Supplementary information

Device fabrication

The interdigitated transducers (IDTs) were fabricated by standard photolithography on a 1 mm thick 128° rotated Y-cut lithium niobate wafer using 20 nm chromium, 80 nm gold deposition and lift-off. The widths of the electrode fingers and spacing were 75 µm, resulting in an acoustic wavelength of 300 µm and theoretical operating frequency of 12.6 MHz. For each IDT, 20 finger pairs have been used. The polydimethylsiloxane (PDMS) channel was fabricated via soft lithography. The 2 cm long microchannel is 50 µm in height, 240 µm in width to allow for the presence of two acoustic pressure nodes. The width of the PDMS between channel and the IDT is 2 mm on each side. The two parts of the device were bonded after a mixed oxygen plasma treatment, using a corona gun and reactive ion etching.[1] The best bonding performance was obtained when the substrate was treated in the reactive ion etch machine for 10 s at 60 W power and 100 mTorr pressure.[2] The surface of the PDMS channel was activated for 30 s at medium power using the discharge gun. A small amount of methanol was applied on the substrate prior to bonding to allow positioning of the PDMS.[3]

Radiation force in a surface acoustic wave device due to phase modulated acoustic fields

The velocity potential, Φ, of the two travelling waves in the fluid (Φ_1 and Φ_2), is used to obtain the acoustic pressure and acoustic velocity in the medium, and consequently the radiation force. This potential can be expressed as:

$$\Phi_1 = \frac{u_0}{k}\exp\left(i(\omega t - k_y y - k_z z)\right) = \frac{u_0}{k}\exp(i\alpha) \tag{1}$$

$$\Phi_2 = \frac{u_0}{k}\exp\left(i\left(\omega t + k_y y - k_z z - s(t - t_s)\right)\right) = \frac{u_0}{k}\exp(i\delta) \tag{2}$$

where u_0 is the amplitude of the velocity field. The wavenumbers are $k = \omega/c_f = \sqrt{k_y^2 + k_z^2}$, $k_y = k\sin\theta_r$ and $k_z = k\cos\theta_r$. The rate, s, at which the phase of one of the travelling waves is modulated is $s = 2\pi/t_{ramp}$ or $s = 2\pi/t_{slope}$ (depending on the pattern as shown in Fig. 1a or b in the main body of the article). The start of the phase shift is defined as t_s.

The acoustic force can be generally expressed for any acoustic field as:[4]

$$\mathbf{F}_{ac} = -\pi R^3\left[\frac{2\kappa_0}{3}\mathrm{Re}\{f_1^* p_{in}^* \nabla p_{in}\} - \rho_0 \mathrm{Re}\{f_2^* \mathbf{v}_{in}^* \cdot \nabla \mathbf{v}_{in}\}\right] \tag{3}$$

where R is the particle radius, κ_0 is the compressibility of the medium, f_1 and f_2 are constants depending on the density and compressibility of the particles and medium. $\mathrm{Re}\{z\}$ denotes the real part of a complex quantity z, and z^* stands for the complex conjugate of z. Finally, the pressure and velocity fields can be obtained from the velocity potential using $p_{in} = -\rho_0\,\partial\Phi/\partial t$ and $\mathbf{v}_{in} = \nabla\Phi$.[5]

Therefore the pressures are

$$p_1 = -\rho_0\frac{\partial\phi_1}{\partial t} = -i\rho_0\omega\frac{u_0}{k}\exp(i\alpha) = -ip_A\exp(i\alpha) \tag{4}$$

$$p_2 = -\rho_0\frac{\partial\phi_2}{\partial t} = -i\rho_0(\omega - s)\frac{u_0}{k}\exp(i\delta) = -ip_B\exp(i\delta) \tag{5}$$

and the total pressure is

$$p_{in} = p_1 + p_2 = -i\left(p_A \exp(i\alpha) + p_B \exp(i\delta)\right) \tag{6}$$

Similarly for the velocity field

$$\mathbf{v} = \nabla\phi \tag{7}$$

and substitution of (1) and (2) into (7) leads to

$$\mathbf{v}_1 = \nabla\phi_1 = -ik_y \frac{u_0}{k} \exp(i\alpha)\hat{\mathbf{y}} - ik_z \frac{u_0}{k} \exp(i\alpha)\hat{\mathbf{z}} \tag{8}$$

$$\mathbf{v}_2 = \nabla\phi_2 = ik_y \frac{u_0}{k} \exp(i\delta)\hat{\mathbf{y}} - ik_z \frac{u_0}{k} \exp(i\delta)\hat{\mathbf{z}} \tag{9}$$

and therefore the total velocity field is

$$\mathbf{v}_{in} = \mathbf{v}_1 + \mathbf{v}_2 = -ik_y \frac{u_0}{k}\left[\exp(i\alpha) - \exp(i\delta)\right]\hat{\mathbf{y}} - ik_z \frac{u_0}{k}\left[\exp(i\alpha) + \exp(i\delta)\right]\hat{\mathbf{z}} \tag{10}$$

To evaluate the first term of (3) the complex conjugate and gradient of the pressure have to be obtained from (6):

$$p_{in}^{*} = i\left(p_A \exp(-i\alpha) + p_B \exp(-i\delta)\right) \tag{11}$$

$$\nabla p_{in} = -k_y\left(p_A \exp(i\alpha) - p_B \exp(i\delta)\right)\hat{\mathbf{y}} - k_z\left(p_A \exp(i\alpha) + p_B \exp(i\delta)\right)\hat{\mathbf{z}} \tag{12}$$

Furthermore note that

$$\exp(i\alpha)\exp(-i\alpha) = \exp(i\delta)\exp(-i\delta) = \exp 0 \equiv 1 \tag{13}$$

$$\exp(i\alpha)\exp(-i\delta) = \exp\left(i\left(\omega t - k_y y - k_z z - \omega t - k_y y + k_z z + s(t - t_s)\right)\right) = \exp\left(-i\left(2k_y y - s(t - t_s)\right)\right) \tag{14}$$

$$\exp(-i\alpha)\exp(i\delta) = \exp\left(-(i\alpha - i\delta)\right) = \exp\left(i\left(2k_y y - s(t - t_s)\right)\right) \tag{15}$$

and therefore multiplying (11) by (12) and using identities from (13) to (15) gives

$$\left(p_{in}^{*}\nabla p_{in}\right)_y = -ik_y\left[p_A^2 - p_B^2 + p_A p_B \exp\left(-i\left(2k_y y - s(t - t_s)\right)\right) - p_A p_B \exp\left(i\left(2k_y y - s(t - t_s)\right)\right)\right]\hat{\mathbf{y}} \tag{16}$$

$$\left(p_{in}^{*}\nabla p_{in}\right)_z = -ik_z\left[p_A^2 + p_B^2 + p_A p_B \exp\left(-i\left(2k_y y - s(t - t_s)\right)\right) + p_A p_B \exp\left(i\left(2k_y y - s(t - t_s)\right)\right)\right]\hat{\mathbf{z}} \tag{17}$$

The well-known Euler's formula, and the assumption that $p_0 = p_A \approx p_B$ since the modulation rate is much smaller than the frequency, $s \ll \omega$, gives the following simplified forms for (16) and (17):

$$\left(p_{in}^{*}\nabla p_{in}\right)_y \approx -2k_y p_0^2 \sin\left(2k_y y - s(t - t_s)\right)\hat{\mathbf{y}} \tag{18}$$

$$\left(p_{in}^{*}\nabla p_{in}\right)_z \approx -2k_z p_0^2\left[1 + \cos\left(2k_y y - s(t - t_s)\right)\right]\hat{\mathbf{z}} \tag{19}$$

A similar procedure can be performed to obtain the second term of (3). Firstly the Jacobian matrix of the partial derivatives can be expressed as

$$\nabla \mathbf{v}_{in} = \begin{bmatrix} \frac{\partial v_{in,y}}{\partial y} & \frac{\partial v_{in,y}}{\partial z} \\ \frac{\partial v_{in,z}}{\partial y} & \frac{\partial v_{in,z}}{\partial z} \end{bmatrix} \tag{20}$$

where

$$\frac{\partial v_{in,y}}{\partial y} = -\frac{u_0}{k} k_y^2 \left[\exp(i\alpha) + \exp(i\delta)\right] \tag{21}$$

$$\frac{\partial v_{in,y}}{\partial z} = \frac{\partial v_{in,z}}{\partial y} = -\frac{u_0}{k} k_y k_z \left[\exp(i\alpha) - \exp(i\delta)\right] \tag{22}$$

$$\frac{\partial v_{in,z}}{\partial z} = -\frac{u_0}{k} k_z^2 \left[\exp(i\alpha) + \exp(i\delta)\right] \tag{23}$$

Finally the complex conjugate of the velocity field (10) is

$$\mathbf{v}_{in}^* = ik_y \frac{u_0}{k} \left[\exp(-i\alpha) - \exp(-i\delta)\right]\hat{\mathbf{y}} + ik_z \frac{u_0}{k} \left[\exp(-i\alpha) + \exp(-i\delta)\right]\hat{\mathbf{z}} \tag{24}$$

Combining (20) to (23) with (24) and using again the identities (13) to (15) and the Euler's formula we have

$$\begin{aligned} \left(\mathbf{v}_{in}^* \cdot \nabla \mathbf{v}_{in}\right)_y &= v_{in,y}^* \frac{\partial v_{in,y}}{\partial y} + v_{in,z}^* \frac{\partial v_{in,y}}{\partial z} = \\ &- i \frac{u_0^2}{k^2} k_y^3 \left[1 + \exp\left(i\left(2k_y y - s(t-t_s)\right)\right) - \exp\left(-i\left(2k_y y - s(t-t_s)\right)\right) - 1\right] \\ &- i \frac{u_0^2}{k^2} k_y k_z^2 \left[1 - \exp\left(i\left(2k_y y - s(t-t_s)\right)\right) + \exp\left(-i\left(2k_y y - s(t-t_s)\right)\right) - 1\right] \\ &= 2\frac{u_0^2}{k^2} k_y \left(k_y^2 - k_z^2\right) \sin\left(2k_y y - s(t-t_s)\right) \end{aligned} \tag{25}$$

for the y component of the inner product. Since both (18) and (25) are real, when substituting into (3), only the real parts of f_1 and f_2 will play a role:

$$F_{ac,y} = -\pi a^3 \left[\frac{2\kappa_0}{3} f_1^r \left(-2k_y p_0^2 \sin\left(2k_y y - s(t-t_s)\right)\right) - \rho_0 f_2^r 2\frac{u_0^2}{k^2} k_y \left(k_y^2 - k_z^2\right) \sin\left(2k_y y - s(t-t_s)\right)\right] \tag{26}$$

On rearranging and converting to velocity amplitude, and noting that $\kappa = 1/(\rho c^2)$ and $\omega = kc$ we have

$$F_{ac,y} = \frac{4\pi a^3}{3} \frac{p_0^2}{\rho_0 c_0^2} k_y \left[f_1^r + \frac{3}{2} f_2^r \frac{\left(k_y^2 - k_z^2\right)}{k^2}\right] \sin\left(2k_y y - s(t-t_s)\right) \tag{27a}$$

for the radiation force in the horizontal direction.

For a bulk device with phase shift this equation would take the form[6]

$$F_{ac,y} = \frac{4\pi a^3}{3} \frac{p_0^2}{\rho_0 c_0^2} k_y \left[f_1^r + \frac{3}{2} f_2^r\right] \sin\left(2k_y y - s(t-t_s)\right) \tag{27b}$$

Particle trajectories in phase modulated acoustic standing wave fields

Using the formula obtained for the radiation force, and combined with the viscous force, Newton's second law can be written as

$$-c_{ac}\sin\left(2k_y y - s(t-t_s)\right) - c_{visc}\dot{y} = m\ddot{y} \tag{28}$$

Applying the inertial approximation,[7] the $m\ddot{y}$ term can be neglected. Moreover since

$$\frac{\partial}{\partial t}\left(2k_y y - s(t-t_s)\right) = 2k_y\dot{y} - s = -2k_y\frac{c_{ac}}{c_{visc}}\sin\left(2k_y y - s(t-t_s)\right) - s \tag{29}$$

using the substitution $u = 2k_y y - s(t-t_s)$ and $\gamma = 2k_y c_{ac}/c_{visc}$ the resulting differential equation

$$\frac{\partial u}{\partial t} = -\gamma\sin u - s \tag{30}$$

can be separated and solved:

$$\frac{du}{\gamma\sin u + s} = -dt \tag{31}$$

$$\frac{2}{\sqrt{s^2-\gamma^2}}\tan^{-1}\left(\frac{\gamma + s\tan\frac{u}{2}}{\sqrt{s^2-\gamma^2}}\right) = -t + c_1 \tag{32}$$

Afterwards direct rearrangement for *y(t)* yields

$$y(t) = \frac{s(t-t_s)}{2k_y} - \frac{1}{k_y}\tan^{-1}\left(\frac{\gamma - Q\tan\left((c_1Q - tQ)/2\right)}{s}\right) \tag{33a}$$

where $Q = \sqrt{s^2-\gamma^2}$ and the c_1 constant can be determined from the initial particle position. The inverse tangent function is taken to be monotonic during the ramping. For the constant phase shift case without the jump, t_s is simply zero, for the 90° jump case it is quarter of the slope time.

For the rest phase the particle trajectories can be described by[8]

$$y(t) = \frac{1}{k_y}\tan^{-1}\left(c_2\exp(-\gamma t)\right) \tag{33b}$$

where c_2 is a constant depending on initial conditions.

Numerical implementation of the differential equation of motion

The particle motion predicted by the analytical equations are compared with those obtained by a finite-difference scheme. From the position $y(t)$ of a given particle and its velocity $u(t)$ at time t, the position $y(t+\Delta t)$ and the velocity $u(t+\Delta t)$ after a small time step, Δt, are calculated by solving the differential equation of motion using the Euler method:

$$y(t+\Delta t) = y(t) + u(t)\Delta t + \frac{1}{2}a(t)\Delta t^2 \tag{34}$$

$$u(t+\Delta t) = u(t) + a(t)\Delta t \tag{35}$$

where the acceleration $a(t)$ is given by:

$$a(t) = \frac{F_{ac}(t)+F_{drag}(t)}{m} \tag{36}$$

The parameters used in the numerical simulation can be seen in Table I.

TABLE I
PARAMETERS USED FOR NUMERICAL SIMULATIONS

Symbol	Quantity	Value
κ_p	compressibility of particle	2.16 x 10^{-10} Pa^{-1}
κ_0	compressibility of medium	4.56 x 10^{-10} Pa^{-1}
ρ_p	density of particle	1.05 $g{\cdot}cm^{-3}$
ρ_0	density of medium	1 $g{\cdot}cm^{-3}$
c_s	sound velocity on $LiNbO_3$ surface	3780 $m{\cdot}s^{-1}$
c_f	sound velocity in medium (water)	1480 $m{\cdot}s^{-1}$
η_{medium}	dynamic viscosity of medium	1 mPa·s
f	frequency	13.3 MHz
p_0	pressure amplitude	96 kPa
R_A, R_B	particle radii	5 and 7.5 μm
Δt	time step for numerical method	1 μs
t_{slope}	slope time parameter	0.85 s
t_{ramp}	ramping time parameter	0.64 s
t_{rest}	rest time parameter	1 s

Force measurement methodology

To measure the acoustic radiation force acting on the particles and to obtain the acoustic energy density in the device, a modified version of the curve fitting method[9] is used. Firstly, the two IDTs were activated, and the particles trapped at the pressure nodes. The phase of one IDT was suddenly changed by 130°, and the particles translated with the shifted node. The resulting trajectories follow (33b) and can be used to obtain the acoustic energy densities. A phase jump of 130° was used instead of a phase jump of 180° to avoid the unstable position of the particles at antinodes (Fig. 3c). The curve fitting was performed on the position-speed plots that follow sinusoidal curves and offer easier fitting rather than on the time-position plots that can be modelled by a more complex equation (33b) only.

Since the radiation force is balanced by the drag force, considering the absolute value of the particle speed gives:

$$\dot{y} = \frac{c_{ac}}{c_{visc}}\sin\left(2k_y y\right) = A\sin\left(2k_y y\right) \tag{37}$$

and the speed was calculated as the central finite difference quotient $u_i = (y_{i+1} - y_{i-1})/(t_{i+1} - t_{i-1})$ at each point of the trajectory. The only fitting parameter is the amplitude A that can be approximated by minimizing the overall squared error sum. From this approximated maximum speed A, the acoustic energy density can be simply obtained by

$$E_{ac} = \frac{9\eta_{medium}A}{2R^2 k_y \phi} \tag{38}$$

where R is the radius of the particle, k_y is the apparent wavenumber in the y direction, ϕ is the acoustic contrast factor and η_{medium} is the dynamic viscosity of the medium. Furthermore the pressure amplitude within the cavity is given by[4]

$$p_a = \sqrt{4E_{ac}/\kappa_0} \tag{39}$$

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