EQUILIBRIUM COORDINATION WITH DISCRETIONARY POLICYMAKING*

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Abstract

Discretionary policymakers cannot manage private-sector expectations and cannot coordinate the actions of future policymakers. As a consequence, coordination failures can occur and multiple equilibria can arise. In this paper we employ notions of self-enforceability and learnability to motivate and identify equilibria of particular interest in discretionary policy problems exhibiting multiple equilibria. Central among these criteria are whether an equilibrium is robust to the formation of coalitions, and whether it is learnable by private agents and jointly learnable by private agents and the policymaker. Unless the Pareto-preferred equilibrium is learnable by private agents we find little reason to expect coordination on that equilibrium.

Key Words: Discretionary policymaking, multiple equilibria, coordination, equilibrium selection.

JEL References: E52, E61, C62, C73

Discretionary policymakers can fall foul of expectations traps and coordination failures. When private agents are forward-looking their expectations, shaped by anticipations about future policy, influence how policy today is conducted. The discretionary policymaker's Achilles heel is that when formulating policy it is unable to manage private sector expectations, and this inability leads to policies that are sub-optimal—exhibiting inflation and/or stabilisation bias—and leaves ajar the door to multiple equilibria. When expectations cannot be managed, private agents can form expectations that, although unwelcome from the policymaker's perspective, lead private

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agents to react in a manner that traps the policymaker into implementing a policy that validates those expectations. The trap is closed when a policy that renders those unwelcome expectations without foundation is more costly and hence less attractive to the discretionary policymaker than a policy that accommodates them.

It is not unusual for economies to transition between periods of high and low inflation or high and low economic volatility, a phenomenon that expectations traps have the potential to explain (Albanesi *et al.*, 2003). Similarly, transitions from one equilibrium to another offers an explanation for policy regime changes, like those analysed by Davig and Leeper (2006). Accordingly, one explanation for the change in U.S. inflation's behavior between the 1970s and the 1980s could be that Volcker's appointment to Federal Reserve Chairman served to coordinate expectations and behavior, switching the economy from one discretionary equilibrium to another. However, in order to utilise the explanatory power of multiple equilibria it is necessary to first consider how an economy arrives at a particular equilibrium. In the words of Benhabib and Farmer (1999, pp. 438) "in any model with multiple equilibria one must address the issue of how an equilibrium comes about".

We study a model in which monetary policy is conducted under discretion and for which there are multiple equilibria; these equilibria are finite in number, distinct, and can be welfare ranked. In this context we consider the issue of how the agents residing in the model might coordinate on an equilibrium. We suggest two approaches. First, following Oudiz and Sachs (1985) we note that feedback equilibria of the discretionary control problem correspond to Markov-perfect Nash equilibria of a corresponding dynamic game. Although unilateral deviations from a Nash equilibrium are not beneficial, any particular Nash equilibrium may not be meaningful in the sense that it is not robust to the formation of coalitions that deviate jointly from equilibrium play. Pursuing this idea, we examine whether the Nash equilibria that we obtain are robust to coalition-formation, drawing on the notion of self-enforceability (Bernheim, Peleg, and Whinston, 1987). Specifically, we ask whether Pareto-dominated equilibria survive coalition-formation or instead whether a coalition may form to achieve a switch to the Pareto-preferred equilibrium. In our model, once coordination on the Pareto-preferred equilibrium occurs no coalition will choose to deviate. Informal justification for coalition-formation as an equilibrium selection mechanism comes from the fact that monetary policy is invariably conducted by committees whose members have tenures that are over-lapping and span multiple decision periods, providing ample opportunity for (costless) discussions of strategy and for the formation of non-binding coalitions.

Second, we consider learning as a coordinating mechanism for equilibrium selection (Evans, 1986), drawing on the large literature that employs learning to analyse coordination in rational expectations models (see Guesnerie and Woodford (1992), Evans and Guesnerie (1993; 2005), and Evans and Honkapohja (2001), among others). We extend the learning literature by developing iterative E-stability conditions in discretionary policy problems involving multiple agents that determine whether private agents and/or the policymaker might reasonably learn and coordinate on a particular equilibrium. Moreover, we prove that iterative E-stability and coalition formation are related. A sufficiently large coalition of policymakers should be able to generate the switch in private sector expectations needed to coordinate on the Pareto-preferred equilibria, provided the Pareto-optimal equilibrium is iteratively E-stable under private-sector learning.

We discuss these coordination mechanisms using a sticky price model with government debt adapted from Leeper (1991) by Blake and Kirsanova (2012) that in known to exhibit multiple discretionary equilibria. In this model the task confronting the central bank is to stabilise inflation without impacting unduly the real economy. Inflation is determined by the expected path of real marginal costs, so the policy challenge is to generate an appropriate path for real marginal costs. Because inflation depends on the entire expected path for real marginal costs while the discretionary policymaker can choose only today's policy, the policy chosen today depends necessarily on expected future policy. At the same time, the decisions that future policymakers make depend materially on the economic circumstances that they find themselves in, and hence on the choices that previous policymakers have made. This interaction between policymakers over time can lead to multiple equilibria.

The discretionary equilibria that we examine are all examples of sustainable equilibria. In this sense our research is related to the work on sustainable plans initiated by Chari and Kehoe (1990). Like ourselves Chari and Kehoe (1990) consider the design of optimal policy in the absence of commitment. However we examine policies that are Markovian and focus on the equilibrium selection issue whereas they examine the set of equilibria that can be supported by trigger strategies.

The remainder of the paper is structured as follows. In Section 1 we describe the simple New Keynesian model with government debt. In Section 2 we formulate the discretionary policy problem and describe the properties of the resulting equilibria. In Section 3 we use the model to motivate and describe mechanisms by which agents might coordinate on an equilibrium. Section 4 concludes. We leave the formal treatment of the coordination mechanisms in the linear-quadratic class of models to Appendices.

1 A simple New Keynesian model

To illustrate the equilibrium coordination process, we study a simple New Keynesian model with government debt that is based on Leeper (1991). This model was used previously by Blake and Kirsanova (2012) to establish the existence of multiple discretionary equilibria in linear-quadratic models. The economy is populated by a representative household, by a unit-continuum of monopolistically competitive firms, and by a single large government that performs separately monetary policy and fiscal policy. Fiscal policy is undertaken via a mechanistic rule that relates the income tax rate to the debt-to-output ratio. Monetary policy, in contrast, is conducted under discretion with the central bank choosing the nominal rate on a one-period non-state-contingent nominal bond. Importantly, when formulating monetary policy the central bank takes the fiscal policy rule into account. Monopolistically competitive firms produce differentiated goods according to a production function that depends only on labor, and these goods are combined via a Dixit and Stiglitz (1977) technology to produce an aggregate output good that is allocated to either private consumption or government spending. Households choose their consumption and leisure and can transfer income over time through purchasing government bonds. The government issues bonds period-by-period in order to pay the principal and interest on its existing debt and to fund any primary budget deficit. Firms set prices subject to a Calvo (1983) nominal price rigidity, and aggregation across prices leads to a New Keynesian Phillips curve relating inflation to expected future inflation, to real marginal costs, and to a serially correlated markup shock.

When approximated about an efficient zero-inflation non-stochastic steady state, the Phillips curve can be expressed as[1]

$$b_{t+1} = \rho b_t - \eta c_t, \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda c_t + \nu b_t + u_t, \tag{2}$$

where b_t represents the ratio of real government debt to output, c_t represents real consumption, π_t represents inflation, and u_t represents the markup shock. In equations (1) and (2), $\beta \in (0,1)$ denotes the discount factor, while $\rho \in (0,1)$, $\eta \in (0,\beta^{-1})$, $\lambda \in (0,\infty)$, and $\nu \in (0,\infty)$ are convolutions of behavioral parameters—preference and technology parameters. Underlying equations (1) and (2) is a fiscal policy rule that relates the income-tax rate positively to the economy's debt-to-output ratio.

Following (Clarida *et al.*, 1999), we assume that the central bank's policy instrument is consumption, c_t. The central bank's intertemporal welfare criterion is described by the quadratic

[1] The model's complete derivation and first-order approximation can be found in Blake and Kirsanova (2012).

loss function

$$L_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\pi_s^2 + \alpha c_s^2\right), \tag{3}$$

where $\alpha \in (0, \infty)$ is a convolution of behavioral parameters, derived under the assumption that the monopolistic distortion is offset by a labor subsidy, financed by a lump-sum tax.

Currie and Levine (1985a) and Backus and Driffill (1986) show certainty equivalence for this class of models. As a consequence, we focus on the deterministic component of the model in what follows, setting the markup shock aside.

2 Coordination failures and multiple discretionary equilibria

When formulating the optimal monetary policy, the central bank takes into account the evolution of the aggregate economy, equations (1) and (2). The current-period policymaker determines their policy, knowing that all future policymakers will solve an equivalent decision problem. Consider the period-t optimisation problem. Because b_t is the only state variable and the model is linear-quadratic, in a Markov-perfect equilibrium expectations of π_{t+1} can depend only on b_{t+1}, and the dependence must be linear

$$E_t \pi_{t+1} = \pi_b b_{t+1}. \tag{4}$$

With a quadratic policy objective, the cost-to-go from $t+1$ is $V b_{t+1}^2$, where V is a non-negative scalar. Accordingly, the period-t central bank's problem is to choose c_t to minimise

$$\pi_t^2 + \alpha c_t^2 + \beta V b_{t+1}^2,$$

subject to equations (1), (2), and (4). Combining the three constraints yields

$$\pi_t = (\beta \rho \pi_b + \nu)\, b_t + (\lambda - \beta \eta \pi_b)\, c_t,$$

which allows the optimal discretionary policy problem to be expressed in terms of the Bellman equation

$$Vb_t^2 = \min_{c_t} \left\{ \left[\left(\beta\rho\pi_b + \nu \right) b_t + \left(\lambda - \beta\eta\pi_b \right) c_t \right]^2 + \alpha c_t^2 + \beta V b_{t+1}^2 \right\}. \tag{5}$$

Optimising equation (5) yields a solution of the form

$$\begin{aligned} c_t &= c_b b_t, \\ \pi_t &= \pi_b b_t, \\ b_{t+1} &= b_b b_t, \\ L_t &= V b_t^2, \end{aligned}$$

where

$$c_b = -\frac{\left(\lambda - \beta\eta\pi_b\right)\left(\beta\rho\pi_b + \nu\right) - \eta\beta\rho V}{\beta\eta^2 V + \left(\lambda - \beta\eta\pi_b\right)^2 + \alpha}, \tag{6}$$

$$\pi_b = \beta\rho\pi_b + \nu + \left(\lambda - \beta\eta\pi_b\right) c_b, \tag{7}$$

$$b_b = \rho - \eta c_b, \tag{8}$$

$$V = \left[\left(\beta\rho\pi_b + \nu\right) + \left(\lambda - \beta\eta\pi_b\right) c_b \right]^2 + \beta V \left(\rho - \eta c_b\right)^2 + \alpha c_b^2. \tag{9}$$

Employing the Fisher equation, the form of this solution can be used to determine the coefficient on debt in the equilibrium reaction function for the nominal interest rate

$$i_t = i_b b_t. \tag{10}$$

For this decision problem, any set of coefficients $\{c_b, \pi_b, V\}$ that satisfy equations (6), (7), and (9) represents a discretionary equilibrium (with b_b and i_b determined residually by equations (8) and (10)). As discussed in Blake and Kirsanova (2012) this model possesses three discretionary equilibria, which we label A, B, and C in what follows. Some characteristics of these three equilibria are reported in Table 1.[2]

Table 1
Three Discretionary Equilibria

Eqm.	Policy Reaction c_b	Private sector Reaction π_b	Implied response of interest rate i_b	Speed of adjustment b_b	Value function V
A	−0.0343	0.0066	0.0103	0.9408	0.0004
B	0.0155	0.0430	0.0380	0.9314	0.0131
C	1.6403	0.2561	−1.0748	0.6237	0.1449

Table 1 shows that equilibria A and B share certain characteristics while equilibrium C appears very different. In particular, the feedback coefficients on consumption, inflation, and the nominal interest rate in equilibrium C are all much larger in magnitude than those for equilibria A and B, suggesting greater volatility in a stochastic economy. In addition, where the nominal interest rate is raised in response to higher debt in equilibria A and B it is lowered markedly in equilibrium C. These three equilibria produce qualitatively and quantitatively different economic dynamics, as shown in Figure 1 which plots the responses of key variables to an initial debt-level that is one percent higher than steady state.

Looking at Figure 1 and focusing first on equilibrium A (Panel A), if initial debt is higher than its steady state value, then the fiscal authority raises the tax rate according to its fiscal rule which relates the tax rate positively to the stock of debt. Higher tax revenues lead to a slow reduction in debt while the resulting cost-push inflation is addressed through monetary policy. The nominal interest rate is raised slightly, which lowers consumption and tax revenues, but this

[2] The parameterisation follows Blake and Kirsanova (2012). The model's frequency is quarterly. The subjective discount factor β is set to 0.99, ρ is set to 0.9343, η is set to 0.1894, λ is set to 0.0582, ν is set to 0.0025, and α is set to 0.0087. Underlying this parameterisation is a fiscal policy rule for the tax rate that has a coefficient on the debt-to-output ratio of 0.075.

reduction in tax revenues only slows slightly the rate at which government debt declines. In subsequent periods, although the nominal interest rate is lowered gradually back to steady state, debt is brought back to steady state predominantly through fiscal surpluses, rather than through a decline in the cost debt-financing. In equilibrium B (Panel B), the nominal interest rate is raised by more than it is in equilibrium A. This difference arises because inflation is perceived to be higher in the future, which leads to an ongoing negative real interest rate. A consequence of this persistently negative real interest rate is that consumption rises slightly, which is consistent with higher marginal costs and higher inflation, while the increased tax revenues contribute to debt-stabilisation, which occurs more rapidly than in equilibrium A.

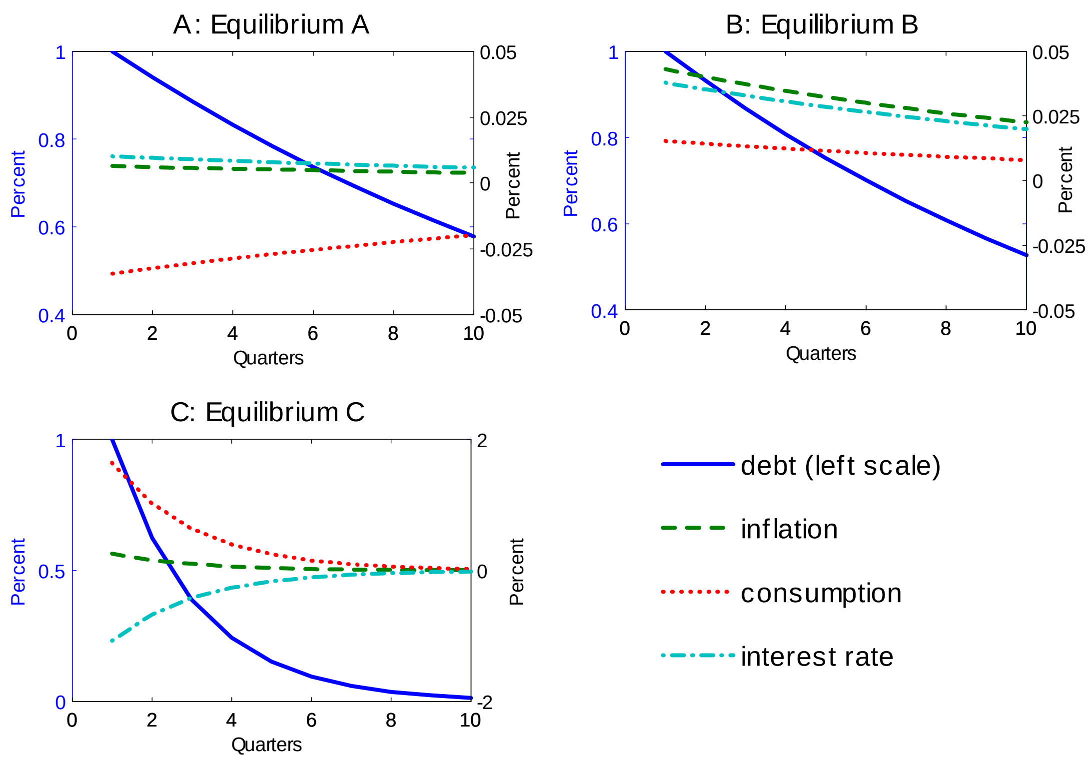

Fig. 1. *Transition to steady state from a high-debt initial state*

As suggested by Table 1, where inflation, consumption, debt, and the nominal interest rate behave similarly in equilibria A and B, their behavior in equilibrium C (Panel C) is very different. In particular, the central bank responds to the higher debt level by stimulating consumption and output, which raises real marginal costs and causes inflation to rise by more than it otherwise would. Such monetary policy causes tax revenues to rise, which leads to a rapid decline in government debt. In this equilibrium, in order to stabilise debt the central bank lowers the cost of financing the debt, which causes consumption, output, and real marginal costs to rise and places upward pressure on inflation. With the real interest rate decreasing in response to higher initial debt, in the spirit of Leeper (1991) monetary policy can be thought of as being passive in equilibria B and C and active in equilibrium A (where the real interest rate rises).

Importantly, Table 1 shows that the three equilibria can be welfare ranked and that equilibrium A is the Pareto-preferred equilibrium. Ideally, then, although this model admits three distinct Markov-perfect equilibria, the most desirable equilibrium, the equilibrium that all agents residing in the model would like to prevail, is equilibrium A. To understand better why multiple equilibria arise in this model, it is useful to recognise that the challenge facing the central bank is to return the economy to steady state without creating too large of a recession. According to the Phillips curve, in any stationary equilibrium inflation depends on the entire expected future path of real marginal costs,

$$\pi_t = E_t \sum_{j=t}^{\infty} \beta^{(j-t)} \left(mc_j + u_j \right),$$

where real marginal costs are given by

$$mc_t = \lambda c_t + \nu b_t.$$

Notice that when the discount factor, β, is large mc_t and mc_{t+1} are highly substitutable in terms of their effect on period-t inflation. Clearly, if inflation is above target, then there are multiple paths for real marginal costs that will return inflation to target. Each of these paths for real marginal costs is associated with a different monetary policy and each has a different cost in terms of loss. Faced with debt higher than steady state, the central bank might choose a policy that involves higher real marginal costs (consumption) and inflation if that policy stimulates tax revenues and future central banks are expected to raise the nominal interest rate in response to declining debt.

3 Equilibrium coordination

In the previous section we studied the three equilibria possessed by the simple model with debt and emphasised that they can be welfare ranked. Of these three equilibria, equilibrium C, in particular, exhibits many undesirable properties, such as the prescription that monetary policy should be kept very loose when debt levels are high in order to reduce debt by lowering the government's borrowing costs. In this section we consider two mechanisms through which coordination on a particular equilibrium might occur: coalition forming and learnability.

3.1 Coalition forming

We first approach the coordination problem by asking whether an equilibrium is "self-enforceable" (Bernheim *et al.*, 1987; Bernheim and Whinston, 1987), robust to the potential formation of coalitions containing consecutive policymakers. The central idea is that in non-cooperative decision problems in which policymakers can communicate strategy and make non-binding commitments a "meaningful" Nash equilibria should be coalition-proof. Intuitively, policymakers can more easily coordinate on an equilibrium if no group—or coalition—of policymakers finds beneficial to deviate from equilibrium play. In our context, central bankers are often appointed on multi-year contracts and policy itself is commonly determined by committees whose composition changes gradually over time as some committee members have their terms expire and new members are appointed. At any policy meeting, therefore, there is ample opportunity for committee members to communicate strategy and make non-binding agreements. The coalitions that we envisage are motivated by the fact that policy is often undertaken by committees whose members have tenures spanning multiple decision periods.[3]

There are three discretionary equilibria in the New Keynesian model with debt. Importantly, because the economic environment is one in which there is complete and perfect information, the existence and nature of all three equilibria is known to all agents. Moreover, the three equilibria can be welfare ranked and agents are not indifferent as to which equilibrium prevails.

We treat the policy rules associated with the three equilibria as a set of policy actions, $\{c_b^A, c_b^B, c_b^C\}$. Because the equilibria are Nash, if policymakers in periods $s = t+1, ..., \infty$ are known to play $\{c_b^j\}$, $j \in \{A, B, C\}$, then the period-t policymaker's best response is to also play $\{c_b^j\}$. However, although it is never beneficial for the period-t policymaker to unilaterally deviate from Nash play, the period-t policymaker can potentially benefit from deviations that involve multiple consecutive policymakers. With this in mind, we consider the possibility that a finite number of consecutive policymakers might form a coalition that deviates from playing $\{c_b^j\}$.

For the sake of concreteness, consider equilibria A and C. Suppose that equilibrium C is the prevailing equilibrium and that the economy enters period t having experienced outcomes from equilibrium C in period $t-1$. The policy-loss associated with staying in equilibrium C permanently is summarised by the value function coefficient, V^C, determined by equation (9). Because equilibrium A is Pareto-preferred to equilibrium C a switch to equilibrium A would be Pareto improving. Now, suppose the period-t policymaker implements policy $\{c_b^A\}$, but both the period-t policymaker and private agents anticipate that policy $\{c_b^C\}$ will be implemented from period $t+1$ onwards. In other words, the period-t policymaker attempts a unilateral deviation. Private agents in period t will react according to

$$\pi_{b,t} = \beta \left(\rho - \eta c_{b,t}^A\right) \pi_{b,t+1}^C + \lambda c_{b,t}^A + \nu,$$

[3] In the US, Federal Reserve chairmen are appointed to a four year term, but the average tenure is somewhat longer; governors have 14 year terms. In the UK, external monetary policy committee members have once-renewable three-year contracts that overlap to prevent members from retiring simultaneously; internal committee members have terms that last between $5-8$ years. With most external member serving two terms, the UK MPC completely renews after about ten years.

and the level of debt in period $t+1$ will be given by

$$b_{t+1} = \left(\rho - \eta c_{b,t}^{A}\right) b_t.$$

Now, the intertemporal loss function in period t associated with the period-t policy switch from $\{c_b^C\}$ to $\{c_b^A\}$ is fully determined by the value function, V_t, and the initial condition, b_t,

$$L_t^{CA} = V_t b_t^2,$$

where

$$V_t = (\pi_{b,t})^2 + \alpha \left(c_{b,t}^{A}\right)^2 + \beta V_{t+1}^{C} \left(\rho - \eta c_{b,t}^{A}\right)^2.$$

For a one-player "coalition" the loss associated with a (one-period) policy switch from C to A, which we denote $L_t^{CA}(1)$, is necessarily greater than the loss of staying in equilibrium C, L_t^C, (because a unilateral deviation from equilibrium play is not optimal in a Nash equilibrium). But a coalition of two consecutive policymakers might find it beneficial if both deviate. We can compute the period-t loss associated with a 2-player coalition switching from equilibrium C to A using the following equations

$$\begin{aligned}
\pi_{b,t+1} &= \beta \left(\rho - \eta c_{b,t+1}^{A}\right) \pi_{b,t+2}^{C} + \lambda c_{b,t+1}^{A} + \nu, \\
\pi_{b,t} &= \beta \left(\rho - \eta c_{b,t}^{A}\right) \pi_{b,t+1} + \lambda c_{b,t}^{A} + \nu, \\
b_{t+2} &= \left(\rho - \eta c_{b,t+1}^{A}\right) b_{t+1}, \\
b_{t+1} &= \left(\rho - \eta c_{b,t}^{A}\right) b_t, \\
V_{t+1} &= (\pi_{b,t+1})^2 + \alpha \left(c_{b,t+1}^{A}\right)^2 + \beta V_{t+2}^{C} \left(\rho - \eta c_{b,t+1}^{A}\right)^2, \\
V_t &= (\pi_{b,t})^2 + \alpha \left(c_{b,t}^{A}\right)^2 + \beta V_{t+1} \left(\rho - \eta c_{b,t}^{A}\right)^2.
\end{aligned}$$

If V_t is less than V_t^C (implying $L_t^{CA}(2) < L_t^C$), then a switch from equilibrium C to equilibrium A could be achieved by a coalition containing the policymakers in periods t and $t+1$.

Continuing in this way, we can compute the period-t loss associated with any coalition size, k. If $L_t^{CA}(k)$ becomes less than L_t^C as k increases, then the period-t policymaker will find it beneficial to switch to $\{c_b^A\}$. For this simple model we obtain an analytical formula for the loss $L_t^{CA}(k)$

$$L_t^{CA}(k) = \zeta^k L_t^C + \left(1-\zeta^k\right) L_t^A + \left[\left(1-\beta^k\right)\zeta^k \frac{\beta\left(\pi_b^C - \pi_b^A\right)^2}{1-\beta} + \left(1-\xi^k\right)\delta^k \frac{2\pi_b^A\left(\pi_b^C - \pi_b^A\right)}{1-\xi}\right] b_0^2,$$

where $\zeta = \beta\left(b_b^A\right)^2 < \beta < 1$ and $\delta = \beta b_b^A < b_b^A = \xi < 1$. Of course, $L_t^{CA}(0) = L_t^C$. The loss is monotonically decreasing to L_t^A as k tends to infinity, $\lim_{k\to\infty} L_t^{CA}(k) = L_t^A$.[4] This asymptotic monotonicity implies that if a coalition of size k exists, and the size of this coalition is sufficiently big such that $L_t^{CA}(k+1) < L_t^{CA}(k)$, then a coalition of size greater than k must also exist. In this respect if a coalition of size k can be convinced to switch from $\{c_b^C\}$ to $\{c_b^A\}$ then so too should a coalition of size greater than k and that $\{c_b^A\}$ will be the prevailing policy with the equilibrium given by $\{c_b^A, \pi_b^A, V^A\}$.

Reintroducing the markup shock to our baseline model,[5] Figure 2 plots $L_t^{CA}(k)$ and L_t^C as a function of the coalition size, k.

[4] The first two terms are a linear combination of L_t^C and L_t^A, which decreases monotonically to L_t^A as the coalition size, k, increases. The terms in the square brackets first increase but then decrease monotonically to zero as k increases. Each of these terms has the functional form $\left(1-a^k\right)b^k$, $0 < b < a < 1$, hence the asymptotic behavior.

[5] The markup shock is assumed to follow an AR(1) process with the persistence parameter set to 0.3 and the standard deviation for the innovation set to 0.0046, so that the markup shock itself has a standard deviation of 0.005.

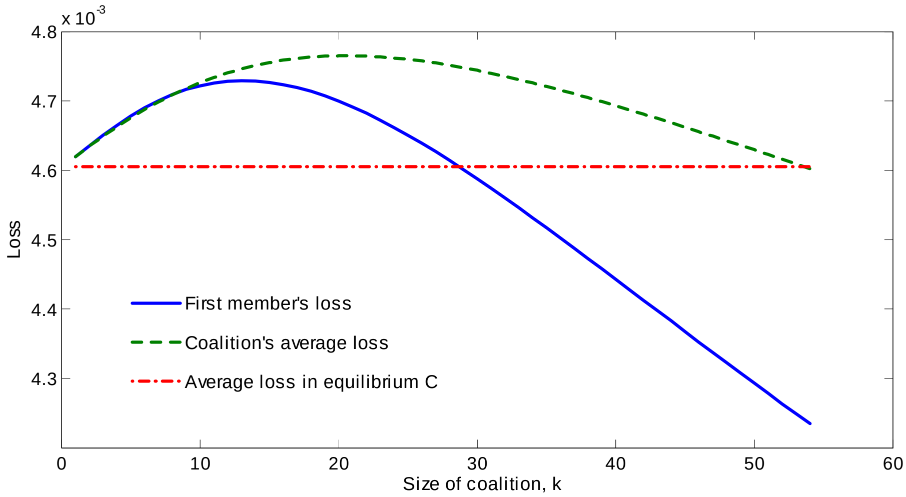

Fig. 2. *Coalition size needed to switch to equilibrium A*

Figure 2 shows that it requires a coalition containing 29 consecutive policymakers in order for $L_t^{CA}(k)$ to fall below L_t^C. In other words, for this model a coalition containing 29 consecutive policymakers would be necessary to make the first member of the coalition better off from a switch to equilibrium A from equilibrium C. But, of course, the fact that the first member of the coalition is better off by the switch is not sufficient to ensure that the coalition will form. We now consider the following two possibilities. The first possibility recognizes that policymakers have tenures spanning multiple decision periods. Suppose that tenures are long enough that all coalition members are the same policymaker and this policymaker is interested in the average payoff over their tenure. To consider this possibility, we also plot in Figure 2 the coalition's average loss as a function of the coalition size, k. Where the first member of the coalition is better off from the switch to equilibrium A with a coalition size of 29, Figure 2 shows that the average coalition member is better off from the switch to equilibrium A when the coalition size reaches 54.[6] The implication is that a switch to equilibrium A becomes more feasible when policymakers have longer tenures.

[6]The corresponding numbers for a switch to equilibrium A from equilibrium B are 2 players and 3 players, respectively.

The second possibility supposes that prospective coalition members and the private sector are uncertain of the actions to be taken by the remaining policymakers. Thus, rather than assuming that policymakers in periods $t+k+1$ onward play c_b^C they place probabilities over the three possible actions c_b^C, c_b^B, and c_b^A and correspondingly over the three continuation values L_{t+k+1}^C, L_{t+k+1}^B, and L_{t+k+1}^A, where k is the conjectured coalition size. In this framework, the results in Figure 2 correspond to the case where the probability assigned to $\left(c_b^C, L_{t+k+1}^C\right)$ equals 1 and the probabilities assigned to $\left(c_b^B, L_{t+k+1}^B\right)$ and $\left(c_b^A, L_{t+k+1}^A\right)$ both equal 0. While $L_t^{CA}(k)$ exhibits a hump as k increases, a consequence of the private sector's "updating dynamic", it is notable that $L_t^{CA}(1)$ is not much larger than L_t^C. Undertaking a numerical examination, we find that even a probability of just 0.005 assigned to equilibrium A as the continuation equilibrium is sufficient to induce the period-t policymaker to switch to play equilibrium A.

Our analysis of coalition forming suggests that the Pareto-preferred equilibrium A is more likely to prevail either when policymakers have long tenures or when a switch to equilibrium A by some policymakers leads to (even slight) uncertainty about the resulting continuation equilibrium.

3.2 Eductive learnability

In this section we complement our analysis of coalition forming by considering learnability as a mechanism for discerning among equilibria. Evans (1986) motivates expectational stability as an equilibrium selection criterion in rational expectations models. A rational expectations equilibrium is expectationally stable if, following small perturbations to the expectation formation process, the system returns to that equilibrium under a "natural revision rule". The relevant revision rule emerges naturally from the thought process whereby agents undertake to revise how they form expectations based on how those expectations would effect the actual economy, seeking to rationalise, or equate, a perceived law-of-motion with the actual law-of-motion.[7] If this

[7] Although the revisions occur in meta-time, there is a close connection between expectational stability and real-time least-squares learnability of a rational expectations equilibrium (Marcet and Sargent, 1989; Evans and Honkapohja, 2001).

"natural revision rule" returns the system to the perturbed equilibrium, then that equilibrium is said to be "expectationally stable" (Evans, 1986).

Like Evans (1986) and Evans and Guesnerie (2005), we view learning as a mechanism through which agents might coordinate upon an equilibrium. Unlike their studies, however, the models that we analyse are populated by both private agents and a policymaker, one or both of which might be learning. As a consequence, we extend the learning literature by considering cases where either the private sector or both the private sector and the policymaker are learning and we derive two new expectational stability conditions.[8] In each case, the learning that we entertain is eductive in nature with agents revising their behavior in meta-time based on the outcomes of thought experiments.[9] The notion of stability under learning that we consider is iterative expectational stability (IE-stability).[10] Finally, we establish an important connection between self-enforceability and IE-stability under private sector learning.

3.2.1 Eductive joint learning

We begin with the case where both the private sector and the central bank are learning, the case we call joint learning. Recall that a discretionary equilibrium is fully characterised by the set $\{\pi_b, c_b, V\}$. We assume that all agents form expectations about the discretionary equilibrium and that the perceived rules held by the private sector and policymaker must be supported by the economy's actual evolution in order to be consistent with a discretionary equilibrium. We further assume that all agents know the beliefs (perceived reactions) of the other agents in the model. In other words, the private sector and the central bank both know that the other is learning, and they know the other's perceived decision rules.

[8] We have also analysed the case where only the policymaker is learning. It turns out that this case is uninteresting as all equilibria are always IE-stable under policymaker learning.

[9] Although we focus on eductive learning, we recognise that in an adaptive learning environment realtime learning of E-stable equilibria need not happen quickly. We leave to future work an application of real-time learnability in models that have multiple discretionary equilibria.

[10] See Evans (2001) for a very useful discussion of adaptive versus eductive learning and of expectational stability (E-stability) versus iterative expectational stability (IE-stability).

Suppose that private agents and the central bank employ the following perceived decision rules

$$\pi_t = \bar{\pi}_b b_t, \tag{11}$$

$$c_t = \bar{c}_b b_t, \tag{12}$$

respectively. These perceived decision rules will be consistent with a discretionary equilibrium if they are supported by the economy's actual evolution over time. Using equation (11), equations (1)—(2) imply

$$\begin{aligned} \beta E_t \pi_{t+1} &= \beta \bar{\pi}_b (\rho b_t - \eta c_t), \\ &= \pi_t - \lambda c_t - \nu b_t, \end{aligned}$$

so that

$$\pi_t = (\beta \rho \bar{\pi}_b + \nu) + (\lambda - \beta \eta \bar{\pi}_b) c_t. \tag{13}$$

Given the perceived decision rules for the private sector and the central bank, in order to be a best response the central bank's actual decision rule must solve

$$V b_t^2 = \min_{c_t} \left\{ \left[(\beta \rho \bar{\pi}_b + \nu) b_t + (\lambda - \beta \eta \bar{\pi}_b) c_t \right]^2 + \alpha c_t^2 + \beta \bar{V} (\rho b_t - \eta \bar{c}_b b_t)^2 \right\},$$

where $\bar{V}$ is the perceived future loss. The central bank's revised decision rule

$$c_t = c_b b_t, \tag{14}$$

with coefficient

$$c_b = -\frac{(\lambda - \beta \eta \bar{\pi}_b)(\beta \rho \bar{\pi}_b + \nu) - \eta \beta \rho \bar{V}}{\beta \eta^2 \bar{V} + (\lambda - \beta \eta \bar{\pi}_b)^2 + \alpha} = c_b \left(\bar{V}, \bar{\pi} \right), \tag{15}$$

implements the best policy response. Similarly, and the private sector's decision rule is revised according to

$$\pi_t = \pi_b b_t,$$

where

$$\pi_b = \beta\rho\bar{\pi}_b + \nu + (\lambda - \beta\eta\bar{\pi}_b)\, c_b\left(\bar{V}, \bar{\pi}\right) = \pi_b\left(\bar{V}, \bar{\pi}\right), \tag{16}$$

Finally, the value function is revised according to

$$V = \pi_b\left(\bar{V}, \bar{\pi}\right)^2 + \beta\bar{V}\left[\rho - \eta c_b\left(\bar{V}, \bar{\pi}\right)\right]^2 + \alpha c_b\left(\bar{V}, \bar{\pi}\right)^2. \tag{17}$$

Equations (15)—(17) define a $\mathbb{T}$-mapping, $x = \mathbb{T}(\bar{x})$, where $x = \{\pi_b, c_b, V\}$. A fixed point, x^*, of the $\mathbb{T}$-map, $x = \mathbb{T}(\bar{x})$ is said to be locally IE-stable under joint learning if

$$\lim_{k\to\infty} \mathbb{T}^k(\bar{x}) = x^*,$$

for all $\bar{x}$ in a neighborhood of x^*, $\bar{x} \neq x^*$. It follows that x^* is locally IE-stable under joint-learning if and only if it is a locally stable fixed-point of the difference equation

$$x^{k+1} = \mathbb{T}\left(x^k\right).$$

To establish local stability we compute the eigenvalues of the derivative $\mathbb{T}$-map and determine whether they have modulus less than one. Substituting equation (15) into equations (17) and (16) to create a two-equation $\mathbb{T}$-map, the two eigenvalues of the derivative $\mathbb{T}$-map can be written as

$$z_{1,2} = \frac{\delta^2 + \varsigma \pm \sqrt{\left(\delta^2+\varsigma\right)^2 - 4\alpha\rho\delta^2\left[\alpha + \beta\eta^2 V + (\lambda - \beta\eta\pi_b)^2\right]}}{2\left(\delta^2+\varsigma\right)},$$

where

$$\begin{aligned}
\delta &= \alpha\rho + \lambda^2\rho + \lambda\nu\eta - \beta\nu\eta^2\pi_b - \beta\lambda\eta\rho\pi_b,\\
\varsigma &= \left\{\alpha\left[(\lambda - \beta\eta\pi_b)(\lambda\rho + 2\nu\eta + \beta\eta\rho\pi_b) + \alpha\rho + V\beta\eta^2\rho\right] + 2V\beta\eta^2(\lambda - \beta\eta\pi_b)(\lambda\rho + \nu\eta)\right\}.
\end{aligned}$$

A numerical examination of these eigenvalues, evaluated at equilibria A, B, and C, establishes that only equilibria A and C are jointly learnable.

3.2.2 Eductive private-sector learning

Simplifying, we now consider the case where only private agents are learning. We want to examine whether private agents can learn the reaction function $\{\pi_b\}$, given the policy rule and payoffs described by $\{c_b, V\}$. Suppose that private agents know that the policymaker implements the policy $c_t = c_b b_t$, with known coefficient c_b. Also suppose that the private sector employs the perceived decision rule (11). Equations (1)—(2) imply

$$\begin{aligned} \beta E_t \pi_{t+1} &= \beta \bar{\pi}_b (\rho - \eta c_b) b_t, \\ &= \pi_b b_t - \lambda c_b b_t - \nu b_t, \end{aligned}$$

and equating coefficients yields

$$\pi_b = \beta \bar{\pi}_b (\rho - \eta c_b) + \lambda c_b + \nu. \tag{18}$$

Equation (18) defines the $\mathbb{T}$-mapping from the perceived decision rule, $\{\bar{\pi}_b\}$, to the actual decision rule, $\{\pi_b\}$, and can be summarised in the form $\pi_b = \mathbb{T}(\bar{\pi}_b)$. A fixed point of this $\mathbb{T}$-mapping results in a perceived decision rule that is consistent with the economy's actual decision rule in a discretionary equilibrium.

Now, a fixed point, $\pi_b^* = \{\pi_b^*\}$ of the $\mathbb{T}$-map, $\pi_b = \mathbb{T}(\bar{\pi}_b)$ is locally IE-stable if and only if it is a stable fix-point of the difference equation

$$\pi_b^{k+1} = \mathbb{T}\left(\pi_b^k\right). \tag{19}$$

In this model, all three discretionary equilibria turn out to be locally IE-stable under private sector learning. To see this, we linearise equation (19) around π_b^* to yield

$$\pi_b^{k+1} = \beta (\rho - \eta c_b) \pi_b^k.$$

Applying standard results for linear difference equations, π_b^* is locally stable if and only if all of the eigenvalues of the derivative map $\mathbb{DT}(\pi_b^*) = \beta(\rho - \eta c_b)$ have modulus less than one. To see that $\beta(\rho - \eta c_b) < 1$ must hold for all three equilibria, note that if a discretionary equilibrium exists, then the rate at which debt increases over time, $\rho - \eta c_b$, cannot exceed $\beta^{-1/2}$. Therefore, the existence of a discretionary equilibrium implies $\beta(\rho - \eta c_b) < 1$.

3.3 A connection between private-sector learnability and coalition forming

We note that the IE-stability properties associated with private sector learning and joint learning, although connected, are distinct. Joint learnability of an equilibrium neither implies nor is implied by private sector learnability of that equilibrium. However, IE-stability under private-sector learning and coalition forming are related. Using the general linear-quadratic rational expectations framework we prove in an online Appendix that accompanies this paper that a finite coalition can exist to bring about a switch to the Pareto preferred equilibrium if and only if the Pareto-preferred equilibrium is locally IE-stable under private sector learning. The intuition for this result is that in order to generate a successful switch to the Pareto-preferred equilibrium the coalition of policymakers has to induce and sustain a switch in private sector expectations.

4 Conclusion

Discretionary policymakers can manage neither the expectations of private agents nor the actions of future policymakers. As a consequence, discretionary policymakers are susceptible to expectations traps and coordination failures and discretionary control problems can have multiple equilibria. Recognising this potential for multiple equilibria, this paper explores the important issue of equilibrium coordination. Using a simple New Keynesian model in which government debt is an endogenous state variable and that is known to possess multiple discretionary equilibria, we motivate and develop a range of equilibrium coordination/selection mechanisms. The central coordination mechanism that we focus on is coalition forming, the idea that a meaningful Nash equilibrium ought to be robust to the formation of coalitions in environments in which decisionmakers can costlessly communicate strategy and enter into non-binding agreements. Our two main results relating to coalition forming are that coordination upon the Pareto-preferred equilibrium becomes more feasible as the policymaker's tenure lengthens and as the probability that the Pareto-preferred equilibrium will be the continuation equilibrium increases. Our treatment of coalition forming is complemented and enhanced by our analysis of learnability, an equilibrium coordination mechanism employed increasingly in the rational expectations literature. We extend existing work on learnability by developing conditions for IE-stability in models in which multiple agents are learning. We show that the Pareto preferred equilibrium needs to be joint-learnable in order for a switch to the Pareto-preferred equilibrium driven by coalition forming to occur. Our experience across a range of models is that the Pareto-preferred equilibrium is jointly learnable, but that it is not necessarily private-sector learnable. It is entirely possible, therefore, that in other models these coordination mechanisms could point toward an equilibrium that is Pareto-dominated.

Finally, while we have focused on coalition forming and learnability in this paper, there are, of course, other vehicles for selecting among equilibria. One such approach might be to determine an equilibrium of interest using minimax-loss or minimax-regret, another might be to identify an equilibrium from the limiting behavior of quasi-commitment policies. We leave the study and application of these criteria, and an investigation into whether multiple discretionary equilibria is a general feature of New Keynesian monetary policy models, to future work.

A Coordination mechanisms in LQ RE Models

A.1 The discretionary control problem

In this appendix, we outline the control problem facing a discretionary policymaker in the general linear-quadratic rational expectations framework.

A.1.1 Constraints and objectives

The economic environment is one in which n_1 predetermined variables, $\mathbf{x}_t$, and n_2 nonpredetermined variables, $\mathbf{y}_t$, $t = 0, 1, ..., \infty$, evolve over time according to

$$\mathbf{x}_{t+1} = \mathbf{A}_{11}\mathbf{x}_t + \mathbf{A}_{12}\mathbf{y}_t + \mathbf{B}_1\mathbf{u}_t + \mathbf{v}_{\mathbf{x}t+1}, \tag{20}$$

$$E_t\mathbf{y}_{t+1} = \mathbf{A}_{21}\mathbf{x}_t + \mathbf{A}_{22}\mathbf{y}_t + \mathbf{B}_2\mathbf{u}_t, \tag{21}$$

where $\mathbf{u}_t$ is a $p \times 1$ vector of control variables, $\mathbf{v}_{\mathbf{x}t} \sim i.i.d.\,[\mathbf{0}, \mathbf{\Sigma}]$ is an $s \times 1$ $(1 \leq s \leq n_1)$ vector of white-noise innovations, and E_t is the mathematical expectations operator conditional upon period t information. Equations (20) and (21) capture aggregate constraints and technologies and the behavior (aggregate first-order conditions) of private agents. For their part, private agents are comprised of households and firms who are ex ante identical, respectively, infinitely lived, and atomistic. The matrices $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are conformable with $\mathbf{x}_t$, $\mathbf{y}_t$, and $\mathbf{u}_t$ as necessary and contain the parameters that govern preferences and technologies. Importantly, the matrix $\mathbf{A}_{22}$ is assumed to have full rank.

In addition to private agents, the economy is populated by a large player, a policymaker. For each period t, the period-t policymaker's objectives are described by the loss function

$$L_t = E_t \sum_{k=t}^{\infty} \beta^{(k-t)} \left[\mathbf{z}_k^{'}\mathbf{W}\mathbf{z}_k + 2\mathbf{z}_k^{'}\mathbf{U}\mathbf{u}_k + \mathbf{u}_k^{'}\mathbf{Q}\mathbf{u}_k\right], \tag{22}$$

where $\beta \in (0, 1)$ is the discount factor and $\mathbf{z}_k = \left[\begin{array}{cc} \mathbf{x}_k^{'} & \mathbf{y}_k^{'} \end{array} \right]^{'}$. We assume that the weighting matrices $\mathbf{W}$ and $\mathbf{Q}$ are symmetric and, to ensure that the loss function is convex, that the matrix $\left[\begin{array}{cc} \mathbf{W} & \mathbf{U} \\ \mathbf{U}^{'} & \mathbf{Q} \end{array} \right]$ is positive semi-definite.[11] We assume that the policymaker is a Stackelberg leader and that private agents are followers; we further assume that the policymaker does not have

[11] It is standard to assume that the weighting matrices, $\mathbf{W}$ and $\mathbf{Q}$, are symmetric positive semi-definite and symmetric positive definite, respectively (see Anderson, Hansen, McGrattan, and Sargent (1996), for example). However, since many economic applications involve a loss function that places no penalty on the control variables, we note that the requirement of $\mathbf{Q}$ being positive definite can be weakened to $\mathbf{Q}$ being positive semi-definite if additional assumptions about other system matrices are met (Clements and Wimmer, 2003).

access to a commitment technology and that policy is conducted under discretion.[12] With policy conducted under discretion, the policymaker sets its control variables, $\mathbf{u}_t$, each period to minimise equation (22), taking the state, $\mathbf{x}_t$, and the decision rules of all future agents as given. Since the policymaker is a Stackelberg leader, the period-t policy decision is formulated taking equation (21) as well as equation (20) into account.

A.1.2 Characterising equilibrium

For the decision problem summarised by equations (20)—(22), any linear Markov-perfect equilibrium will have the form

$$\mathbf{u}_t = \mathbf{F}\mathbf{x}_t, \quad (23)$$

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t, \quad (24)$$

with the law-of-motion for the predetermined variables given by

$$\mathbf{x}_{t+1} = \mathbf{M}\mathbf{x}_t + \mathbf{v}_{\mathbf{x}t+1},$$

where the spectral radius of $\mathbf{M}$ is less than $\beta^{-\frac{1}{2}}$. Further, since the loss function is quadratic and the constraints are linear, the payoff to the policymaker is given by the quadratic state-contingent value function

$$V(\mathbf{x}_t) = \mathbf{x}_t'\mathbf{V}\mathbf{x}_t + d,$$

where $\mathbf{V}$ is symmetric positive semi-definite.

As is well-known from Backus and Driffill (1985), Currie and Levine (1985b), and Söderlind (1999), among others, Markov-perfect equilibria can be computed by finding the fix-point of the

[12] Events within a period occur as follows. After observing the state, $\mathbf{x}_t$, decisions are made first by the incumbent policymaker and subsequently by private agents. At the end of the period the shocks $\mathbf{v}_{\mathbf{x}t+1}$ are realized.

system

$$\mathbf{F} = -\left(\widehat{\mathbf{Q}} + \beta\widehat{\mathbf{B}}^{\prime}\mathbf{V}\widehat{\mathbf{B}}\right)^{-1}\left(\widehat{\mathbf{U}}^{\prime} + \beta\widehat{\mathbf{B}}^{\prime}\mathbf{V}\widehat{\mathbf{A}}\right), \tag{25}$$

$$\mathbf{0} = \mathbf{H}\mathbf{A}_{12}\mathbf{H} - \mathbf{A}_{22}\mathbf{H} + \mathbf{H}\left(\mathbf{A}_{11} + \mathbf{B}_1\mathbf{F}\right) - \mathbf{A}_{21} - \mathbf{B}_2\mathbf{F}, \tag{26}$$

$$\mathbf{V} = \widehat{\mathbf{W}} + 2\widehat{\mathbf{U}}\mathbf{F} + \mathbf{F}^{\prime}\widehat{\mathbf{Q}}\mathbf{F} + \beta\left(\widehat{\mathbf{A}} + \widehat{\mathbf{B}}\mathbf{F}\right)^{\prime}\mathbf{V}\left(\widehat{\mathbf{A}} + \widehat{\mathbf{B}}\mathbf{F}\right), \tag{27}$$

$$d = \beta tr\left(\mathbf{V}\boldsymbol{\Sigma}\right) + \beta d, \tag{28}$$

where

$$\mathbf{J} = \left(\mathbf{A}_{22} - \mathbf{H}\mathbf{A}_{12}\right)^{-1}\left(\mathbf{H}\mathbf{A}_{11} - \mathbf{A}_{21}\right), \tag{29}$$

$$\mathbf{K} = \left(\mathbf{A}_{22} - \mathbf{H}\mathbf{A}_{12}\right)^{-1}\left(\mathbf{H}\mathbf{B}_1 - \mathbf{B}_2\right), \tag{30}$$

$$\widehat{\mathbf{W}} = \mathbf{W}_{11} + \mathbf{W}_{12}\mathbf{J} + \mathbf{J}^{\prime}\mathbf{W}_{21} + \mathbf{J}^{\prime}\mathbf{W}_{22}\mathbf{J}, \tag{31}$$

$$\widehat{\mathbf{U}} = \mathbf{W}_{12}\mathbf{K} + \mathbf{J}^{\prime}\mathbf{W}_{22}\mathbf{K} + \mathbf{U}_1 + \mathbf{J}^{\prime}\mathbf{U}_2, \tag{32}$$

$$\widehat{\mathbf{Q}} = \mathbf{Q} + \mathbf{K}^{\prime}\mathbf{W}_{22}\mathbf{K} + 2\mathbf{K}^{\prime}\mathbf{U}_2, \tag{33}$$

$$\widehat{\mathbf{A}} = \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{J}, \tag{34}$$

$$\widehat{\mathbf{B}} = \mathbf{B}_1 + \mathbf{A}_{12}\mathbf{K}. \tag{35}$$

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