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ABSTRACT

Mathematical self-similar fractals manifest identical replicated patterns at every scale. Recently, fractals have found their way into a myriad of applications. In optics, it has been shown that manipulation of unstable resonator parameters such as cavity length, curvatures of mirrors, the design of aperture and its transverse position can reveal self-similar fractal patterns in the resonators eigenmodes. Here, we present a novel laser resonator that can generate self-similar fractal output modes. This resonator has a special plane termed self-conjugate, during each round trip inside the cavity, is imaged upon itself with either a magnification or demagnification depending on the direction of beam propagation inside the cavity. By imaging an aperture placed in the self-conjugate plane inside the cavity, we qualitatively show the fractal behaviour occurring at various scales which, are given by powers of the magnification at the self-conjugate plane. We computed the fractal dimension of the patterns we generated and obtained non-integer values, as is expected for fractals.

Keywords: laser resonator, fractal, frenesel numeber, differaction limits, unstable resontor

1. INTRODUCTION

Fractal is a mathematical set which is described by a continuous but nowhere differentiable function that exhibit a repeated pattern at every scale. Fractal is known as expanding symmetry or evolving symmetry.[1,2] However, if the pattern reassembles itself at different scales, this fractal characterized by self-similarity property. A self-similar fractal means that the object is similar or exactly the same to a part of the same shape which is characteristic of that particular fractal.

One of the most striking characteristics of fractals is their non-integer dimensionality. Classical geometry deals with objects of integer dimensions: zero dimensional points, one dimensional lines and curves, two dimensional plane figures such as squares and circles, and three dimensional solids such as cubes and spheres. However, many natural phenomena are better described using a dimension between two whole numbers. So while a straight line has a dimension of one, a fractal curve will have a dimension between one and two, depending on how much space it takes up as it twists and curves. The more the flat fractal fills a plane, the closer it approaches two dimensions.

The dimensionality can be determined by using the so called box-count method:[3] We cover the curve with a grid of boxes of size $d \times d$ and count, for each box size d, the number of boxes n that are needed to cover the curve fully. Then the fractal dimension follows as

$$D = \frac{\log n}{\log\left(1/d\right)}. \tag{1}$$

If this recipe is applied to a smooth curve one will find D = 1, since n = 1/d for small d, but for a fractal one will find a non-integer value for D, since a fractal has structure on any scale.

Fractal has permeated many area of science, such as astrophysics,[4] biological sciences,[5] and has become one of the most important techniques in computer graphics.[6] Many image compression schemes use fractal algorithms to compress computer graphics files to less than a quarter of their original size.

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In optics, the light within unstable laser resonaors possess a self-similar and therefore fractal structure.[7,8] The fundamental question should be asked is; how come that a simple optical system possess such unexpected property? The answer of this question have been argues by Woerdman and McDonald.[9,10] They concluded that diffraction plays crucial role which renders unstable cavities modes not rigorously but only statistically self-similar. This conclusion can be explained as follows; unstable laser resonator are defined using two numbers, the round trip resonator magnification M and Frensel number N which can be determined by

$$N = \frac{1}{2}(M+1)\frac{a^2}{\lambda L}, \tag{2}$$

where a is the mirror radius, λ is the wavelength and L is the laser cavity. The eigenmodes of the unstable cavities are fully determined by the N and M numbers. If we restricted our discussion into only the lowest order mode. We find that in confocal unstable resonator consists of two spherical mirrors with focal lengths f_1 and f_2 facing each other at distance d, there is a self-conjugate plane S. At this plane, each round trip leads to magnified image of the intensity distribution of the imaged plane. This is an essential property of unstable-cavity resonators that is absent in stable cavity resonators. The eigenmode intensity distribution in the aperture plane does not change after one round trip (otherwise it would not be an eigenmode).[11] But a round trip amounts to magnification, so the Eigen mode must be invariant under magnification, and this is exactly the definition of a fractal.

2. EXPERIMENT

Here, we represent the experimental realisation of self-similarity inside the laser cavity. The cavity consisted of an Nd doped YAG rod as a gain medium (6.35×76 mm) that was flash-lamp pumped. The unstable cavity was constructed in a L-shape which consisted of two concave high reflectors acting as the end mirrors with a 45° output coupler (OC) of 99.8% reflectivity positioned at the apex of the L as shown in Fig. 1.

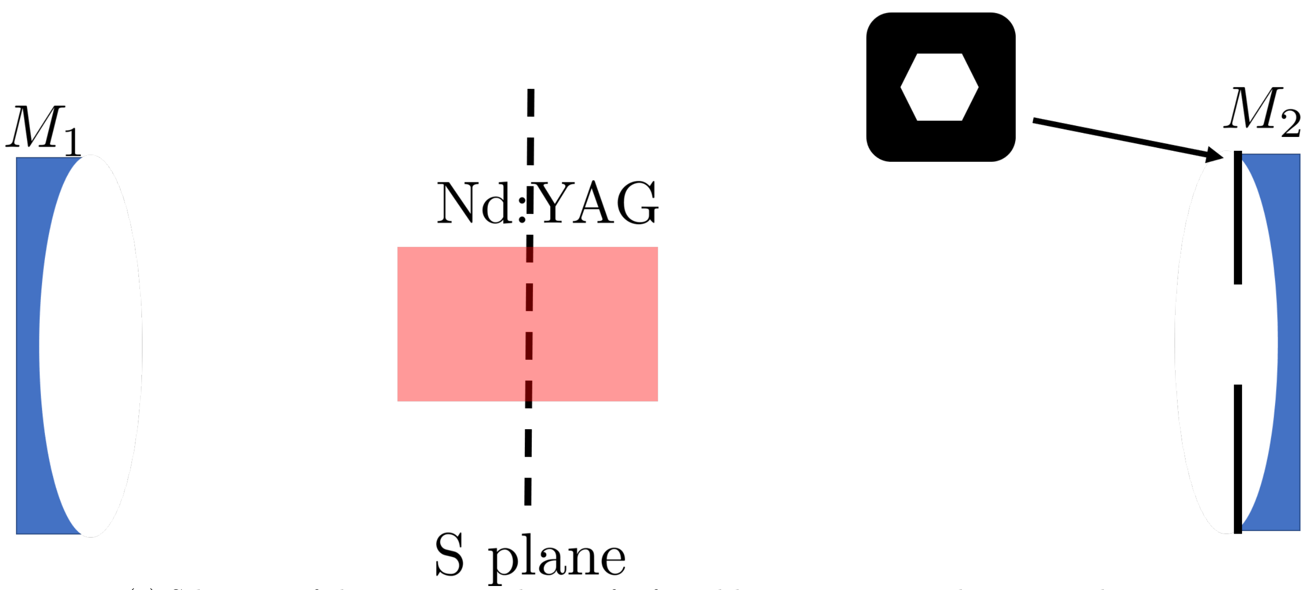

Figure 1: (a) Schematic of the experimental setup for fractal beam generation where M1 and M2 are concave high reflectors with a 45° OC of 99.8% reflectivity. The length of the cavity is selected as $R_1/2 + R_2/2$ with the S Plane positioned at the centre of the Nd:YAG gain medium. A hexagon shaped aperture is positioned at M2 to induce a fractal pattern. The S plane is captured outside the cavity at a distance R2/2 from M2 using a CCD camera.

During each round trip in the fractal resonator, light is imaged geometrically by the resonators two curved mirrors. Geometrical imaging of planes is the main reason behind the evolution of fractal structure in the resonators eigenmodes. The self-similar patterns were formed by positioning an aperture which induces the self-similarity;[7,8] therefore, in our setup a hexagonal polygon-shaped aperture was inserted at the larger of the two end mirrors. The aperture is much smaller than the other mirror size; as a result, only diffraction from this mirror contributes to the fractal pattern. The output beam is captured using a CCD camera (Spiricon SPU260 BeamGage).

The S plane represents the self-conjugate plane which, during each round trip inside the cavity, is imaged upon itself with either a magnification or demagnification of M or $1/M$, respectively. Resonator parameters manipulate the position of the self-conjugate plane S. Here, we restricted our cavity to the unstable confocal resonator *i.e.* the resonators where the length of the cavity corresponded to the curvature of the mirrors $L = R_1/2 + R_2/2$ with R_2 R_1. Consequently, the S plane existed in the common focal plane. The S plane was positioned at the centre of the gain medium. This position was selected to prevent any aperturing effects on the oscillating mode by the gain medium with only the polygon-shaped aperture acting on the mode.

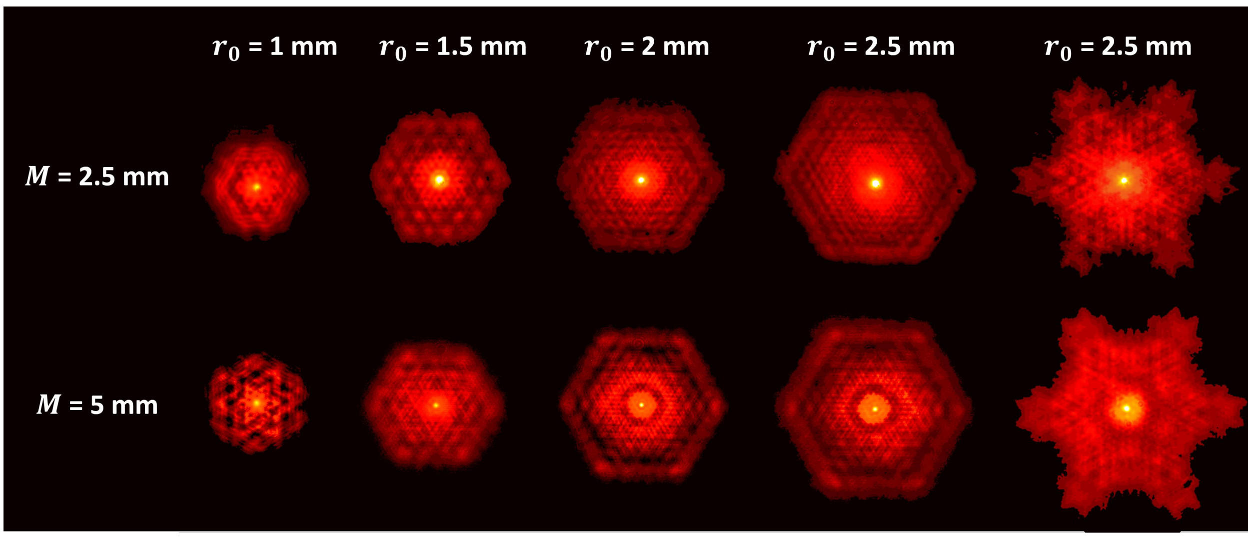

Figure 2: Experimental comparison between fractal beams with different magnifications generated with a hexagonal aperture of radius 2mm.

The fractal laser cavity, represented in Fig. 1, generated two versions of the self-similarity pattern. Both exhibiting the same pattern structure, but different magnifications: one version is built up from a successive magnification of the original pattern of the aperture while the second is built up from a de-magnification of the original aperture pattern.[12] The self-similarity pattern version depends mainly on the beam propagation direction inside the laser cavity. For example, if the beam inside the cavity propagates from plane S to M_1, M_1 to M_2,and M_2 to plane S, the fractal pattern is magnified by a factor $M(= R_2/R_1)$.

In this case, our resonator considered as an analogue to the monitor outside a monitor effect where the monitor here is represented by the self-conjugate plane. The self similarity is seen in the magnified output and was subsequently captured using the CCD camera at a distance of $R_2/2$ from M_2 as illustrated in Fig. 1. Consequently, if the direction of propagation is the opposite (plane S to M_2, M_2 to M_1, and to plane S), the output fractal pattern is de-magnified by a factor $1/M(= R_1/R_2)$ and the cavity exhibits an analogue to the monitor inside a monitor effect.

Several fractal cavities corresponded to different magnifications. The fractal cavities parameters are shown in Table 1. A snowflake-shaped aperture as well as hexagon polygon-shaped aperture of diameter of varied from 2 mm to 5 mm were tested inside the fractal cavities. The output fractal beams are shown in Fig. 2.

Table 1: shows Resonator parameters used to design fractal cavities of different magnifications.

R_1	200 mm	150 mm	100 mm
R_2	500 mm	500 mm	500 mm
$g_1 g_2$	1.523	1.65	1.9
M	2.5	3.33	5

3. CONCLUSION

In this work, we have presented a novel laser resonator that can generate self-similar fractal Output modes. The measurement of the onaxis intensity of the propagting modes has been investigated to confirm the position of the self-conjgate, S, plane. By imaging an aperture placed in the self-conjugate plane inside the cavity, we qualitatively show the fractal behaviour occurring at various scales which, are given by powers of the magnification at the self-conjugate plane. We computed the fractal dimension of the patterns we generated and obtained non-integer values, as is expected for fractals.

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