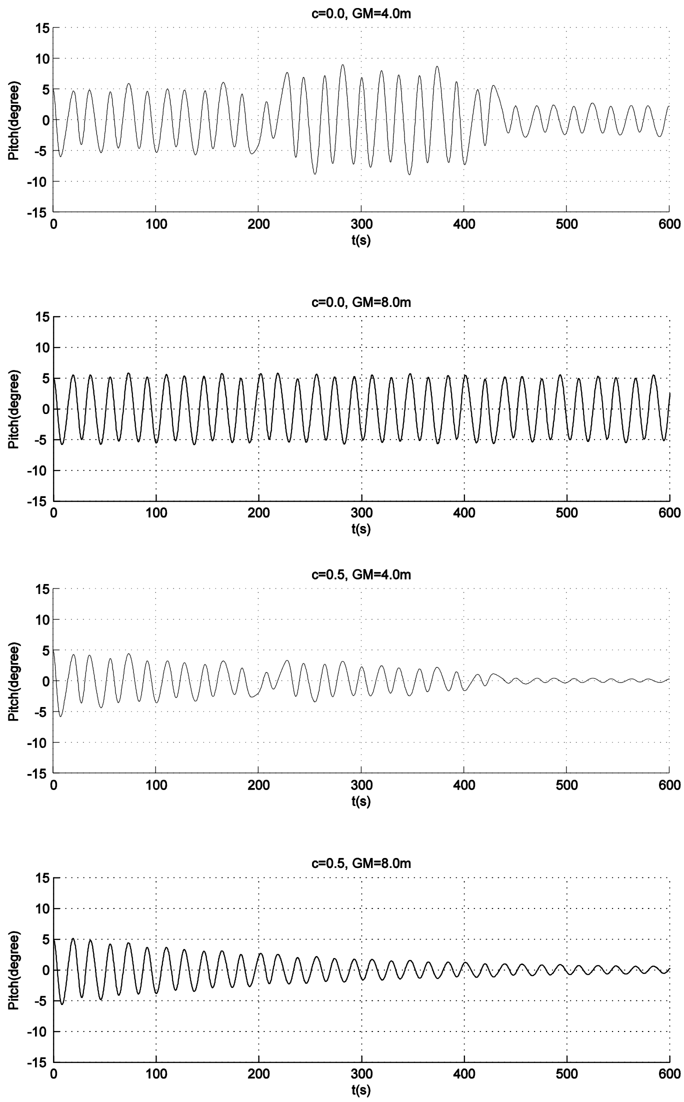


Fig.14. Time history of pitch motion with multi-frequency heave

It can be concluded from the above that increasing both the damping and the metacentric height can make the spar platform more stable. However, for a stable case, the two methods have different effects.

5. Conclusions

This paper presented a methodology for a parametric instability analysis of a spar platform in irregular waves. A mathematical model was derived in the form of a Hill equation. The three-dimensional stability charts of irregular waves not only display information about the stability boundaries, but the damping changes are also considered. Stability charts and simulations under different damping coefficients were also investigated. From the presented work, the following conclusions can be drawn:

1) Through a numerical simulation of the pitch motion, it could be confirmed that parametric instability may occur in irregular waves. However, its occurrence is more random compared to a regular wave condition. The stability charts showed a wider distribution of the regions of instability under an irregular wave condition, whereas the distribution was more concentrated under a regular wave condition.

2) By comparing the analyses using a Mathieu equation in regular waves and a Hill equation in irregular waves, it could be observed that the parametric instability properties of a spar in regular and irregular waves have obvious differences. Thus, it is necessary to predict the parametric resonance of a spar in irregular waves.

3) Predicting the parametric instability of a spar platform by using a Hill equation and stability charts is an effective method. With a given heave motion, the method only requires the natural period of the pitch and the initial value of metacentric height GM. This makes it very convenient to use for design.

4) Smaller metacentric height changes make it easier to avoid instability (when the damping is not zero). For spar platforms, this means a decrease in the amplitude of the heave motion. Additional heave damping can be used to decrease the peak value of RAO, such as through the use of damping plates. It is also important for the peak RAO to avoid the peak frequency of the waves.

5) Increasing the initial value of metacentric height GM is also an effective method to reduce the possibility of parametric instability. The most effective way is to make the center of gravity lower. However, it is not easy to make a large change in GM in an engineering design, and some other parameters may also change with it.

6) Increasing the damping of the pitch motion has a great effect under most conditions for both regular and irregular waves. In the stability charts, this is shown as a reduction in the instability areas. However, for the instability regions determined by the wave element with the largest amplitude, the effect of damping is smaller than in other regions. Thus, it is recommended that the design point be kept away from these regions.

The methodology presented herein may help designers estimate parametric instability at the preliminary design stage for spar platforms in a more practical way. Unfortunately, it is not yet supported by experimental results. Thus, experimental tests are necessary in the future. To thoroughly determine the parametric resonance of a spar platform, some investigations can be carried out in the future using a simulation model that includes the influences of the hull geometrical form and an analysis of stochastic stability in irregular waves.

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