A simpler puzzle of ground

(Word count: 1892; Category: Logic)

Abstract: Metaphysical grounding is standardly taken to be irreflexive: nothing grounds itself. Kit Fine has presented some puzzles that appear to contradict this principle. I construct a particularly simple variant of those puzzles that is independent of several of the assumptions required by Fine, instead employing quantification into sentence position. Various possible responses to Fine's puzzles thus turn out to apply only in a restricted range of cases.

In the recent debate on metaphysical grounding and its logic,[1] it has generally been accepted that grounding is irreflexive, i.e. that nothing grounds itself, and that true existential quantifications are grounded in their true instances. Call this second principle *EG* (Existential Grounding). Kit Fine (2010) has shown that given a number of plausible auxiliary assumptions, *EG* yields counterexamples to irreflexivity.[2] It turns out that an extremely simple derivation of a counterexample to irreflexivity from *EG* is available if we (i) take grounding to be expressed by an operator on sentences (or lists of sentences), (ii) avail ourselves of (non-substitutional[3]) quantification into sentence position, and (iii) assume that *EG* extends to this kind of quantification. It is safe to assume that

(1) $\exists p\ p$

[1] I am thinking in particular of Fine 2010, 2012a, 2012b, Schnieder 2011, Correia 2011, Rosen 2009.

[2] Fine's paper also establishes analogous points for a counterpart to *EG* for universal quantifications.

[3] This qualification will be taken as understood henceforth.

– a theorem of any standard logic with sentential quantification. Now note that (1) is a true instance of itself, so that by *EG*, we immediately obtain a case of self-grounding (writing ‘$\prec$’ for partial[4] grounding):

(2) $\exists p\, p \prec \exists p\, p$

Fine’s own arguments differ from this one in that they do not rely on sentential quantification, but instead quantify first-order over facts, sentences, or propositions, and additionally appeal to a number of principles relating grounding to these kinds of entities. A Finean counterpart to the present puzzle might go as follows.[5] In place of (1), we assume that some proposition is true (read the variables as restricted to propositions):

(F1) $\exists x\, x$ is true

Next, we assume a claim of Propositional Existence: that there is such a thing as the proposition that some proposition is true. Letting ‘<*p*>’ abbreviate ‘the proposition that *p*’:

(PE) $\exists y\, y = {<}\exists x\, x \text{ is true}{>}$

Now let ‘*s*’ abbreviate ‘$\exists x\, x$ is true’, i.e. sentence (F1). Using the suitable instance of a Truth-Introduction principle

(TI) If p & $\exists x\, x{=}{<}p{>}$, then ${<}p{>}$ is true

we infer from (F1) and (PE):

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[4] All of the grounding claims to follow are standardly taken to hold for both partial and complete grounding. For simplicity, take ‘$\prec$’ as binary; arguably, for some purposes we need an operator that takes an arbitrary number of sentences in the left-hand argument place (cf. e.g. Fine 2012a, 46f).

[5] Fine does not state or discuss this exact argument, which is something of a hybrid of Fine’s Particular Argument for Facts and his Universal Argument for Propositions. The latter rests on essentially the same premises as the present one, except that in place of (F1), it assumes that every proposition is either true or false. See Fine 2010, sec. 5.

(F2) $<s>$ is true

Next, we make use of the following plausible principle, which is often associated with Aristotle (cf. e.g. Schnieder 2006, 35f):

(A) If $<p>$ is true then $p \prec <p>$ is true

We infer from (F2) and the relevant instance of (A):

(F3) $s \prec <s>$ is true

But '[*s*] is true' is a true instance of '$\exists x\, x$ is true', i.e. '*s*'. So from (F2) and *EG* we obtain:

(F4) $<s>$ is true $\prec s$

The claims (F3) and (F4) jointly violate the principle that grounding is asymmetric. If we assume moreover the Transitivity of Grounding

(TG) If $p \prec q$ and $q \prec r$ then $p \prec r$

we obtain a violation of irreflexivity.

The only responses to this Finean argument that straightforwardly apply to my version of the puzzle as well are to reject irreflexivity or *EG*. Some possible reasons for denying (F1) may also motivate a rejection of (1), but since both moves appear extremely unattractive I shall set them aside. Since none of the other auxiliary assumptions (PE), (TI), (A), and (TG) are used in my argument, rejecting any of these will not by itself help with the latter.[6]

I shall now briefly comment on the assumptions (i)—(iii), stated at the beginning of the paper, that are required in my, but not Fine's argument. I have nothing interesting to say here about the claim (i) that grounding is adequately expressed by a sentential operator. Like

[6] Correia (forthcoming), in the context of this puzzle, argues that irreflexivity should be given up. Skiles (ms.) advocates rejecting (A) as well as my assumption (i) from the beginning of the paper. I should also note that Correia 2011, 7f formulates a version of the Finean puzzle that, like mine, employs sentential quantification, but also assumes a version of (A) and of (TI).

Fine and many others, I accept the claim, but it is not uncontroversial – dissenters include, e.g. Rosen (2009) and Audi (2012).

Re (ii): If one thinks that quantification into sentence position is meaningless one can of course very simply reject the purported counterinstance to irreflexivity on that ground. If one thinks that quantification into sentence position is meaningless unless understood as merely abbreviating first-order quantification over proposition, the argument will turn out to be a notational variant on its Finean counterpart, tacitly relying on the same premises. Both claims are highly controversial, though, so it is interesting to see what happens if they are denied. Moreover, given (i), there are special reasons for friends of grounding not to dismiss quantification into sentence position. For some interesting and important structural theses about grounding – like irreflexivity, transitivity, and well-foundedness – then seem most naturally expressed by means of such quantification. The most natural statement of transitivity, for instance, is as the claim that $\forall p\ \forall q\ \forall r\ ((p \prec q\ \&\ q \prec r) \rightarrow p \prec r)$ rather than the schematic (TG) above. For in contrast to the schema, the quantificational claim can be properly embedded, and its import does not inappropriately depend on the linguistic resources available to instantiate it. By way of analogy, expressing the irreflexivity of grounding by saying that every instance of ‘$p \prec p$’ is false seems as unsatisfactory as stating the reflexivity of identity by saying that every instance of ‘$a = a$’ is true, instead of saying that $\forall x\ x=x$.

Re (iii): Here is a natural statement of a rule capturing *EG* for the case of sentential quantification (‘α’ stands for an arbitrary sentence, ‘$\phi(\ )$’ for a suitable sentential context):

(EG-S) From $\phi(\alpha)$, infer $\phi(\alpha) \prec \exists p\ \phi(p)$.

The notion of (1) being an instance of itself may perhaps seem weird, so let me try to briefly dispel that impression. The instance of (EG-S) that legitimizes the move from (1) to (2) – i.e.

(X) From $\exists p\ p$, infer $\exists p\ p \prec \exists p\ p$.

– can be obtained by putting '$\exists p\, p$' for α and the null context, i.e. nothing, for ϕ. Note that it seems perfectly appropriate to take (EG-S) as allowing the null instantiation of ϕ, for an analogous reading is required for the corresponding rule of existential generalization – from $\phi(\alpha)$, infer $\exists p\, \phi(p)$ – if it is to legitimize the clearly valid inference to '$\exists p\, p$' from an atomic sentence letter as a premise. Moreover, on the standard conception of how to construct an instance of a given quantification, we proceed by deleting the initial quantifier phrase and then systematically replacing, in the remaining expression, the formerly bound variable by an expression of the suitable grammatical category. Deleting the '$\exists p$' from'$\exists p\, p$' yields 'p', and systematically replacing 'p' in 'p' by the sentence '$\exists p\, p$' then yields '$\exists p\, p$', as desired.

So let us look for reasons to reject (EG-S) while still accepting a rule capturing *EG* for the case of first-order quantification. *Simply* rejecting (EG-S) and insisting that *EG* holds for first-order quantification is unsatisfactory, for the motivation usually offered for *EG* is not specific to the case of first-order quantification. The principle is sometimes held to command intuitive support (cf. e.g. Rosen 2009, 117); to the extent that it does so, it seems to me, it does so equally for any sort of quantification. More substantively, the principle is sometimes motivated by appeal to the highly plausible principle of disjunctive grounding – if p, then $p \prec (p \vee q)$ – and the analogy between existential quantification and disjunction (cf. e.g. Schnieder 2011, 460; Fine 2012a, 60). This analogy extends to the case of sentential quantification.

Can we give reasons to reject (X) while retaining a *restricted* version of (EG-S)? The only potentially plausible suggestion I can think of is to blame the *impredicativity* exhibited by (X). Thus we might say, firstly, that in the sense that is relevant to *EG*, being an instance of a quantification is not a purely syntactic matter. Rather, the expression generalized upon also has to satisfy a semantic condition: roughly, that of determining, or picking out, a value in the range of the corresponding existential quantifier. Secondly, we say that a sentence that itself contains a given sentential quantifier does not determine a value in the range of that quantifier. A simple implementation of that idea restricts (EG-S) to cases in which α is free of sentential quantifiers; a less restrictive option is to introduce a hierarchy of sentential quantifiers and postulating a version of (EG-S) for each of them, requiring in each case that α contain only quantifiers lower in the hierarchy.

A rejection of impredicative definition is also one of the possible motivations to reject (PE) in Fine's arguments. So in this way the predicativists' response to Fine's puzzles may also generalize to my version. But if my contention is correct and impredicativity provides the only potentially plausible ground for rejecting instance (X) of (EG-S), then there are simply no analogues available to the rejection of (TI), (A), or (TG) above. A wholesale rejection of *EG*, a ban on at least certain sorts of impredicative instances of *EG*, and the admission of counterexamples to irreflexivity then are the only options left.[7]

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[7] I would like to thank Fabrice Correia, Kit Fine, Stefan Roski, Benjamin Schnieder, and Alex Skiles for helpful feedback and discussion.

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