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A New Methodology for Designing PD Controllers

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SUMMARY

It is known that it is impossible to select fixed gains for a PD controller that will critically damp the response to disturbances for all configurations of a given robot system. Because of this the potential for overshoot is always present and cannot be avoided unless the system is severely overdamped. This is not necessarily a practical solution and can be an economically unacceptable approach. On the other hand, however, if overshoot is permissible to some degree for some systems in the case of conventional *Serial robots* it is still prohibited in the case of *Parallel robots* as it may easily bring the robot to one of its possible singular configurations, causing damage to the system. This paper introduces a new algorithm for the design of PD controllers that ensures uniform and fast dynamic responses, which are free from overshoots for all robot configurations. The technique also satisfies general stability requirements for the system.

KEYWORDS: Robot control; PD controllers; Overshoot; Serial robots; Parallel robots; Redundant-actuated parallel robots.

1. INTRODUCTION

It is well known that a very simple PD controller, with gravity compensation, can be an adequate solution for set-point control of robot manipulators and that it can satisfy general stability requirements.[1–2] The practical significance of this control technique lies in the fact that it requires no detailed knowledge of the manipulator dynamics except the gravity-loading vector. In practice an integral action is added (to give PID Control) in order to reject constant perturbations, at the cost of a reduced system bandwidth. In fact with the existing PD and/or PID control structures it is impossible to select fixed gains that can cope with all robot configurations. Average gains are always chosen which approximate critical damping at the centre of the manipulator workspace.[3] This inevitably results in overshoot, or even instability, at other positions within the workspace.

To cope with these problems, Seraji[4–5] suggests using a sector-bounded nonlinear gain in cascade with a linear PID (PD) controller (without gravity compensation). This gain represents an even function of the error to give high outputs at high inputs, and vice versa. From the author's point of view, the gain allows a large corrective action when the error is large. As the error diminishes the gain is reduced in order to prevent large overshoots in the response. Following this a stability analysis is performed based on the Popov stability criterion after assuming linear dynamics for the robot with restrictions to single-input single-output conditions. This does not necessarily guarantee full stability of a real system. It is also important to refer to the fact that the method is restricted only to systems with output feedback (velocity feedback is not allowed). It appears that no simulations have been performed to clarify the idea.

Armstrong[6] used a nonlinear PD controller of P and D gains, each with two terms for force control. The first term represents the smaller control gain that is kept constant. The second term consists of a higher constant gain multiplied by a switching function that controls its application to the system. The modulation of the P gain increases the damping while the modulation of the D gain shortens the rise time. In Armstrong's work both gains are modulated, with the larger values applied at large errors and the smaller values at low errors. The method is also restricted to linear systems with single-input single-output configuration. The idea is different to that of Seraji[4–5] because it can be applied to systems with output and velocity feedback.

A review of the literature in the area of nonlinear PD or PID control demonstrates that all concepts are centred on the one idea in which variable gains are used to improve the system response. This is a very old technique, and the examination of an early textbook[7] shows that it is the idea of the nonlinear servomechanism, as proposed by Lewis and described in that reference. The idea is to have a positioning system with negative, or very small, damping when the error is large. This tends to ensure a more rapid response to large errors than in the corresponding linear system. This desirable effect can be accomplished by using nonlinear velocity feedback which depends on the absolute value of the error. Although the system response can be improved for step inputs, instability was reported for double pulse inputs.

Parallel manipulators[8–13] are known to possess many advantages over conventional serial robots such as high rigidity, high precision, high load carrying capacity, and high-speed capability. Despite this, they suffer from many singular configurations distributed throughout their workspace.[14–16] Therefore any control designer must appreciate this fact and must avoid overshoot to prevent the system being brought in to one of these configurations, thereby causing damage to the robot manipulator. This provides a motivation for the controller design discussed henceforth.

In this paper we introduce a new methodology for the design of PD controllers to ensure fast and uniform system responses, with no overshoot, for all robot configurations. In addition the method is also able to satisfy general stability requirements for nonlinear systems. For theoretical assessment, simulations have been carried out on the SEPA robot[16] to ensure validation for all robot designs (*Series, Parallel and Redundant-Actuated parallel* Robots).

2. THEORETICAL BACKGROUND AND PROBLEM STATEMENT

Using standard assumptions the dynamics of a rigid robotic manipulator, with either prismatic or revolute joints, can be described by the following equation:

$$\tau_c = M(\Theta)\ddot{\Theta} + V_m(\Theta, \dot{\Theta})\dot{\Theta} + F(\dot{\Theta}) + G(\Theta) \qquad (1)$$

Here, Θ is a vector of generalised co-ordinates, $M(\Theta)$ is a symmetric, positive definite, inertia matrix, $V_m(\Theta, \dot{\Theta})\dot{\Theta}$ is a vector of centrifugal and Coriolis forces, $G(\Theta)$ is a vector of gravity forces, $F(\dot{\Theta})$ is a vector of friction forces, and τ_c is a vector of control forces or torques.

With $F(\dot{\Theta}) = 0$, global asymptotic stability of the closed loop system is ensured by the following PD control:[17]

$$\tau_c = K_p E - K_V \dot{\Theta} + G(\Theta) \qquad (2)$$

where K_p and K_v are constant, diagonal, gain matrices of dimensions that depend on the mechanical design of the robot manipulator. The vector E is the regulation error with respect to the reference input Θ_d, which is constant here.

Proper choice of the elements of the gain matrices can critically damp the system response to a specific input. Once this input is changed, the system exhibits either sluggish or oscillatory response depending on the input values. This represents a non-uniform dynamic response that is undesirable and overshoot is the most critical issue that arises because of this. The problem must be avoided in robotic systems, especially in the case of parallel robots that have many singular configurations inside their workspace. The concern is always to avoid the state of singularity which can cause damage to the manipulator as the control is lost.

3. CONTROLLER DESIGN

To prevent the robotic system as defined by equation 1 (under the control of equation 2), from showing overshoot from its desired input, the response of the i^{th} degree of freedom (DOF) must be kept inside the second or the fourth quadrants of the phase plane of Figure 1. However, this is not the strongest condition for the prevention of overshoot. That condition is where the response of all degrees of freedom is maintained inside the two shaded areas in the second and fourth quadrants of Figure 1. The line which divides the second and the fourth quadrants is defined by the following equation:

$$\phi_i = \lambda_i e_i - \dot{\theta}_i = 0 \qquad (3)$$

where λ_i is a gain representing the slope of the line.

After specifying the above line, and because we seek the response to be in the aforementioned shaded areas, the distance from the error axis to the line can be used as a parameter to control the value of the derivative gain. To implement this, the control law of equation 2 will take the following nonlinear form:

$$\tau_c = K_p E - \Omega(E, \dot{\Theta}) K_V \dot{\Theta} + G(\Theta) \qquad (4)$$

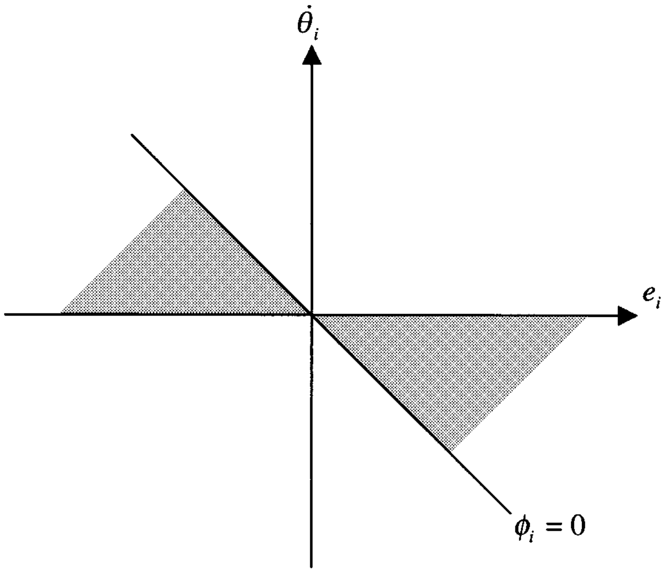

Fig. 1. Phase plane of the i^{th} DOF.

With: $\Omega(E, \dot{\Theta}) = diag[1 + \exp(-k_1\psi_1), 1 + \exp(-k_2\psi_2), \ldots, 1 + \exp(-k_n\psi_n)]$

And, $\psi_i = \lambda_i|e_i| - |\dot{\theta}_i|$.

where, *diag* means diagonal, k_i is a user-defined constant positive gain, and n is the number of degrees of freedom of the robot.

In the control law above the elements of K_v should be chosen to be small so as to represent the smallest derivative gain. The motivation for using this controller cannot be discussed qualitatively. When the system is far away from the line defined by equation 3 the exponential term is reduced drastically causing the derivative term in the control law to be at its lowest value. This ensures rapid response in the early stages. As time passes the system reaches the specified line and the exponential term becomes equal to one, resulting in an increase in the value of the damping term. When the system overshoots the line the damping is increased by a very large amount causing the system to converge to the line. It is important to refer here to the fact that, despite this potential overshoot, we are still inside the desirable areas that are defined by the second and the fourth quadrants of the phase plane. This process is repeatedly applied to any trajectory on the phase plane. This inevitably results in the desired uniform dynamic performance of the system.

The next section examines the stability of the system defined by equation 1 under the control law of equation 4.

4. STABILITY ANALYSIS

Assuming only viscous friction, equation 1 becomes as follows:

$$\tau_c = M(\Theta)\ddot{\Theta} + V_m(\Theta, \dot{\Theta})\dot{\Theta} + B\dot{\Theta} + G(\Theta) \qquad (5)$$

where, B is a diagonal matrix with positive gains representing the viscous friction coefficients.

Theorem

The closed-loop system (5) and (4) is globally asymptotically stable with respect to E and $\dot{E}$:

$$E, \dot{E} \to 0 \quad \text{as } t \to \infty$$

Proof:
Consider the following scalar Lyapunov function,

$$V = \frac{1}{2}\dot{\Theta}^T M(\Theta)\dot{\Theta} + \frac{1}{2}E^T K_p E \tag{6}$$

Differentiating with respect to time yields,

$$\dot{V} = \frac{1}{2}\dot{\Theta}^T \dot{M}(\Theta)\dot{\Theta} + \dot{\Theta}^T M(\Theta)\ddot{\Theta} + E^T K_p \dot{E} \tag{7}$$

Substituting from equations 4 and 5 into equation 7 gives,

$$\dot{V} = \frac{1}{2}\dot{\Theta}^T \dot{M}(\Theta)\dot{\Theta} + \dot{\Theta}^T\{K_P E - \Omega(E, \dot{\Theta})K_V\dot{\Theta} - V_m(\Theta, \dot{\Theta})\dot{\Theta} - B\dot{\Theta}\} - E^T K_P \dot{\Theta}$$

Due to the skew symmetry of $\frac{1}{2}\dot{M}(\Theta) - V_m(\Theta, \dot{\Theta})$, and $\dot{\Theta} = -\dot{E}$ for set-point control, the time derivative of the Lyapunov function is reduced to the following equation,

$$\dot{V} = -\dot{\Theta}^T\Omega(E, \dot{\Theta})K_V\,\dot{\Theta} - \dot{\Theta}^T B\,\dot{\Theta} \tag{8}$$

Since K_V, B and Ω are positive definite, $\dot{V} \le 0$ is satisfied and the system is globally stable.

5. AN EXAMPLE APPLICATION

5.1 The SEPA Robot

The SEPA robot [16] is a 2-DOF manipulator that can be operated in all known mechanical modes (Serial, Parallel and Redundant-Parallel). The mechanical schematic of the manipulator is shown in Figure 2. A detailed description of the system can be obtained from reference 16.

Assuming that the manipulator is moving in the horizontal plane, and using the following design parameters $m = 1$, *kg*, $2L = 1$ m and $m_m = 0.5$ *kg*, where m_m is the mass of the actuator at joint C, the dynamics of the different operating modes are as follows:

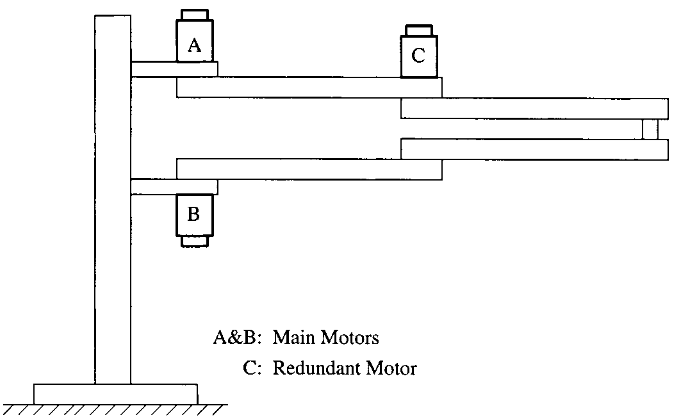

Fig. 2. Illustration of SEPA robot manipulator.

- **Normal Parallel Mode**

$$\begin{bmatrix}\tau_A \\ \tau_B\end{bmatrix} = \begin{bmatrix}1.66 & 0.996\cos(\theta_2-\theta_1) \\ 0.996\cos(\theta_2-\theta_1) & 1.66\end{bmatrix} \times \begin{bmatrix}\ddot{\theta}_1 \\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix}-0.996\sin(\theta_2-\theta_1)\dot{\theta}_2^2 \\ 0.996\sin(\theta_2-\theta_1)\dot{\theta}_1^2\end{bmatrix} \tag{9}$$

- **Redundant Parallel Mode**

$$\begin{bmatrix}\tau_A + \tau_C \\ \tau_B - \tau_C\end{bmatrix} = \begin{bmatrix}1.66 & 0.996\cos(\theta_2-\theta_1) \\ 0.996\cos(\theta_2-\theta_1) & 2.16\end{bmatrix} \times \begin{bmatrix}\ddot{\theta}_1 \\ \ddot{\theta}_2\end{bmatrix} + \begin{bmatrix}-0.996\sin(\theta_2-\theta_1)\dot{\theta}_2^2 \\ 0.996\sin(\theta_2-\theta_1)\dot{\theta}_1^2\end{bmatrix} \tag{10}$$

- **Serial Mode**

$$\begin{bmatrix}\tau_A + \tau_B - \tau_C \\ \tau_C\end{bmatrix} = \begin{bmatrix}3.156 & 0.996\cos(\theta_3-\theta_1) \\ 0.996\cos(\theta_3-\theta_1) & 1.164\end{bmatrix} \times \begin{bmatrix}\ddot{\theta}_1 \\ \ddot{\theta}_3\end{bmatrix} + \begin{bmatrix}-0.996\sin(\theta_3-\theta_1)\dot{\theta}_3^2 \\ 0.996\sin(\theta_3-\theta_1)\dot{\theta}_1^2\end{bmatrix} \tag{11}$$

5.2 Simulations

To examine the uniformity in the dynamic responses of all robot modes under the control law of interest the arm has been subjected to a set of step inputs of different magnitudes. The design parameters of the controller (for all dynamic modes) are as follows:

$$K_P = \begin{bmatrix}9 & 0 \\ 0 & 9\end{bmatrix}, \; K_V = \begin{bmatrix}4.5 & 0 \\ 0 & 4.5\end{bmatrix}, \; \lambda_1 = \lambda_2 = 2, \text{ and } k_1 = k_2 = 2$$

Because friction only increases the stability of the system it is ignored in the simulation. The simulations have been performed using *Simulink.*

5.2.1 Simulation Results.. The simulation results, in the form of system responses and motors control signals, for the *normal parallel mode*, the *redundant parallel mode* and the *serial mode* are shown in Figures 3–6, 7–11 and 12–15, respectively. The results show that the new controller is able to reject all the disturbances with the same effectiveness. The phase planes for the various operating modes are shown in Figures 16–21, and explain the uniformity of the system responses to all step inputs. In addition the controller is able to achieve fast system responses with no overshoots.

Despite the fact that fixed design parameters have been used for the controller in all the dynamic modes the controller has succeeded in dealing with the differences

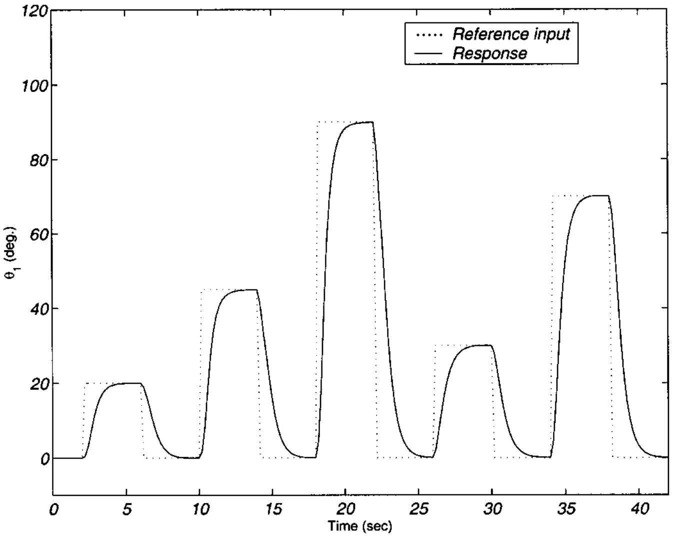

Fig. 3. Angular position of link BD (*Normal Parallel Mode*).

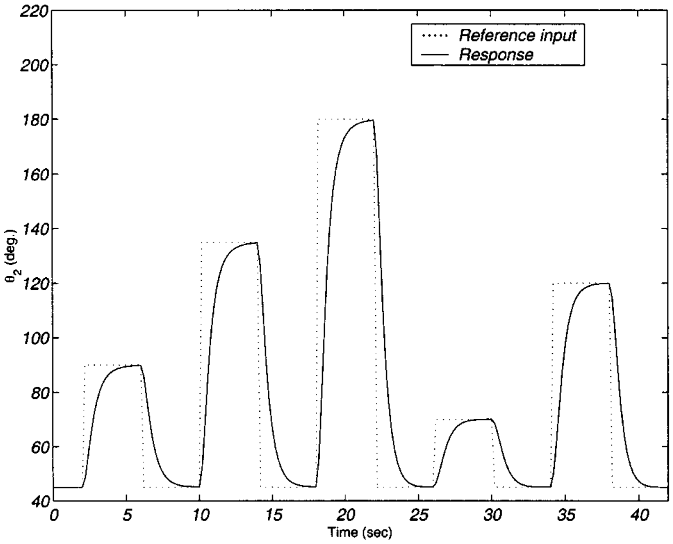

Fig. 4. Angular position of link AC (*Normal Parallel Mode*).

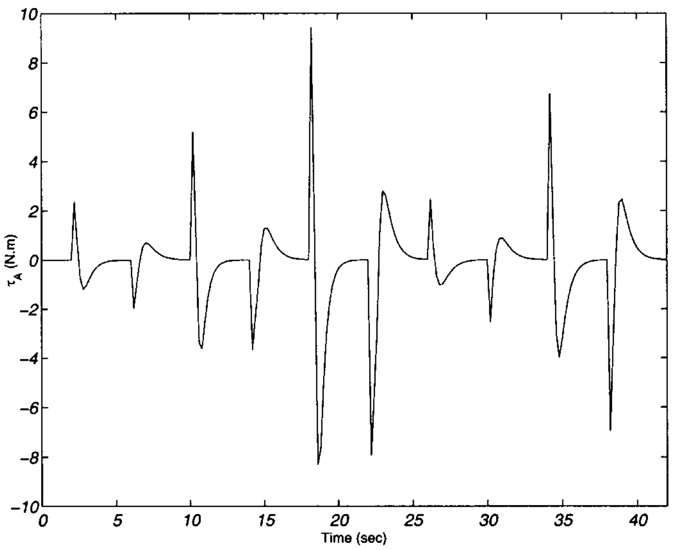

Fig. 5. Control signal of motor A (*Normal Parallel Mode*).

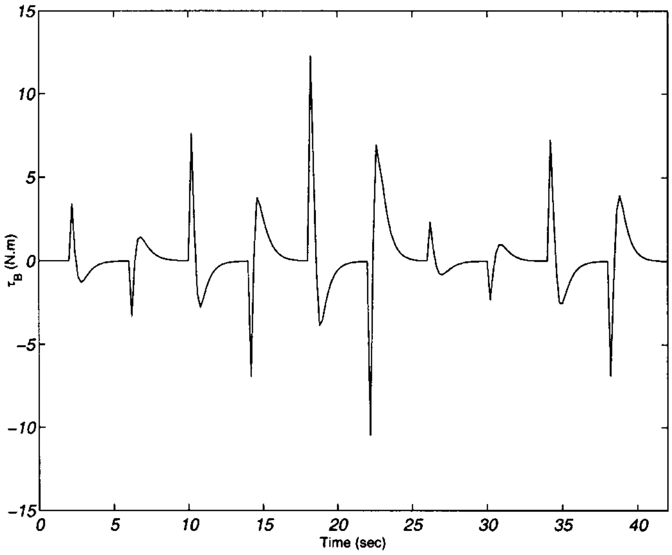

Fig. 6. Control signal of motor B (*Normal Parallel Mode*).

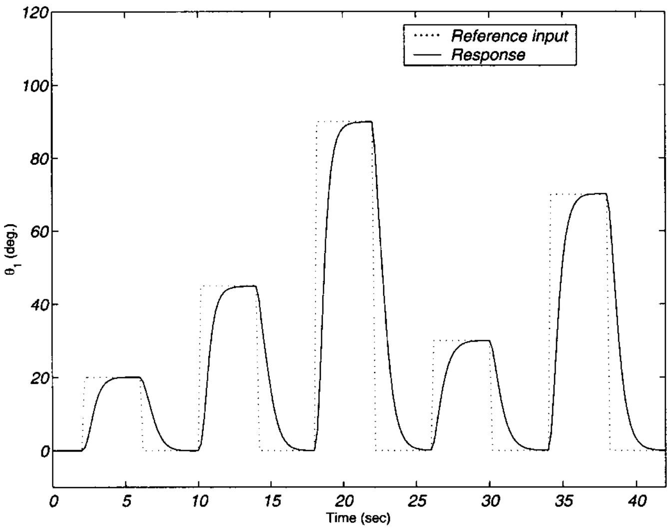

Fig. 7. Angular position of link BD (*Redundant Parallel Mode*).

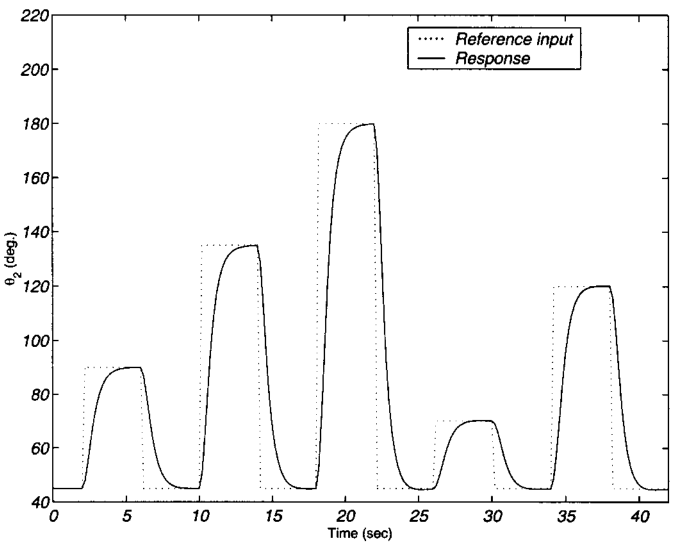

Fig. 8. Angular position of link AC (*Redundant Parallel Mode*).

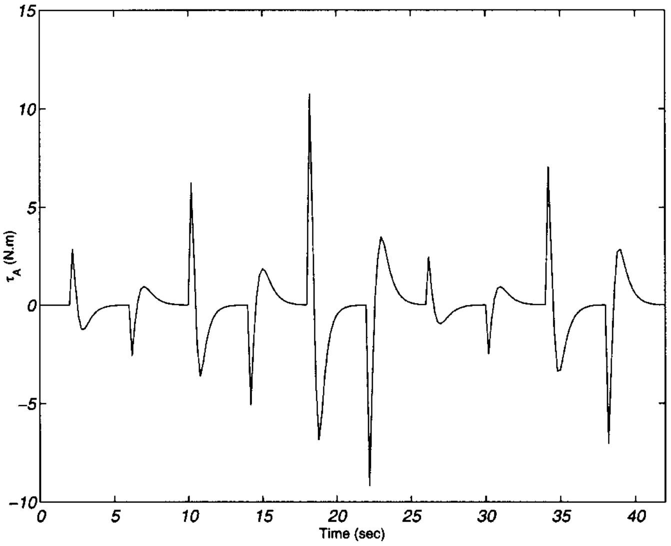

Fig. 9. Control signal of motor A (*Redundant Parallel Mode*).

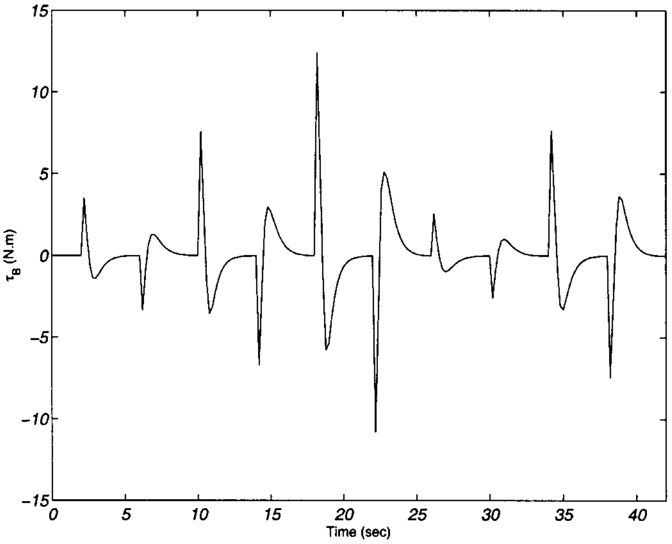

Fig. 10. Control signal of Motor B (Redundant Parallel Mode).

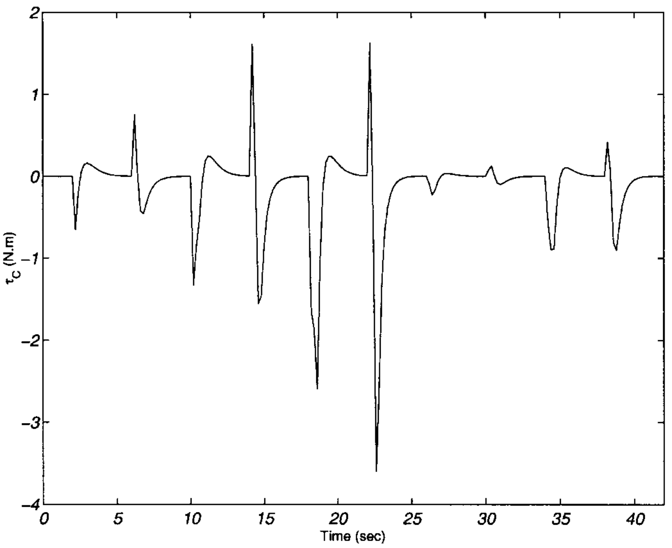

Fig. 11. Control signal of Motor C (Redundant Parallel Mode).

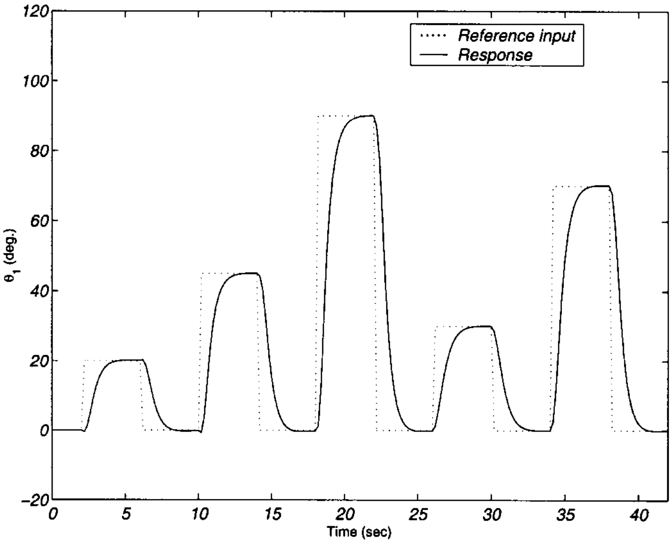

Fig. 12. Angular position of links BD and AC (*Serial Mode*).

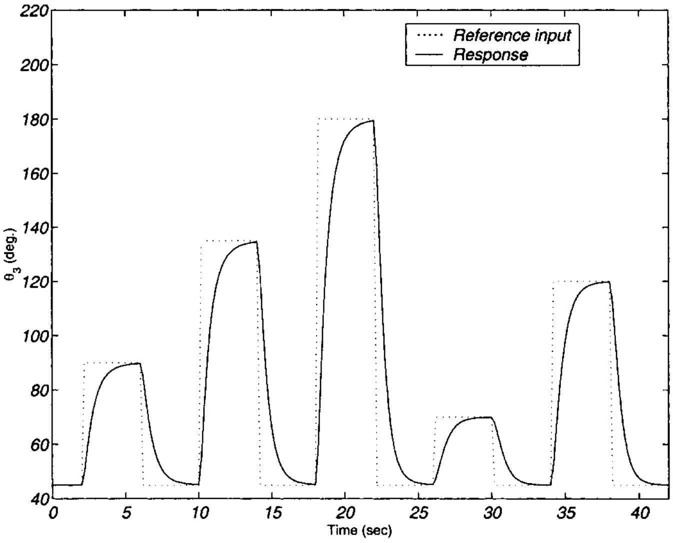

Fig. 13. Angular position of links DP and CP (*Serial Mode*).

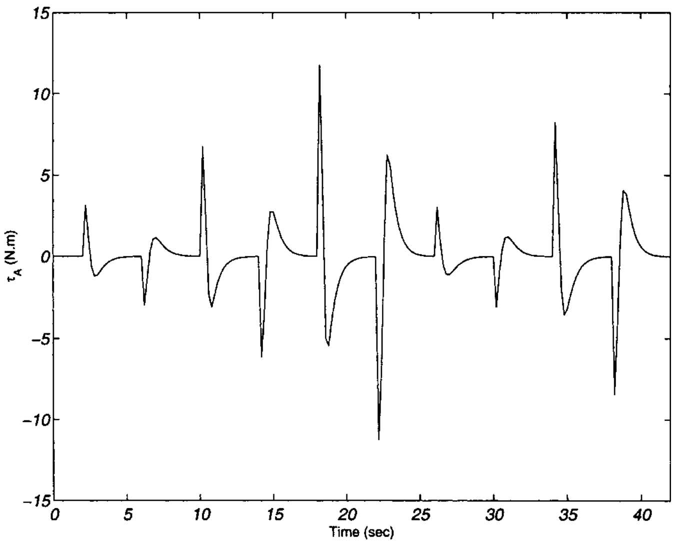

Fig. 14. Control signal of Motor A and B (Serial Mode).

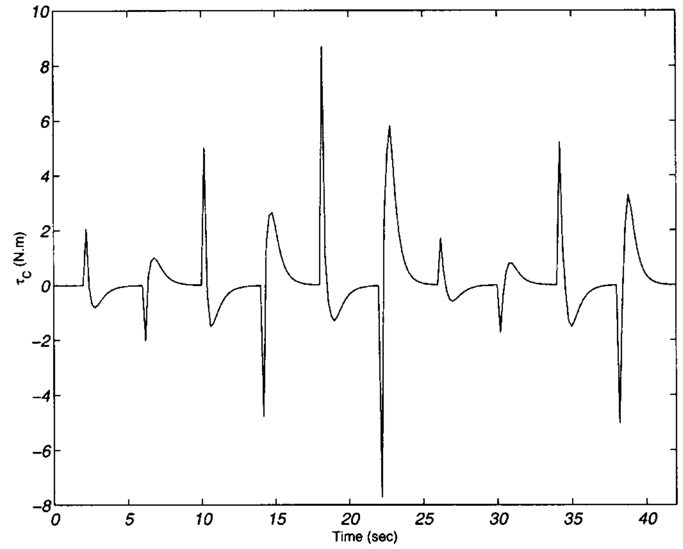

Fig. 15. Control signal of Motor C (Serial Mode).

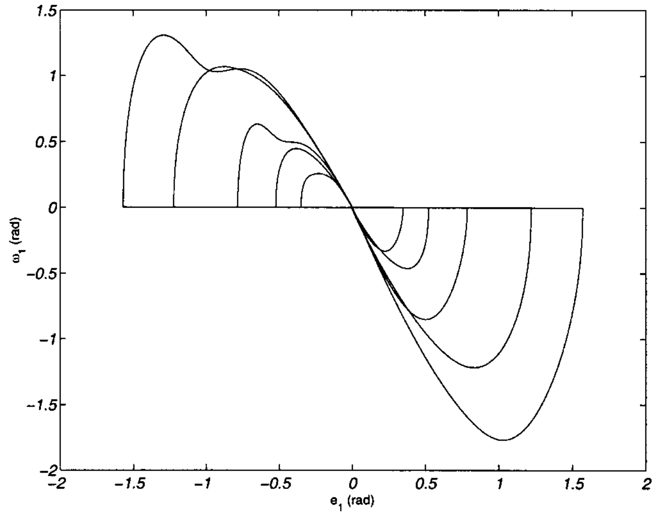

Fig. 18. Phase plane of link BD (Redundant Parallel Mode).

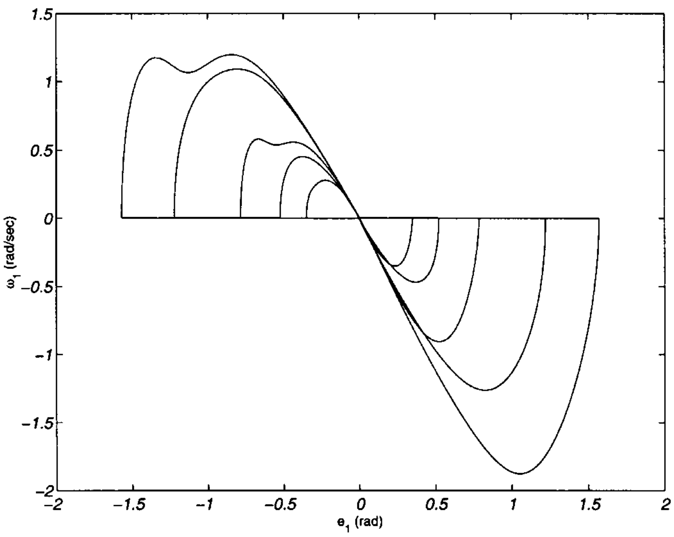

Fig. 16. Phase plane of link BD (Normal Parallel Mode).

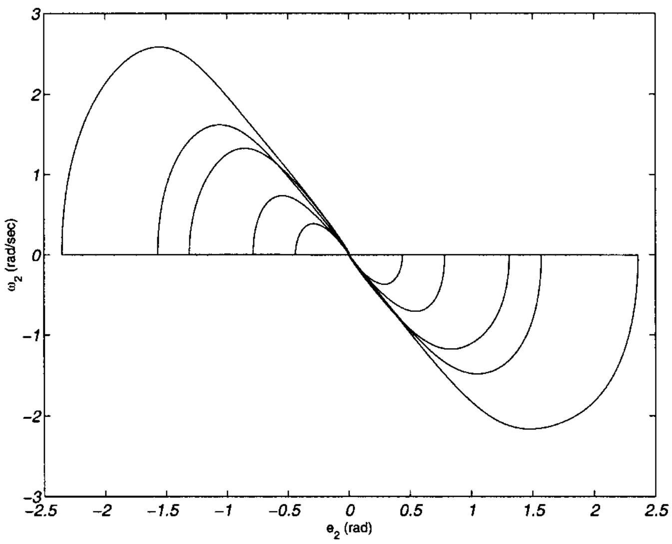

Fig. 19. Phase plane of link AC (Redundant Parallel Mode).

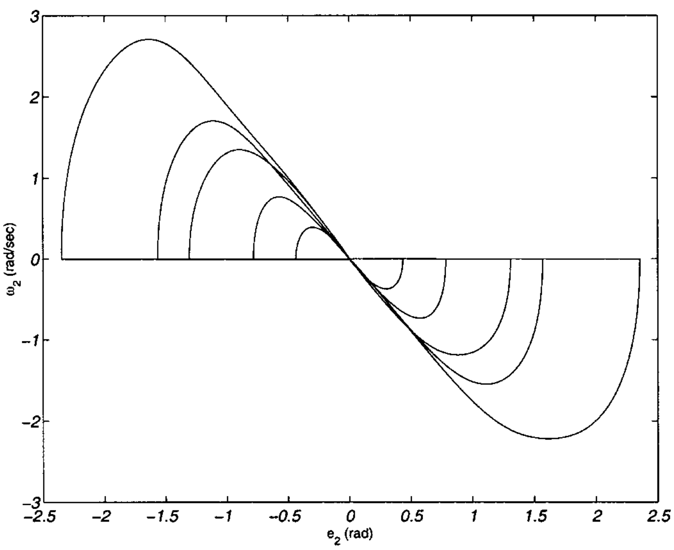

Fig. 17. Phase plane of link AC (Normal Parallel Mode).

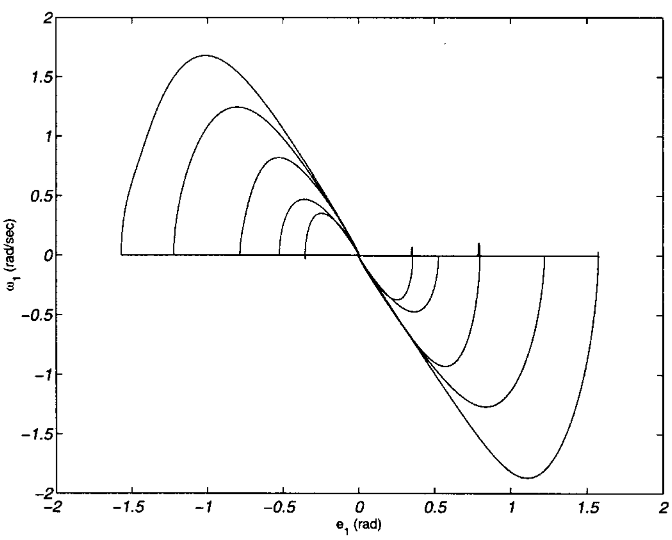

Fig. 20. Phase plane of link AC and BD (Serial Mode).

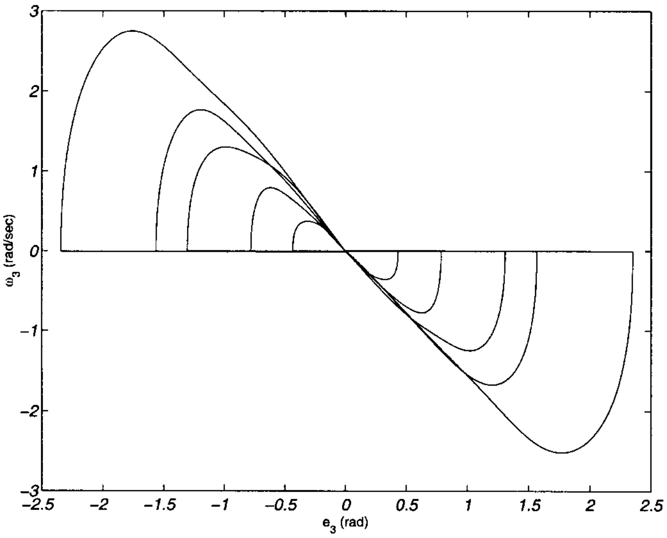

Fig. 21. Phase plane of link DP and CP (Serial Mode).

between the dynamic models. This helps to underpin the robustness properties of the PD controller with gravity compensation.[17]

6. CONCLUSIONS

In this paper, a new technique for the design of PD controllers has been introduced. The design allows fast system responses without overshoots. In addition the system using this control law exhibits a uniform dynamic performance. These facts have been discussed theoretically. The results that have been obtained are valid for all robotic systems. Stability requirements have been met for general nonlinear systems. Due to these results the new PD control law can be easily attached to any nonlinear model based control, as a servo controller, to enhance its performance.

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