Spectrum Utilization Efficiency Analysis in Cognitive Radio Networks

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Abstract—In cognitive radio network, secondary (unlicensed) users (SUs) are allowed to utilize the licensed spectrum when it is not used by the primary (licensed) users (PUs). Because of the dynamic nature of cognitive radio network, the activities of SUs such as "how long to sense" and "how long to transmit" significantly affect both the service quality of the cognitive radio networks and protection to PUs. In this work, we formulate and analyze spectrum utilization efficiency problem in the cognitive radio network with various periodic frame structure of SU, which consists of sensing and data transmission slots. Energy detection is considered for spectrum sensing algorithm. To achieve higher spectrum utilization efficiency, the optimal sensing and data transmission length are investigated and found numerically. The simulation results are presented to verify the our analysis and to evaluate the interference to the PU which should be controlled into tolerable level.

Index Terms—Cognitive radio network; spectrum utilization efficiency; spectrum sensing; energy detection; frame structure.

I. INTRODUCTION

As one of finite resources in wireless communication, spectrum bands are becoming scarcer and should be utilized more efficiently. However it is reported by Federal Communications Commission(FCC) that 70% of the allocated spectrum in US is not fully utilized [1]. Under this motivation, the concept of cognitive radio was firstly provided by [2] in 1999. With cognitive radio technology, SUs are allowed to use free spectrum bands licensed to PUs [3–5]. In order to protect PUs, the interference brought by the activity of SU should be controlled into tolerable level.

The traditional definition of spectrum efficiency is defined as the information rate that can be transmitted over a given bandwidth, which has been discussed in many precious works. In this work, we focus on spectrum utilization efficiency which is different from spectrum efficiency. Spectrum utilization efficiency is defined as, for specific licensed frequency band, the ratio of occupation time by the SU to the total free time. For example, if the spectrum utilization efficiency is 50%, it means that the half of the total free time of licensed spectrum is utilized by the SU. In this context, a typical frame structure is considered for the SU which comprises of the sensing and the data transmission slots. To achieve higher spectrum utilization efficiency defined above, the sensing slot and the data transmission slot are required to be coordinated in a unit frame such that the licensed band occupation time by SU increases and at the same time the collided transmission time with the PUs decreases.

Multiple trade-off problems exist in the frame structure optimization. From the SUs' perspective, the lower the probability of sensing errors occur, the more chances the channel can be reused when it is available, thus the higher the throughput of the SU could be achieved. Therefore, a tradeoff exists between the sensing length and throughput, which was formulated by using this frame structure of SUs [6, 7]. Following each sensing period, the secondary transmission starts when the licensed channel is considered as idle by the SU. Otherwise, the SU has to wait until the next frame to sense the licensed channels again before any secondary usage. In [7], the optimization of spectrum sensing length has been studied using the sensing-throughput tradeoff metric. Specifically, the paper studied the design of the sensing length to maximize the achievable throughput of a single channel cognitive radio network, under the constraint of the probability of detection. To provide better service for SUs, it is advisable to aggregate the perceived spectrum opportunities obtained through simultaneous sensing over multiple channels. In [8] the design of the sensing time has been investigated so as to maximize the average achievable throughput of the multiple channels in cognitive radio network without causing harmful interference to the PUs or exceeding the power limit of the secondary transmitter. The optimal sensing length is identified for the above problem under average power constraint. As an extended work of [8], [9] also studied the problem of designing the optimal sensing length that maximizes the throughput of a wideband sensing-based spectrum sharing cognitive radio network and a wideband opportunistic spectrum access cognitive radio network.

Compared with sensing length, transmission duration length also impacts the extent of interference between the PUs and the SU. Therefore the optimal transmission duration length should be also investigated for higher quality networks. With the same frame structure, [10] considered a cognitive radio network that a SU makes opportunistic access to a spectrum band licensed to a PU according to the sensing result. Based on the required sensing time and the traffic pattern of PU, an optimal value for transmission duration of SU was found so that the throughput of the SU is maximized, yet the collision probability of the PU is not greater than a threshold.

Unlike majority of the current research which has focused on improving throughput/energy efficiency of the SU, in this

paper we are interested in the spectrum utilization efficiency of the licensed spectrum band. The contributions of this work are: (i) we provide the analytical analysis to spectrum utilization efficiency with both various sensing and data transmission length and write the expression of spectrum utilization efficiency for single SU and PU cognitive radio network. (ii) Optimal sensing and frame length of the SU are investigated and found numerically. (iii) To enhance the spectrum utilization efficiency of the licensed band at the same time to control the interference levels by the secondary activity, simulation results of achievable spectrum utilization efficiency and interference level are provided to determine how to design the frame structure subject to the different requirements of proposed cognitive radio networks.

The rest of this paper is organized as follows. Section II defines the system model and explains how the SU detects and accesses the spectrum bands; In Section III, we formulate and analyze the spectrum utilization efficiency with energy detection spectrum sensing. Simulation results and discussions to validate the analytical analysis are presented in Section IV. Finally, we conclude the paper in Section V.

II. SYSTEM MODEL

In this paper, we consider the SU makes opportunistic access to a spectrum band licensed to a PU in the cognitive radio network. The SU may be considered as a secondary terminal such that the SU collects N samples from each PU during the sensing phase for each sensing. The collected samples will be forwarded to a fusion center for combined processing and decision. A typical frame structure of the SU is shown in Fig.1 where each frame with length T consists of the sensing slot with length τ and the data transmission slot with length $T - \tau$. The SU performs spectrum sensing to determine the status of each channel. The data transmission of the SU is activated subject to the spectrum sensing results based on the following two hypotheses for each channel

$$\mathcal{H}_0 : \boldsymbol{y}(n) = \boldsymbol{w}(n), \tag{1}$$
$$\mathcal{H}_1 : \boldsymbol{y}(n) = \boldsymbol{h}(n)\, s(n) + \boldsymbol{w}(n), \tag{2}$$

where $\boldsymbol{y}(n)$ is the observed complex time series received at instant n; $\boldsymbol{w}(n)$ for all $n = \{1, 2, \cdots, N\}$ represents an independent and identically distributed (i.i.d) circularly symmetric complex Gaussian (CSCG) with zero mean and N_0 variance. Hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1$ stand for the spectrum band detected are idle and occupied respectively. In (2), the vector $\boldsymbol{h}(n)$ typically represents the propagation channel between the corresponding PU and the SU and the signal $s(n)$ for all $n = \{1, 2, \cdots, N\}$ denotes a standard scalar i.i.d random process and stands for the source signal to be detected. In this work, the PU's signals are assumed to be complex-value PSK signals.

Once the channels has been confirmed as idle, the SU is allowed to transmit on the channel and we assume (i) the SU is heavily loaded and always has data to transmit, (ii) the traffic loads of the PUs are exponentially distributed with the mean of the occupied and the idle durations denoted by α_1
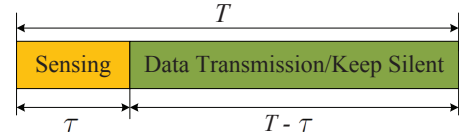

Fig. 1. A graphical structure of a typical frame structure of SU cognitive radio transmission.

and α_0 respectively. Otherwise the SU keeps silent until the next frame.

III. SPECTRUM UTILIZATION EFFICIENCY ANALYSIS

In order to derive the spectrum utilization efficiency, which is defined as the ratio of occupation time by the SU to the total free time, the mean of both occupation time and collision time per frame should be calculated firstly. In practice, spectrum sensing is always imperfect and the sensing errors due to missed detection lead to interference between the SU and the PU where the channel is wrongly considered idle while the false alarm makes the SU keep silent even if the idle channel is available to SU. Moreover, another type of spectrum sensing errors which is referred to as false alarm would not cause interference but will reduces the spectrum utilization efficiency of the licensed frequency band. Because we assume that the traffic loads of the PUs are exponentially distributed with the mean of the occupied and the idle durations denoted by α_1 and α_0 respectively, in the condition that missed detection occurs, the percentage of transmission with collisions out of data transmission duration is given by [11]:

$$P_{ip} = \frac{\alpha_1}{T - \tau} \left(1 - \exp\left(-\frac{T - \tau}{\alpha_1} \right) \right). \tag{3}$$

which is illustrated in Fig. 2. In this work, the SU preforms energy detection for spectrum sensing and transmits data on the frequency bands based on the decision made during the sensing phase. With any given sensing length, the probability of detection and false alarm for the channel under energy detection scheme are given by:

$$\mathscr{P}_d(\tau, \epsilon) = \mathcal{Q}\left(\left(\frac{\epsilon}{N_0} - \gamma - 1 \right) \sqrt{\frac{\tau f_s}{2\gamma + 1}} \right). \tag{4}$$

$$\mathscr{P}_{fa}(\tau, \epsilon) = \left(\left(\frac{\epsilon}{N_0} - 1 \right) \sqrt{\tau f_s} \right). \tag{5}$$

respectively in [7], where ϵ denotes the decision threshold of the energy detector on the licensed channel, γ is received signal-to-noise ratio (SNR) from the PU at the secondary detector on the licensed channel and f_s represents the sampling frequency, $\mathcal{Q}(\cdot)$ is the complementary distribution function of the standard Gaussian. To control the interference to PUs, the target detection probability $\overline{\mathscr{P}_d}$ should be guaranteed. With this condition, (5) could be further expressed as

$$\mathscr{P}_{fa}(\tau, \epsilon) = \mathcal{Q}\left(\sqrt{2\gamma + 1}\, \mathcal{Q}^{-1}(\mathscr{P}_d) + \sqrt{\tau f_s}\, \gamma \right). \tag{6}$$
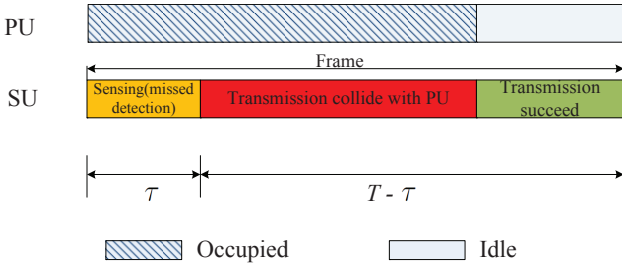
Fig. 2. A graphical illustration of a typical cognitive radio transmission with missed detection.
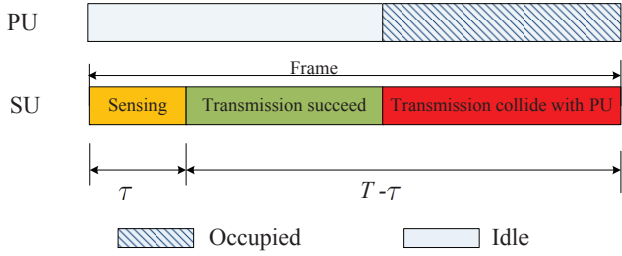

Fig. 3. A graphical illustration of data packets collision when PU becomes active in a typical cognitive radio transmission.

Please note that the probability of the interference (collisions) shown by (3) is only due to the sensing errors. The PU may change its status anytime and thus it is possible that one PU suddenly becomes active but the corresponding frequency band is used by the SU, which brings extra collisions (interference) to both the PU and the SU. In this case, the percentage of transmission with collisions out of data transmission duration has been given in [11]

$$P_p = 1 - \frac{\alpha_0}{T - \tau} \left(1 - \exp\left(-\frac{T - \tau}{\alpha_0} \right) \right). \quad (7)$$

Based on the analysis above, the ratio of average successful data transmission time to the total data transmission length per frame could be expressed using chain rule of conditional probability [12] as

$$P_s = \left(\beta \left(1 - \mathscr{P}_d \right) \left(1 - P_{ip} \right) + \left(1 - \beta \right) \left(1 - \mathscr{P}_{fa} \right) \left(1 - P_p \right) \right) \quad (8)$$

where $\beta = \left(\frac{\alpha_1}{\alpha_0 + \alpha_1} \right)$ is the traffic load factor which is defined as the amount of the traffic that each PU generates on the respective channel. As a consequence, the licensed spectrum band utilization efficiency could be expressed as

$$
\begin{aligned}
S &= \frac{\frac{1}{T} P_s (T - \tau)}{1 - \beta} \\
&= \frac{(T - \tau) \left(\beta \left(1 - \mathscr{P}_d \right) \left(1 - P_{ip} \right) + \left(1 - \beta \right) \left(1 - \mathscr{P}_{fa} \right) \left(1 - P_p \right) \right)}{T(1 - \beta)}
\end{aligned}
\quad (9)
$$

Similarly, if we define interference level as the percentage of collided transmission out of total PU busy time, the inter-

ference level could be expressed as

$$I = \frac{(T - \tau) \left(\beta \left(1 - \mathscr{P}_d \right) P_{ip} + \left(1 - \beta \right) \left(1 - \mathscr{P}_{fa} \right) P_p \right)}{T\beta} \quad (10)$$

IV. SIMULATION RESULT

In this section, simulation results are presented to evaluate the achieved spectrum utilization efficiency with various sensing and data transmission length in this cognitive radio network. 20000 Monte Carlo simulations are performed. The PU is assumed to be a QPSK modulated signal with bandwidth 6MHz. The sampling frequency is same as the bandwidth. The noise is additive white Gaussian noise with zero mean. The traffic load is Voice over Internet Protocol (VOIP) traffic with $\alpha_0 = 650$ms, $\alpha_1 = 352$ms and $\beta = 0.35$, i.e. 35% traffic load. The target probability of detection $\overline{\mathscr{P}}_d = 0.9$ and the received signal-to-noise ratio (SNR) of PU $SNR_P = -15$dB.

A. Various frame length

With given sensing length $\tau = 1$ms, Fig. 4 shows the spectrum utilization efficiency of the licensed frequency band as function of various frame length for VOIP network traffic. It can be observed clearly that the spectrum utilization efficiency increases dramatically and then decreases after it reach the maximum value, which is approximately 74% under VOIP traffic. The corresponding optimal frame length which gives the best spectrum utilization efficiency could be found numerically at $T_{opt} = 38$ms. This means 74% of the total free time is occupied by the secondary transmission if the frame length of the SU is set to be 38ms.

Fig. 5 illustrates the interference to the licensed frequency band brought by the secondary activity as function of various frame length. It can be seen that the interference level increases with the increase in the frame length and the interference at the optimal frame length is around 8% under VOIP traffic. This is because the longer of the frame is, the higher probability that the PU would be reactivated and thus the more collisions would occur.

B. Various sensing length

The impact of various sensing length is also investigated and verified via simulation. With given fixed frame length $T = 100$ms, Fig. 6 shows the achievable spectrum utilization efficiency of the licensed frequency band as function of various sensing length for VOIP network traffic. The trend of spectrum utilization efficiency varying as the SU's sensing length varying is similar as that varying as frame length. This is because that in order to reach the maximum spectrum utilization efficiency, on one hand the sensing length should be long enough to collect sensing samples and to provide sufficient sensing accuracy. On the other hand, the sensing length should not be too long, which would waste the time left for data transmission. Thus there exists an optimal sensing length which provides best spectrum utilization efficiency.
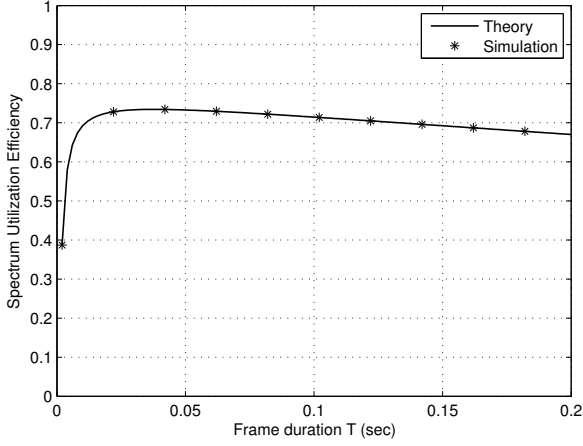
Fig. 4. Spectrum utilization efficiency of the licensed frequency band as function of various frame duration with fixed sensing length $\tau = 1$ ms.

Fig. 6. Spectrum utilization efficiency of the licensed frequency band as function of various sensing duration with energy detection and fixed frame length $T = 100$ ms.
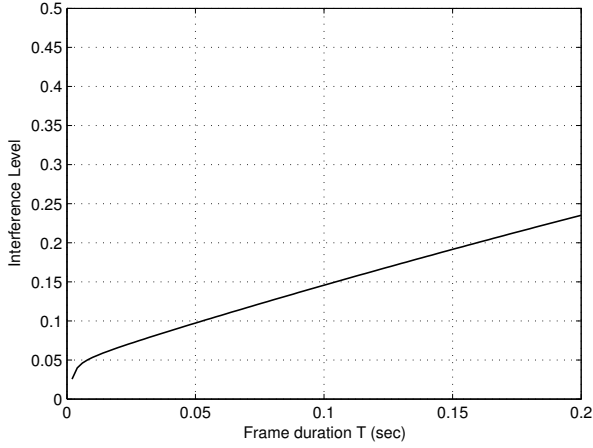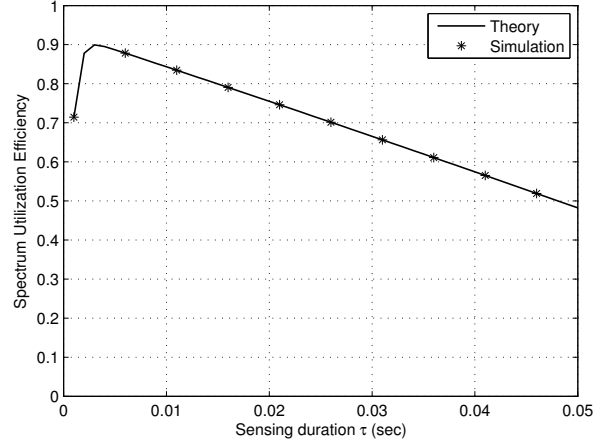

Fig. 5. Interference level to PU by the secondary opportunistic access activity as function of various frame duration with fixed sensing length $\tau = 1$ ms.
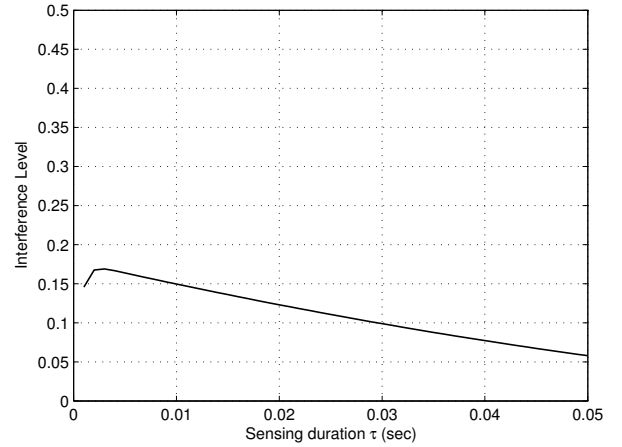

Fig. 7. Interference level to PU by the secondary opportunistic access activity as function of various sensing duration with energy detection and fixed frame length $T = 100$ ms.

In Fig. 6 this optimal value is found numerically located at $\tau_{opt} = 3$ms.

Similarly as various frame length, the interference to the licensed frequency band as function of various sensing length is illustrated in Fig. 7. Note that there is another non-intuitive result shown by Fig. 7, which is that the interference level is not monotonously decreasing with the increase in the sensing length. Intuitively, the longer of the sensing is, the more accurate result should be obtained, which should provide less missed detection and false alarm, and the interference level should be lower. But in this work because we consider practical system model, besides the collisions due to sensing errors (missed detection), the extra collisions (P_p) due to the unpredictable activity of PU is also counted and included. When the sensing length increases, it is possible that the number of these extra collision exceeds the collisions avoided by increasing sensing accuracy. In this case the total interference level may increase with the increase in the sensing length. This is reason why in Fig. 7 the interference level is

not monotonously decreasing with the increase in the sensing length.

Finally, it is illustrated that the simulation results and the analytical results are in perfect agreement with both the various sensing and frame length.

V. CONCLUSION

In this paper, we analyzed the spectrum utilization efficiency in a cognitive radio network based on the frame structure which consists of sensing and data transmission slots. The achievable spectrum utilization efficiency of the licensed frequency band has been analytically expressed with energy detection spectrum sensing. The optimal sensing and data transmission length which give maximum spectrum utilization efficiency has been found numerically. It has been illustrated that the analytical and the simulation results are in

perfect agreement. Moreover, the interference brought by the secondary activity has also been evaluated which should be controlled into tolerable level.

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