Reduced-Order Aeroelastic Model for Limit-Cycle Oscillations in Vortex-Dominated Unsteady Airfoil Flows

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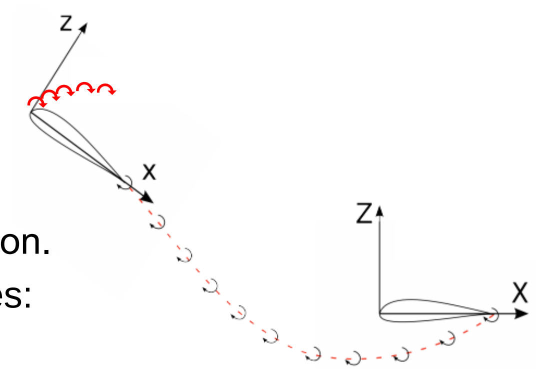
Background: LDVM method

- A low-order method for unsteady airfoil flows with intermittent vortex shedding from rounded leading edges
- LDVM = LESP-modulated Discrete Vortex Method
- LESP = Leading-Edge Suction Parameter



- A discrete TEV is shed at every time step to satisfy Kelvin's condition.
- The circulation distribution on the airfoil is taken to be Fourier series:

$$\gamma(\theta, t) = 2U\left[A_0(t)\frac{1+\cos\theta}{\sin\theta} + \sum_{n=1}^{\infty} A_n(t)\sin(n\theta)\right]$$

Where θ is related to the chordwise coordinate x as: $x = \frac{c}{2}(1-\cos\theta)$

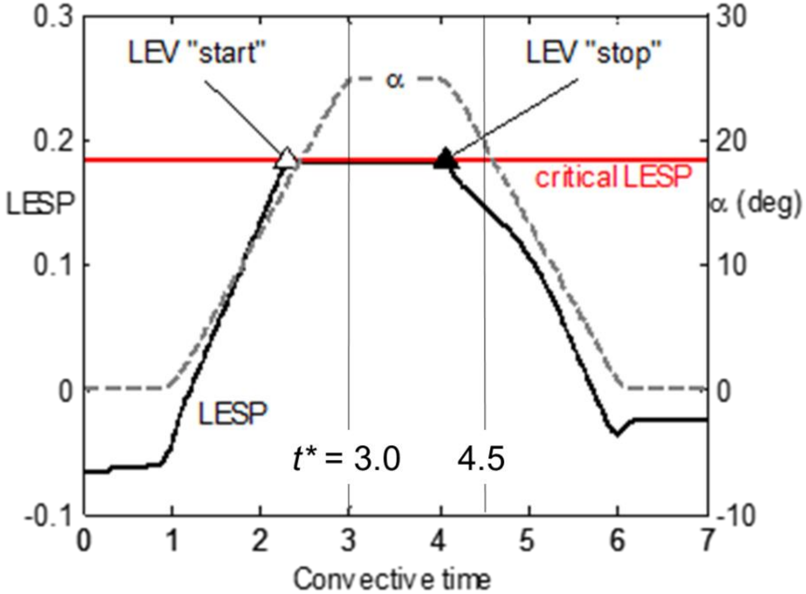
- It was observed by Ramesh et. al [1] that the vortex shedding at LE is related to a Leading Edge Suction Parameter reaching a critical value.

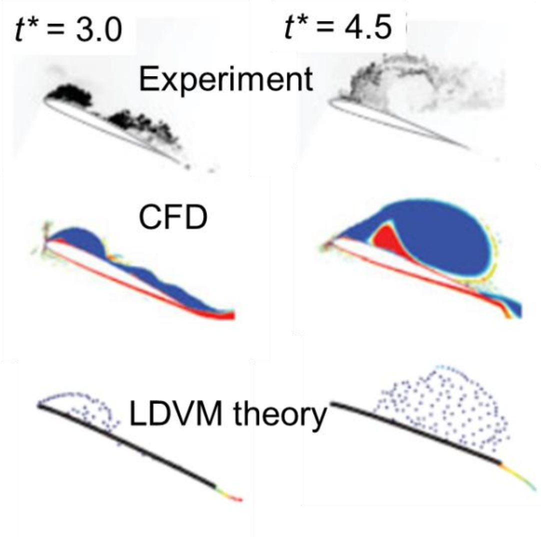
$$\boldsymbol{LESP(t) = A_0(t)}$$

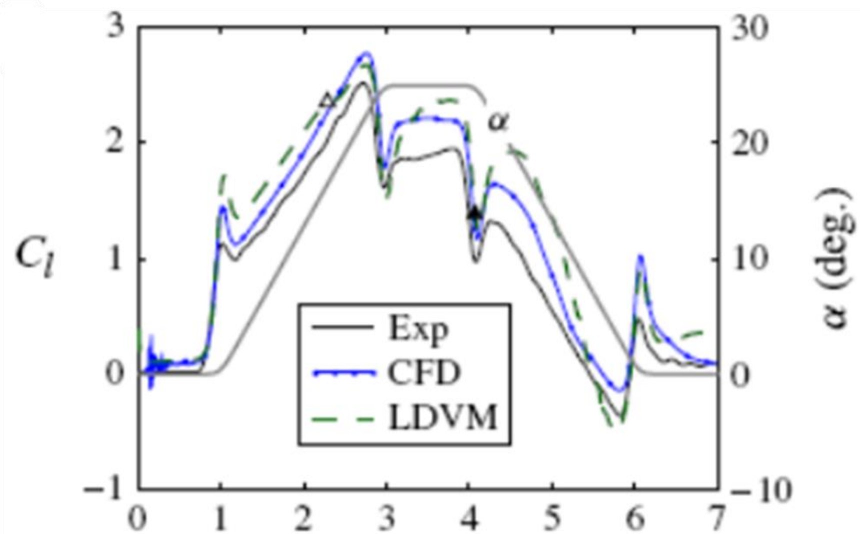
- When LESP is above a critical value (LESPcrit) at a time step, a discrete LEV is shed so as to bring it down to the critical value.

[1] Ramesh, Gopalarathnam, Granlund, Ol, and Edwards, "Discrete-vortex method with novel shedding criterion for unsteady aerofoil flows with intermittent leading-edge vortex shedding," *Journal of Fluid Mechanics*, Volume 751, July 2014, pp 500-538.

Background: LDVM sample results



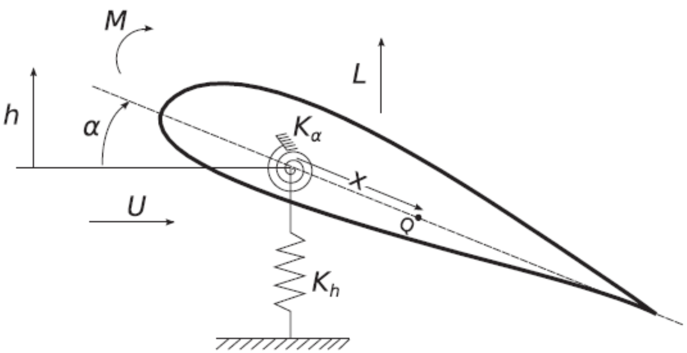




Comparison of results from experiment, RANS CFD, and LDVM theory for a SD 7003 airfoil undergoing pitch up-hold-return motion at Re = 30,000.

Background:
Study of Aeroelastic Limit Cycle Oscillations Using LDVM

- The LDVM framework is coupled with a structural model to investigate high-frequency limit-cycle oscillations in flows dominated by leading-edge vortex shedding [2].
- The structural model has degrees of freedom in pitch and plunge, and allows for large amplitudes and cubic stiffening.



The spring forces have the form:

$$F_h(h) = k_h f_h(h) = k_h\left(h + \beta_h h^3\right),$$
$$F_\alpha(\alpha) = k_\alpha f_\alpha(\alpha) = k_\alpha\left(\alpha + \beta_\alpha \alpha^3\right),$$

The non-dimensional equations of motion are:

$$\frac{2}{c}\ddot{h} - x_\alpha \ddot{\alpha}\cos\alpha + x_\alpha \dot{\alpha}^2 \sin\alpha + \frac{2}{c}\omega_h^2 f_h = \frac{4}{\pi}\kappa\frac{U^2}{c^2}C_l,$$
$$-\frac{2}{c}x_\alpha \cos\alpha \ddot{h} + r_\alpha^2 \ddot{\alpha} + (r_\alpha \omega_\alpha)^2 f_\alpha = \frac{8}{\pi}\kappa\frac{U^2}{c^2}C_m,$$

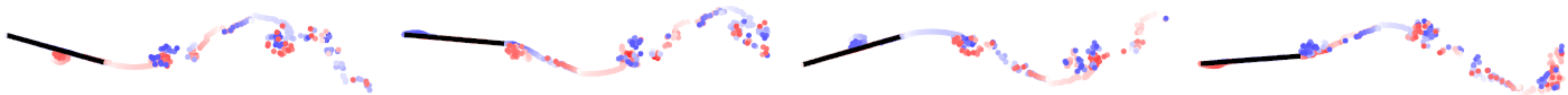
[2] Ramesh, K., Murua, J., and Gopalarathnam, A., “Limit-cycle oscillations in unsteady flows dominated by intermittent leading-edge vortex shedding,” Journal of Fluids and Structures, Vol. 55, No. Supplement C, 2015, pp. 84 – 105.

Background:
Study of Aeroelastic Limit Cycle Oscillations Using LDVM

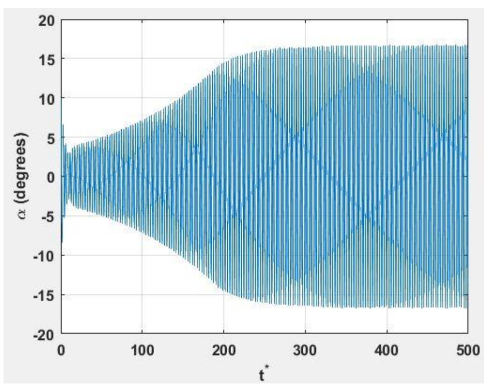
LCO’s for a flat plate at Re=1000

Initial conditions: • $U = 0.467$ • $\alpha_0 = 10^0$ • $h_0 = 0$

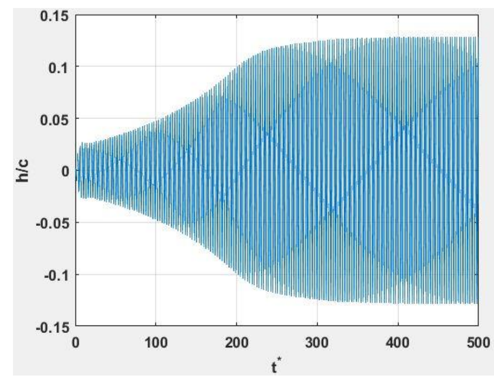
Flowfield at different time instants



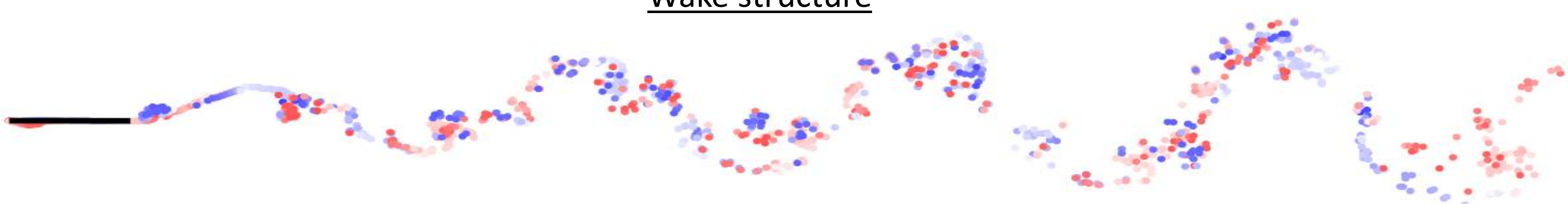
Pitch-amplitude variation



Plunge-amplitude variation



Wake structure



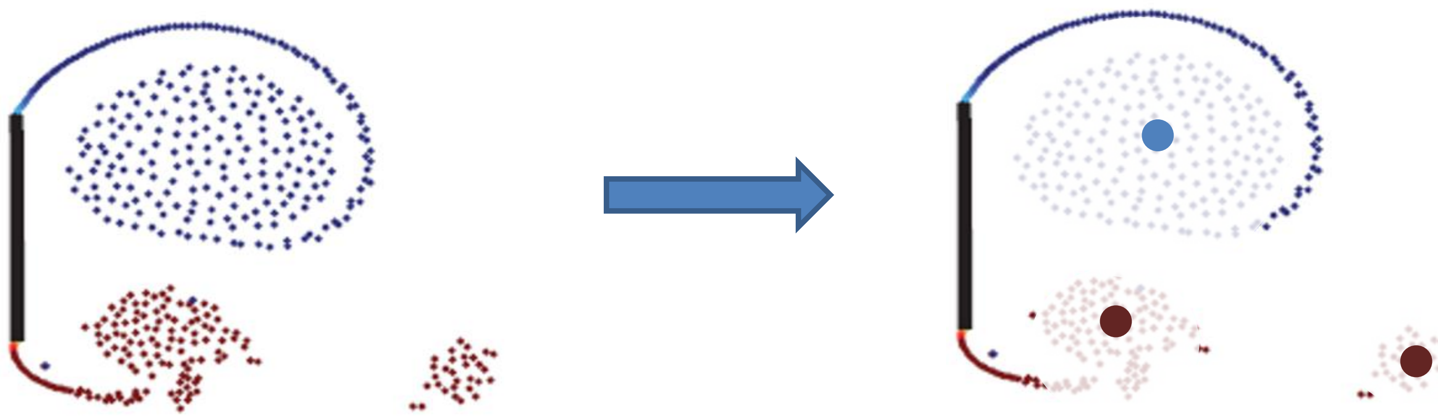
[2] Ramesh, K., Murua, J., and Gopalarathnam, A., “Limit-cycle oscillations in unsteady flows dominated by intermittent leading-edge vortex shedding,” Journal of Fluids and Structures, Vol. 55, No. Supplement C, 2015, pp. 84 – 105.

Motivation for model reduction

- Thousands of time steps are required for aero-elastic case studies.
- This leads to a huge number of discrete vortices in the flow-field.
- The computational complexity increases as $O(n^2)$ where n is the number of vortices in the flowfield.
- The consequence is that the CPU time can increase tremendously if the number of discrete vortices in the flow field keeps increasing.

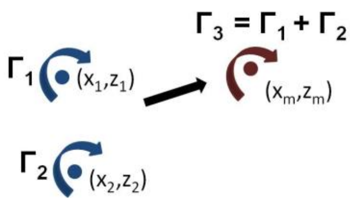
Full model with a large number of discrete vortices

Equivalent model with fewer discrete vortices



Methodology: DV Amalgamation

- Discrete vortex count is reduced by amalgamating suitable pair of vortices at their centroid.



- Discrete vortex pairs are identified using a slightly modified version of Spalart's criterion [3]:

$$\frac{|\Gamma_j \Gamma_k|}{|\Gamma_j + \Gamma_k|} < 2.5E - 3 \qquad \text{AND} \qquad \frac{|z_j - z_k|^2}{(D_0 + d_j)^{3/2}(D_0 + d_k)^{3/2}} < 5.0E - 3$$

d_j and d_k are the distances of the DVs from the leading edge of the airfoil and $D_o = 0.1c$.

- Besides, it is also required that the errors in the Fourier coefficients A_0 and A_1 due to amalgamation are less than 10^{-6}.
- This ensures that the leading edge suction and bound circulation are not affected.
- At most one pair of LEVs and one pair of TEVs are amalgamated at every time step.
- The tolerance values are constant for all cases studied.

[3] Spalart, P., "Vortex methods for separated flows," In VKI, Computational Fluid Dynamics, Volume 1 64 p (SEE N89-17818 10-34), Vol. 1, 1988

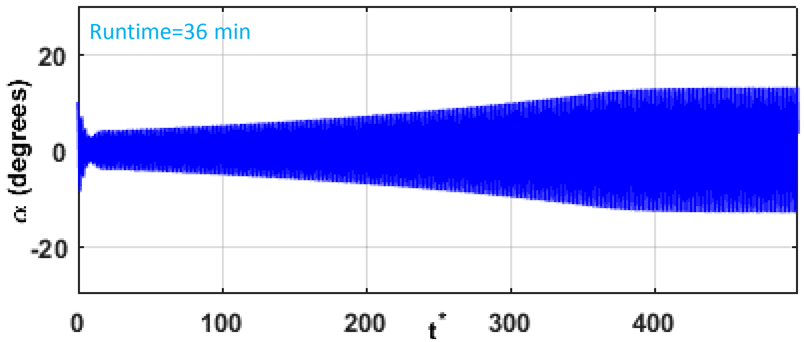
Results

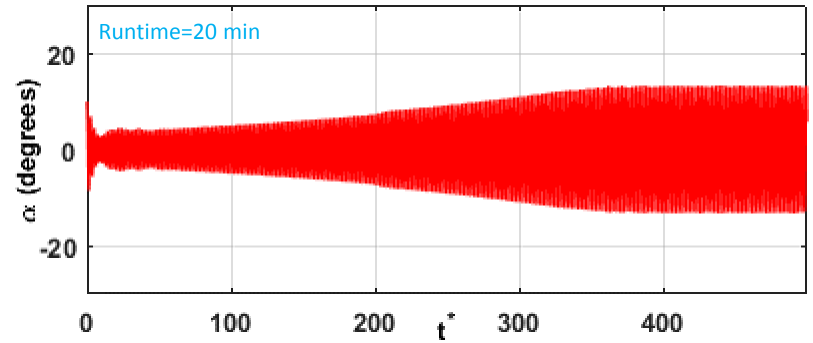
Case 1: Effect of varying freestream velocity : $\alpha\ vs\ t^*$

Full model

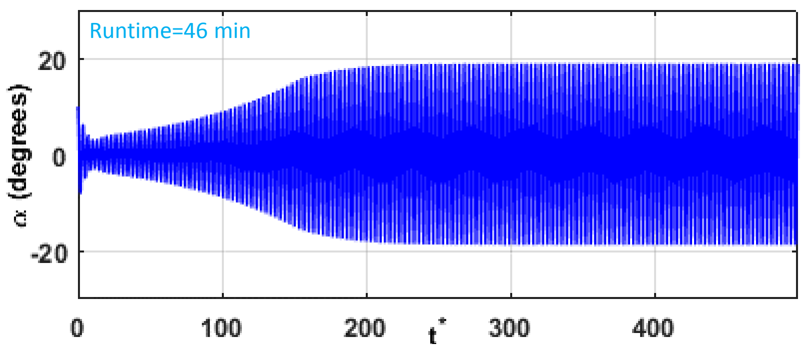
Reduced order model

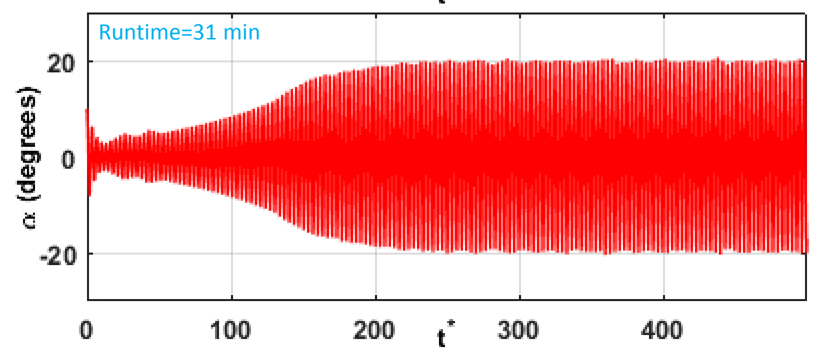
$U = 0.30$



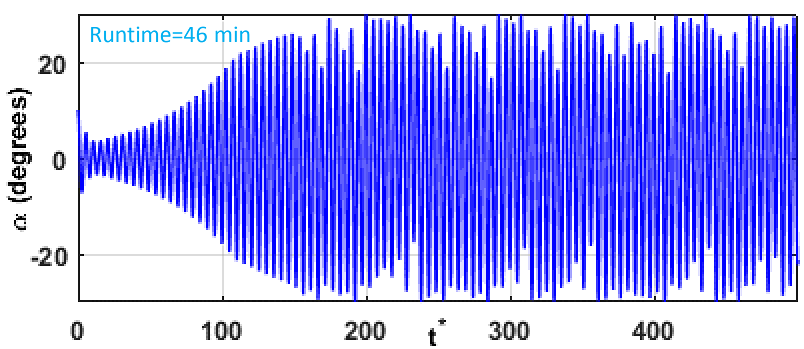


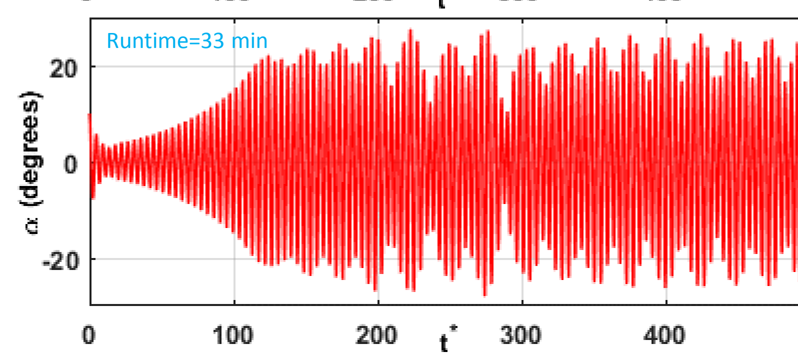
$U = 0.44$





$U = 0.80$





Results

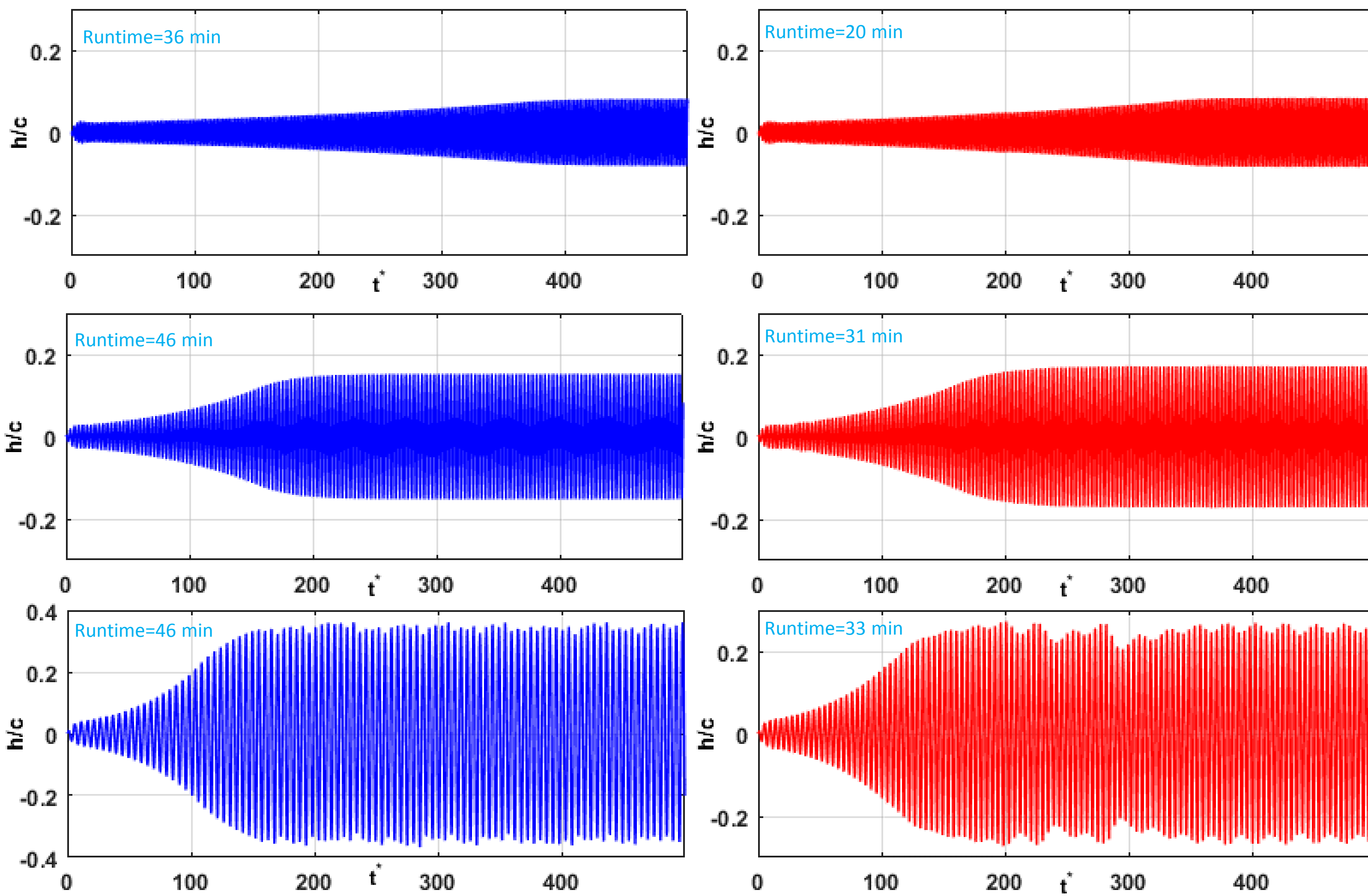
Case 1: Effect of varying freestream velocity : $h/_c \; vs \; t^*$

Full model | **Reduced order model**

$U = 0.30$

$U = 0.44$

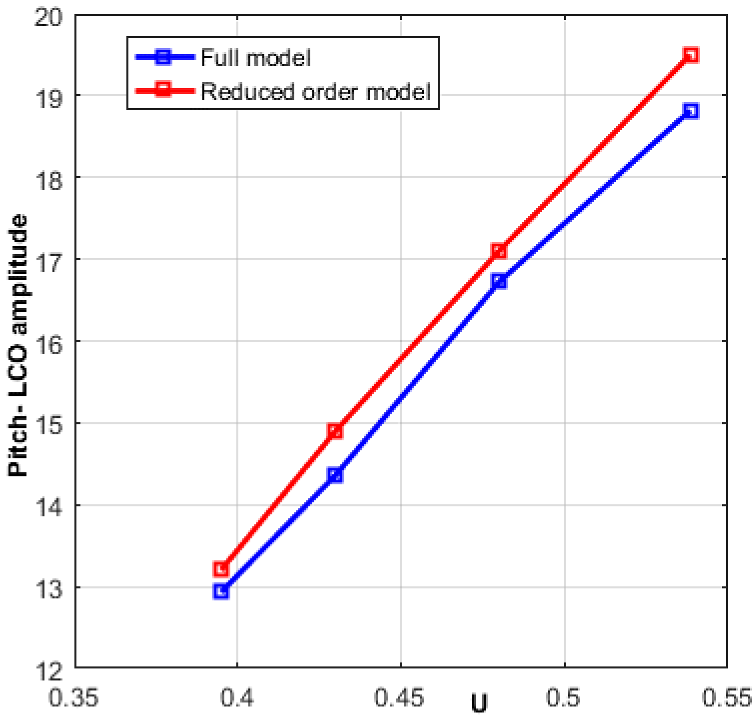
$U = 0.80$

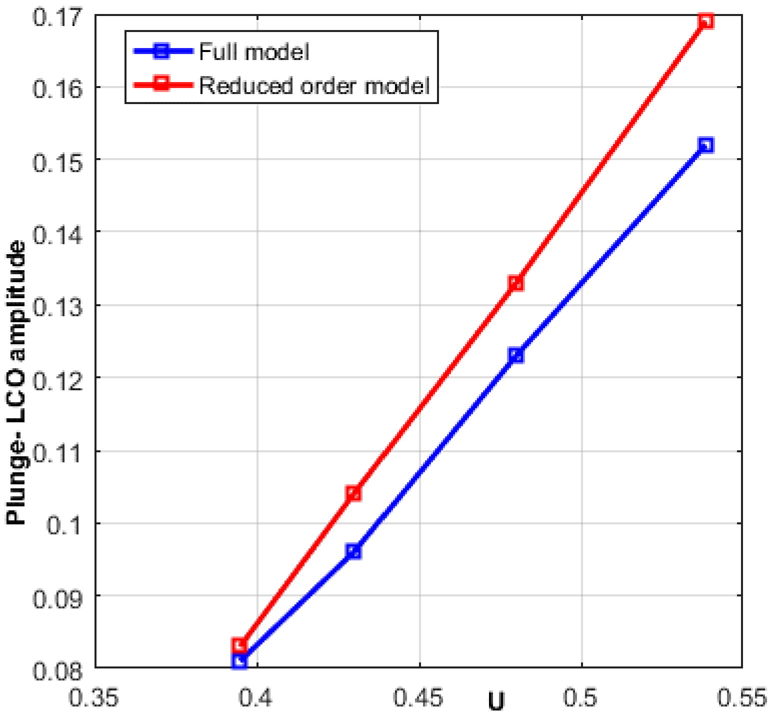


Results

Case 1: Effect of varying freestream velocity

Comparison of predictions of the two models



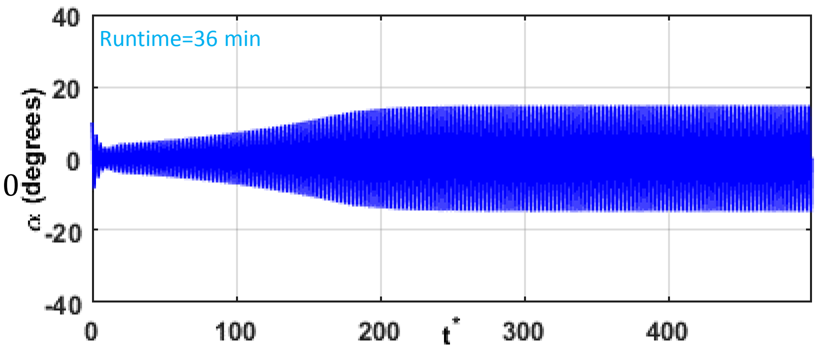


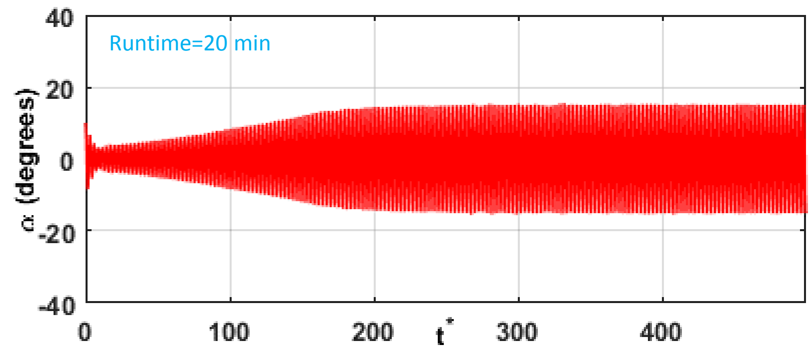
Results

Case 2: Effect of varying $\text{LESP}_{\text{critical}}$: $\alpha\ vs\ t^*$

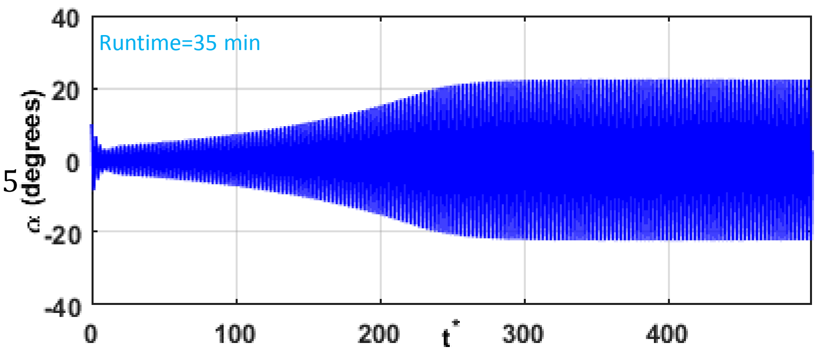
Full model | **Reduced order model**

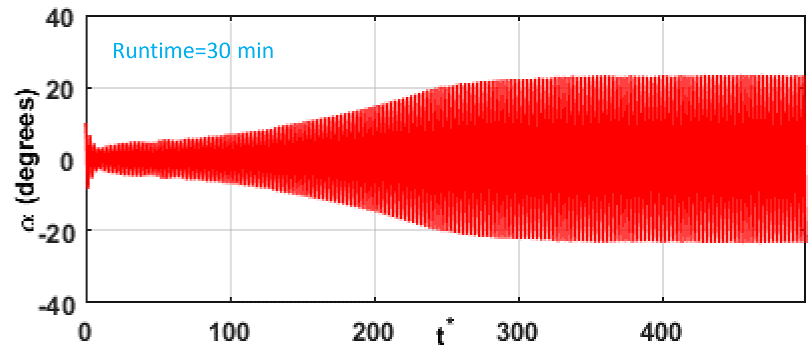
$LESP_{crit} = 0.10$



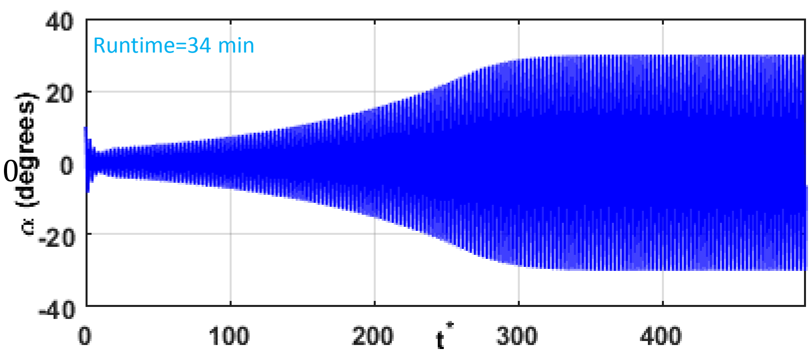


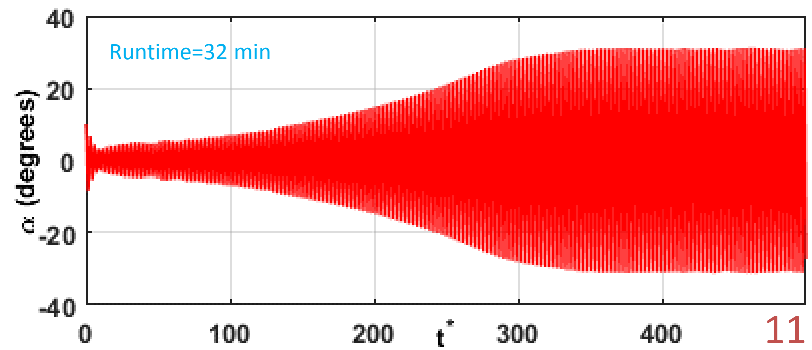
$LESP_{crit} = 0.15$





$LESP_{crit} = 0.20$



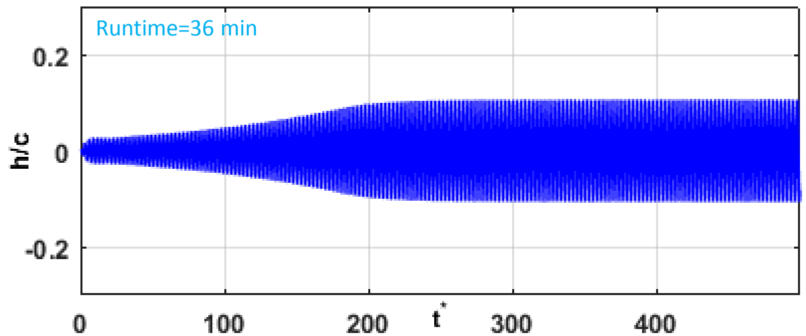


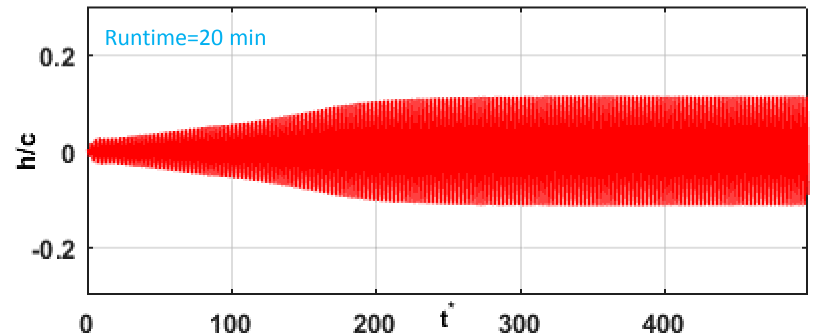
Results

Case 2: Effect of varying $\text{LESP}_{\text{critical}}$: ${}^{h}/_{c}\ vs\ t^{*}$

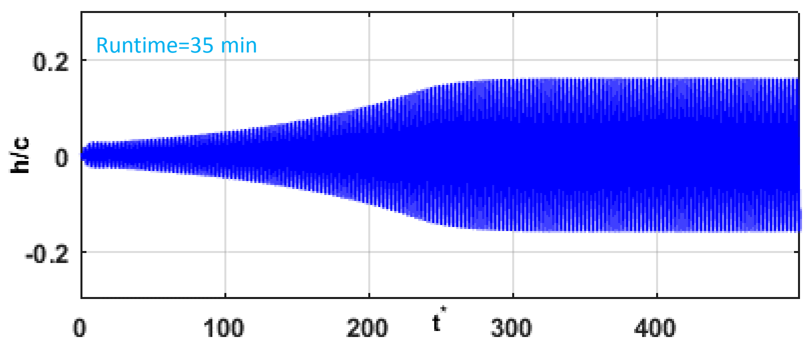
Full model | Reduced order model

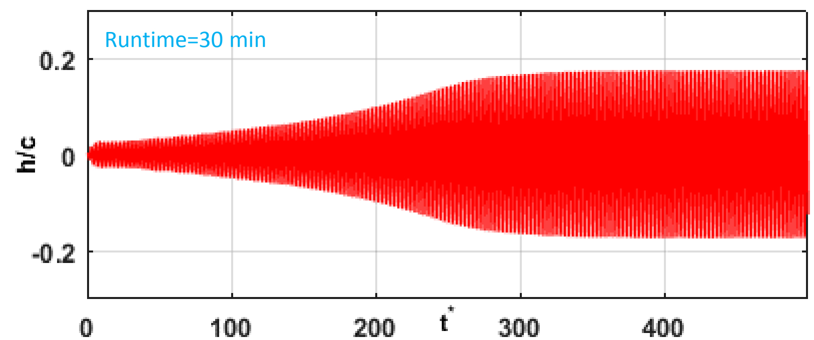
$LESP_{crit} = 0.10$



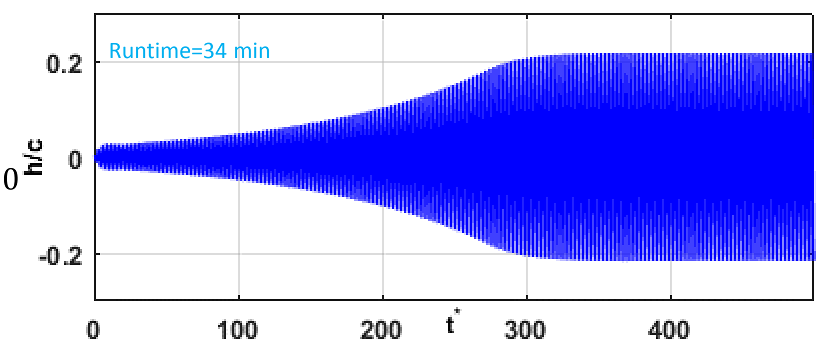


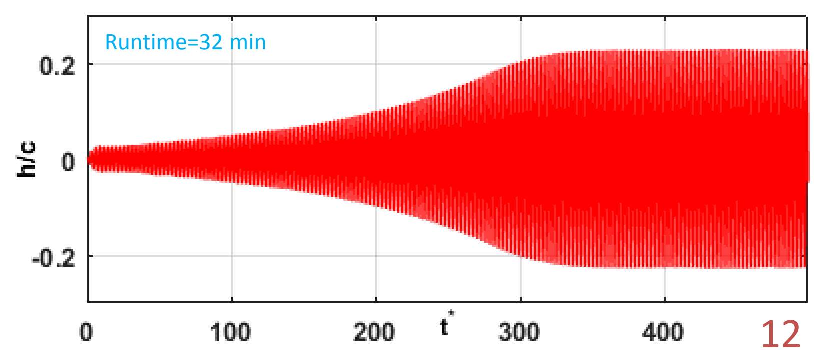
$LESP_{crit} = 0.15$





$LESP_{crit} = 0.20$

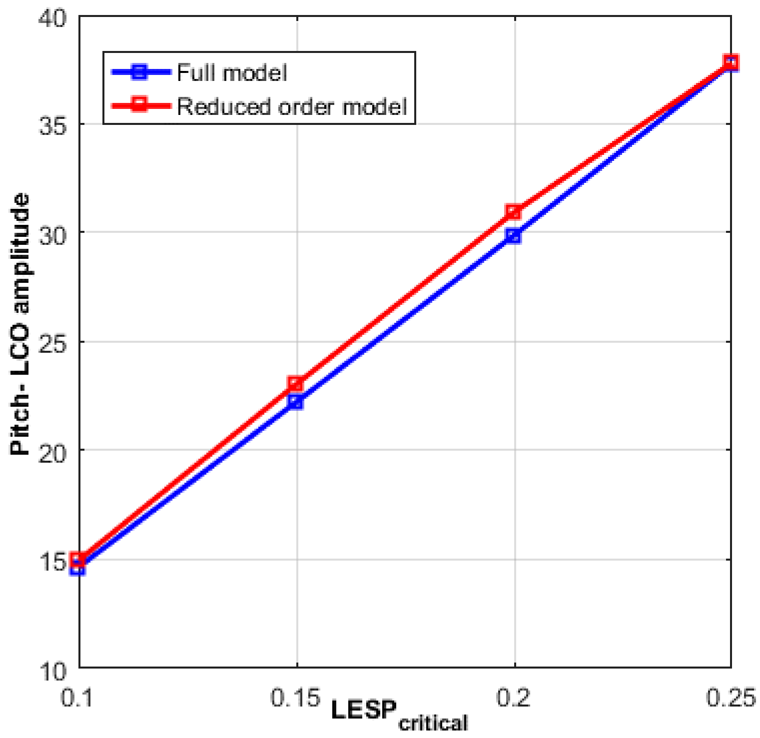


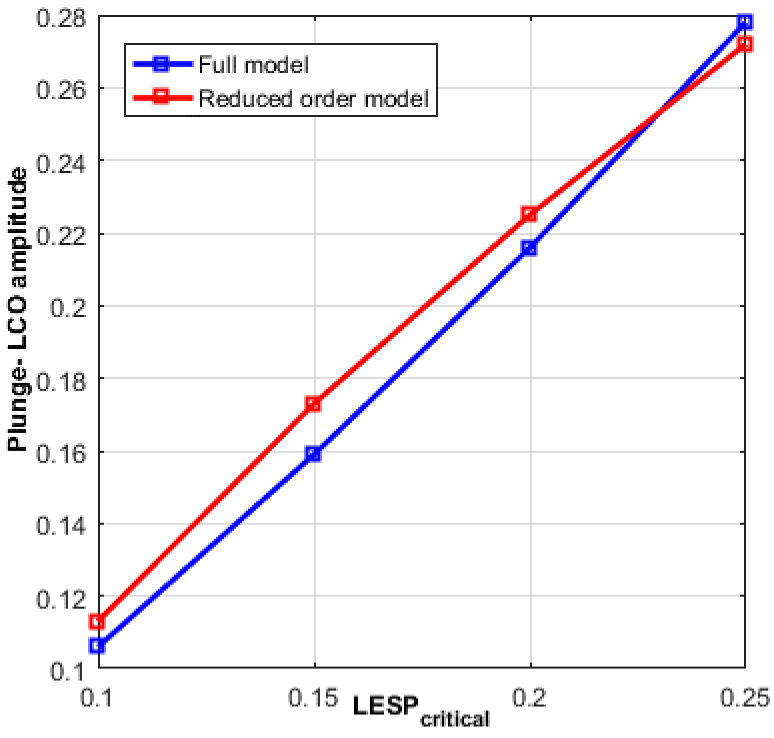


Results

Case 2: Effect of varying $LESP_{critical}$

Comparison of predictions of the two models





Conclusions

- The amalgamation algorithm shows promise in general.
- The LCO behavior predicted by the reduced order model is in good agreement with that predicted by the full model.
- Run-time savings vary from case to case:

  - CPU-time reductions vary from 44% to 6% for the cases presented.

  - The tolerance values have been kept constant for all cases.

  - It will be useful to explore the possibility of allowing the amalgamation scheme to adjust the tolerances during program execution.
- Future work will include multiple amalgamations at one time step.
- The broader objective is to extend the current study to tandem airfoils.

Wake effects on LCO characteristics for airfoils in tandem

THANK YOU!