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The Market for Salmon Futures: An Empirical Analysis of Fish Pool using the Schwartz Multifactor Model

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Abstract

Using the popular Schwartz 97 two-factor approach, we study future contracts written on fresh farmed salmon, which have been actively traded at the Fish Pool Market in Norway since 2006. This approach features a stochastic convenience yield for the salmon spot price. We connect this approach with the classical literature on fish-farming and aquaculture using first principles, starting by modeling the aggregate salmon farming production process and modeling the demand using a Cobb-Douglas utility function for a representative consumer. The model is estimated by means of Kalman filtering, using a rich data set of contracts with different maturities traded at Fish Pool between 12/06/2006 and 22/03/2012. The results are then discussed in the context of other commodity markets, specifically live cattle which acts as a substitute.

Keywords: Futures, Commodities, Aquaculture, Fisheries Economics, Agricultural Economics, Risk Management

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1 Introduction

Fish Pool is a new derivatives market, where futures and options on fresh farmed salmon are traded in large quantities since 2006. Located in Bergen (Norway), contract volumes traded at this market have reached 102.295 tons, equivalent to 440 million Euro, during 2013. These numbers continue a strong upwards trend from previous years. Following its great success in the start-up phase the Oslo Stock Exchange acquired 71% of Fish Pool in December 2012.

Bergfjord (2007), Dalton (2005) and Bulte and Pennings (1997) provide possible explanations for this trend. In short, markets for forwards and futures on fresh salmon help companies which use fresh salmon in their production, for example, food processing companies, to hedge the price risk and plan ahead, by fixing the price in advance. In the same way, they help producers, i.e. salmon farmers, to reduce their (selling) price risk. An analysis of the welfare effects of futures markets in a rather general context is presented in Hirschleifer (1988). He discusses a two period model which includes consumers, processors, producers and speculators. In fact speculative investors at Fish pool play a more and more important role[1], which in consequence urges the issue of finding appropriate, theoretical well-founded and sound pricing formulas for the futures and options traded there.

In this article we discuss the valuation of futures on fresh farmed salmon as traded on the Fish Pool exchange. Our major concern is the accurate and market consistent pricing of the futures contracts, taking into account at least some of the key-elements describing the salmon farming process as well as the demand for farmed salmon and combining these coherently with the methodology of arbitrage free pricing developed in the derivatives pricing literature. More specifically we are connecting the Schwartz (1997) multi-factor approach with stochastic convenience yield to the classical literature in fish-farming and aqua-culture. We estimate the parameters in our model on the basis of an extensive data-set obtained from the Fish Pool market covering the period from 12/06/2006 until 22/03/2012. Solibakke (2012) presents

[1] Compare Fish Pool News Archive, March 20th, 2012.

an approach using stochastic volatility to model the Fish Pool market. However, only front months contracts are considered and the term structure, which can only be obtained from contracts with longer maturities, is not accounted for. In fact, it is well known that stochastic volatility alone cannot produce realistic term structures. While stochastic volatility is without doubt an important feature, modeling the term structure of the future contracts and identifying the stochastic convenience yield is generally considered to be more important.

The classical salmon farming literature, e.g. Bjorndal (1988), Arnason (1992), Heaps (1995), Cacho (1997), Yu and Leung (2006) as well as Guttormsen (2008) focuses on the harvesting behavior of one individual salmon farmer. In contrast to this, our focus is on the aggregate salmon production, as the aggregate production alone will affect the market price, which features prominently in our financial model. In order to get there, we assume that at any given time, a constant proportion of salmon farmers (or farming units) will harvest. This assumption accurately reflects how salmon farming companies operate world wide and salmon can be harvested at any time, reflecting consumer demand. The demand for farmed salmon is then modeled in a rather classical way by attaching a Cobb Douglas type utility function to a representative consumer, who chooses between farmed salmon and an alternative consumption good. The market clearing price will then be used in the analysis of future contracts within the Schwartz (1997) framework.

To place our study into context and compare the estimated parameters of our model with those obtained for other commodities, we have also included a data set for live-cattle future contracts as traded on the Chicago Mercantile Exchange into our analysis.

A problem related to pricing farmed salmon futures and options has been discussed in Ewald (2013). The difference there, is that the population is assumed to be wild and not farmed, and managed as an open access fishery. Further the driving dynamics, e.g. the biomass of the wild population in the sea, is assumed to be of different type. Ewald (2013) uses stochastic logistic growth, which is mainly motivated by the classical fishery economics as well as population ecology literature such as Beddington and May (1977), May

(1973), Lande (1995), Alvarez (1998) as well as Alvarez and Shepp (1998). This specification however does only allow for approximate pricing formulas for futures and options, and hence causes problems in the calibration of the model. A mean variance approach in the context of optimizing sustainable yields under uncertainty in the same dynamic setup has been presented in Ewald and Wang (2010).

The rest of the paper is structured as follows. In section 2 we will briefly review the Schwartz (1997) multi-factor approach, while in section 3 we discuss farmed salmon supply and demand leading to an equilibrium price. Section 4 contains our empirical analysis, using Kalman filtering to estimate the parameters within our model for different sub-samples of our data-set. In section 5 we draw comparisons with live cattle futures and identify subtle differences in the two markets. Our main conclusions are summarized in section 6. The appendix contains a number of figures which support the findings in the main text.

2 The Schwartz (1997) multi-factor framework

Let us denote with $P(t)$ the price of a commodity at time t. In the Schwartz (1997) framework the state variables $P(t)$, $\delta(t)$ and $r(t)$ are given by

$$\begin{aligned} dP(t) &= (\mu - \delta(t))P(t)dt + \sigma_1 P(t)dZ_1(t) && (1)\\ d\delta(t) &= \kappa(\alpha - \delta(t))dt + \sigma_2 dZ_2(t) && (2)\\ dr(t) &= a(m - r(t))dt + \sigma_3 dZ_3(t) && (3) \end{aligned}$$

with constants μ, κ, α, a, m, σ_1, σ_2 and σ_3 under the real world probability $\mathbb{P}$. The Brownian motions $Z_1(t)$, $Z_2(t)$ and $Z_3(t)$ are assumed to be correlated, according to

$$dZ_1(t)dZ_2(t) = \rho_1 dt, dZ_2(t)dZ_3(t) = \rho_2 dt, dZ_1(t)dZ_3(t) = \rho_3 dt. \quad (4)$$

We assume $\kappa, a \geq 0$. The process $r(t)$ denotes the stochastic interest rate. Under the assumption $\sigma_3 = 0$ and $a = 0$, the interest rate remains constant and the model in fact becomes a two-factor model, also known as Schwartz (1997) two-factor model. The process $\delta(t)$ represents the stochastic convenience yield and can be recognized as a mean reverting Ornstein Uhlenbeck process. It reflects the utility that an agent receives when holding the commodity, or storage/maintenance costs that the agent needs to pay. The price dynamics (1) has an implicit mean reversion feature. If $\rho_1 > 0$, then the instantaneous correlation between $P(t)$ and $\delta(t)$ is positive. Hence $P(t)$ is likely to be large when $\delta(t)$ is large and in this case $\delta(t)$ is likely to be larger than μ. The drift term in (1) will then push $P(t)$ downwards. The opposite happens if $P(t)$ is small, pushing $P(t)$ upwards. If in fact one chooses $\delta(t) = \kappa \ln(P(t))$, one obtains the dynamics of a geometric Ornstein-Uhlenbeck process in (1), and $\delta(t)$ defined in this way satisfies (2) with $\rho = 1$. In this case we obtain the so called Schwartz (1997) one-factor model. In its full generality, i.e. without any coefficient restrictions other than $\kappa, a \geq 0$ the model is known as Schwartz (1997) three-factor model.

A forward contract in this context is an agreement established at a time $s < T$ to deliver or receive the renewable resource at time T for a price K, which is specified at time s. In financial terms, the payoff at time of maturity T of such a forward contract is

$$H = P(T) - K. \tag{5}$$

The value K that lets this contract have a value zero under a no-arbitrage assumption is given by

$$F_P^{forw}(s,T) = \frac{1}{B(s,T)} \mathbb{E}_{\mathbb{Q}} \left(e^{-\int_s^T r(t)dt} \cdot P(T) | \mathcal{F}_s \right), \tag{6}$$

where $B(s,T) = \mathbb{E}_{\mathbb{Q}} \left(e^{-\int_s^T r(t)dt} \right)$ denotes the prize of a zero coupon bond maturing at time T at current time s. This is called the forward price at time s. The symbol $\mathcal{F}_s$ denotes the information available at time s and we denote in the following with $\mathbb{F} = (\mathcal{F}_s)$ the associated filtration which represents the information flow.[2]

The expectation in (6) is taken with respect to the pricing measure $\mathbb{Q}$, which takes into account a market price of convenience yield risk λ, i.e.

$$\begin{aligned} dP(t) &= (r - \delta(t))P(t)dt + \sigma_1 P(t)d\tilde{Z}_1(t) && (7) \\ d\delta(t) &= (\kappa(\alpha - \delta(t)) - \lambda)dt + \sigma_2 d\tilde{Z}_2(t) && (8) \\ dr(t) &= a(m^* - r(t))dt + \sigma_3 d\tilde{Z}_3(t) && (9) \end{aligned}$$

with

$$d\tilde{Z}_1(t)d\tilde{Z}_2(t) = \rho_1 dt, d\tilde{Z}_2(t)d\tilde{Z}_3(t) = \rho_2 dt, d\tilde{Z}_1(t)d\tilde{Z}_3(t) = \rho_3 dt. \quad (10)$$

Here m^* denotes the risk adjusted long-term mean interest rate.

A futures contract is basically a type of forward contract which is centrally cleared on a daily basis. The clearing exchange then usually requires the agent to set up a margin account, the amount held reflecting price movements in the market, protecting buyer and seller from possible default of the other party. The mechanism of the margin account affects the price as determined above and in fact the futures price is then provided via

$$F_P^{fut}(s,T) = \mathbb{E}_{\mathbb{Q}}\left(P(T)|\mathcal{F}_s\right). \quad (11)$$

It is a direct consequence from equations (6) and (11), that if the interest rate process $r(t)$ and the commodity price $P(t)$ are uncorrelated, the forward and futures prices coincide. This is in particular the case, if the interest rate is assumed to be deterministic, which is the case in the Schwartz (1997) two-factor model. While until 19/07/2007 contracts traded at Fish Pool had been exclusively bilateral and of forward type, the majority of contracts traded after that date had been cleared, and in fact close to 100% of contracts are nowadays cleared daily via Fish Pool's link with NASDAQ, hence are of futures type. This will be reflected in our empirical analysis. To simplify the notation, we write $F_P(s,T) = F_P^{fut}(s,T)$.

[2] More precisely, $\mathbb{F} = (\mathcal{F}_s)$ denotes the augmented and completed filtration generated by the Brownian motions $Z_1(s)$, $Z_2(s)$ and $Z_3(s)$.

Let us assume initially that the interest rate is constant and equal to r, corresponding to the case $a = \sigma_3 = 0$. As indicated above, in this case, forward prices and futures prices coincide, and we do not need to distinguish these any further. In fact we use the notion forwards and futures as synonymous here.

We can always assume that current time is normalized to 0 and that the time of maturity T is relative to this, hence the same as time to maturity. Since our model is Markovian, we can then denote the futures price in (11) as $F(P, \delta, T)$ depending on current spot price, level of convenience yield and time to maturity T. With this notation, Schwartz (1997) refers to Jamshidian and Fein (1990) and Bjerksund (1991) for an explicit expression for (11)

$$\begin{aligned} F(P,\delta,T) &= P \cdot \exp\left(-\delta \cdot \left(\frac{1-e^{-\kappa T}}{\kappa}\right) + A(T)\right) \\ A(T) &= \left(r - \alpha + \frac{\lambda}{\kappa} + \frac{1}{2}\frac{\sigma_2^2}{\kappa^2} - \frac{\sigma_1\sigma_2\rho}{\kappa}\right) T + \frac{1}{4}\sigma_2^2 \left(\frac{1-e^{-2\kappa T}}{\kappa^3}\right) \\ &\quad + \left(\alpha\kappa - \lambda + \sigma_1\sigma_2\rho - \frac{\sigma_2^2}{\kappa}\right)\left(\frac{1-e^{-\kappa T}}{\kappa^2}\right). \end{aligned} \tag{12}$$

Note, that the futures price (12) has a log-normal distribution, which makes the analytical pricing of options in this framework possible. On the other hand note that at least one of the state variables, the convenience yield $\delta(t)$ is unobservable. In fact Schwartz (1997) assumes that both the commodity price $P(t)$ and the convenience yield $\delta(t)$ are unobservable, and only the future prices (12) are observable. In order to estimate the model, Schwartz (1997) then applies Kalman filtering techniques.

The case of stochastic interest rates is slightly more involved, but more of notational means rather than mathematical complexity, as the futures prices remain log-normal. The futures price in the Schwartz three-factor model is given as

$$F(P,\delta,r,T) = P \cdot \exp\left(-\delta \cdot \left(\frac{1-e^{-\kappa T}}{\kappa}\right) + r \cdot \left(\frac{1-e^{-aT}}{a}\right) + C(T)\right), \tag{13}$$

with

$$
\begin{aligned}
C(T) \quad = \quad & \frac{(\kappa(\alpha - \frac{\lambda}{\kappa}) + \sigma_1\sigma_2\rho_1)(1 - e^{-\kappa T} - \kappa T)}{\kappa^2} \\
& - \frac{\sigma_2^2(4(1 - e^{-\kappa T}) - (1 - e^{-2\kappa T}) - 2\kappa T)}{4\kappa^3} \\
& - \frac{(am^* + \sigma_1\sigma_3\rho_3)(1 - e^{-aT} - aT)}{a^2} \\
& - \frac{\sigma_3^2(4(1 - e^{-aT}) - (1 - e^{-2aT}) - 2aT)}{4a^3} \\
& + \sigma_2\sigma_3\rho_2 \left(\frac{(1 - e^{-\kappa T}) + (1 - e^{-aT}) - (1 - e^{-(\kappa+a)T})}{\kappa a(\kappa + a)} \right. \\
& + \left. \frac{\kappa^2(1 - e^{-aT}) + a^2(1 - e^{-\kappa T}) - \kappa a^2 T - a\kappa^2 T}{\kappa^2 a^2(\kappa + a)} \right).
\end{aligned}
$$

The empirical analysis in section 4 predominantly focuses on the application of the two factor model. The function of the three factor model in the context of this paper lies mainly in assessing how robust the results from the two factor model are in light of stochastically fluctuating interest rates, in particular when longer term contracts are used in the analysis.

3 Farmed Salmon Production and Demand

Aggregate salmon supply and demand in the context of market interactions on a global level has been discussed in Asche et al (1999) and Asche et al (2001), but from a mostly exogenous and empirical point of view. We attempt to provide a micro founded model of aggregate salmon supply and demand.

Let us look at the farmed salmon production. We follow a more or less classical approach, which is outlined in Cacho (1997) for example, and presents a consensus of many models that are available in the literature. The total number of salmon in all pens contributing to the salmon production process is denoted with $n(t)$. We assume that mortality $m(t)$ follows an adapted stochastic process on $(\Omega, \mathbb{P}, \mathbb{F})$, and therefore at any time before harvesting

$$dn(t) = -m(t) \cdot n(t)dt. \tag{14}$$

Note that salmon does not reproduce in the pens, and therefore the number of salmon in each pen has to decrease over time. However, salmon gain in weight and it is assumed that the average weight of one fish is assumed to follow the dynamic

$$dw(t) = (\Theta - \beta(t))\, w(t)dt + \sigma_w w(t)dB(t), \tag{15}$$

where $B(t)$ represents a standard Brownian motion on $(\Omega, \mathbb{P}, \mathbb{F})$ and $\beta(t)$ an arbitrary adapted stochastic process, such that the dynamics (15) is well defined. In fact $\beta(t)$ represents the weight saturation, and should be positively correlated with $w(t)$, introducing a mean reversion feature in the weight dynamics towards the mean reversion level Θ, which is assumed to be constant. We denote with

$$X(t) = n(t)w(t) \tag{16}$$

the total biomass at time t. The dynamics of $X(t)$ in the absence of harvesting can be easily derived and follows

$$dX(t) = (\Theta - m(t) - \beta(t))\, X(t)dt + \sigma_w X(t)dB(t). \tag{17}$$

An individual salmon farmer would now try to optimize the time of harvest, so as to achieve an optimal profit. The classical aquaculture literature around Bjorndal (1988), Cacho (1997), Yu and Leung (2006), Guttormsen (2008), Heaps (1995) and Arnason (1992) focuses on this and adopts the methodology of optimal stopping and control. In the present context however, it is the aggregate farmed salmon production that matters. Assuming that salmon farmers are heterogeneous and that because of limited market demand it cannot be optimal for all salmon farmers to harvest at the same time, no unique harvesting time can be identified.[3] We assume that at each instant of time t a proportion $\nu(t)$ of salmon farmers will harvest. Assum-

[3]The oligopolistic aquaculture harvesting problem does not seem to have been discussed in the literature.

ing that salmon farmers own equally sized portions of the total biomass, the biomass will then evolve according to the equation

$$dX(t) = (\Theta - (m(t) + \nu(t)) - \beta(t))\, X(t)dt + \sigma_w X(t)dB(t). \tag{18}$$

which is of the same type as (17). [4] The salmon supply in each infinitesimal time interval dt will then be $\nu(t)X(t)dt$.

Let us now look at the consumer side. We assume that a representative consumer chooses between farmed salmon and an alternative consumption good, and that the utility from consumption is of Cobb-Douglas type. The consumer's problem is at each time t to maximize utility

$$\max \quad \left(x(t)^{\alpha(t)} y(t)^{1-\alpha(t)}\right) \tag{19}$$

$$\text{subject to:} \quad P(t)\cdot x(t) + y(t) = c(t), \tag{20}$$

where $x(t)$ denotes the amount of farmed salmon and $y(t)$ the amount of the alternative consumption good consumed. The total budget of the consumer is limited to $c(t)$ and can vary stochastically over time, while $P(t)$ denotes the price of farmed salmon and the price of the alternative consumption good is normalized to one. The preference parameter $\alpha(t)$ is also assumed to be stochastic at this point, taking into account changes in the consumer preferences, which are known to effect the price of salmon significantly.

The solution of the consumer problem is then given by

$$x(t) = \frac{\alpha(t)c(t)}{P(t)}. \tag{21}$$

In equilibrium we must have $x(t) = \nu(t)X(t)$ and hence we obtain the inverse demand function

$$P(t) = \frac{\epsilon(t)}{X(t)}, \tag{22}$$

[4]Note that while individual farmers still do complete harvests rather than continuously harvesting a proportion of the biomass, in aggregation the affect is like continuous harvesting. Even for a single salmon farming unit consisting of multiple pens, it would be unwise to harvest all pens at once.

where

$$\epsilon(t) = \frac{\alpha(t)c(t)}{\nu(t)}. \tag{23}$$

This price functional will be used in the following, and interpreted as the Fish Pool Index, which in turn corresponds to the salmon spot price.[5] Without further specifying the functional forms of $\alpha(t)$,$c(t)$ and $\nu(t)$ it is however impossible to obtain any explicit pricing formulas. However, rather than looking at each factor individually, we assume that the various effects of $\alpha(t)$,$c(t)$ and $\nu(t)$ aggregate to

$$d\epsilon(t) = \epsilon(t)\left(\gamma(t)dt + \eta dW(t)\right) \tag{24}$$

where $W(t)$ is a second Brownian motion, which is correlated with $B(t)$ according to the relationship

$$dB(t)dW(t) = \rho_D dt, \tag{25}$$

and $\gamma(t)$ is as yet unspecified.[6]
A simple application of the Ito-formula yields

$$\begin{aligned} dP(t) &= P(t)\big(m(t) + \nu(t) + \sigma_w^2 - \eta\sigma_w\rho_D + (\beta(t) + \gamma(t)) - \Theta\big)\,dt \\ &\quad + P(t)\left(\eta dW(t) - \sigma_w dB(t)\right). \end{aligned} \tag{26}$$

Noticing that $var\left(\eta dW(t) - \sigma_w dB(t)\right) = \left(\eta^2 + \sigma_w^2 - 2\eta\sigma_w\rho_D\right)dt$, this can be rewritten as

$$\begin{aligned} dP(t) &= P(t)\big(\sigma_w^2 - \eta\sigma_w\rho_D - \Theta - \delta(t)\big)\,dt \\ &\quad + P(t)\big(\eta^2 + \sigma_w^2 - 2\eta\sigma_w\rho_D\big)\,dZ_1(t), \end{aligned} \tag{27}$$

[5] The Fish Pool price index is based on a weighted weekly average of salmon categories 3-4 kg: 30 %, 4-5 kg: 40 %, 5-6 kg: 30 %, superior quality, head-on gutted. Further details are available on $http://fishpool.asp.manamind.com/?page\_id = 65$.

[6] As $\gamma(t)$ at this point can be an arbitrary stochastic process, the only assumption made here is that the volatility of $\epsilon(t)$ is proportional to its level, which is a simplifying but intuitive assumption.

where $Z_1(t)$ is a standard Brownian motion and

$$\delta(t) = -\left(m(t) + \nu(t) + \beta(t) + \gamma(t)\right). \tag{28}$$

Now, taking into account that $\delta(t)$ is an aggregation of four seemingly unrelated processes of which at least some feature mean-reversion, we are led to assume that $\delta(t)$, at least in approximation, follows an Ornstein-Uhlenbeck process, as described in (2). As for the dynamics of $P(t)$, we see that it exactly matches the dynamics (1), with the following choice of parameters

$$\mu = \sigma_w^2 - \eta\sigma_w\rho_D - \Theta \tag{29}$$

$$\sigma_1 = \eta^2 + \sigma_w^2 - 2\eta\sigma_w\rho_D. \tag{30}$$

With this parametrization it is worthwhile to keep in mind, what generates the uncertainty here: σ_W takes account of volatility generated by the fluctuations in weights of individual fish, due to sources such as nutrition, weather and disease, while η takes account of volatility generated by fluctuations in consumer income and preferences. At most times, it will be the case that $\sigma_W < \eta$.

4 Empirical Estimates

The data used to test the model developed so far consist of daily observations of futures prices in Fish Pool ASA from 12/06/2006 to 22/03/2012. For the whole sample period, complete data on the first 29 futures contracts sorted by different maturities are available. We use a similar notation as in Schwartz (1997) and denote with F1 the contract closest to maturity (with average maturity of 0.041 year) counting up to F29 which represents the contract farthest to maturity (with average maturity of 2.427 years). We further divide the whole sample period into three different regimes according to the level of Norwegian interest rates as shown in Table 1 leading to sub-samples $Data_1$, $Data_2$ and $Data_3$.[7] Under each regime, contracts in Panel A, Panel

[7]Average interest rate r over the whole sample time period is 2.13%.

	Time Period	Interest Rate	Observations	Description
$Data_1$	12/06/2006-1/11/2006	2.88%	103	Medium interest regime
$Data_2$	2/11/2006-17/12/2008	4.00%	545	High interest regime
$Data_3$	18/12/2008-22/03/2012	1.93%	849	Low interest regime

Table 1: Sub Data Sets

B and Panel C are chosen as proxies for short-term, medium-term and long-term futures contracts respectively. In each test, five contracts (i.e., N=5) are used for the estimation. More precisely, Panel A contains F1, F3, F5, F7 and F9; Panel B contains F12, F14, F16, F18, F20 and Panel C contains F24, F25, F26, F28 and F29. A summary statistics on the contracts being used can be found in tables 7-9 in the appendix. In this paper we use an approach based on Kalman filtering in order to estimate the parameters in the model. To place our empirical results better into context we also include a comparison involving live-cattle data.

4.1 Data

As shown in Table 1, $Data_1$ ranges from 12/06/2006 to 1/11/2006 with average interest rate of 2.88%; $Data_2$ ranges from 2/11/2006 to 17/12/2008 with average interest rate of 4.00%; $Data_3$ ranges from 18/12/2008 to 22/03/2012 with average interest rate of 1.93%. Contracts used for tests in each data set are described in tables 7-9 respectively. Naturally, for each contract with a fixed maturity, the time-to-maturity changes as time progresses.

Parameter+	Panel A F1, F3, F5, F7, F9 (Short Term)	Panel B F12, F14, F16, F18, F20 (Medium Term)	Panel C F24, F25, F26, F28, F29 (Long Term)
μ	0.299 (0.446)	0.567 (0.567)	0.832 (0.345)**
κ	2.348 (0.203)***	1.009 (0.373)***	1.035 (0.277)***
α	0.084 (1.106)	1.311 (0.976)	1.484 (0.562)***
σ_1	0.236 (0.027)***	0.135 (0.031)***	0.128 (0.014)***
σ_2	1.444 (0.136)***	0.185 (0.095)**	0.162 (0.047)***
ρ	0.624 (0.103)***	0.866 (0.050)***	0.847 (0.030)***
λ	0.097 (2.615)	1.240 (1.364)	1.507 (0.809)*
Log-Likelihood	-1238	-1914.3	-2600.5

+ Standard errors in parentheses
*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level

Table 2: Estimation Results for $Data_1$, 12/06/2006-1/11/2006

4.2 Empirical Results for $Data_1$, 12/06/2006-1/11/2006

Table 2 shows the results for the estimation of the two-factor model based on $Data_1$. It can be observed that the correlation coefficient $\rho = \rho_1$ is large[8]; the speed of mean-reversion of the convenience yield κ, the expected return on the spot commodity μ, the mean-level of convenience yield α and the market price of convenience yield risk λ are all positive and reasonable. For Panel A and B however, the parameters μ, α and λ are not significant. This changes for panel C, where all coefficients are significant, most at the 1% level. Besides, it is also worth to note that the expected return on the spot commodity μ increases while the speed of mean-reversion κ decreases as the term of contracts increases. The Kalman filter based estimation is an iterative procedure. Figures 11 in the appendix shows the parameter evolution for Panel A exemplary. The convergence is good in all cases.
Figure 1 shows the filtered state variables, i.e. the spot price and the instantaneous convenience yield along with a number of selected futures prices for Panel A.[9] Prices of futures contracts contained in Panel A are also included in the figure. The figure seems to indicate strong correlation between state variables as well as a strong relationship between futures prices and spot

[8]In the context of the two-factor model, where there is only one relevant correlation, we omit sub-indices and denote $\rho = \rho_1$.

[9]The figures for Panel B and C look similar, but are omitted due to space limitations.

price. As one would expect the ability of futures contracts to proxy spot prices becomes weaker when maturity increases. The futures prices determined by the model are at most times within 2% of the market prices, which presents a good fit. Figures 2 represents the term structure, where the left part shows the actual term structures and the right part shows the model generated term structures. In general, the model makes a good prediction for the short-term panel but finds it more difficult to capture the shapes of longer-term panels, where the actual term structure appears to be rather unconventional, see figures 12-13 in the appendix.[10]

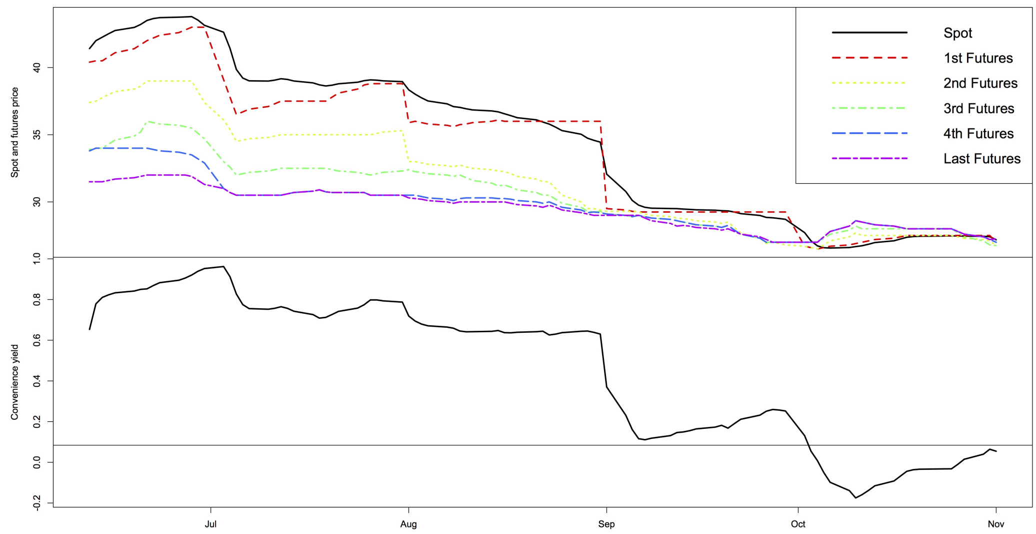


Figure 1: State Variables for Panel A in $Data_1$, 12/06/2006-1/11/2006

4.3 Empirical Results for $Data_2$, 2/11/2006-17/12/2008

Table 3 shows the results for the two-factor model obtained from $Data_2$. Similar as before the correlation coefficient ρ is large; the expected return on the spot commodity μ, the mean-reversion level of the convenience yield α and the market price of convenience yield risk λ are all positive and reasonable.

[10]The slightly odd looking actual term structure for longer dated salmon future contracts is likely to be caused by the rather low trading volume of these contracts.

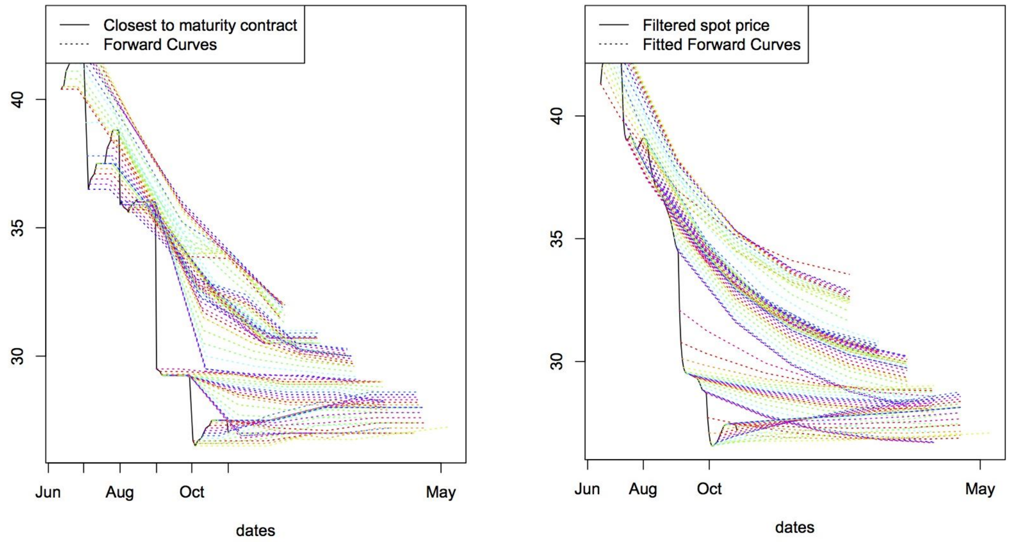


Figure 2: Term Structures for Panel A in $Data_1$, 12/06/2006-1/11/2006

However, the speed of mean-reversion of the convenience yield κ for Panel A is significantly larger than before, and volatilities σ_1 and σ_2 are significantly lower. For Panel A, the parameters μ, α and λ are not significant. This changes for panel B and C though, where all coefficients are highly significant at the 1% level. Furthermore, it is worth mentioning that the expected return on the spot commodity μ increases while the speed of mean-reversion κ decreases as the terms of contracts increase. For all cases, the convergence of the Kalman filter is very good.

Figures 3 shows the filtered state variables for Panel A, i.e. the spot price and the instantaneous convenience yield, along with selected futures prices. As before, we observe strong correlation between state variables as well as a close relationship between futures price and spot price. The ability of futures contracts to proxy spot prices becomes weaker when maturity extends. Again, the model presents a good fit, with model prices at most times being within 2% of market prices. Figure 4 presents the term structures for Panel

Parameter^{+}	Panel A F1, F3, F5, F7, F9 (Short Term)	Panel B F12, F14, F16, F18, F20 (Medium Term)	Panel C F24, F25, F26, F28, F29 (Long Term)
μ	0.214 (0.160)	0.747 (0.177)***	0.854 (0.122)***
κ	5.776 (0.616)***	1.387 (0.155)***	0.660 (0.018)***
α	0.216 (0.257)	0.951 (0.216)***	1.356 (0.069)***
σ_1	0.109 (0.006)***	0.141 (0.003)***	0.159 (0.023)***
σ_2	0.651 (0.059)***	0.223 (0.018)***	0.142 (0.022)***
ρ	0.580 (0.108)***	0.811 (0.021)***	0.895 (0.038)***
λ	0.818 (1.402)	1.290 (0.427)***	0.865 (0.098)***
Log-Likelihood	-8279.7	-9822.6	-12745

$^{+}$ Standard errors in parentheses
*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level

Table 3: Estimation Results for $Data_2$, 2/11/2006-17/12/2008

A contracts, where once more the left part shows the real term structures while the right part shows the model generated term structures. In general, the model makes a good prediction for the short-term panel but again finds it difficult to capture the shapes of longer-term panels, which show the rather odd looking actual term structure already observed in the first case, compare figures 14-15 in the appendix.

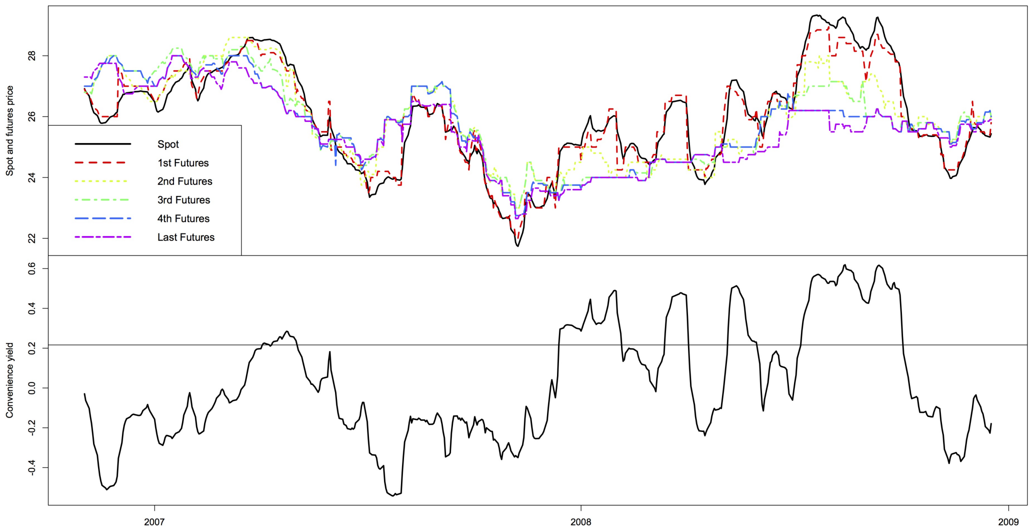


Figure 3: State Variables for Panel A in $Data_2$, 2/11/2006-17/12/2008

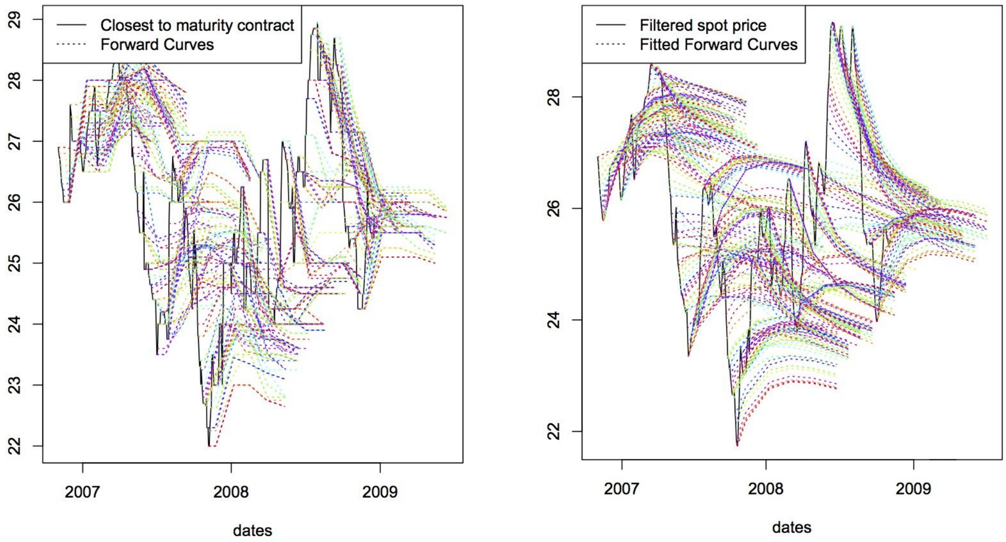


Figure 4: Term Structures for Panel A in $Data_2$, 2/11/2006-17/12/2008

4.4 Empirical Results for $Data_3$, 18/12/2008-22/03/2012

Table 4 shows the results for the two-factor model obtained from $Data_3$. As in the other two cases, the correlation coefficient ρ is large; the expected return on the spot commodity μ, the mean-reversion level of convenience yield α and the market price of convenience yield risk λ are all positive and reasonable. The speed of mean-reversion of the convenience yield κ for Panel A is significantly larger than for the other two panels. However, α and λ are insignificant for Panel's A and B, and μ is insignificant for Panel B. As before, all parameters are significant at 1% level for Panel C. Further, it is worth to mention that the expected return on the spot commodity μ increases while the speed of mean-reversion κ decreases as the terms of the contracts increase. As in the previous cases, the convergence of the Kalman filter is very good.

Figure 5 shows the filtered state variables for Panel A, i.e. the spot price and the instantaneous convenience yield, along with selected futures prices.

Parameter+	Panel A F1, F3, F5, F7, F9 (Short Term)	Panel B F12, F14, F16, F18, F20 (Medium Term)	Panel C F24, F25, F26, F28, F29 (Long Term)
μ	0.255 (0.113)**	0.398 (0.323)	0.917 (0.167)***
κ	3.554 (0.191)***	0.347 (0.125)***	0.232 (0.032)***
α	0.181 (0.134)	1.000 (1.066)	1.821 (0.261)***
σ_1	0.182 (0.020)***	0.188 (0.040)***	0.189 (0.004)***
σ_2	0.698 (0.099)***	0.161 (0.020)***	0.104 (0.004)***
ρ	0.740 (0.156)***	0.905 (0.065)***	0.908 (0.007)***
λ	0.297 (0.476)	0.351 (0.251)	0.418 (0.101)***
Log-Likelihood	-9341.1	-11804	-12870

+ Standard errors in parentheses
*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level

Table 4: Estimation Results of $Data_3$, 18/12/2008-22/03/2012

As before, we observe strong correlation between the state variables as well as a close relationship between futures price and spot price, which however becomes weaker as maturities extends. Model prices are still within 2% of market prices at most times, however, in particular for panel A, fall out of the 2% range more frequently, than for $Data_1$ and $Data_2$. Figure 6 shows the actual and model generated term structures as before. Similar as in the previous two cases the model makes a good prediction for the short-term panel but cannot capture the shapes of longer-term panels which as in the previous cases show odd looking actual term structures, most likely to be caused by the illiquidity of these contracts, compare figures 16-17 in the appendix.

4.5 Three-Factor Model

Accounting for stochastic interest rates and their term structure is of particular importance for longer term contracts. The longest maturity contract included in our study has a 2 1/2 year time to maturity. The longest maturities currently traded at Fish Pool are 5 years. In both cases it makes sense to consider stochastic rates and to assess in how far this effects the results obtained in the previous sections. We therefore consider the full three factor model represented as in equations (1)-(3) under $\mathbb{P}$ and (7)-(9) under the pricing measure $\mathbb{Q}$.

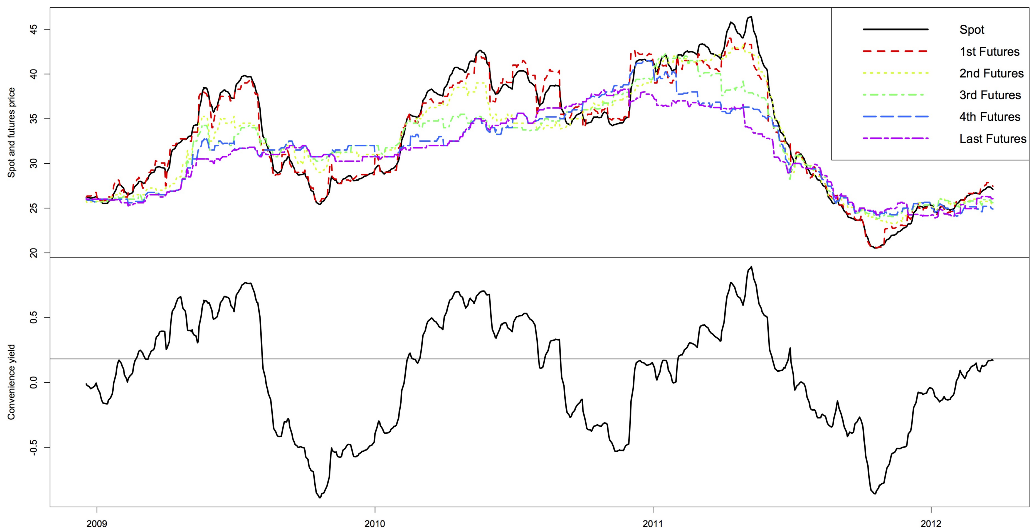


Figure 5: State Variables for Panel A in $Data_3$, 18/12/2008-22/03/2012

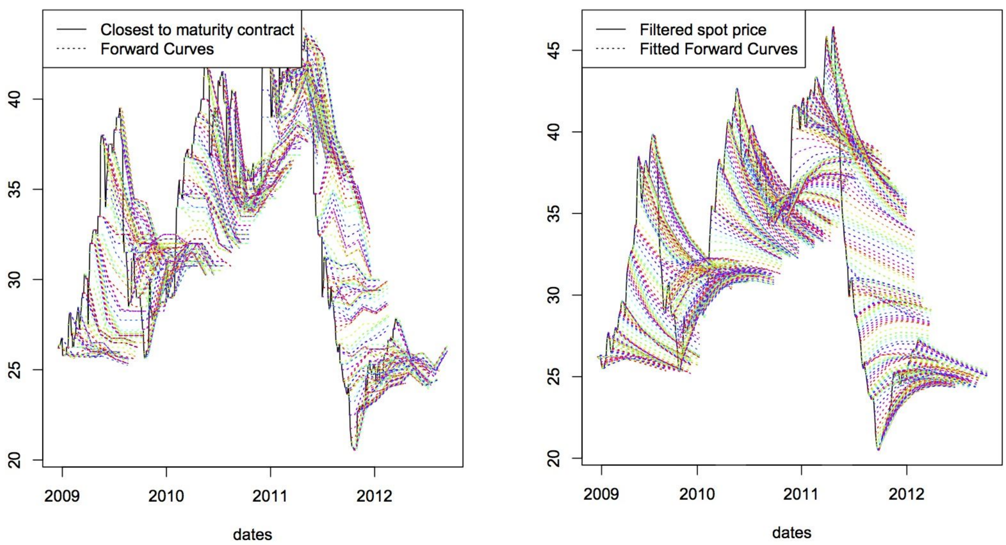


Figure 6: Term Structures for Panel A in $Data_3$, 18/12/2008-22/03/2012

Once the three-factor model has been cast in state space form, the Kalman Filter can be applied to estimate model parameters. Although ideally parameters in all three processes should be estimated simultaneously, here we will follow Schwartz (1997) by first estimating the interest rate process by fitting to the term structure of interests and then using the full three-factor model in order to determine the other two processes.

In this paper, Norwegian Treasury Bill yields are used to estimate the interest rate process over the whole sample period. The Euler discretion of equation (9) can be expressed as

$$r(t_{n+1}, \psi) = r(t_n, \psi) + a(m^* - r(t_n, \psi))\Delta t + \sigma_3 \Delta \tilde{Z}_3(t_n), \qquad (31)$$

where ψ stands for Norwegian Treasury Bill with different maturities. We can estimate parameters by rewriting (31) and solving the equation below

$$(\hat{a}, \hat{m}^*) = \underset{a,m^*}{\arg\min} \sum_{n=1}^{T-1} (r(t_{n+1}, \psi) - r(t_n, \psi) - am^* \Delta t + ar(t_n, \psi)\Delta t)^2 \qquad (32)$$

Once we have solved (32), $\hat{\sigma_3}$ can also be obtained by $\frac{\sigma_\epsilon}{\sqrt{\Delta t}}$, where σ_ϵ is the standard deviation of residuals. Since (9) is only capable of describing the short-term behavior, the 3-month, 6-month, 9-month and 12-month Norwegian Treasury Bills yields during the sample period are selected to estimate the interest rate process, accordingly only short-term futures contracts, i.e., Panel A consisting of F1, F3, F5, F7, and F9 in each data-set, are used to test the three-factor model. Moreover, ρ_2 and ρ_3 are approximated by the correlations between the 3-month Norwegian Treasury Bill yields and the filtered state variables, i.e. spot price and convenience yield, obtained from the corresponding two-factor model. The estimation results are displayed in Table 5.

As shown in Table 5, the estimated coefficients for the three-factor model are very close to those obtained from using the two-factor approach. However some of the estimates, which had been insignificant with the two-factor approach, now appear as significant. Specifically, the coefficients μ, α and

Parameter^{+}	$Data_1$	$Data_2$	$Data_3$
μ	0.102 (0.466)	0.294 (0.109)***	0.647 (0.143)***
κ	2.520 (0.200)***	5.950 (0.209)***	3.429 (0.142)***
α	1.884 (1.181)	0.402 (0.121)***	0.681 (0.164)***
σ_1	0.280 (0.032)***	0.143 (0.008)***	0.228 (0.012)***
σ_2	1.792 (0.171)***	0.935 (0.069)***	0.878 (0.061)***
ρ	0.843 (0.038)***	0.857 (0.023)***	0.901 (0.016)***
λ	4.646 (2.95)	1.978 (0.748)***	2.046 (0.551)***
Log-Likelihood	-1241.7	-8357.7	-9381.7
a	0.543	0.543	0.543
m^*	0.027	0.027	0.027
ρ_2	-0.926	0.560	0.127
ρ_3	-0.961	0.031	0.277
σ_3	0.017	0.017	0.017

$^{+}$ Standard errors in parentheses
*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level

Table 5: Estimation Results of Three Factor Model: Panel A

λ for Panel A $Data_2$ now become highly significant at the 1% level, while being insignificant before, compare Table 3. Some problems however remain within the analysis of $Data_1$. Besides insignificant μ, α and λ, the absolute values of ρ_2 and ρ_3 in $Data1$ are close to 1, which suggests that the three factor model used might be inappropriate to deal with this particular data-set. Most likely, the fact that the data-set $Data_1$ contains much fewer data points than the other two is to blame for this. By and large, the three factor approach confirms the results from the two-factor approach.

5 Comparison between Cattle and Salmon

How do the salmon futures compare to futures traded on other related commodities? Live-cattle seems to reflect some of the properties of farmed salmon as a commodity and futures on live-cattle are traded in high volume on the Chicago Mercantile Exchange. Based on data availability for both the Fish Pool market and the live-cattle futures market, we have chosen 6 live-cattle

Parameter[+]	$Data_1$		$Data_2$		$Data_3$	
	Cattle	Salmon	Cattle	Salmon	Cattle	Salmon
μ	0.224 (0.250)	0.241 (0.608)	0.108 (0.108)	0.195 (0.085)**	0.103 (0.106)	0.570 (0.121)***
κ	0.770 (0.179)***	2.844 (0.059)***	0.975 (0.082)***	1.139 (0.133)***	0.444 (0.180)***	4.257 (0.135)***
α	1.488 (0.934)	0.209 (0.667)	0.191 (0.143)	0.289 (0.120)**	0.060 (0.232)	0.233 (0.099)**
σ_1	0.145 (0.019)***	0.292 (0.023)***	0.149 (0.010)***	0.116 (0.006)***	0.136 (0.009)***	0.152 (0.007)***
σ_2	0.426 (0.054)***	1.595 (0.085)***	0.188 (0.018)***	0.174 (0.017)***	0.130 (0.018)***	0.527 (0.032)***
ρ	0.505 (0.107)***	0.636 (0.172)***	0.797 (0.034)***	0.884 (0.019)***	0.889 (0.021)***	0.864 (0.030)***
λ	0.819 (0.793)	0.368 (1.782)	0.113 (0.139)	0.247 (0.127)**	0.056 (0.102)	0.787 (0.419)*
Log-Likelihood	-1574.8	-1806	-8112.1	-10098	-6670.9	-6642

[+] Standard errors in parentheses
*** Significant at 1% level; ** Significant at 5% level; * Significant at 10% level

Table 6: Estimation Results: Comparison between Cattle and Salmon

contracts covering the period from 12/06/2006 to 07/09/2010. In analogy to our previous analysis, we divide the whole sample period into three different regimes as described in Table 1, but cut off at 07/09/2010. We continue to use Norwegian interest rates for the salmon contracts, but use the corresponding 3-month U.S treasury bill rates for each of the periods, i.e., 4.9%, 3.07% and 0.14%, for cattle contracts, which are traded in the US. Further, we select 6 salmon contracts F2, F5, F7, F10, F13 and F16 which have similar maturities as the live-cattle contracts. The average maturity of these contracts is 0.126 years, 0.383 years, 0.554 years, 0.810 years, 1.065 years and 1.321 years respectively. The empirical results of our analysis are shown in Table 6. We observe that in general, there are no significant differences between the expected returns on the spot commodity μ of salmon and cattle contracts. More interesting perhaps is that salmon contracts show significantly higher mean-reversion speeds κ and mean-reversion level of the convenience yield α as compared to cattle contracts.[11] In addition, the market price of convenience yield risk in the case of salmon is notably higher, at least for the time periods corresponding to $Data_2$ and $Data_3$.

As before Convergence of the Kalman filter is very good in all cases. Figures 7 and 8 show the filtered state variables, i.e. the spot price and the instantaneous convenience yield, along with selected futures prices. The model fit

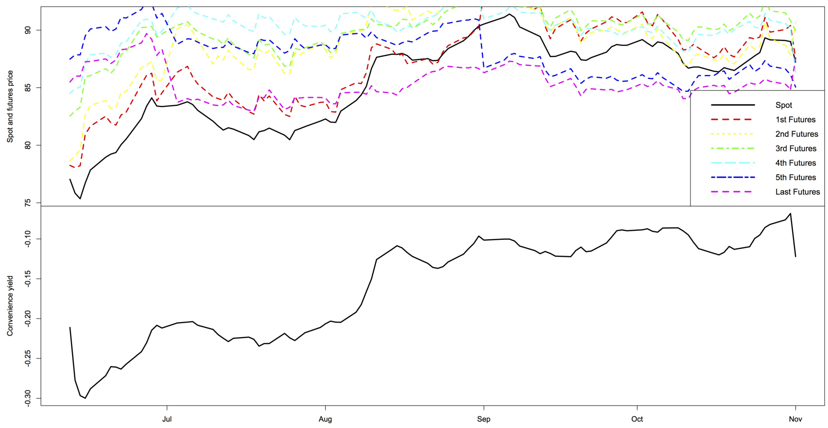
[11]Note that for $Data_1$ the α's for both cattle and salmon are insignificant.

is about the same, slightly better for salmon than for live-cattle where the relative error remains within 3% for most times. Figures 9 and 10 plot the term structures for both cattle and salmon.

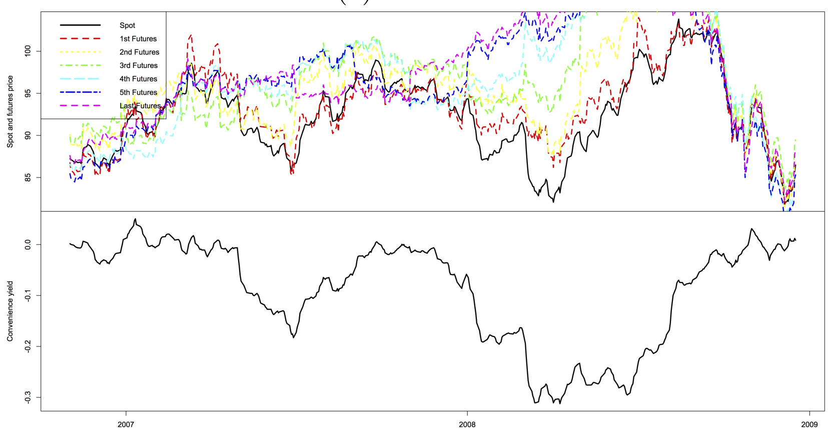
We observe from Figures 7 and 8 that the convenience yields are notably different in cattle than in salmon. While the convenience yield for cattle is negative almost all of the time, the convenience yield for salmon changes signs relatively frequently and is relatively equally balanced between positive and negative. This maybe attributed to storage issues and costs reflecting that fresh salmon is a highly perishable good, more so than cattle. It may also point towards liquidity issues and the fact that salmon farming is still far less developed than cattle farming, which may affect supply. In this case, the benefits for holding salmon in storage in the short term and hence being able to provide liquidity are higher than for cattle. Looking at the term structures in figures 9 and 10 it appears that the model captures the salmon contracts much better than the cattle contracts. This fact is confirmed numerically by tables 10 and 11 in the appendix, which show the residual mean square errors and MAE.

6 Conclusions

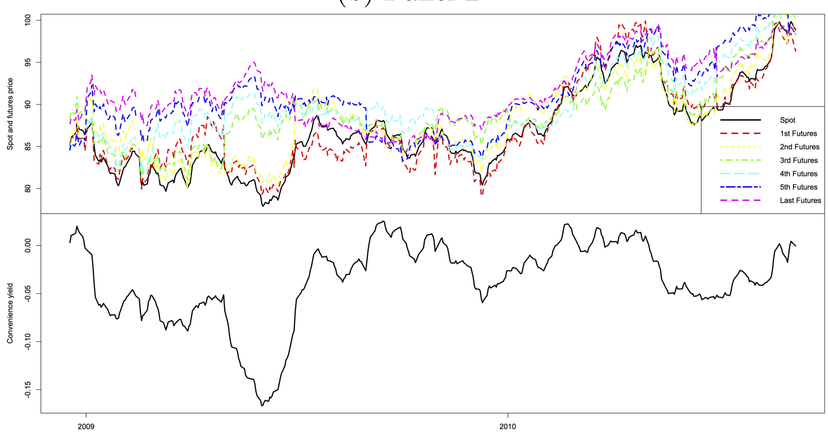
In this paper we established a link between the popular Schwartz (1997) multi-factor models used for the pricing of commodity derivatives and classical models originating from the aquaculture/fish farming literature. Specifically we looked at future contracts written on fresh farmed salmon, which have been actively traded at the Fish Pool Market in Norway since 2006. The link with the fish farming literature, has been established following first principles, starting by modeling the aggregate salmon farming production as well as modeling salmon demand using a Cobb-Douglas utility function for a representative consumer. We estimated our model using a rich data set of futures contracts with different maturities traded at Fish Pool between 12/06/2006 and 22/03/2012 by means of Kalman filtering. Our results show that the framework presented is able to produce an excellent fit to the actual term structure of salmon futures. A comparison with live cattle futures



(a) Panel A

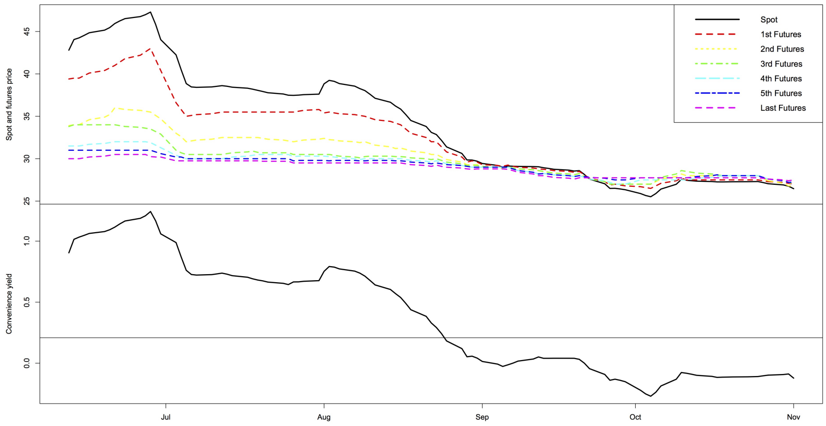


(b) Panel B

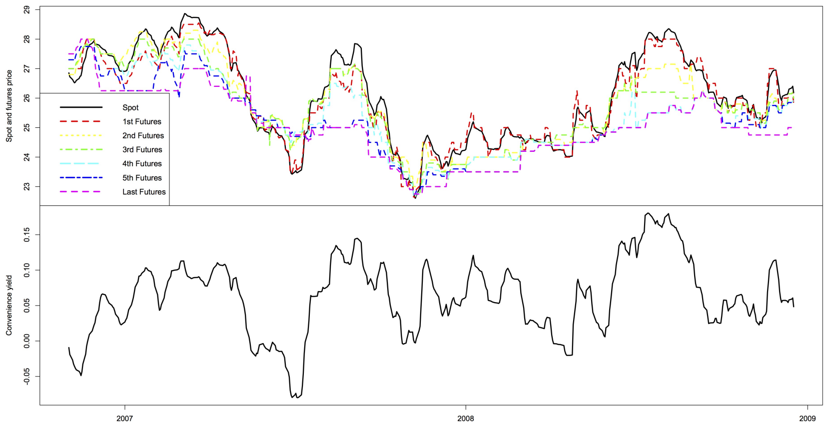


(c) Panel C

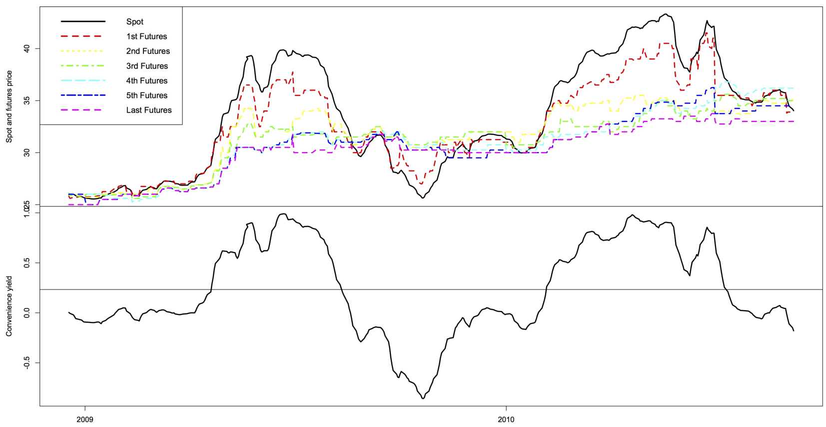
Figure 7: State Variable in Cattle Contracts



(a) Panel A

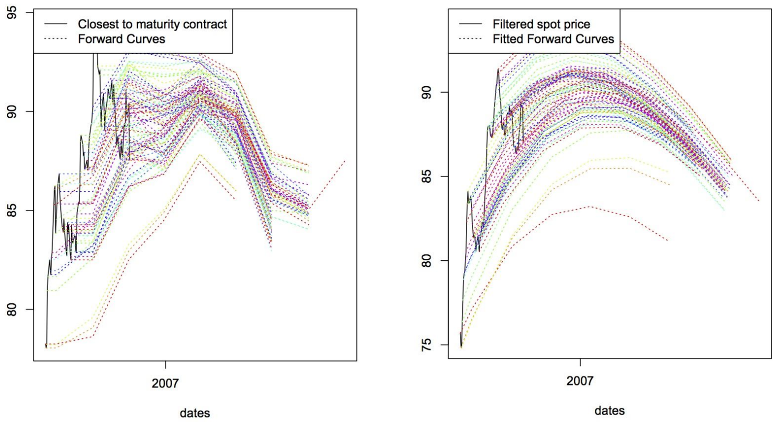


(b) Panel B

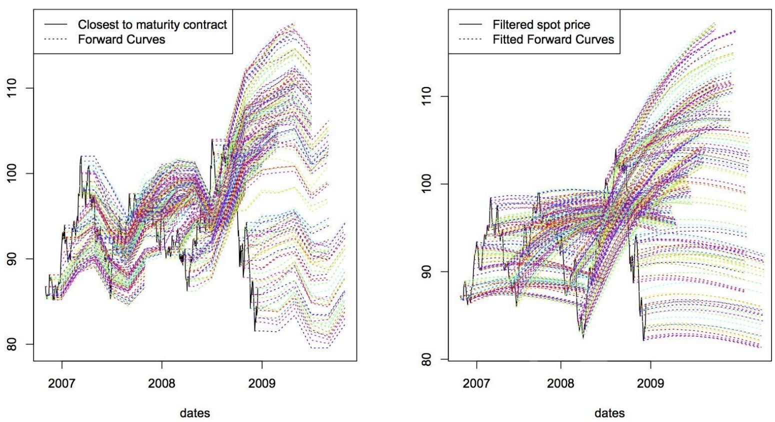


(c) Panel C

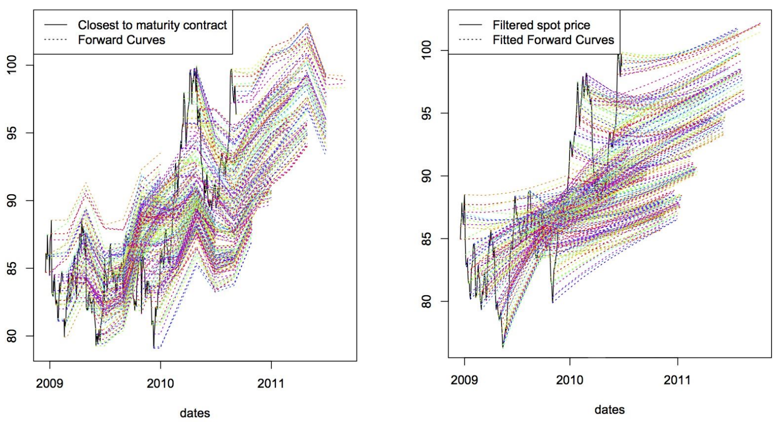
Figure 8: State Variable in Salmon Contracts



(a) Panel A

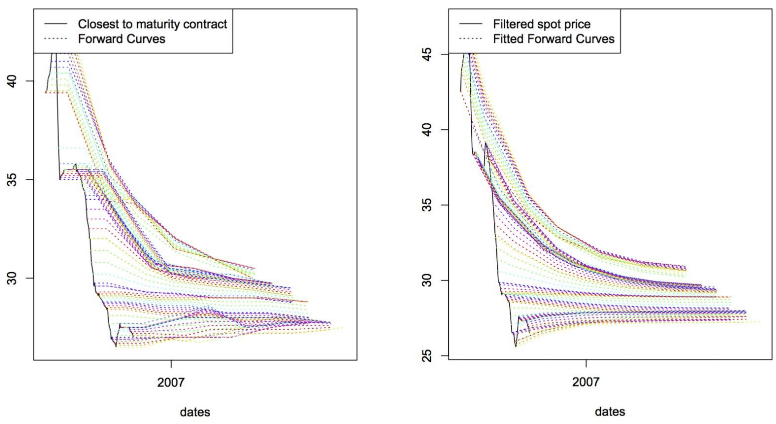


(b) Panel B

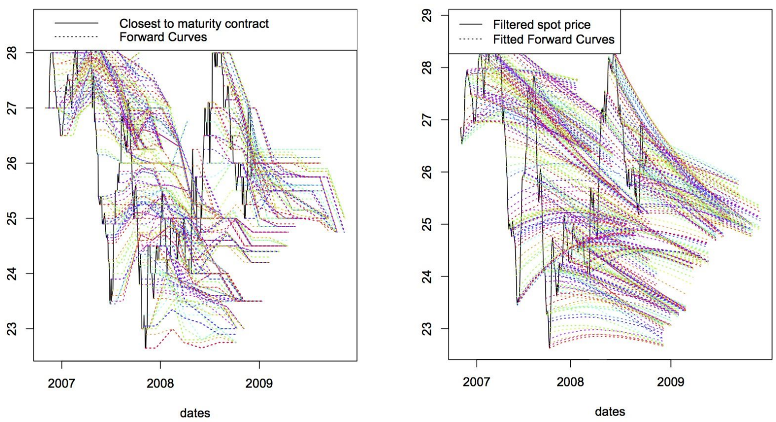


(c) Panel C

Figure 9: Term Structures in Cattle Contracts



(a) Panel A



(b) Panel B

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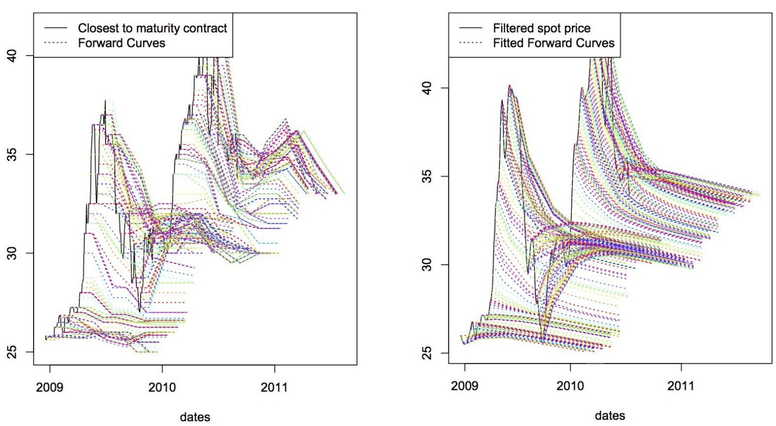

(c) Panel C

Figure 10: Term Structures in Salmon Contracts

traded within the same period reveals subtle difference, for example within the level of the convenience yield, the speed of mean reversion of the convenience yield and the convenience yield risk premium. Overall, the Schwartz (1997) multi factor approach appears to fit the salmon data better than the live cattle data.

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Appendix

Contract	Mean Price (Standard Deviation)	Mean Maturity (Standard Deviation)
Panel A: From 12/06/2006 to 1/11/2006: 103 Daily Observations		
F1	33.86 (5.32) *NOK*	0.040 (0.025) *years*
F3	31.68 (4.02)	0.212 (0.025)
F5	30.53 (2.68)	0.382 (0.025)
F7	29.82 (2.03)	0.551 (0.025)
F9	29.45 (1.51)	0.717 (0.025)
Panel B: From 12/06/2006 to 1/11/2006: 103 Daily Observations		
F12	29.20 (1.25) *NOK*	0.968 (0.025) *years*
F14	29.05 (1.05)	1.141 (0.025)
F16	28.91 (0.98)	1.315 (0.025)
F18	28.74 (0.89)	1.485 (0.025)
F20	28.57 (0.79)	1.650 (0.025)
Panel C: From 12/06/2006 to 1/11/2006: 103 Daily Observations		
F24	28.53 (0.80) *NOK*	1.984 (0.025) *years*
F25	28.53 (0.78)	2.072 (0.025)
F26	28.53 (0.78)	2.158 (0.025)
F28	28.53 (0.78)	2.327 (0.025)
F29	28.53 (0.78)	2.410 (0.025)

Table 7: Contracts in $Data_1$, 12/06/2006-1/11/2006

Contract	Mean Price (Standard Deviation)	Mean Maturity (Standard Deviation)
	Panel A: From 2/11/2006 to 17/12/2008: 545 Daily Observations	
F1	25.96 (1.59) *NOK*	0.041 (0.025) *year*
F3	25.92 (1.42)	0.210 (0.025)
F5	25.85 (1.39)	0.378 (0.026)
F7	25.71 (1.33)	0.547 (0.026)
F9	25.53 (1.28)	0.717 (0.025)
	Panel B: From 2/11/2006 to 17/12/2008: 545 Daily Observations	
F12	25.30 (1.24) *NOK*	0.973 (0.025) *years*
F14	25.12 (1.18)	1.143 (0.026)
F16	25.04 (1.18)	1.312 (0.026)
F18	24.94 (1.12)	1.483 (0.026)
F20	24.90 (1.10)	1.654 (0.027)
	Panel C: From 2/11/2006 to 17/12/2008: 545 Daily Observations	
F24	24.89 (1.12) *NOK*	1.997 (0.027) *years*
F25	24.89 (1.12)	2.083 (0.028)
F26	24.88 (1.13)	2.169 (0.028)
F28	24.86 (1.14)	2.341 (0.029)
F29	24.86 (1.14)	2.427 (0.028)

Table 8: Contracts in $Data_2$, 2/11/2006-17/12/2008

Contract	Mean Price (Standard Deviation)	Mean Maturity (Standard Deviation)
	Panel A: From 18/12/2008 to 22/03/2012: 849 Daily Observations	
F1	32.93 (6.28) *NOK*	0.041 (0.025) *year*
F3	32.47 (5.53)	0.213 (0.025)
F5	32.01 (4.99)	0.386 (0.025)
F7	31.51 (4.66)	0.558 (0.026)
F9	31.07 (4.31)	0.729 (0.026)
	Panel B: From 18/12/2008 to 22/03/2012: 849 Daily Observations	
F12	30.77 (3.91) *NOK*	0.986 (0.026) *years*
F14	30.45 (3.59)	1.157 (0.027)
F16	30.15 (3.16)	1.328 (0.028)
F18	30.12 (2.97)	1.498 (0.029)
F20	30.00 (2.81)	1.668 (0.031)
	Panel C: From 18/12/2008 to 22/03/2012: 849 Daily Observations	
F24	29.29 (2.38) *NOK*	2.007 (0.033) *years*
F25	29.17 (2.26)	2.092 (0.034)
F26	29.08 (2.15)	2.176 (0.035)
F28	28.99 (1.90)	2.345 (0.036)
F29	28.89 (1.82)	2.430 (0.037)

Table 9: Contracts in $Data_3$, 18/12/2008-22/03/2012

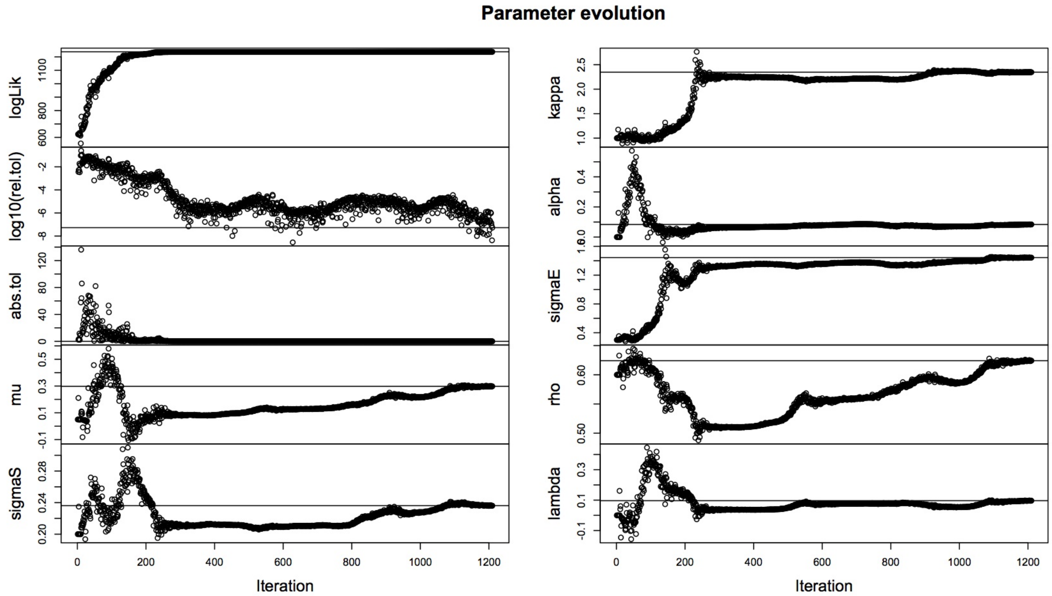

Figure 11: Parameter Evolution for Panel A in $Data_1$, 12/06/2006-1/11/2006

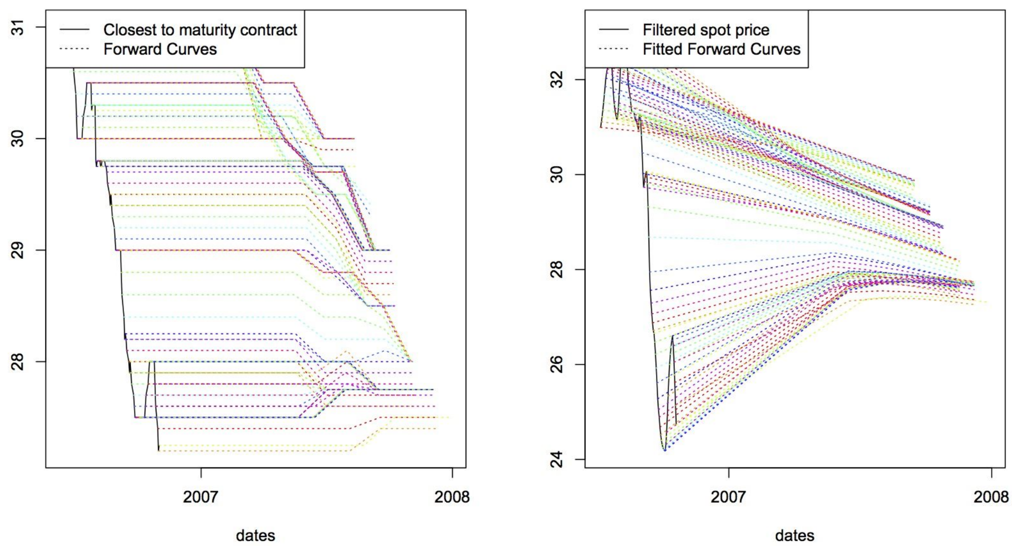

Figure 12: Term Structures for Panel B in $Data_1$

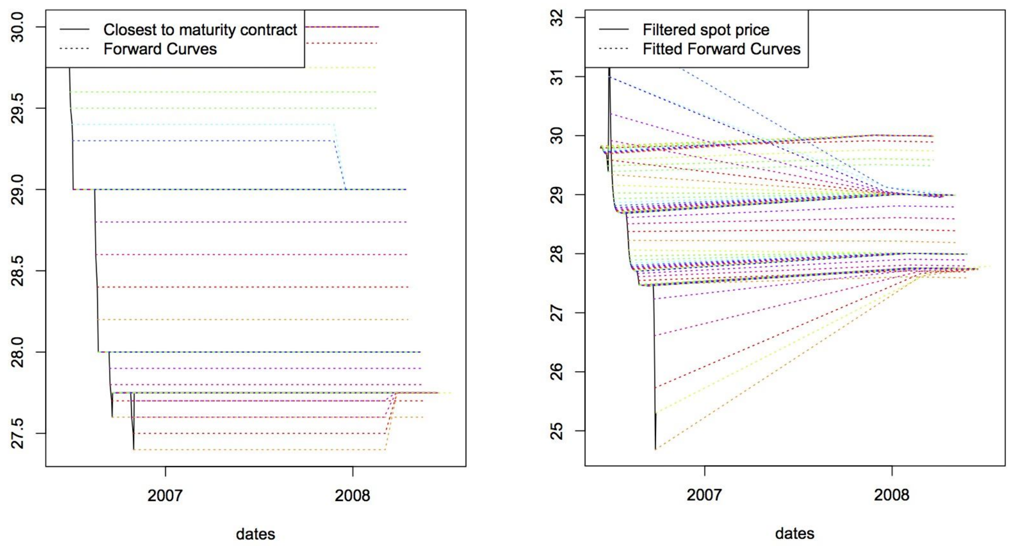

Figure 13: Term Structures for Panel C in $Data_1$, 12/06/2006-1/11/2006

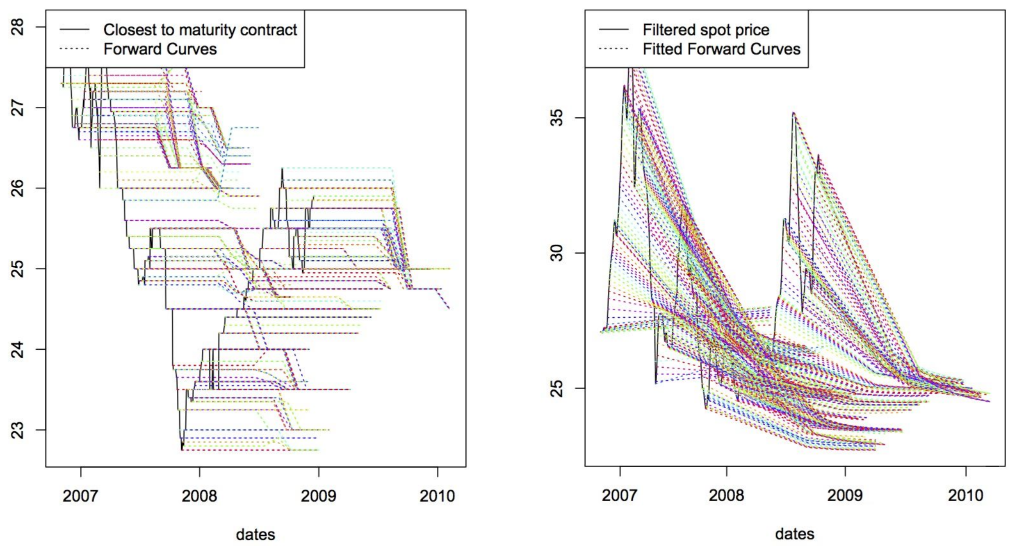

Figure 14: Term Structures for Panel B in $Data_2$, 2/11/2006-17/12/2008

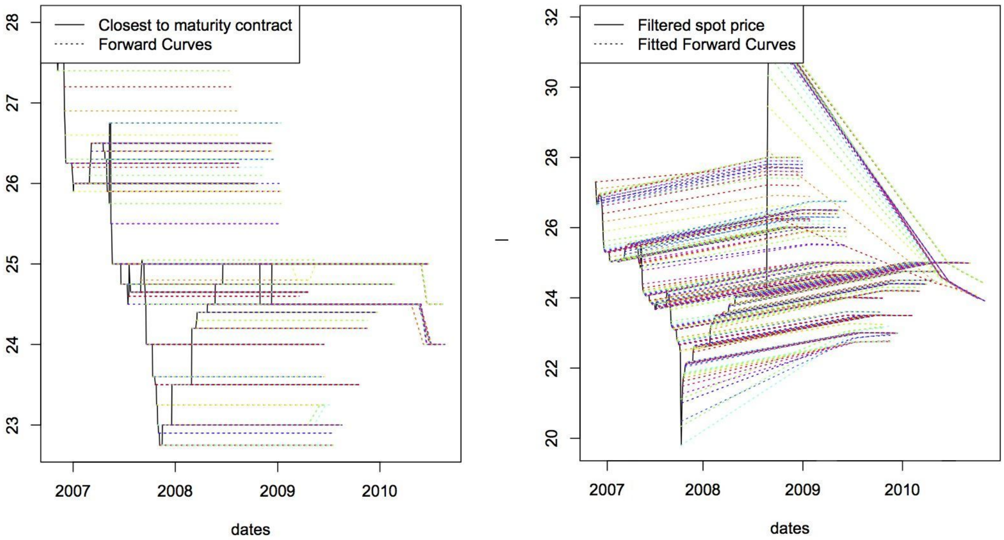

Figure 15: Term Structures for Panel C in $Data_2$, 2/11/2006-17/12/2008

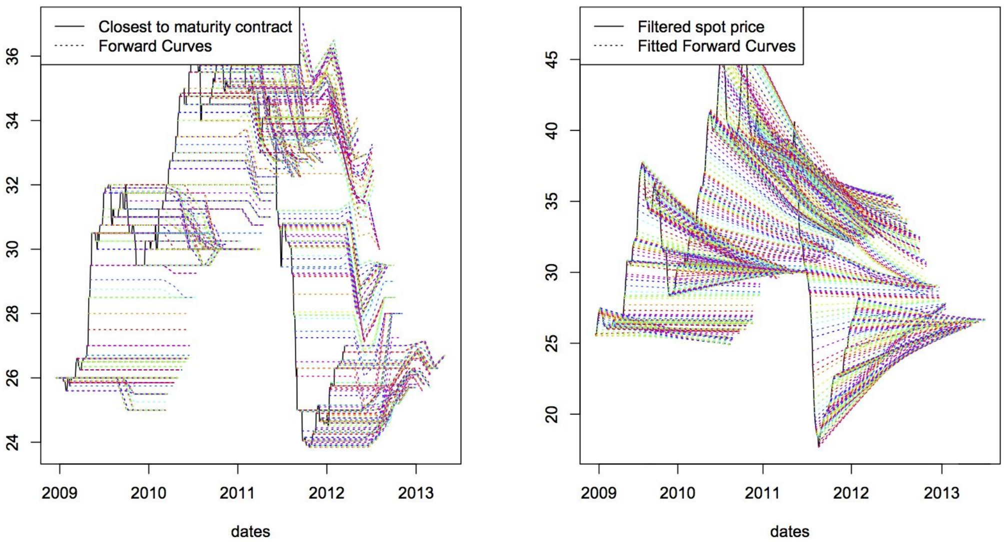

Figure 16: Term Structures for Panel B in $Data_3$, 18/12/2008-22/03/2012

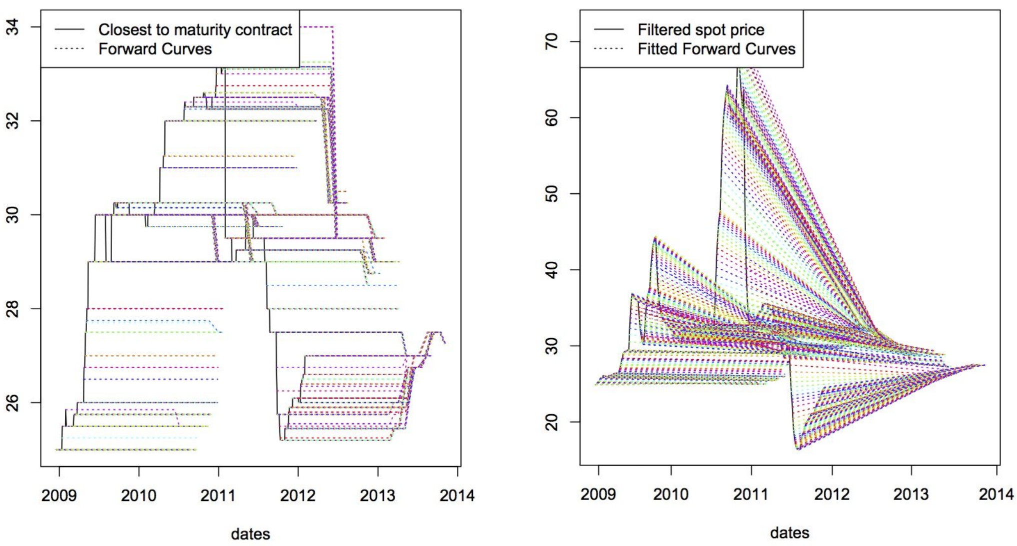

Figure 17: Term Structures for Panel C in $Data_3$, 18/12/2008-22/03/2012

Contracts	$Data_1$		$Data_2$		$Data_3$	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
F2	0.0124	0.0102	0.0150	0.0128	0.0175	0.0141
F5	0.0144	0.0123	0.0149	0.0123	0.0169	0.0141
F7	0.0054	0.0042	0.0208	0.0181	0.0196	0.0165
F10	0.0133	0.0114	0.0170	0.0139	0.0149	0.0119
F13	0.0140	0.0129	0.0144	0.0123	0.0126	0.0110
F16	0.0164	0.0134	0.0191	0.0164	0.0164	0.0137
ALL	0.0131	0.0107	0.0170	0.0143	0.0165	0.0136

Table 10: RMSE and MAE of Log Price: Cattle

Contracts	$Data_1$		$Data_2$		$Data_3$	
	RMSE	MAE	RMSE	MAE	RMSE	MAE
F2	0.0043	0.0033	0.0107	0.0087	0.0174	0.0131
F5	0.0059	0.0048	0.0095	0.0073	0.0231	0.0187
F7	0.0134	0.0106	0.0104	0.0083	0.0166	0.0129
F10	0.0047	0.0036	0.0085	0.0068	0.0190	0.0135
F13	0.0051	0.0044	0.0078	0.0056	0.0177	0.0132
F16	0.0078	0.0060	0.0090	0.0072	0.0144	0.0110
ALL	0.0075	0.0054	0.0094	0.0073	0.0182	0.0137

Table 11: RMSE and MAE of Log Price: Salmon