Effect of Forward Error Correction Codes on the Performance of LDS-OFDM

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Abstract— **In this paper, selection criteria of Forward Error Correction (FEC) codes, in particular, the convolutional codes are evaluated for a novel air interface scheme, called Low Density Signature Orthogonal Frequency Division Multiple Access (LDS-OFDM). In this regard, the mutual information transfer characteristics of turbo Multiuser Detector (MUD) are investigated using Extrinsic Information Transfer (EXIT) charts. LDS-OFDM uses Low Density Signature structure for spreading the data symbols in frequency domain. This technique benefits from frequency diversity in addition to its ability of supporting parallel data streams more than the number of subcarriers (overloaded condition). The turbo MUD couples the data symbols' detector of LDS scheme with users' FEC decoders through the message passing principle.**

Index Terms—**Low density signature, Multiuser detection, Iterative decoding.**

I. Introduction

In recent years, there has been considerable interest in improving the efficiency of modulation and coding techniques to be used for broadband wireless services. Future wireless communication systems are expected to provide a range of high speed services with different Quality of Service (QoS) requirements. Multi carrier code division multiple access (MC-CDMA) is a promising approach to the challenge of providing high data rate wireless communication [1], [2]. On the other hand it has been shown that the maximum *a posteriori* (MAP) and maximum likelihood (ML) multi-user detection (MUD) are the optimum detection schemes that minimize the detection errors [3], [4]. However, implementation of these optimum MUDs is prohibited by their computational complexity; even for a moderate number of interfering users, the complexity becomes significant (the complexity increases exponentially with the number of users). To address this problem, low density signatures (LDS) for code division multiple access (CDMA) and MC-CDMA systems have been proposed in [5], [6]. These low density signatures, when created using suitable rules [7], allow the application of a belief-propagation algorithm [8] at the MUD. It was shown in [5] and [6] that these systems have promising performances for overloaded systems, i.e., systems with more spreading sequences than the number of available chips. In terms of detection errors, LDS schemes can achieve a performance level close to single user bound, even for a load of 200%, i.e., the number of symbols equal to twice the number of chips. This can be achieved with a moderate complexity using MAP-based and chip-level iterated MUD [5], [6].

In OFDMA (Orthogonal Frequency Division Multiple Access) systems, the set of subcarriers is divided into several mutually exclusive subsets that are assigned to different users for simultaneous transmission [9]. As in OFDMA, user-data symbols are assigned directly to sub-channels, the frequency domain diversity will not be achievable at modulation symbol level. Thus this will be crucial to incorporate properly designed error correction coding and interleaving schemes to obtain this diversity at a later stage [10].

LDS-OFDM approach combines benefits of OFDM based multi-carrier transmission with the idea of Low Density Signature based spreading proposed for CDMA systems in [5]. In LDS-OFDM, due to low density signature structure, every data symbol will only be spread over a small subset of subcarriers (effective processing gain) and also every subcarrier will only be used by a small subset of data symbols that could belong to different users. The LDS structure can be captured by a low density graph, thus, similar to the application of LDS for CDMA system, the detection of LDS-OFDM could be based on message passing algorithm (MPA) presented in [5] for LDS-CDMA systems. This new technique can be viewed as a system which applies LDS as multiple access technique and OFDM for multi-carrier modulation. In other words, LDS-OFDM is a special case of MC-CDMA which its signature is sparse matrix. In [6], it was shown that LDS-OFDM has improved performance compared to OFDMA but with the cost of increased complexity. It is noticeable that the complexity of LDS-OFDM is higher than the conventional OFDMA but it is still affordable. In [11] sets of spreading sequences that are specifically designed to suit a belief-propagation multiuser detection structure were presented. It is shown that the performance improvements can be achieved using this structured approach in contrast to the LDS with random signatures. A turbo multiuser detector/decoder is proposed in [12] for this technique to improve the performance.

Inspired by the message passing analysis presented in [13] [14], this paper evaluates the extrinsic information transfer characteristics to describe the flow of information through the Soft-Input Soft-Output (SISO) components of the turbo MUD for the LDS-OFDM scheme. In [15], the Extrinsic Information Transfer (EXIT) chart has been used for evaluating the effect of overloading on the performance of turbo LDS-OFDM. Considering that LDS-OFDM has shown to be a suitable multiple access technique for the next generation of cellular systems, an analysis on the effect of FEC codes on its performance becomes necessary. Therefore, in this paper we used the EXIT charts to analyse the effect of channel coding on convergence of turbo MUD which gives us insights about the selection criteria of FEC codes. From literature it has been well-studied that not only the code generator, but also the constraint length and the coding rate can affect the performance of convolutional codes. Using the EXIT charts, we can compare the behaviour of three convolutional codes with different rates and constraint lengths. For both components, a detection/decoding trajectory is derived to visualize the evolution of extrinsic information in their detection/decoding process. Simulation results suggest that the derived EXIT charts are able to accurately predict the convergence behavior of the turbo MUD used for LDS-OFDM.

The rest of the paper is organized as follows: Section II presents the LDS-OFDM system architecture. Section III depicts the EXIT chart analysis of the turbo MUD of LDS-OFDM. Numerical results and analysis are provided in Section IV. Conclusions are drawn in section V.

II. LDS-OFDM System Model

We consider a multiple access channel (MAC), corresponding to the uplink communications from multiple users to a single base station in a practical system. The block diagram of LDS-OFDM for an uplink system is shown in 1. It can be noted that the main blocks are similar to an MC-CDMA system. Similar to an MC-CDMA spreading process, we multiply the modulated symbol with a spreading signature (a random sequence of chips). However, in the LDS-OFDM case, the main difference is that the spreading signature has a low density (a large number of chips in the sequence are equated to zero). In this section, we first explain the system model of LDS-OFDM, which is similar to the one published in [12]. Then the iterative receiver for this system is described.

We consider an LDS-OFDM system with K users and user indices k = 1, ..., K. Without loss of generality all users are assumed to take their symbols from the same binary constellation alphabet $\mathcal{X} \in \{\pm 1\}$ [1]. Also users are assumed to have the same number of data symbols M so the spreading signature for user k will be $\mathbf{S}_k = (\mathbf{s}_{k,1}, ..., \mathbf{s}_{k,M}) \in \mathbb{C}^{N_c \times M}$ that has only d_v nonzero components on each column. We define d_v as the effective spreading factor. Let us also define $\mathbf{S} = (\mathbf{S}_1, ..., \mathbf{S}_K) \in \mathbb{C}^{N_c \times MK}$ as the overall low density signature matrix. Also d_c is defined as the number of symbols that are allowed to interfere to each other at each chip. Let $\mathbf{A} = \text{diag}(A_1, ..., A_K)$ and $\mathbf{G}_k = \text{diag}(g_{k,1}, ..., g_{k,N_c})$ represent the users' transmit gain and the corresponding channel gain for user k respectively. Each user's generated chip will be transmitted over a subcarrier of OFDM system. Then the received spreading signature for data symbol m of user k will be $\mathbf{h}_{k,m} = A_k \mathbf{G}_K \mathbf{s}_{k,m}$. In particular, the received signature at chip n of data symbol m of user k is $h^n_{k,m} = A_k g_{k,n} s^n_{k,m}$.

To further explain and clarify, in this system, each chip represents a subcarrier of OFDM modulation and the data symbols using the same subcarrier will interfere with each other. The amount of interference will depend on the allocated power of data symbols on each subcarrier and user's corresponding channel gain.

Let $\mathcal{J}_n = \{(k,m) : s^n_{k,m} \neq 0\}$ be the set of different users' data symbols that share the same chip n or in other words be the set of nonzero positions in the n^{th} row of the signature matrix $\mathbf{S}$. So the received signal at the n^{th} chip (subcarrier) can be written as:

$$y_n = \sum_{(k,m) \in \mathcal{J}_n} h^n_{k,m} x_{k,m} + v_n, \tag{1}$$

where v_n is the additive white Gaussian noise of subcarrier n and $x_{k,m}$ is the m^{th} data symbol of user k. At the receiver side, after performing OFDM demodulation operations the signal is passed to a near-optimum MUD based on Message Passing Algorithm [5]. An LDS system with K users and N chips can be shown using factor graph $\mathcal{G}(\mathcal{U}, \mathcal{C})$ where users' symbols are represented by variable nodes $u \in \mathcal{U}$ and chips are represented by function nodes $c \in \mathcal{C}$. The connections between the received chip and its related users are represented by edges. The basic form of chip level iterated MUD can be explained as follows. Using MPA, messages containing the reliability values of adjacent variable node are exchanged between the function nodes and the variable nodes. The messages of the j^{th} iteration sent by variable nodes are updated using the following rule:

$$L^j_{l,out} = \sum_{m \neq l} L^{j-1}_{m,in}, \tag{2}$$

where $L_{m,in}$ is the m^{th} *a priori* L-value (Log Likelihood Ratio) going into the variable node, $L_{l,out}$ is the l^{th} extrinsic L-value coming out of the variable node. For the n^{th} function node the message at the j^{th} iteration is calculated as follow:

$$L^j_{l,out} = f\left(x_l | y_n, L^j_{m,in}, \forall m \in \mathcal{J}_n \setminus \{l\}\right). \tag{3}$$

To approximate the optimum MAP detector, the function $f(\cdot)$ in (3) represents marginalization function. This function calculates extrinsic values for all the constituents bits involved in (1), based on (1) and observed chip y_n and *a priori* input information. Having a small number of interferers in each subcarrier allows applying Maximum *A Posteriori* based Chip-Level iterated (MAP-CLi) multiuser detection [5]. After appropriate number of iterations the soft output which is the calculated log likelihood ratio at each variable node will be sent to the channel decoders. More details regarding the LDS

[1]It can be extended to any modulation order but here for the convenience of presentation we assume BPSK modulation.

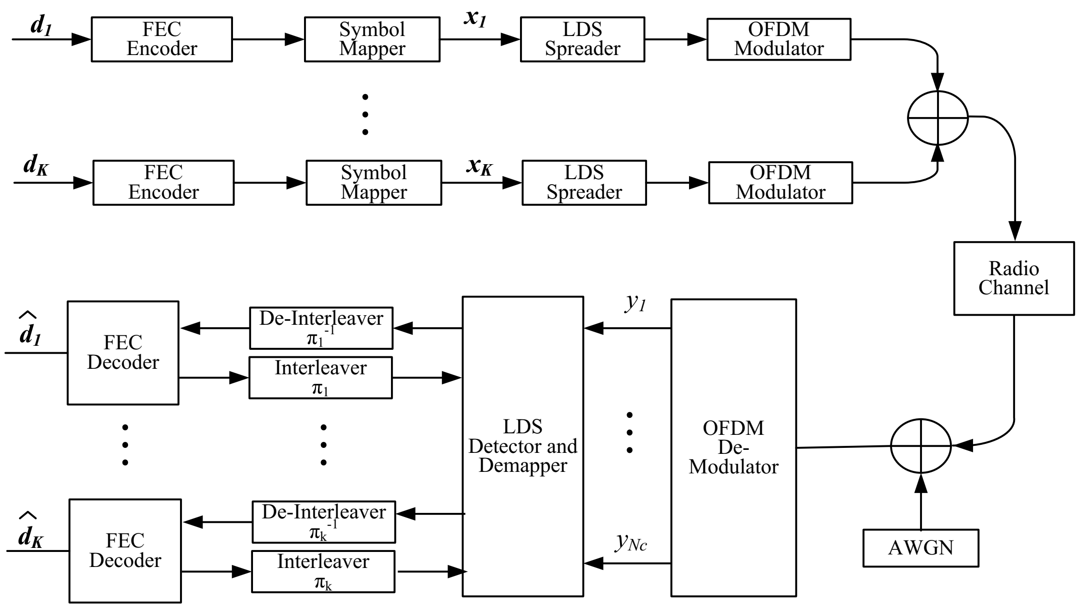

Fig. 1. LDS-OFDM block diagram.

MUD can be found in [5].

The turbo receiver of LDS-OFDM is based on iterative detection/decoding between the LDS data symbol detector and users' FEC decoders. This is realized by iterative exchange of extrinsic information between detection and FEC decoding stages. The block diagram of LDS-OFDM's turbo MUD is illustrated in Fig. 1. In the turbo MUD, there exist two iterative processes: inner and outer iterative processing for MUD and the turbo-style processing, respectively. Considering now that two iterative processes are involved, the message update algorithm should be addressed accordingly. Let t be the turbo iteration index, for inner iteration the message updates in (2) and (3) can be modified as:

$$L_{l,out}^{j,t} = \sum_{m \neq l} L_{m,in}^{j-1,t} + L_{d,out}^{t-1}, \tag{4}$$

$$L_{l,out}^{j} = f\left(x_l | y_n, L_{m,in}^{j,t}, \forall m \in \mathcal{J}_n \setminus \{l\}\right), \tag{5}$$

where $L_{d,out}^{t-1}$ is the extrinsic information coming from an FEC decoder to its corresponding variable node in the previous turbo iteration. So considering message passing algorithm [16] the messages that variable nodes send to decoder are as follows:

$$L_{v,out}^{j,t} = \sum_{m \neq l} L_{m,in}^{j-1,t}. \tag{6}$$

As mentioned earlier m represents the index of all the function nodes connected to the variable node. In other words the variable node with degree d_v has $d_v + 1$ incoming messages; d_v from function nodes and 1 from decoder which gives *a priori* L-value about variable node. Following message passing rule for the new graph, the variable node must consider the messages it receives from all the connected function nodes when calculating its message to be sent out to its corresponding FEC decoder. On the other hand when sending message to a function node it must consider all the messages it has received from the rest of the connected function nodes plus the message that has come from the corresponding FEC decoder. The message update for function nodes remain the same as for conventional LDS-OFDM MUD.

III. EXIT Chart Analysis of Turbo Multiuser Detector

Extrinsic Information Transfer (EXIT) chart is a useful tool to analyse the information transfer between the two components of a decoder with iterations. Therefore in this paper we will use EXIT charts for analysing the effect of different types of codes on the behaviour of the turbo receiver. The LDS-OFDM's turbo receiver is made of two basic components: LDS multiuser symbol detector and a bank of users' SISO FEC decoders. The extrinsic information is iteratively exchanged between the two components towards a solution with less number of bit errors. Noticeable performance improvement is observed compared to the conventional receiver that does not benefit from this iterative exchange of information. Inspired by the message passing analysis, in this section the extrinsic information transfer characteristics are evaluated to describe the flow of information through SISO components of the turbo multiuser detection algorithm for the turbo receiver of the LDS-OFDM scheme. In this context, the extrinsic L-values are passed on and interpreted as *a priori* information by the other detector or decoder. We use the notation of [13] and write I_A for the average mutual information between the bits sent to the detector/decoder (which are the bits about which extrinsic L-values are exchanged) and the *a priori* L-values. Similarly, I_E refers to the average mutual information between the bits sent to the detector/decoder and the extrinsic L-values.

In order to compute an EXIT function, $L^{j-1}_{m,in}$ in (6) is modelled as the output L-value of an AWGN channel whose input is the m^{th} interleaver bit transmitted using BPSK. According to [13], we can write *a priori* L-value by applying an independent Gaussian random variable n_A with variance σ_A^2 and mean zero in conjunction with known bits on the detector graph edges $x \in \pm 1$ as follows:

$$A = \mu_A x + n_A, \tag{7}$$

where

$$\mu_A = \frac{\sigma_A^2}{2}. \tag{8}$$

The mutual information $I_A = I(X; A)$ is calculated as follows [13]:

$$I_A = \frac{1}{2} \sum_{x=-1,1} \int_{-\infty}^{+\infty} p_A(\xi | X = x) \log_2 \frac{2 p_A(\xi | X = x)}{p_A(\xi | X = -1) + p_A(\xi | X = 1)} d\xi. \tag{9}$$

Considering that the conditional probability density function $p_A(\xi | X = x)$ is related to L-value A, with Gaussian distribution and with properties mentioned in (6) we have:

$$I_A(\sigma_A) = 1 - \int_{-\infty}^{+\infty} \frac{e^{-\left((\xi - \sigma_A^2/2)^2 / 2\sigma_A^2\right)}}{\sqrt{2\pi}\sigma_A} \log_2[1 + e^{-\xi}] d\xi. \tag{10}$$

For abbreviation we define:

$$J(\sigma) := I_A(\sigma_A = \sigma), \tag{11}$$

with

$$\lim_{\sigma \to 0} J(\sigma) = 0, \quad \lim_{\sigma \to \infty} J(\sigma) = 1, \quad \sigma \geq 0. \tag{12}$$

Therefore, to create the EXIT chart, a constituent decoder (either MUD or FEC decoder) is modelled as a device, mapping a sequence of observations and the input a priori information to a new sequence of extrinsic information. Therefore, the mutual information for both MUD and decoder can be calculated by firstly estimating the PDF from the histogram of the output L-values of a constituent decoder and then using (9) we can calculate the mutual information numerically.

TABLE I
SIMULATION PARAMETERS

Number of Users	10
Number of data sub-channels	60
FFT size	64
Multipath channel model	ITU Pedestrian Channel B
Channel coding	1/2 & 1/3 rate convolutional code
Modulation	BPSK
Data streams per user	12
Effective spreading factor (LDS)	$d_v = 3$

IV. SIMULATION RESULTS

In this section, we present the simulation results for LDS-OFDM systems with different convolutional codes. LDS-OFDM's signatures are generated randomly. The simulation parameters are listed in Table I. Since the individual users' signals interfere with each other and influence each other's *a priori* mutual information I_A and extrinsic mutual information contributions I_E, a K-dimensional EXIT chart would be required. In order to circumvent this problem and hence allow us to plot the EXIT curves for a multi-user communication system in a two-dimensional, rather than K-dimensional plane, the average of all the user mutual information is calculated.

We denote the *a priori* information and extrinsic information of the MUD by $I_{A,MUD}$ and $I_{E,MUD}$, respectively, while the corresponding quantities of the channel decoder by $I_{A,FEC}$ and $I_{E,FEC}$, respectively.

From literature it has been well-studied that not only the code generator, but also the constraint length and the coding rate can affect the performance of convolutional codes. Using the EXIT charts, we can compare the behaviour of three convolutional codes with different rates and constraint lengths. The codes used for evaluation are two half-rate convolutional codes with $G = (133, 171)_8$ and $G = (23, 35)_8$ with constraint lengths 6 and 4, respectively. Also the effect of a rate 1 /3 convolutional code with $G = (23, 33, 37)_8$ with constraint length 4 is evaluated.

Figure 2 shows the output mutual information of three classes of convolutional codes according to their input mutual information. As expected the results show that convolutional codes with lower rates have better mutual information output when compared with the ones that have higher rates. Furthermore, this figure also shows that between the two convolutional codes with equal rates the one that has longer constraint length has higher output mutual information except for input mutual information less than 0.2. This result is in line with [17] that shows the convolutional codes with shorter constraint length provide better extrinsic output when the mutual information between actual bits and the a priori input is low. However, when a priori input with higher mutual information is known, convolutional codes with longer constraint lengths are preferred. Furthermore, we should consider that increasing the constraint length impose higher memory and complexity

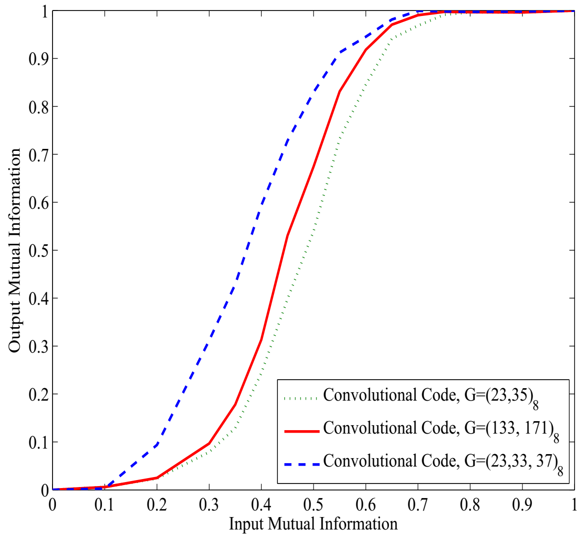

Fig. 2. EXIT charts for convolutional code with different rate and constraint length.

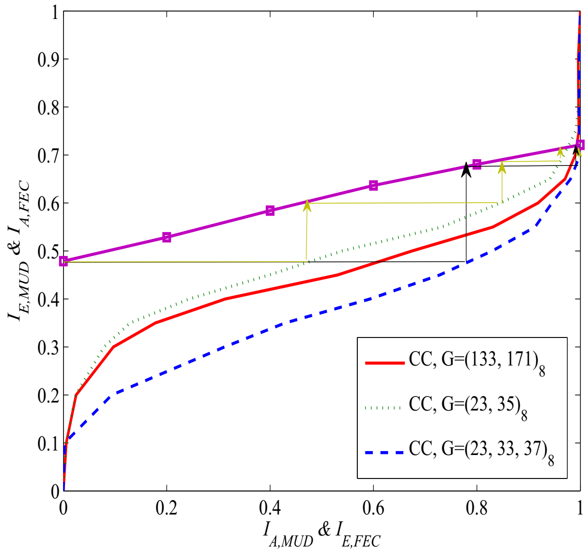

Fig. 4. EXIT charts for turbo MUDD with three inner iterations at $E_b/N_0 =$ 0.5 dB for load 200%.

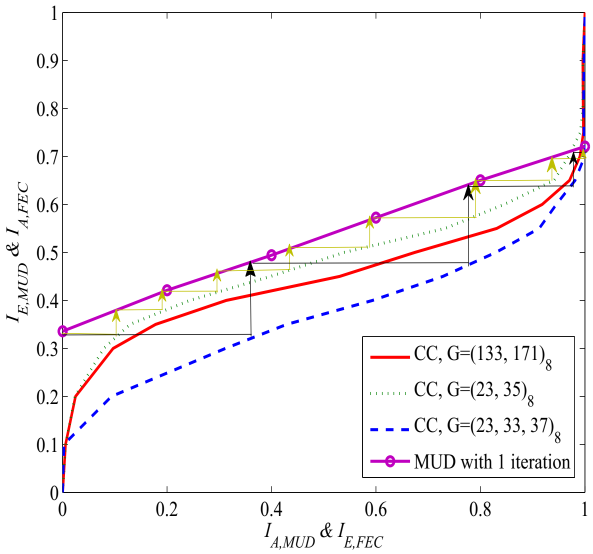

Fig. 3. EXIT charts for turbo MUDD with one inner iteration at $E_b/N_0 =$ 0.5 dB for load 200%.

requirement.

As next step we will evaluate the effect of these different types of convolutional codes on the convergence behavior of turbo MUDD of LDS-OFDM. Figure 3 shows the EXIT charts of turbo MUDD for LDS-OFDM when the MUD has one inner iterations. This figure shows that the tunnel between the FEC curve and MUD curve is wider when we use convolutional code with $G = (23, 33, 37)_8$, therefore, the turbo MUDD will converge with just 4 turbo iterations. On the other hand if the higher rate convolutional code with $G = (23, 35)_8$ is used the turbo MUDD needs 8 turbo iterations to converge which is much higher compare to one for the convolutional code with rate $1/3$. Therefore, it is a trade-off between the rate of the channel encoder and the complexity of the turbo MUDD.

Figure 4 illustrate the EXIT charts for the case when the MUD of LDS-OFDM had three inner iterations. It can be concluded from this figure that the tunnel becomes even wider when we increase the number of inner iterations. Therefore, for the case of convolutional code with $G = (23, 33, 37)_8$ the turbo MUDD will converge after just 2 outer iterations. If we compare the two codes with equal rate but different constraint length, we can see that although the code with higher constraint length requires more number of turbo iterations to converge, for that code, the two curves intersect at a higher mutual information which results in a better BER performance. Therefore, considering the costs and benefits for different schemes we can say that convolutional code with $G = (23, 35)_8$ is a better option from complexity and performance point of view compared with the other types of codes evaluated.

It is also shown that the simulated decoding trajectory evolves within the open detection tunnel between the EXIT curves of the MUD and the channel decoder, until it reaches the intersection of the curves. Since the simulated detection trajectories closely follow the EXIT curves of the receiver components, the validity of EXIT chart analysis can be verified. The analysis can be extended to higher order modulations. A better spectral efficiency can be achieved by finding the trade-off between modulation level and loading factor.

V. Conclusions

Increasing interest in high data rate services demands high spectral efficiency. In this regard LDS-OFDM has recently been introduced as an efficient multiple access technique. In order to improve the performance of LDS-OFDM, the effect of different parameters of the convolutional codes have been evaluated. The turbo receiver of LDS-OFDM operates on iterative decoding principle and its convergence analysis becomes important. In this paper we provide the framework and tools to perform this analysis.

By analysing the convergence behavior of the turbo MUD, we were able to show how different types of convolutional codes affect the performance as the curves intersect at points with different mutual information. Our analysis provides some reference to choose an appropriate convolutional code for practical systems based on the performance and complexity requirements.

ACKNOWLEDGMENT

This work was supported b the UK Engineering and Physical Sciences Research Council (under Grant EP/J017655/1)

REFERENCES

[1] J. Linnartz, "Performance analysis of synchronous MC-CDMA in mobile rayleigh channel with both delay and doppler spreads," *IEEE Transactions on Vehicular Technology*, vol. 50, no. 6, pp. 1375–1387, 2001.

[2] N. Yee, J. Linnartz, and G. Fettweis, "Multi-carrier CDMA in indoor wireless radio networks," in *Proc. Of IEEE PIMRC93*, 1993.

[3] S. Verdu, "Minimum probability of error for asynchronous gaussian multiple-access channels," *IEEE Transactions on Information Theory*, vol. 32, no. 1, pp. 85–96, 1986.

[4] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.

[5] R. Hoshyar, F. P. Wathan, and R. Tafazolli, "Novel low-density signature for synchronous CDMA systems over AWGN channel," *IEEE Transactions on Signal Processing*, vol. 56, no. 4, pp. 1616–1626, 2008.

[6] R. Hoshyar, R. Razavi, and M. Al-Imari, "LDS-OFDM an efficient multiple access technique," in *Proc. IEEE 71st Vehicular Technology Conf. (VTC 2010-Spring)*, pp. 1–5, 2010.

[7] A. Montanari and D. Tse, "Analysis of belief propagation for non-linear problems: The example of CDMA (or: How to prove Tanaka's formula)," in *Proc. ITW '06 Punta del Este Information Theory Workshop IEEE*, 2006.

[8] J. Pearl, *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference*. San Francisco, CA: Morgan Kaufmann Publishers, 1988.

[9] L. Hongxiang and L. Hui, "An analysis of uplink OFDMA optimality," *IEEE Transactions on Wireless Communications*, vol. 6, no. 8, pp. 2972–2983, 2007.

[10] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, 2005.

[11] J. van de Beek and B. M. Popovic, "Multiple access with low-density signatures," in *Proc. IEEE Global Telecommunications Conf. GLOBECOM 2009*, pp. 1–6, 2009.

[12] R. Razavi, M. AL-Imari, M. Imran, R. Hoshyar, and D. Chen, "On receiver design for uplink low density signature OFDM (LDS-OFDM)," *IEEE Transactions on Communications*, vol. 60, no. 11, pp. 3499–3508, 2012.

[13] S. Ten Brink, "Convergence behavior of iteratively decoded parallel concatenated codes," *IEEE Transactions on Communications*, vol. 49, no. 10, pp. 1727–1737, 2001.

[14] S. Ten Brink, G. Kramer, and A. Ashikhmin, "Design of low-density parity-check codes for modulation and detection," *IEEE Transactions on Communications*, vol. 52, no. 4, pp. 670–678, 2004.

[15] R. Razavi, M. Imran, and R. Tafazolli, "EXIT chart analysis for turbo LDS-OFDM receivers," in *Proc. Wireless Communications and Mobile Computing Conference (IWCMC), 2011 7th International*, 2011.

[16] F. R. Kschischang and B. J. Frey, "Iterative decoding of compound codes by probability propagation in graphical models," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 2, pp. 219–230, 1998.

[17] M. H. Moghari and B. Shahrrava, "Convergence behavior of iterative turbo multiuser detection algorithms," in *Proc. IEEE Wireless Communications and Networking Conf*, 2005.